

Embedded systems

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References :

- *An Embedded Software Primer*, David E. Simon, Addison-Wesley, 1999.
- *Mastering the FreeRTOS Real-Time Kernel - A Hands-on Tutorial Guide*, version 1.1.0, Richard Barry, 2024.
- *Real-Time Systems*, Jane W. S. Liu, Prentice Hall, 2000.

Chapter 1

Introduction

Embedded systems

Definition: An **embedded system** is a computer system used as a **component of a more complex entity**.

Typical applications:

- **mobile phones**, televisions, radios, multimedia systems, GPS receivers;
- still and video **cameras**;
- wristwatches, **calculators**, smart cards, RFID tags, remote controls;
- home appliances;
- **computer peripherals**;
- **measurement equipment**, sensors;
- cash dispensers, self-service machines;
- medical devices, **implants**;

- elevators, intrusion-detection devices, domotic systems;
- telephone switches, network routers;
- automotive systems (ABS, ESP, injection controllers, adaptive cruise control, lane-keeping assistance, autonomous drive, ...);
- avionics (fly-by-wire controls, glass cockpits, navigation aids, TCAS, autoland, ...);
- industrial process controllers, robots;
- artificial satellites, spatial probes;
- ...

Advantages

- Moving some functionalities from **hardware** to **software** makes electronic circuits
 - **simpler**,
 - **cheaper** to build,
 - **more powerful**.
- **Complex features** can be implemented.
- Software components can easily be **updated** during the lifetime of a product, and can be **reused** across products.

Developing embedded systems: Main difficulties

- **Low-performance hardware**: Low computing power, small amount of memory . . .
- **Specificity** to a particular application.
- **Concurrency**: Several tasks operate in parallel.
- **Reactivity**: The system must constantly be able to answer solicitations.
- **Real-time** constraints.
- High level of **quality** expected: Reliability, robustness and efficiency are critical.
- Limited **user interface**.
- Adverse **exploitation environment**.
- **Energy management** is often necessary.

Chapter 2

Hardware

Main components of an embedded system

- One or several **processor(s) (CPU)**:
 - **Microcontrollers (MCU)**: 8051 (Intel), PIC, AVR (Microchip), ...
 - **Digital Signal Processors (DSP)**: TMS320 (Texas Instruments), SHARC (Analog Devices), MSC81xx (NXP), ...
 - **Microprocessors** dedicated to embedded applications: ARM [v7, v8, v9], RISC-V, ColdFire (NXP), ARMADA (Marvell), PowerPC (IBM, NXP, STMicroelectronics), x86, x86-64 (Intel, AMD), ...
 - **Generic** microprocessors: Snapdragon (Qualcomm), Xeon, Core, Atom (Intel), ...
 - **Special architectures**: Java Card, multicore processors, reconfigurable processors, ...

- Memory:
 - Static or dynamic RAM, ROM (EEPROM, FLASH, ...).
 - Either internal to the microcontroller, external, or integrated in a System on Chip (SoC).
 - Parallel or serial interface.
 - Possibility of addressing peripherals in memory.
- Internal or external peripherals:
 - Timers,
 - Converters,
 - Communication controllers,
 - ...
- Communication buses.

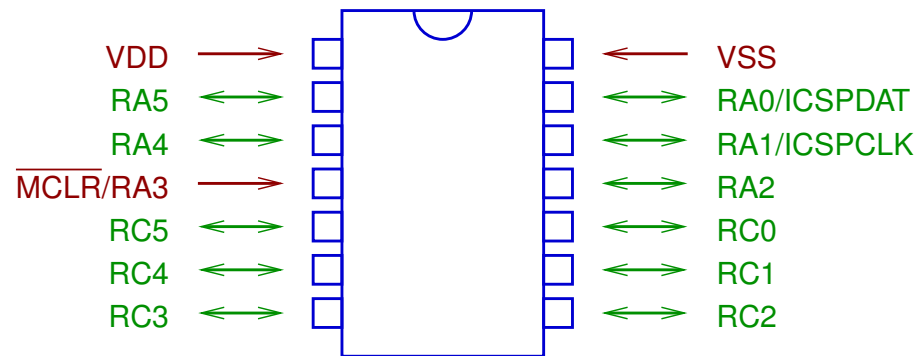
- Interfaces with the circuit environment:
 - Point to point: RS-232, IR, NFC, ...
 - Buses: I²C, SPI, CAN, USB, JTAG, ...
 - Networks: Ethernet, Wi-Fi, ZigBee, Bluetooth,...
- Auxiliary components:
 - Power supply,
 - Clock generator,
 - Bus controllers,
 - ...

Example of embedded microcontroller: Microchip PIC16(L)F18324

Main features:

- **RISC** architecture: Only 48 instructions.
- **Harvard** memory model: 4096 words of (FLASH) **program memory**, 512 bytes of (RAM) **data memory**, 256 bytes of EEPROM, all internal.
- **Reprogrammable** via a serial interface.
- 11 or 12 dynamically configurable **general-purpose input/output pins**.
- **Integrated peripherals**: 7 timers, PWM controllers, analog-to-digital and digital-to-analog converters, ...
- Internal **oscillator** with PLL.
- **Computing power** of up to $8 \cdot 10^6$ instructions per second.
- Low **power consumption**: About $37 \mu\text{A}$ (under 1.8V) at 1 MHz, $8 \mu\text{A}$ at 32 kHz, 40 nA in standby.
- Low price: < 1 €.

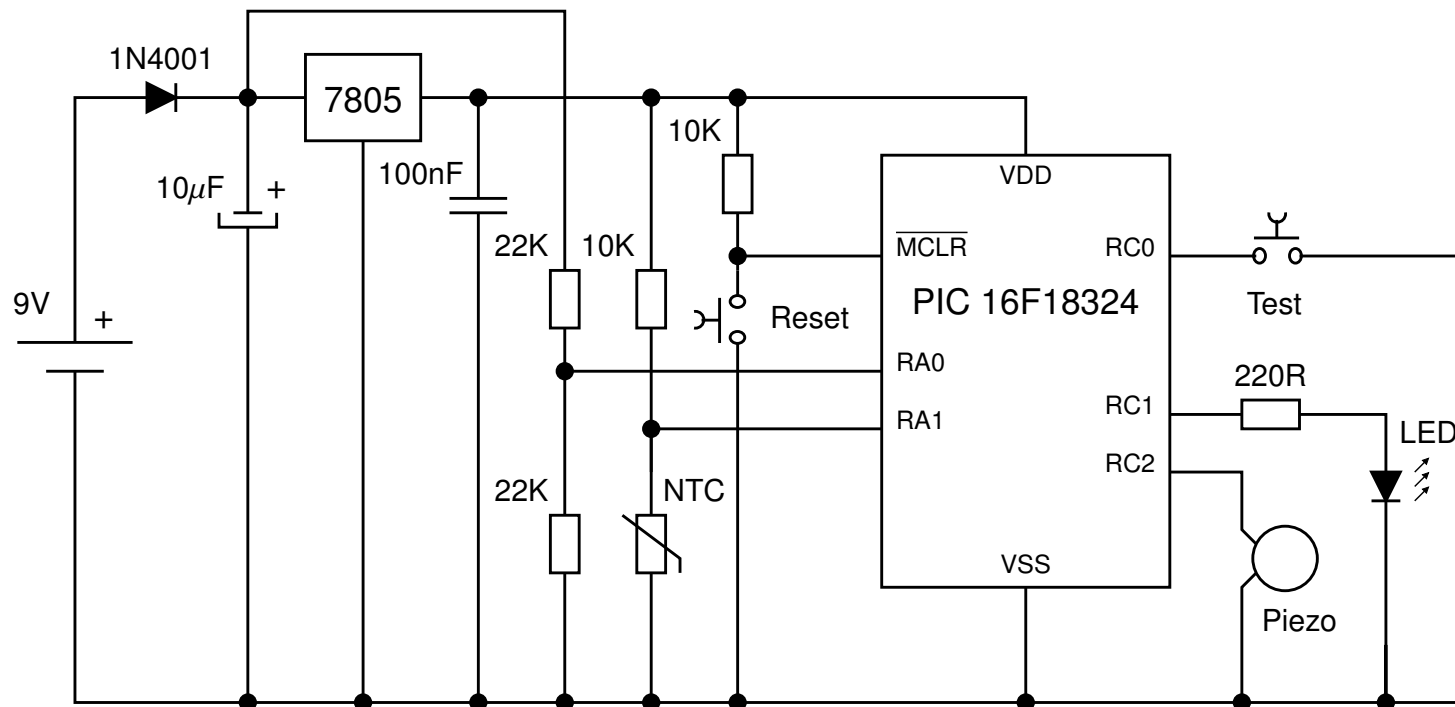
Pinout:



Description:

- **VSS, VDD** : Power supply (1.8 – 3.6 V or 2.3 – 5.5 V depending on version).
- **MCLR** : Operating mode selection (0 V: reset, VDD: program execution, 8 V: programming mode), if enabled by configuration.
- **RA0...RA5, RC0...RC5** : General-purpose input/output pins (TTL/CMOS), dynamically configurable and multiplexed with some peripherals. **RA0** and **RA1** alternatively provide a serial interface in programming mode. **RA3/MCLR** is input only.

Example of application: Temperature alarm



Example of embedded bus

Problem: Managing data transfers between **several devices** (CPU, memory, sensors, peripherals, . . .) using communication hardware that is **as simple as possible**.

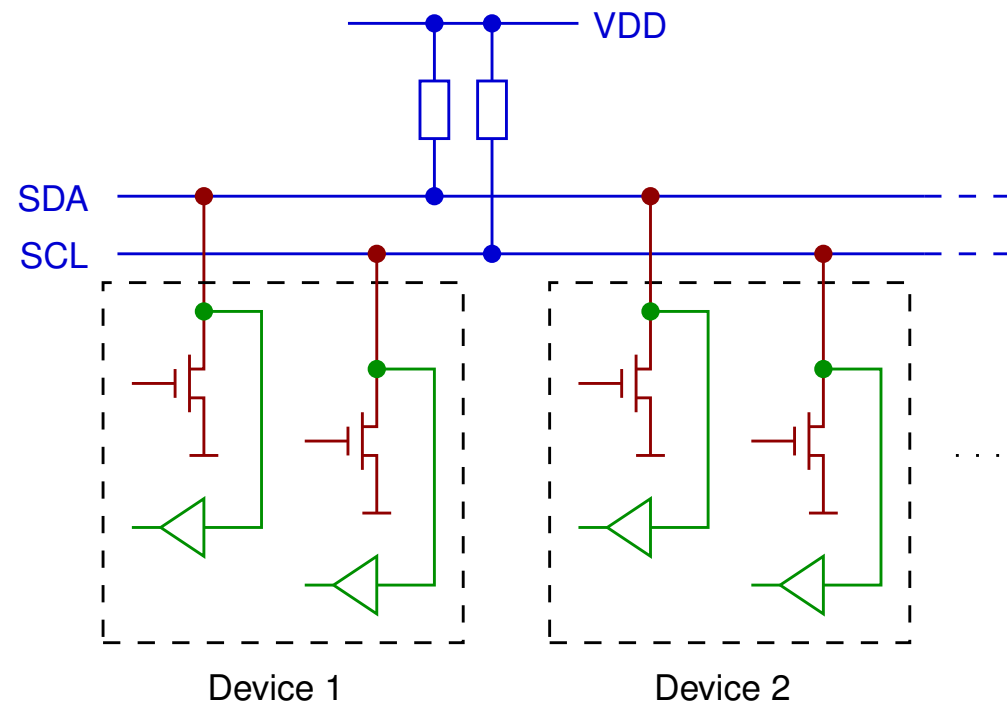
Requirements:

- **Bus** topology.
- Small number of **communication lines**.
- Flexible **configuration**.
- Mechanisms for **addressing** devices, managing **transactions**, for performing **arbitration** and **flow control**.

