

Logic

Tutorial 6

14 November 2019

Resolution Method

1. Five people (a, b, c, d, e) have put their money into the same safe. They however have no confidence in each other and decided therefore that the safe can only be opened in the presence of a and b , or b and c , or b, d and e . How many locks does the safe have? How many keys are needed? And who has them?

Hint: Consider the formula

$$\phi(p_a, p_b, p_c, p_d, p_e) \triangleq \text{“the safe can be opened”},$$

where p_x is true if x is present.

2. Give the disjunctive normal form of the following formulas

$$A = \bigwedge_{1 \leq i < n} (p_i \Rightarrow p_{i+1})$$

$$B = A \wedge (p_n \Rightarrow p_1)$$

$$C = \bigwedge_{1 \leq i, j \leq n, i \neq j} (p_i \Rightarrow \neg p_j)$$

$$D = \bigwedge_{1 \leq i \leq n} \left(\bigvee_{1 \leq j \leq n, j \neq i} p_j \right)$$

3. Give the conjunctive normal form of ϕ and show that it is inconsistent using the resolution method.

$$\phi = \neg((q \Rightarrow r) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r)))$$

4. Using the resolution method, determine whether the following formula is valid, consistent or inconsistent.

$$\phi = (((p \wedge q) \Rightarrow r) \vee ((q \Rightarrow p) \wedge \neg q)) \wedge (\neg p \Rightarrow (q \Rightarrow r))$$

5. Using the resolution method, determine whether the following formula is valid, consistent or inconsistent.

$$\phi = ((p \wedge q) \Rightarrow (\neg q \wedge r)) \Rightarrow (((q \Rightarrow r) \Rightarrow (p \wedge r)) \Rightarrow ((p \wedge q) \Rightarrow (p \wedge r)))$$