

# Logic

## Tutorial 5 - Craig's interpolation theorem 7 November 2019

### Exercises

If  $A \models B$ , then there exists a formula  $C$ , containing only atoms occurring in both  $A$  and  $B$ , such that  $A \models C$  and  $C \models B$ .

1. Consider the following two formulas:

$$A \triangleq p \wedge (r \vee q) \wedge t \quad \text{and} \quad B \triangleq (p \vee r) \wedge (q \vee t)$$

Do we have that

- $A \models B$  ( $\models A \Rightarrow B$ )?
- $B \models A$  ( $\models B \Rightarrow A$ )?

If one of those is true, give an interpolation formula.

2. Consider the following two formulas:

$$A \triangleq [p \vee (q \wedge r)] \wedge (q \vee t) \quad \text{and} \quad B \triangleq (s \vee r) \wedge q \wedge t \wedge p$$

Do we have that  $A \models B$  or  $B \models A$ ?

If one of those is true, give an interpolation formula.

To construct the interpolation formula, use the method presented in the proof of the theorem. Is this interpolation formula unique?

3. Consider the following two formulas:

$$A \triangleq [(q \Rightarrow r) \wedge s] \quad \text{and} \quad B \triangleq (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$$

Do we have that  $A \models B$  or  $B \models A$ ?

If one of those is true, give an interpolation formula.

4. Consider the following two formulas:

$$A \triangleq (p \vee q) \wedge (q \Rightarrow r) \quad \text{and} \quad B \triangleq \neg p \Rightarrow r$$

Do we have that  $A \models B$  or  $B \models A$ ?

If one of those is true, give an interpolation formula.

5. For

$$\begin{aligned} A &\triangleq a \wedge (b \vee c) & \text{and} & & B &\triangleq a \vee (b \wedge c) \\ A &\triangleq (p \Rightarrow q) \Rightarrow r & \text{and} & & B &\triangleq p \Rightarrow (q \Rightarrow r) \\ A &\triangleq (a \vee b \vee c) \wedge d & \text{and} & & B &\triangleq (a \vee b) \wedge c \wedge d \end{aligned}$$

Do we have that  $A \models B$  or  $B \models A$ ?

If one of those is true, give an interpolation formula.