Solution: I²C bus

Principles:

- The bus consists of a pair of two-way lines: **SDA (Serial DAta)** and **SCL (Serial Clock)**.
- The value of each line stays high whenever it is **unused**.
- Each device connected to the bus can **read the value** of SDA and SCL, but is only able to **force them down**, i.e., to impose a low value.



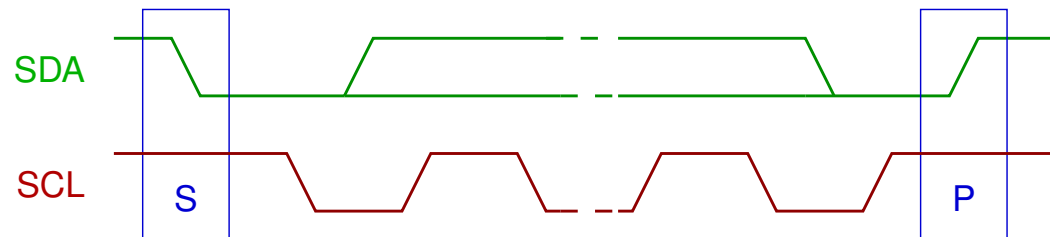
I²C: Transactions

The **master of a transaction** is responsible for

- generating a **clock signal** on **SCL** during the transaction.
- signaling the **beginning (Start, S)** and the **end (Stop, P)** of the transaction. The signals **S** and **P** correspond to the two possible transitions of **SDA** when **SCL** is **high**.

When a transaction is in progress (i.e., between **S** and **P**), transitions of SDA are only allowed when **SCL** is **low**.

Illustration:



I²C: Data transfers

- During a transaction, the **sender** of data can either be the **master** or the **slave**.
- The value of each bit of data sent on the bus corresponds to the **value of SDA** during a **low-to-high transition of SCL**.
- Data is exchanged in **8-bit groups**, the **most significant bit (MSB)** being sent first.
- Each group of 8 bits must be followed by an **acknowledgement**, represented by a **low value** placed on SDA by the receiver.

If a group of bits is not acknowledged, then the master immediately **aborts the transaction**, and the slave stops sending or receiving data.

I²C: Addressing

When a transaction is **initiated**, the master has to specify **which device** is the other participant.

Principles:

- The **first 8 bits** exchanged in a transaction are always **sent by the master**.
- The **first 7 bits** of this group correspond to the **address** of the intended slave.
- The **8th bit** then specifies the **direction** of the following data transfer:
 - **0** : The master is the sender;
 - **1** : The master is the receiver.

Remark: The first group of 8 bits must thus be **acknowledged by the addressed slave**, regardless of the data transfer direction.

I²C: Arbitration

It is possible to have several devices attempting to **initiate transactions** at the same time, by generating **simultaneous Start signals**.

For detecting potential conflicts, each master constantly **monitors the value of SDA** when it sends data. If the observed value **differs from the one being sent**, then the master performing this observation immediately and silently **withdraws from the transaction**.

Remarks:

- A conflict can only be detected by the device that **sends a high value**.
- Transmitting simultaneously two **exactly identical frames** does not lead to a conflict!

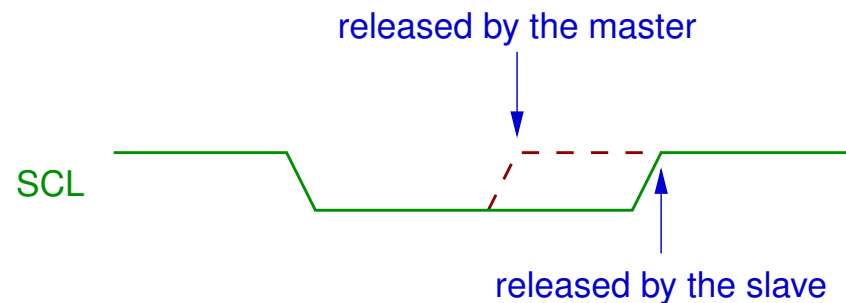
I²C: Flow control

In some cases, the frequency of the clock signal generated is **too high to be followed by the slave**.

In such situations, the slave can request the master to permanently or temporarily **slow down the clock**. This can be done by **stretching the low value of SCL** until the slave is ready again to send or receive data.

When the master **releases SCL** while the clock is stretched, it detects that the value of SCL stays low, and pauses its operations until this line is **released by the slave**.

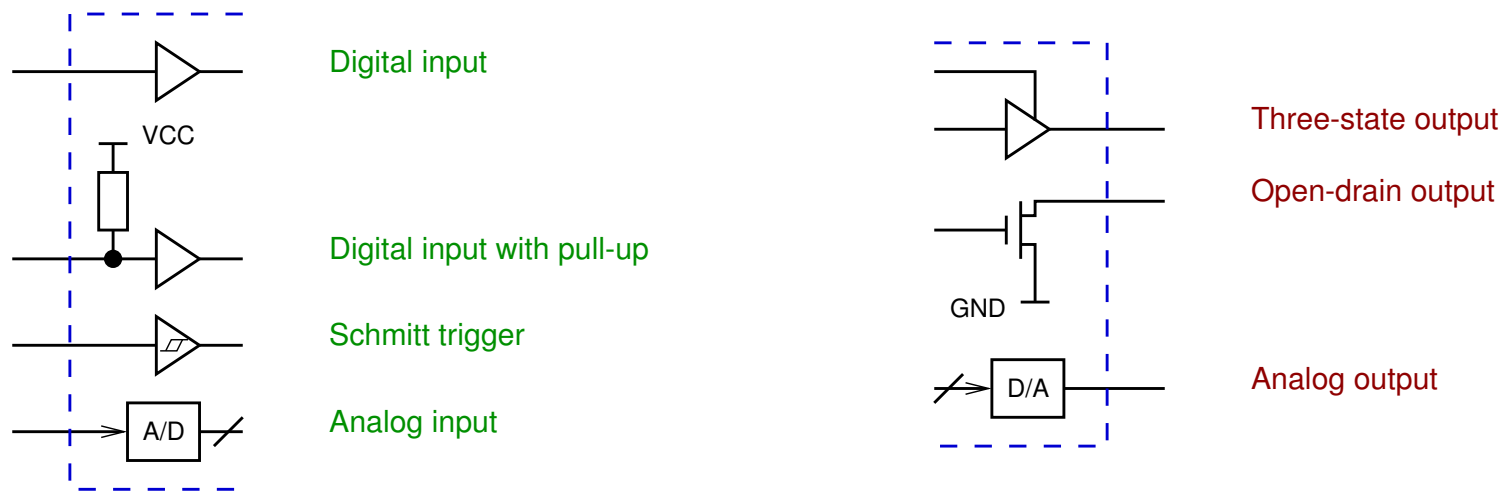
Illustration:



Multiplexed input/output pins

Most microcontrollers allow to **dynamically configure** input/output pins **in software**.

Examples of typical configurations:



This feature makes it possible to build **simple circuits** in which the processor can interact with a **large number of peripherals**.

Example: Digital multimeter

The problem is to interface a microcontroller offering only 12 dynamically configurable input/output pins with:

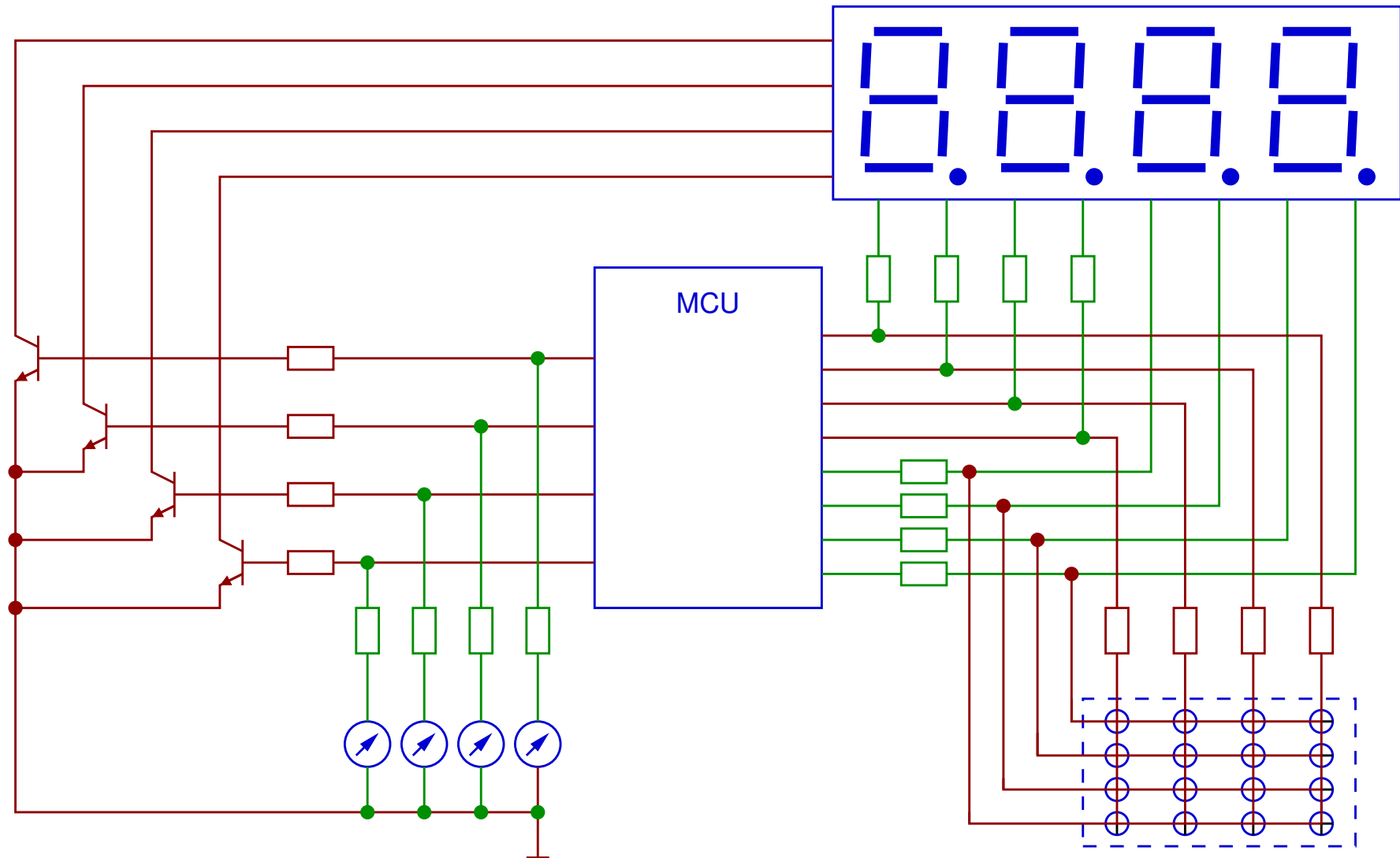
- a screen composed of four 7-segment displays,
- a keyboard organized as a 4×4 matrix,
- 4 analog input channels.

