1 Objectives

At the end of this exercise session you should be able to:

- Apply and decode Hamming (7,4) code

2 Exercises

Channel coding

Exercise 1. [1.5] Considering the Hamming (7,4) code. Decode the received strings:

(a) $\mathbf{r} = 1101011$

(b) $\mathbf{r} = 0110110$

(c) $\mathbf{r} = 0101111$

(d) $\mathbf{r} = 0101110$

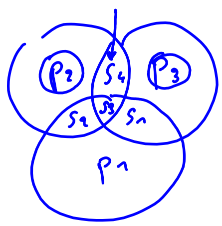
(a) 1101 011 (S, P)

Signal bits 1101 → code word 1101 000

Syndrom: 000 + 011 = 011 (bit per bit)

most probable error pattern: 0001

decoded word: 1100



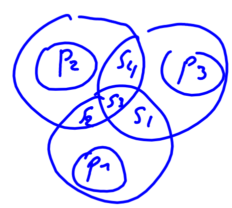
(b) r = 0110 110

Signal 0110 → code word 0110 001

Syndrom 001 + 110 = 111

most probable error pattern: 0010

decoded word: 0100



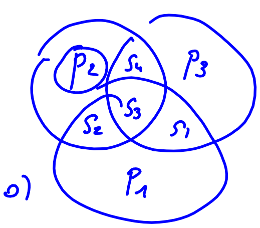
(c) r = 010 1111

code word: 0101 101

Syndrom: 010

most probable err. pattern 0000 (010)

decoded word: 0101



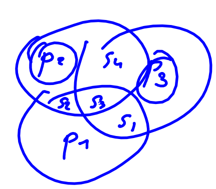
(d) r = 0101 110

→ codeword 0101 101

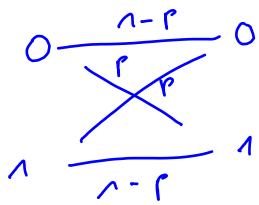
syndrom: 110 + 101 = 011

m.p.e.p: 0001

decoded word: 0100



Exercise 2. [1.2] Show that the error probability is reduced by the use of a repetition code R_3 by computing the error probability of this code for a binary symmetric channel with noise level p.



Source	code
0	000
1	111

An error is made by R^3 if two or more bits are flipped.

all three p^3

exactly 2 bits are flipped: $C_3^2 p^2 (1-p)$, $C_3^2 = 3$

000

$$P_b = P_B = 3p^2(1-p) + p^3 = 3p^2 - 2p^3 < p$$

P_b: probability of error; P_B: probability of block error

$p \ll 3p^2$

Exercise 3. [1.6] Calculate the probability of block error p_B of the Hamming (7,4) code as a function of the noise level p and show that to leading order it goes as $21p^2$.

The probability of block error P_B of the Hamming (7,4) code

is the sum of events:

- 2, 3, 4, 5, 6, 7 errors in <u>one</u> block

$$P_B = \sum_{r=2}^{7} C_7^r \, p^r (1-p)^{7-r}$$

$p \ll$ $\quad P_B \simeq C_7^2 \, p^2 = 21 p^2$

Exercise 4. [8.1] Consider three independent random variables $\mathcal{U}$, $\mathcal{V}$, $\mathcal{W}$ with entropies H_u,H_v,H_w. Let $\mathcal{X} \equiv (\mathcal{U}, \mathcal{V})$ and $\mathcal{Y} \equiv (\mathcal{V}, \mathcal{W})$. What is $H(\mathcal{X}, \mathcal{Y})$? What is $H(\mathcal{X}|\mathcal{Y})$? What is $I(\mathcal{X}; \mathcal{Y})$?