Logic

Tutorial 7

28 November 2019

1. In the definition of the interpretation of a predicate formula, what hypothesis is absolutely necessary in order for $\exists x\,(p(x) \Rightarrow p(x))$ to be valid.

2. Is the following reasoning correct?

   Some students do not work
   All students want to pass
   ———————————————————
   Some people want to pass without working

3. Tony, Mike and John are members of an alpine club. Each member is a skier or an alpinist or both. No alpinist likes rain but all skiers like snow. Mike likes nothing that Tony likes and likes everything that Tony doesn't like. Tony likes rain and snow.

   Is there a member of the alpine club that is an alpinist but not a skier?

4. If $A \models \forall x p(x)$ and $\exists x p(x) \models B$, then $A \Rightarrow \exists x p(x) \models \forall x p(x) \Rightarrow B$.

   Is this statement correct for all formulas A and B?

5. Determine whether the following formulas are valid, consistent or inconsistent.

   (a) $\forall x\,[p(x) \Rightarrow p(a)]$
   (b) $\forall x\,[p(x) \Rightarrow p(x)]$
   (c) $\forall x\,[p(y) \Rightarrow q(x)] \Rightarrow [p(y) \Rightarrow \forall x\,q(x)]$
   (d) $\forall x\,[p(x) \Rightarrow q(x)] \Rightarrow [p(x) \Rightarrow \forall x\,q(x)]$

6. What can you say about the following inference rule?

$$\frac{p(a),\ \forall x\,[p(x) \Rightarrow p(f(x))]}{\forall x p(x)}$$

7. What can you say about the following inference rule?

$$\frac{\forall x p(x,x),\ \forall x \forall y\,[p(x,y) \Rightarrow p(x,f(x))]}{\forall x \forall y p(x,y)}$$