(Source: Microchip application note AN557)

Solution:

- The screen and the keyboard are **scanned**: At a given time, one can only display a **single digit**, or read a **single column** of keys.
- An additional phase is inserted for **reading input channels**.
- 4 pins are associated to both an input channel and a screen digit. They are alternatively configured as **analog inputs** (when **reading channels**) and **digital outputs** (when **displaying a digit** or **reading the keyboard**).
- The 8 remaining pins drive the **screen segments** during display and channel reading phases (8 **digital outputs**), and are also able to **scan the keyboard** (4 **digital outputs** + 4 **digital inputs with pull-up**).

Schematics:



Chapter 3

Interrupts

Introduction

An **interrupt** is a signal that requests the processor to **temporarily suspend** program execution, in order to execute an **interrupt routine** (or **Interrupt Service Routine, ISR**).

Advantages:

- A very short **response time** to solicitations is achievable.
- **Urgent operations** can be programmed **independently** from the main code.

Interrupts can be triggered either by an **exterior component**:

- **Interrupt ReQuest (IRQ)**, received from dedicated input pins,
- **change of logic value** at digital input pins,

or by the processor itself:

- timer expiration,
- completion of a conversion or transmission over a communication channel,
- detection of a bus event, or of reception of data,
- ...
- arithmetic or instruction exception,
- software interrupt request,

The interrupt mechanism

Upon receiving and accepting to service an interrupt request, the processor performs the following operations:

1. The execution of the **current instruction** terminates.
2. A pointer to the **next instruction** to be executed is stored on the **runtime stack**.
3. The address of the **interrupt routine** is read from the appropriate **interrupt vector** (according to the source of the interrupt request).
4. The interrupt routine is **executed**.
5. At the end of the interrupt routine, the processor **resumes program execution**, at the address retrieved from the stack.

Interrupt control

Some critical operations **can never be interrupted**. It is then necessary to temporarily **disable** interrupts prior to their execution, and to **enable** them again afterwards.

Some processors allow to assign specific **priorities** to interrupts originating from **different sources**. Such architectures generally provide a mechanism for disabling the interrupts having a priority **less than some specified threshold**. Interrupt priorities are also used for resolving **simultaneous interrupt requests**.

Enabling and disabling interrupts is performed by executing **specific instructions**, or by setting the value of **dedicated registers**.

Notes:

- At power-on, interrupts are **disabled by default**, in order to allow correct initialization of the program.

- When an interrupt is triggered, some processors **automatically disable** all interrupts of **less or equal priority**. They have to be explicitly reenabled in the interrupt routine if needed.
- When an **interrupt request** is received, the processor sets **interrupt flags**, in order to trigger the interrupt as soon as it becomes enabled. Interrupt flags have to be **cleared explicitly** by the interrupt routine.
- Some architectures provide an interrupt source that **cannot be disabled** (Non Maskable Interrupt, NMI). Its usage is limited to **exceptional situations** (e.g., backing up critical data upon detecting an imminent power failure).

Saving and restoring context

The correct operation of a program **must not be influenced** by interrupts triggered during its execution.

It is thus mandatory for interrupt routines to **leave the processor state unchanged**: values of registers and flags, interface configuration, status of peripherals, . . . , must not be modified.

This is achieved by **saving the context** at the beginning of interrupt routines, and **restoring it** at the end.

Notes:

- The context is either saved on the **execution stack** or in a **specific memory area**.
- Some processors **automatically save the context** (either totally or in part) when an interrupt is triggered.

- Context save and restore operations can sometimes be simplified by using **dedicated instructions**.
- The processors that automatically disable interrupts when branching to an interrupt routine **enable them again** as a side effect of context restoration.

Programming interrupts

The compilers aimed at embedded applications provide **language extension mechanisms** for programming interrupts without going down to assembly language.

- Some functions can be designated as being **interrupt routines** (e.g., **interrupt** keyword, **__attribute__((interrupt))** attribute or **#pragma interrupt** compilation directive in C).

With some compilers, such mechanisms automatically insert **context save and restore** instructions to interrupt routines, and take care of setting **interrupt vectors**.

- **Enabling and disabling** interrupts is performed with the help of macros or specific compilation directives (e.g., **enable()/disable()** or **__enable_irq()/__disable_irq()** macros, **critical** keyword).
- It is sometimes necessary to inform the compiler that the value of a variable can be **modified by interrupt routines**, in order to prevent incorrect optimizations (e.g., **volatile** keyword in C).

Communicating with interrupt routines

Interrupt requests are by nature **unpredictable**. This complicates **data exchange operations** between interrupt routines and the main code.

Example: Industrial controller. The alarm must sound if **two temperature measurements** made by an interrupt routine differ.

Wrong solution:

```
static volatile int temp[2];

interrupt void measure(void)
{
    temp[0] = !! first measurement;
    temp[1] = !! second measurement;
}

void controller(void)
{
    int temp0, temp1;
    for (;;)
    {
        temp0 = temp[0];
        temp1 = temp[1];
        if (temp0 != temp1) !! sound the alarm;
    }
}
```

Notes:

- Carrying out the comparison between the two measurements in a **single C instruction** does not solve the problem:

```
...  
void controller(void)  
{  
    for (;;)   
        if (temp[0] != temp[1]) !! sound the alarm;  
}  
...
```

(Indeed, such an instruction is generally compiled into **several machine instructions**.)

- Even in programs written in assembly language, it is possible for the execution of **individual instructions** to be interrupted before their completion.

This only happens with **specific instructions**, often repeatedly performing a simpler operation (e.g., block copy instructions).

- This type of bug can be **very difficult to detect and to reproduce!**

Correct solution:

The instructions that read the measurements sent by the interrupt routine to the controller form a **critical section**, the execution of which **cannot be interrupted**.

```
static volatile int temp[2];

interrupt void measure(void)
{
    temp[0] = !! first measurement;
    temp[1] = !! second measurement;
}

void controller(void)
{
    int temp0, temp1;
    for (;;)
    {
        disable(); /* Disable interrupts */
        temp0 = temp[0];
        temp1 = temp[1];
        enable(); /* Reenable interrupts */

        if (temp0 != temp1) !! sound the alarm;
    }
}
```

Other solution:

```
static volatile int temp_a[2], temp_b[2];
static int controller_uses_b = 0;

interrupt void measure(void)
{
    if (controller_uses_b)
    {
        temp_a[0] = !! first measurement;
        temp_a[1] = !! second measurement;
    }
    else
    {
        temp_b[0] = !! first measurement;
        temp_b[1] = !! second measurement;
    }
}

void controller(void)
{
    for (;;) controller_uses_b = !controller_uses_b
    if (controller_uses_b)
    {
        if (temp_b[0] != temp_b[1]) !! sound the alarm;
    }
    else
    {
        if (temp_a[0] != temp_a[1]) !! sound the alarm;
    }
}
```

Notes:

- This solution does not require to **disable interrupts**.
- The main code must sometimes perform one **useless iteration** before sounding the alarm.

Improved solution:

```
#define MAX_FIFO 10    /* Must be even ! */
static volatile int temp_fifo[MAX_FIFO];
static volatile int first = 0;
static int last = 0;

interrupt void measure(void)
{
    /* If the buffer is not saturated */
    if (!(first + 2 == last)
        || (first == MAX_FIFO - 2 && last == 0))
    {
        temp_fifo[first] = !! first measurement;
        temp_fifo[first + 1] = !! second measurement;
        first += 2;
        if (first == MAX_FIFO)
            first = 0;
    }
    else    !! discard measurements;
}

void controller(void)
{
    int temp0, temp1;

    for (;;)
        if (first != last)    /* If the buffer is not empty */
        {
            temp0 = temp_fifo[last];
            temp1 = temp_fifo[last + 1];
            last += 2;
            if (last == MAX_FIFO)
                last = 0;
            if (temp0 != temp1)    !! sound the alarm;
        }
}
```

Note: For this solution to be correct, it is necessary that the instruction `last += 2` executes **atomically**.

This kind of solution is thus **very sensitive** to implementation details!

In practice, disabling interrupts during communications with interrupt routines is acceptable in most situations. The more complex solutions are used only when disabling interrupts is **impossible or forbidden**.

Interrupt latency

The delay between an **interrupt request** I and the end of execution of **urgent operations** in an interrupt routine R_I is called the **response time**, or **latency** of the interrupt.

This latency is influenced by **four parameters**:

1. The **longest interval** during which interrupts of priority **larger or equal to** I are disabled.
2. The time needed for executing the interrupt routines with a **higher priority than** R_I .
3. The **maximum delay** between an interrupt trigger and the branch to the corresponding interrupt routine.
4. The **time spent in** R_I before having executed the urgent operations.

A good strategy is therefore to

- disable interrupts for the **shortest possible time** (parameter 1);
- make the interrupt routines **quick and efficient** (parameters 2 and 4).

Parameter 3 is a **feature of the processor**, and cannot be influenced by the programmer.

Example

- A system implements the following interrupt routines, sharing the **same priority**.

Name	Description	Execution time	Period
I_1	Temperature measurement	$100 \mu s$	$500 \mu s$
I_2	Timer expiration	$200 \mu s$	$1000 \mu s$
I_3	Network I/O	$300 \mu s$	$> 1000 \mu s$

- The main program **disables interrupts** during resp. $200 \mu s$ and $250 \mu s$ for exchanging data with I_1 and I_2 .
- The time needed for triggering I_3 and executing the corresponding **urgent operations** is equal to $100 \mu s$.

Question: Is the **latency of I_3** smaller than $1000 \mu s$?

Answer:

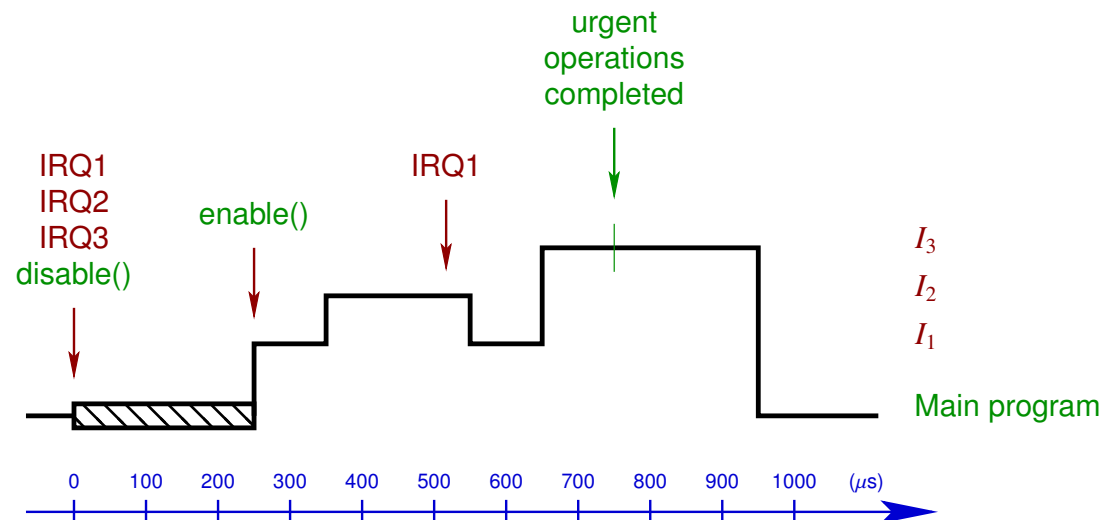
It is sufficient to study the system during an **interval of length equal to $1000\ \mu\text{s}$** . The highest possible latency is obtained with the following delays:

- Interrupts disable time : **$250\ \mu\text{s}$** .
- Executing I_1 : **$2 \times 100\ \mu\text{s}$** .
- Executing I_2 : **$200\ \mu\text{s}$** .
- Triggering and executing the urgent operations of I_3 : **$100\ \mu\text{s}$** .
- $\rightarrow$ Total: **$750\ \mu\text{s}$** .

Notes:

- Only the **largest interval** in which interrupts are disabled needs to be taken into account!

- Example of scenario in which the **maximum latency** is reached:



- The execution of I_3 always terminates **before 1000 μs** .

Chapter 4

Software architectures

The round-robin architecture

Principles:

- **Interrupts** are not used.
- Tasks are invoked **in turn**, and run until their completion.

Illustration:

```
void main(void)
{
    for (;;)
    {
        if (!! task 1 is ready)
        {
            !! operations of task 1;
        }
        if (!! task 2 is ready)
        {
            !! operations of task 2;
        }

        :

        if (!! task n is ready)
        {
            !! operations of task n;
        }
    }
}
```

Advantages:

- Simple solution, but sufficient for some applications.
- Exchanging data between tasks is easy.

Drawbacks:

- The worst-case latency of an external request is equal to the execution time of the entire main loop.
- Implementing additional features can adversely affect the correctness of a system, by increasing latencies beyond acceptable bounds.

Example (multimeter):

```
#include "types.h"
#include "multimeter.h"

static UINT1 phase = 0; /* 0-3: display, 4: keyboard, 5: channels */
static UINT1 display_content[4];
static SINT4 measures[4];

static keyboard_state keys;
static multimeter_state parameters;

void main(void)
{
    !! initialize global data;

    for (;;)
    {
        switch (phase)
        {
            case 4:
                handle_keyboard();
                if (keys.new_keypress)
                {
                    keypress_action();
                    keys.new_keypress= 0;
                }
                break;

            case 5:
                handle_channels();
                update_display_content();
                break;

            default:
                handle_display();
        }
        if (++phase > 5)
            phase = 0;
    }
}
```

```

void handle_display(void)
{
    UINT1 digit, segments;

    !! PORTA: 4 digital outputs;
    !! PORTB: 8 digital outputs;

    digit      = !! compute the digit to be displayed, from the
                 !! values of display_content and phase;
    segments = !! pattern corresponding to digit;

    out(PORTA, 1 << phase);
    out(PORTB, segments);

    delay(DISPLAY_DELAY);
}

void handle_channels()
{
    !! PORTA: 4 analog inputs;
    !! PORTB: 8 digital outputs;

    out(PORTB, 0);
    delay(CHANNELS_DELAY);

    !! read PORTA, and place the result in measures;
}

void handle_keyboard()
{
    static UINT1 column = 0;
    UINT1 row;

    !! PORTA: 4 digital outputs;
    !! PORTB: 4 digital outputs (low nibble),
    !!      4 digital inputs with pull-ups (high nibble);

```

```

    out(PORTA, 0);
    out(PORTB, 1 << column);
    row = in(PORTB) >> 4;

    !! update keys according to the content of row;

    if (++column >= 4)
        column = 0;
}

void keypress_action()
{
    !! update parameters according to the key that has
    !! been pressed (specified in keys);
}

void update_display_content()
{
    !! update display_content according to the values in
    !! measures and parameters;
}

```

Notes: The parameters **DISPLAY_DELAY** and **CHANNELS_DELAY** must be chosen

- **large enough** to ensure an accurate conversion of analog samples, and a good illumination of display segments.
- **small enough** to avoid display flickering, as well as missing key presses.

The round-robin with interrupts architecture

Principles: Tasks are invoked in round-robin fashion, but **interrupt routines** take care of **urgent operations**.

Illustration:

```
volatile BOOL ready1 = 0, ready2 = 0, ...,
               readyn = 0;

interrupt void urgent1(void)
{
    !! urgent operations of task 1;
    ready1 = 1;
}

interrupt void urgent2(void)
{
    !! urgent operations of task 2;
    ready2 = 1;
}

:

interrupt void urgentn(void)
{
    !! urgent operations of task n;
    readyn = 1;
}
```

```

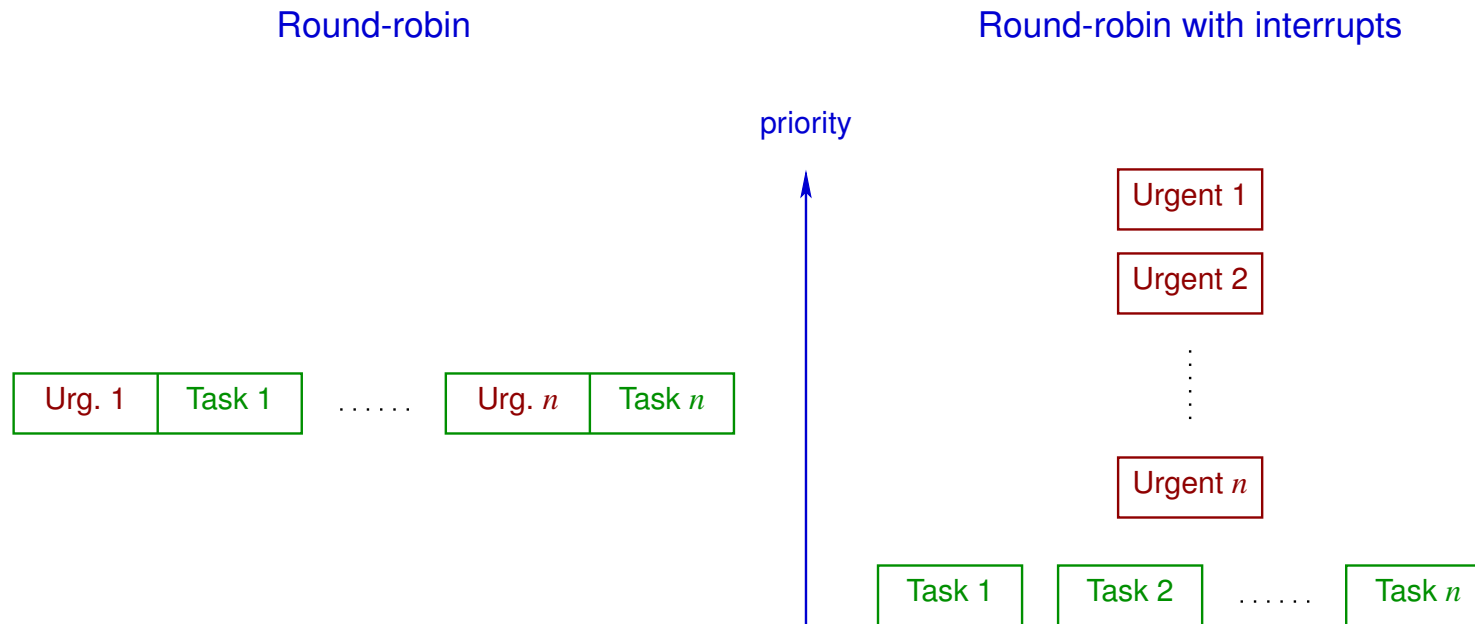
void main(void)
{
    for (;;)
    {
        if (ready1)
        {
            !! non-urgent operations of task 1;
            ready1 = 0;
        }
        if (ready2)
        {
            !! non-urgent operations of task 2;
            ready2 = 0;
        }

        :

        if (readyn)
        {
            !! non-urgent operations of task n;
            readyn = 0;
        }
    }
}

```

Advantage: The urgent operations take priority over the non-urgent ones.



Drawbacks:

- The non-urgent tasks share the same effective priority. This yields high latencies when at least one task has a large execution time (e.g., raster generation in laser printers).

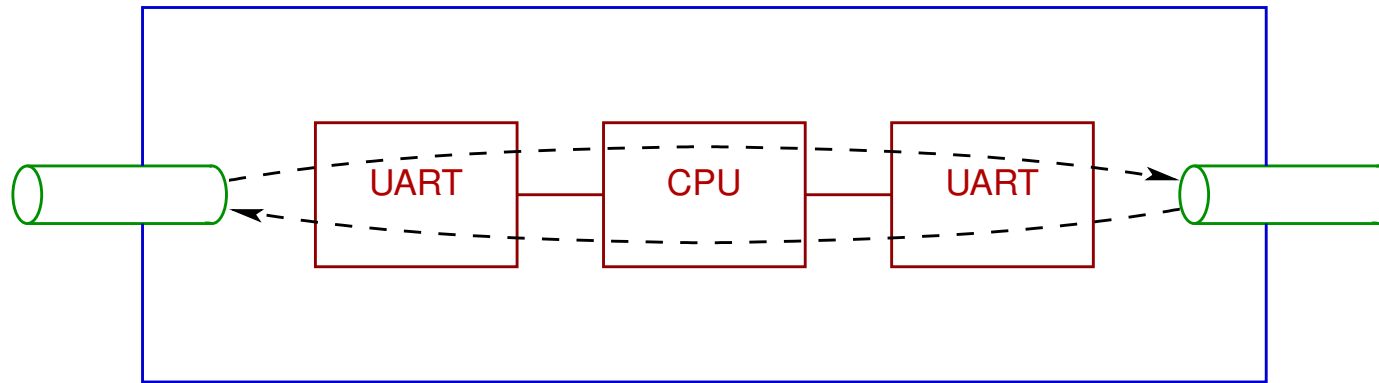
Important note: Moving non-urgent operations from tasks to interrupt routines is not a good solution!

Indeed,

- performing **non-urgent operations** in an interrupt routine **increases the latency** of interrupts with a lower or equal priority;
 - interrupts do not offer flexible **synchronization mechanisms**.
- Data exchange operations between **interrupt routines** and **tasks** have to be correctly implemented (cf. Chapter 3).

Example: Serial filter

The goal is to develop a **two-way filter** connecting two serial lines.



Principles:

- **Incoming bytes** are signaled by interrupt requests, which must be answered as soon as possible (before the next received byte).
- When a UART is **ready to send a byte** on its output line, it requests an interrupt. The processor is then free to wait for an arbitrarily long time before **providing this byte**.

Solution:

```
#include "types.h"
#include "fifo.h"
#include "filter.h"

static volatile BOOL uart1_ready, uart2_ready;
static volatile fifo rx1, tx1,
                    rx2, tx2;

interrupt void uart1_rx(void)
{
    char byte;

    byte = !! reception from UART1;
    fifo_put(rx1, byte);
}

interrupt void uart2_rx(void)
{
    char byte;

    byte = !! reception from UART2;
    fifo_put(rx2, byte);
}

interrupt void uart1_ready_to_send(void)
{
    uart1_ready = 1;
}

interrupt void uart2_ready_to_send(void)
{
    uart2_ready = 1;
}
```

```

void main(void)
{
    !! initialize global data;
    !! initialize interrupt vectors;

    enable();

    for (;;)
    {
        if (fifo_content_size(rx1) >= FILTER_THRESHOLD)
        {
            !! remove data from rx1;
            !! filter;
            !! add the result to tx2;
        }

        if (fifo_content_size(rx2) >= FILTER_THRESHOLD)
        {
            !! remove data from rx2;
            !! filter;
            !! add the result to tx1;
        }

        if (uart1_ready && !fifo_is_empty(tx1))
        {
            char byte;

            byte = fifo_get(tx1);
            disable();
            !! send byte to UART1;
            uart1_ready = 0;
            enable();
        }
    }
}

```

```
    if (uart2_ready && !fifo_is_empty(tx2))
    {
        char byte;

        byte = fifo_get(tx2);
        disable();
        !! send byte to UART2;
        uart2_ready = 0;
        enable();
    }
}
```

Notes:

- Attempting to add data to a **saturated FIFO buffer** cannot be a **blocking operation** (i.e., it must instead discard data).

- The functions for handling FIFO buffers must execute correctly both in the **interrupt routines** and in the **main code**.

Example of implementation:

```
void fifo_put(fifo q, char c)
{
    BOOL intr_enabled;

    ...

    intr_enabled = disable();
    !! critical section;
    if (intr_enabled)
        enable();

    ...
}
```

The waiting-queue architecture

Principles:

- In the same way as the round-robin with interrupts architecture, the operations are partitioned into **urgent** and **non-urgent** tasks.
- **Interrupt routines** perform urgent operations, and then place in a **waiting queue** requests for executing **non-urgent tasks**.
- The main program retrieves **execution requests** from the queue and calls the corresponding functions. These requests are not necessarily processed in FIFO order. (For instance, different **selection priorities** can be assigned to non-urgent tasks.)

Illustration:

```
#include "queue.h"

static volatile queue waiting_queue;

interrupt void urgent1(void)
{
    !! urgent operations of task 1;
    !! add task1 to waiting_queue;
}

interrupt void urgent2(void)
{
    !! urgent operations of task 2;
    !! add task2 to waiting_queue;
}

:

interrupt void urgentn(void)
{
    !! urgent operations of task n;
    !! add taskn to waiting_queue;
}
```

```

void main(void)
{
    !! initialize waiting_queue with an empty content;

    for (;;)
    {
        while (!queue_is_empty(waiting_queue))
        {
            !! extract a function from waiting_queue;
            !! execute this function;
        }
    }
}

void task1(void)
{
    !! non-urgent operations of task 1;
}

void task2(void)
{
    !! non-urgent operations of task 2;
}

:

void task $n$ (void)
{
    !! non-urgent operations of task  $n$ ;
}

```

Advantage: The latency of a **non-urgent high-priority task** can become smaller than the execution time of **all the non-urgent operations**.

Drawbacks:

- The **maximum latency** of a non-urgent task is still at least as large as the execution time of the **slowest task**.
- **Implementing the waiting-queue data structure** can be tricky.

Example of application: A system monitors an industrial process by receiving data from an array of **sensors**, processing this data, and **displaying summarized results**.

With the queue architecture, it is possible to ensure that the values produced by **critical sensors** are always taken into account, even in the case of data saturation caused by a malfunctioning low-priority sensor.

The real-time operating system architecture

Principles:

- Urgent operations are performed by interrupt routines. Those are able to signal to other tasks that non-urgent operations are ready to be carried out.
- The non-urgent tasks are invoked dynamically rather than in a predefined order. The responsibility of calling tasks is assigned to the operating system, implemented as an additional software component.
- The operating system is able to suspend the execution of a task before its completion, in order to transfer the processor to another task.
- The signals exchanged between tasks are handled by the operating system, instead of being implemented with shared variables.

Illustration:

```
#include "signal.h"

interrupt void urgent1(void)
{
    !! urgent operations of task 1;
    !! send signal 1;
}

interrupt void urgent2(void)
{
    !! urgent operations of task 2;
    !! send signal 2;
}

:

void task1(void)
{
    !! wait for signal 1;
    !! non-urgent operations of task 1;
}

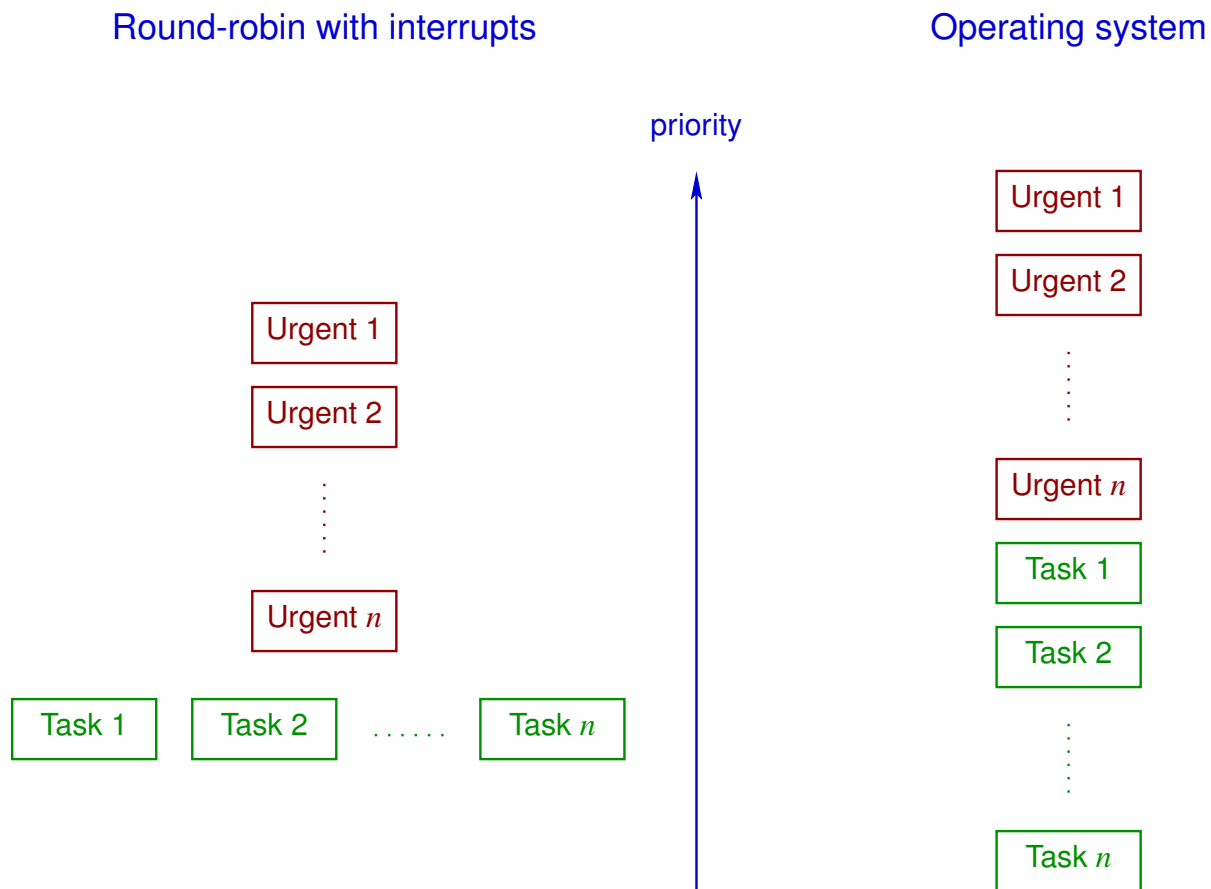
void task2(void)
{
    !! wait for signal 2;
    !! non-urgent operations of task 2;
}

:

void main(void)
{
    !! initialize the operating system;
    !! create and enable tasks;
    !! start task sequencing;
}
```

Advantages:

- One can easily combine **low-latency operations** together with **long computations**.



- The system is **efficient**: When a non-urgent task is waiting for a signal, the processor **remains available** for other computations.
- The structure of the system is **robust**: New features can easily be added without affecting the latencies of urgent operations or of high-priority tasks.
- Operating systems **tailored to embedded applications** are commercially available.

Drawbacks:

- The system is **complex** (but this complexity is mainly located **in the operating system**, which can be reused across projects).
- **Data exchange operations** have to be coordinated between a task and an interrupt routine, but also **between tasks**.
- The operating system **consumes some amount of system resources** (a typical figure is 2 to 4 % of the instructions executed by the processor).

Summary

Task priorities and latencies:

Architecture	Available priorities	Maximum latency
round-robin	none	total execution time of all tasks
round-robin with interrupts	interrupt routines; all tasks share the same priority	total execution time of all tasks + interrupt routines
waiting queue	interrupt routines, then tasks	execution time of the longest task + interrupt routines
operating system	interrupt routines, then tasks	execution time of interrupt routines

Robustness and simplicity:

Architecture	Robustness against modifications	Complexity
round-robin	poor	very simple
round-robin with interrupts	good for interrupt routines, poor for the tasks	must handle data exchanges between tasks and interrupt routines
waiting queue	fair	must handle data exchanges, and implement the waiting queue
operating system	very good	quite complex

Chapter 5

Real-time operating systems

Introduction

An **operating system (OS)** is a software component responsible for coordinating the **concurrent execution** of several **tasks**, by

- managing the system **resources** (processor(s), memory, access to peripherals, ...);
- providing **services** (communication, synchronization, ...).

An OS is implemented by a **kernel** (an autonomous program), together with a **library of functions** for accessing conveniently its services.

Real-time operating systems (RTOS) are operating systems specifically suited for embedded applications:

- They are usable on hardware with **limited resources**.

- The **scheduling strategy** is precisely documented.
- The **internal mechanisms** (e.g., the longest interval during which interrupts are disabled by the kernel, the implementation of system calls, ...) are engineered so as to **minimize latencies**.
- The user can implement urgent operations as **interrupt routines**.
- The OS provides **time-oriented services**: one-shot or periodic timers, periodic execution of tasks, ...
- Complex **protection mechanisms** against invalid user code may be absent.
- **Dynamic memory allocation** is usually optional.
- The **kernel configuration** can be parameterized in detail by the programmer.

Execution levels

At a given time, the instruction **currently executed** by the processor can either be

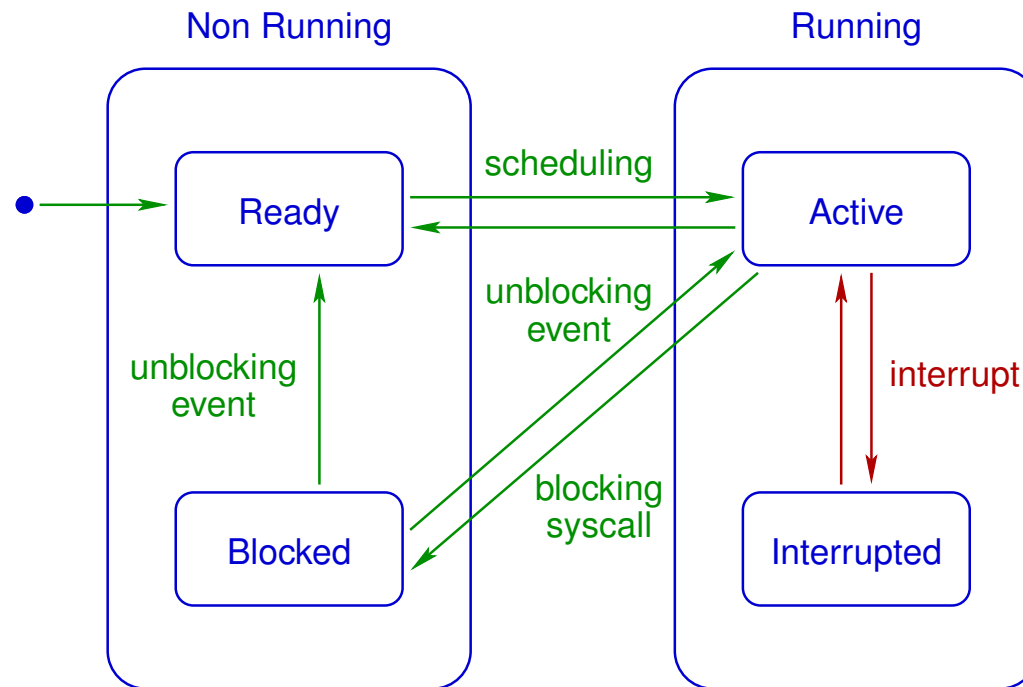
- a **kernel operation** (possibly located in an interrupt routine),
- an instruction belonging to an **interrupt routine** programmed by the user, or
- an instruction of a **user task**.

Process states

Each task managed by the OS is represented by a **process**. At a given time, a process is in one out of four possible **states**:

- **Ready**: The task is **ready to execute instructions**, but is not currently running.
- **Active**: The instructions of the task are now being **executed by the processor**.
- **Blocked**: The execution of the task is **suspended** while waiting for a signal, a timeout, or for a resource to become available.
- **Interrupted**: The task is executing an **interrupt routine** programmed by the user.

Possible transitions between the states of a process:



The scheduler

The **scheduler** is the kernel component responsible for **managing the state of the processes**, i.e., for **assigning the processor** to processes.

Principles:

- Each task is characterized by a **priority** (either constant or variable during its execution).
- The scheduler always assigns the processor to one of the running tasks with the **highest priority**.

If **several tasks** share the highest priority, then the task that is selected can be chosen in several ways:

- The **time slicing** approach consists in assigning the processor **in turn** to each of these tasks, in order to execute a bounded sequence of instructions.

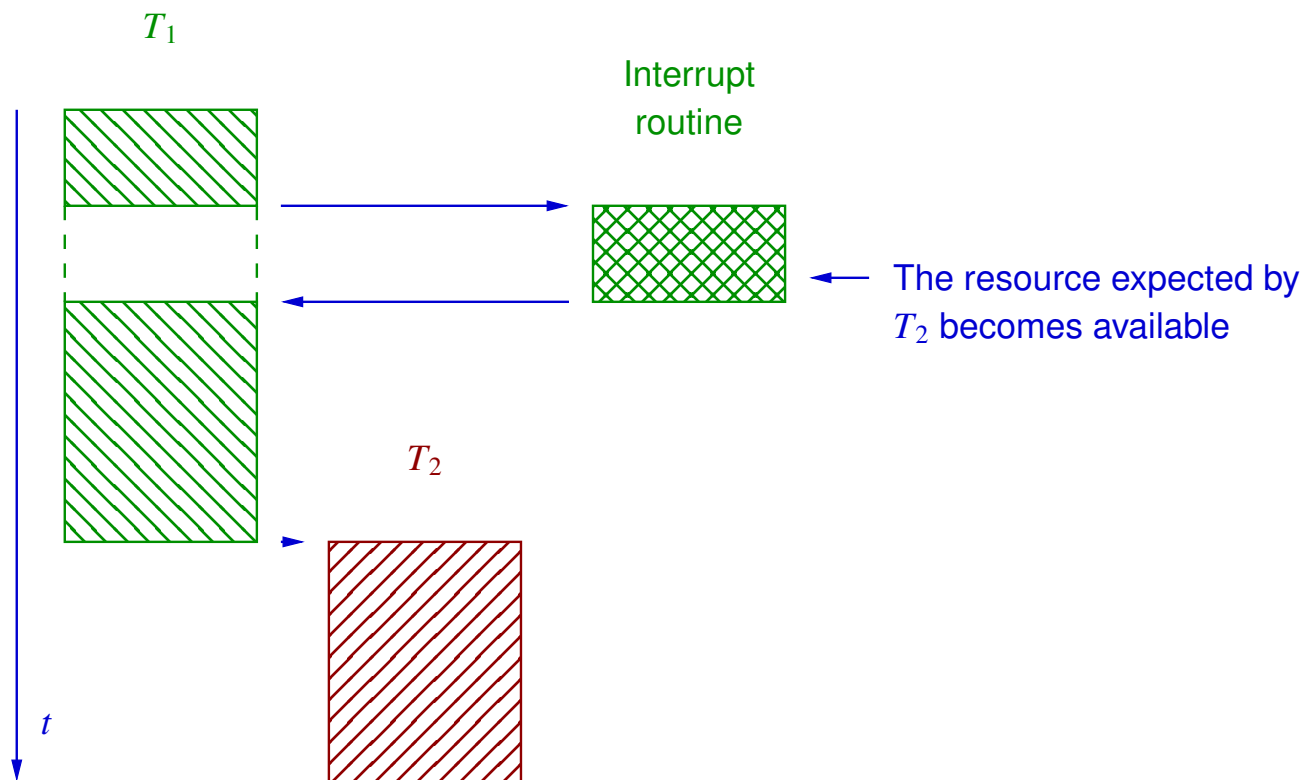
- One can alternatively assign the processor to a task that is **chosen arbitrarily**.
- Another solution is to **forbid different tasks** to share the same priority.

Note: With the first two strategies, **computing the deadline** of a task can become difficult.

Preemption

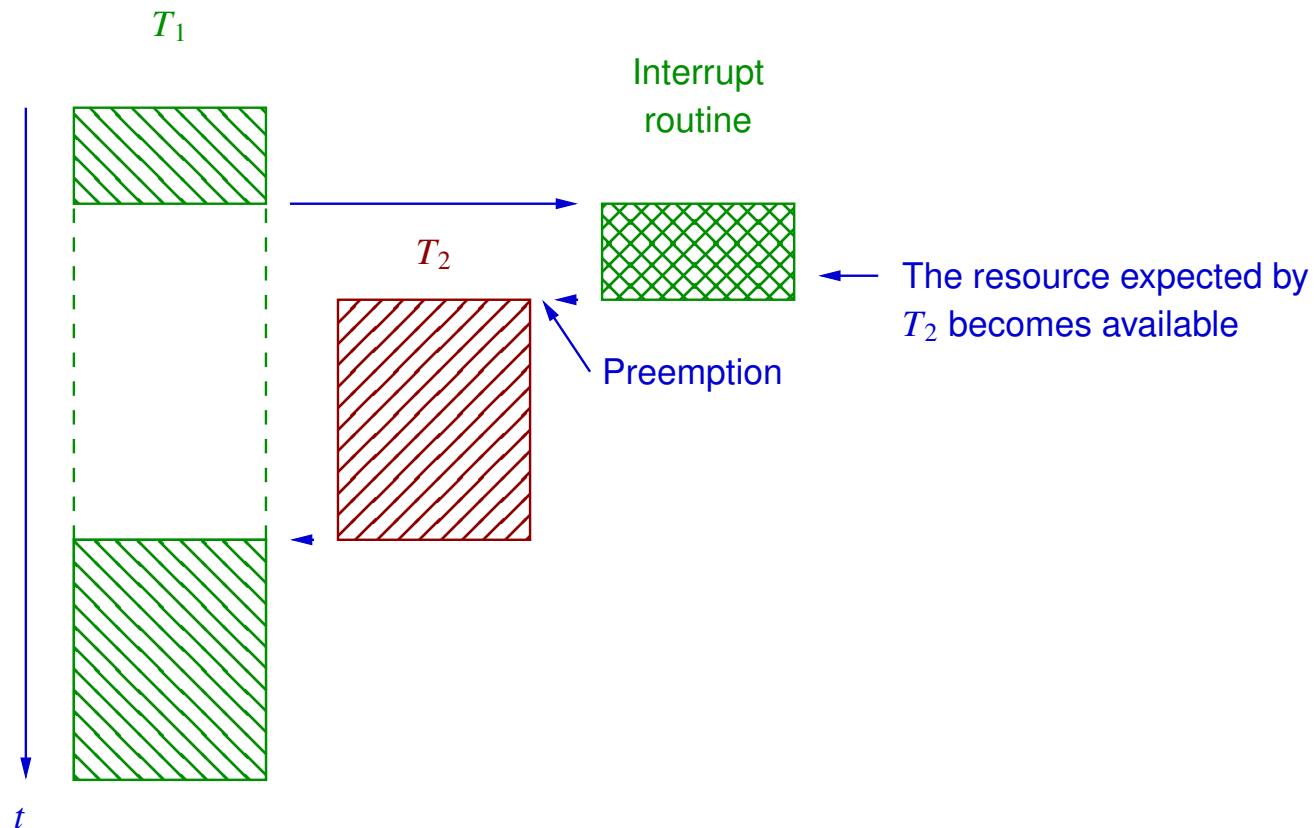
If a task T_2 has a **higher priority** than the active task T_1 and switches from the **blocked** to the **ready** state, then there are two possible scheduling strategies:

- The task T_2 remains **non running** (in ready state) until completion of T_1 . The scheduler is said to be **non-preemptive**.



Drawback: The latency of a task is influenced by the behavior of tasks with a **lower priority**.

- The scheduler turns the task T_1 **ready**, and **assigns the processor** to T_2 . The scheduler is said to be **preemptive**.



Context switching

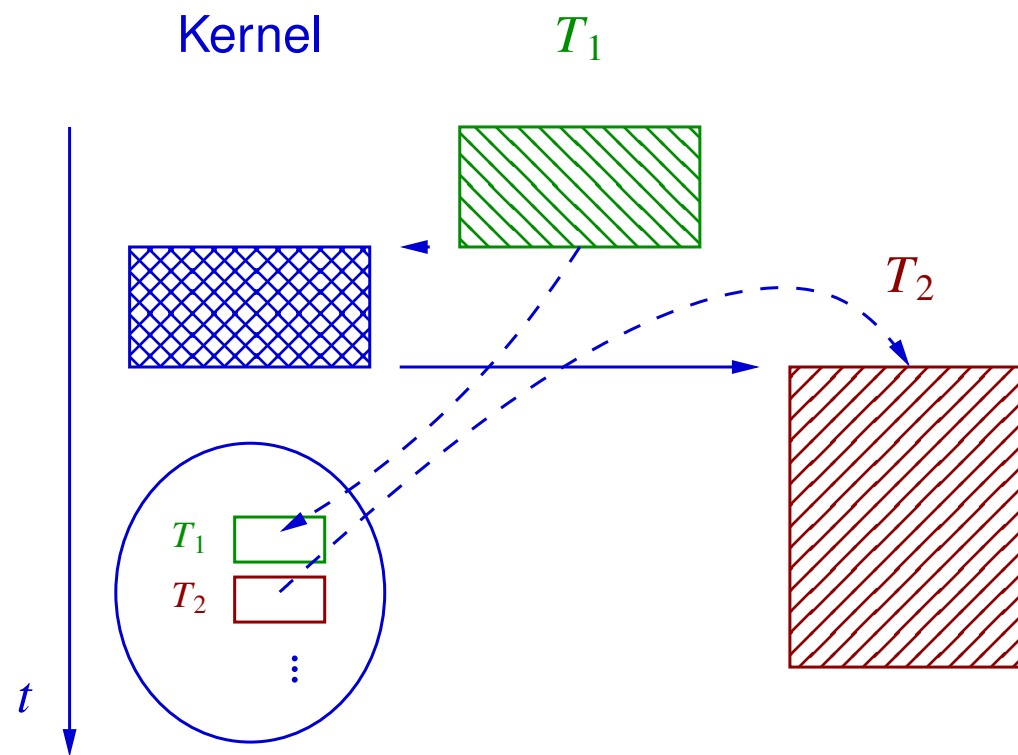
The scheduler performs a **context switch** when it **transfers the processor** from a process to another.

Principles:

- The suspended task must be able to **resume its execution** later. The **state of the processor** thus has to be saved when the task becomes non running.

The **kernel memory** maintains for each process a **context storage area** for this purpose.

Illustration:

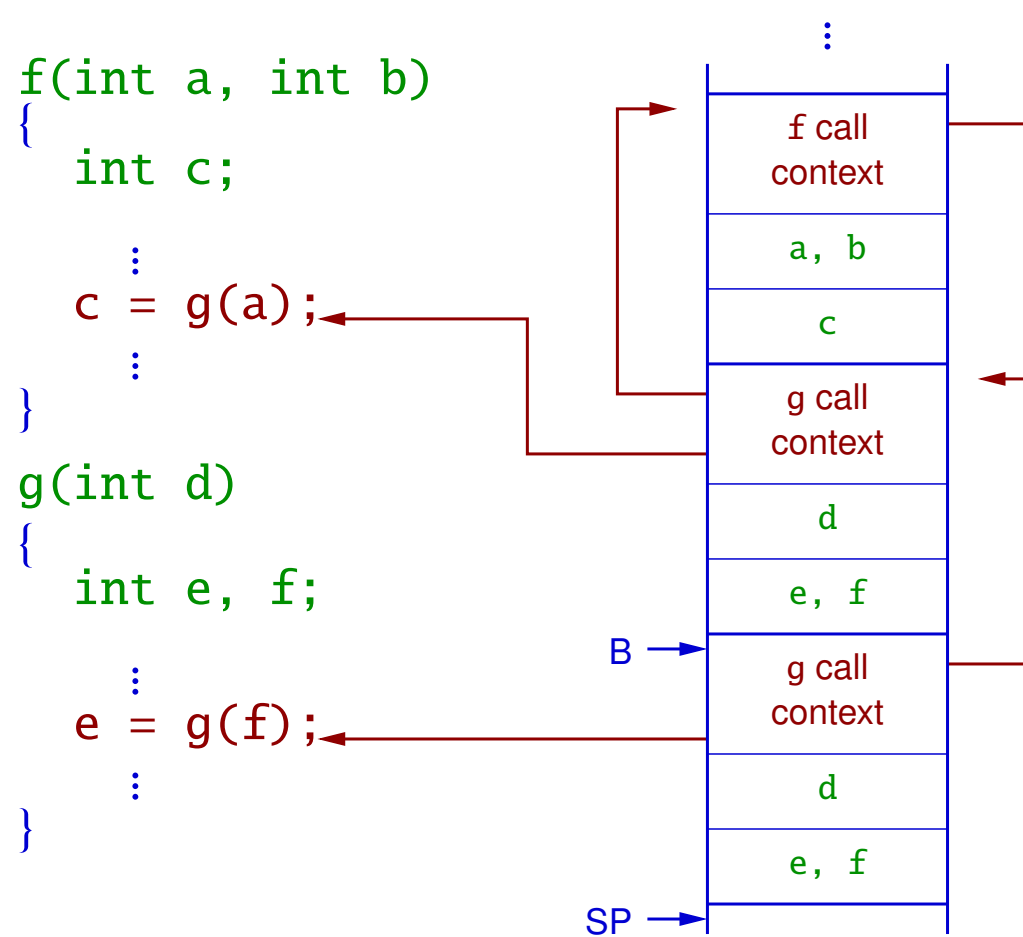


- The **working data** of the suspended task has to be **preserved** until its execution can be resumed.

This data is located on the **runtime stack** of the task, which contains

- the context (return address, stack register values) of the active **function calls**, and
- the **arguments and local variables** of these function calls.

Example:



Notes:

- Since a task can become non running **at any time**, it is necessary for each process to manage **its own stack**.
- In general the **stack pointers** (e.g., top of stack, base of current stack frame) are particular processor registers. Those pointers are therefore saved, together with the other registers, during a context switch.
- The **kernel** also manages its own stack.

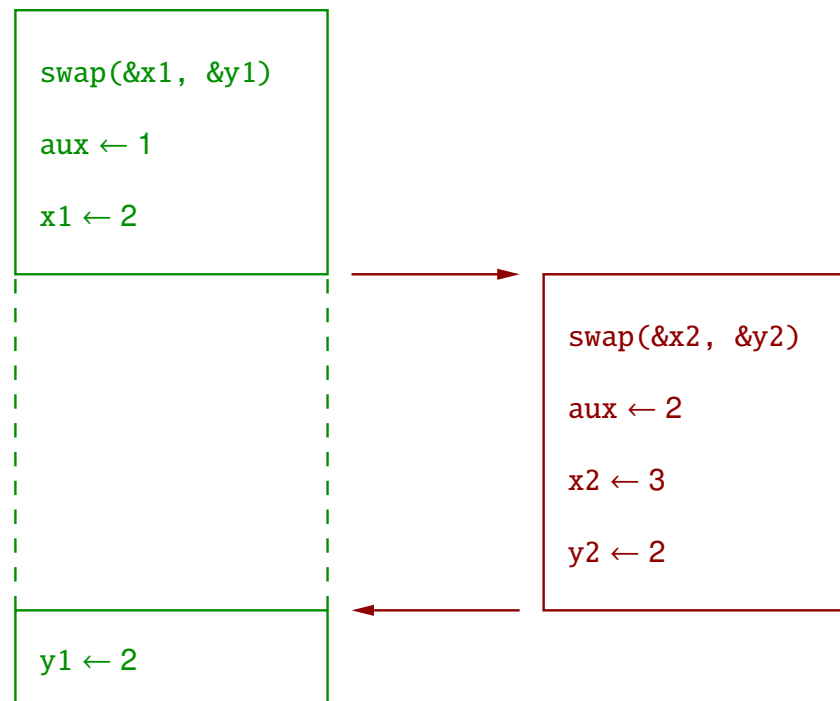
Reentrancy

With a preemptive scheduler, calling **the same function** from **different tasks** can be problematic.

Example:

```
static int aux;  
void swap(int *p1, int *p2)  
{  
    aux = *p1;  
    *p1 = *p2;  
    *p2 = aux;  
}
```

aux	
x1	1
y1	2
x2	2
y2	3



Definition: A function is said to be **reentrant** if it can be **simultaneously called** by several tasks **without possibility of conflict**.

Examples:

- **Reentrant** function:

```
void swap(int *p1, int *p2)
{
    int aux;

    aux = *p1;
    *p1 = *p2;
    *p2 = aux;
}
```

- **Non-reentrant** function:

```
volatile int is_new;  /* modified by another task */
void display(int v)
{
    if (is_new)
    {
        printf(" %d", v);
        is_new = 0;
    }
    else
        printf(" ---");
}
```

Note: The second function is non-reentrant for **three reasons**:

- The **test** and **assignment** operations over the global variable `is_new` are performed by different instructions.
- The operations involving `is_new` are **not necessarily atomic**.
- The function `printf` might **not be reentrant**.

Communication between tasks

Organizing **data transfers** between processes is more difficult than between tasks and **interrupt routines**:

- **Context switches** can occur unpredictably at any time.
- Context switches can only be disabled **in software**, by modifying the scheduling policy.

Solution: One can use **services provided by the kernel**, aimed at

- **synchronizing** the operations of concurrent tasks, and
- coordinating **data transfers** from a process to another.

Note: Using **incorrectly** communication or synchronization services can lead to **deadlocks**, when every task is blocked waiting for resources that can only be provided by **other tasks**.

Semaphores

A semaphore s is an object that

- has a value $v(s) \geq 0$,
- over which the two following operations can be performed:
 - wait(s):
 - * if $v(s) > 0$, then $v(s) \leftarrow v(s) - 1$;
 - * if $v(s) = 0$, the task is suspended (in blocked state).
 - signal(s):
 - * if at least one task is blocked as the result of an operation wait(s), unblock one of them (making it ready or active);
 - * otherwise, $v(s) \leftarrow v(s) + 1$.

Notes:

- The operations that **test and modify** the value of a semaphore must be implemented **atomically**.
- **Binary semaphores** are semaphores with a value restricted to the set $\{0, 1\}$.
- There are several possible strategies for **selecting a task** blocked on a semaphore in order to unblock it: arbitrary choice, FIFO policy, highest priority, ...

In most applications, **acquiring a semaphore** represents the **access right** to a resource.

Example: Mutual exclusion between two tasks (binary semaphore s initialized to 1).

```
void task1(void)
{
    for (;;)
    {
        wait(s);
        !! critical section;
        signal(s);
        !! other operations;
    }
}

void task2(void)
{
    for (;;)
    {
        wait(s);
        !! critical section;
        signal(s);
        !! other operations;
    }
}
```

Message queues

A **message queue** is an object that implements **synchronous or asynchronous** data transfers between tasks.

Principles:

- The **maximum capacity** of a queue (i.e., the maximum number of messages that have been sent to and not yet received from the queue) and the **size of each message** are fixed.
- **Send** and **receive** operations are performed atomically.
- A task that is waiting to receive data from a queue is **suspended** by the scheduler (in blocked state).

Variants:

- Several **data access policies** are possible: FIFO order, arbitrary selection, highest priority.

- Sending data to a **saturated message queue** can either discard the new message, block the sender, block the sender during a bounded amount of time, . . .
- When a task is blocked waiting for data from an **empty queue**, a timeout (i.e., a maximum suspension delay) can be specified.
- The maximum capacity of a queue can be **reduced to zero** (**rendez-vous** synchronization).
- Queues of **capacity one** are sometimes called **mailboxes**.

Programming with interrupts

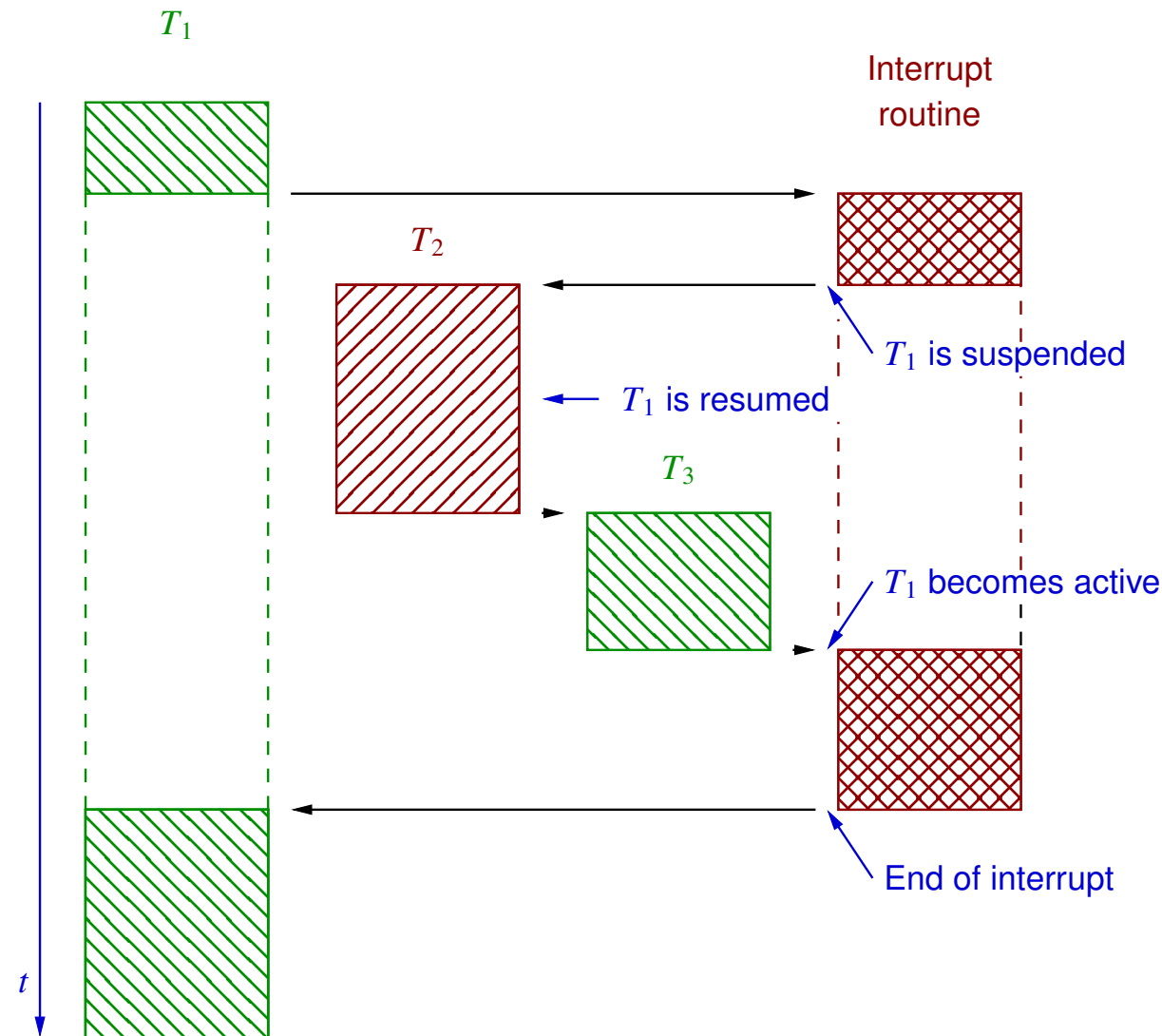
The **scheduler** and the **interrupt mechanism** are both able to **move the control point** from one location in the program code to another. One must take care of **avoiding conflicts** between those mechanisms.

First rule:

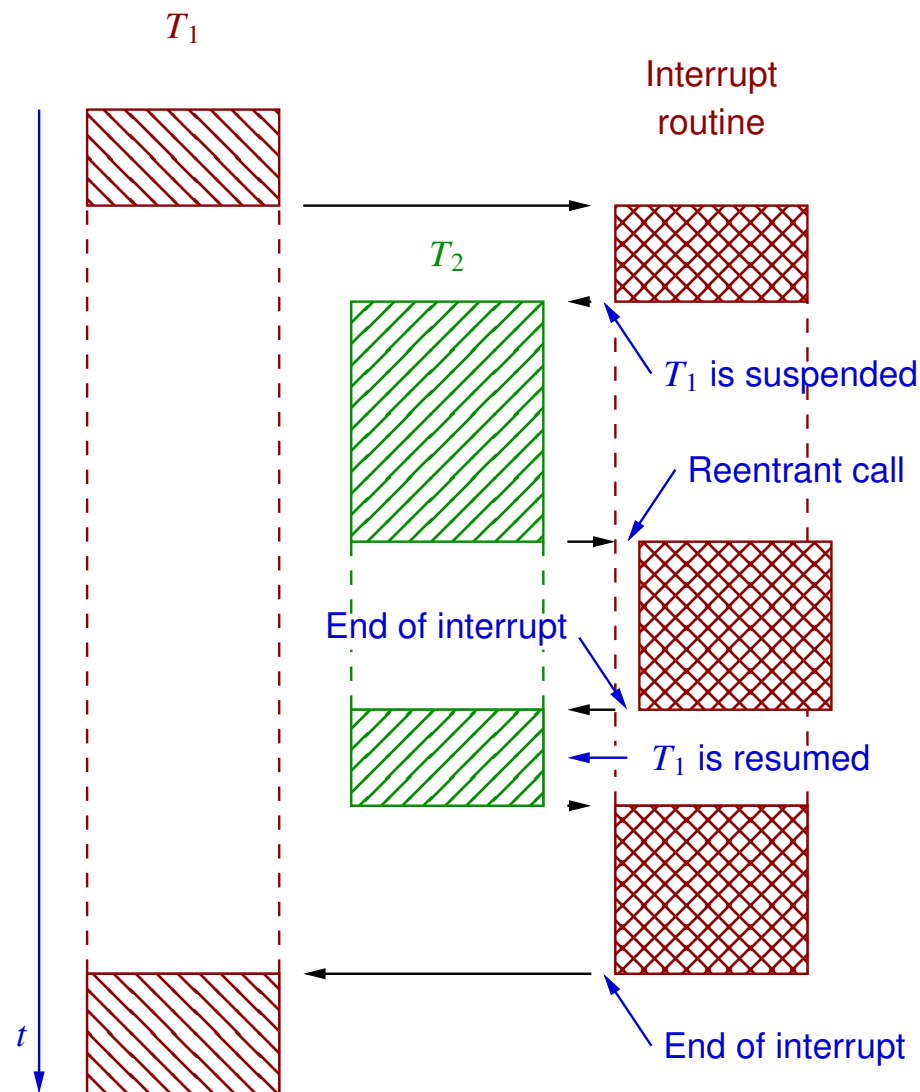
*An **interrupt routine** is not allowed to call an OS service if this service can **block the current task** (e.g., acquiring a semaphore (wait), receiving data from a message queue, waiting for some amount of time, ...).*

- Indeed, if this rule is not respected, then an interrupt routine can get blocked, which amounts to assigning to this interrupt routine an effective priority smaller than the one of a task.

Example:



- Moreover, the interrupt routine might get **called again** before its completion. If this routine is **not reentrant**, then erroneous behaviors are possible (e.g., overwriting a saved processor context).

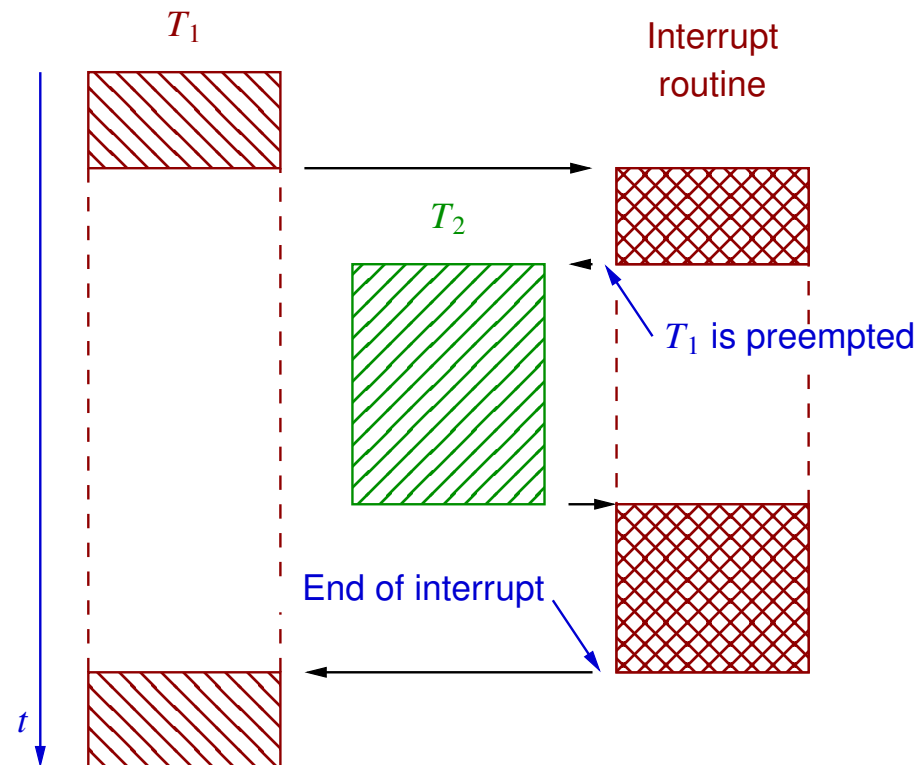


Second rule:

*If an interrupt routine calls an OS service that can lead to a **context switch**, then the **scheduler must be informed** that this system call is performed inside an interrupt routine.*

If this rule is not respected, then the scheduler can **suspend the execution** of an interrupt routine.

Example:

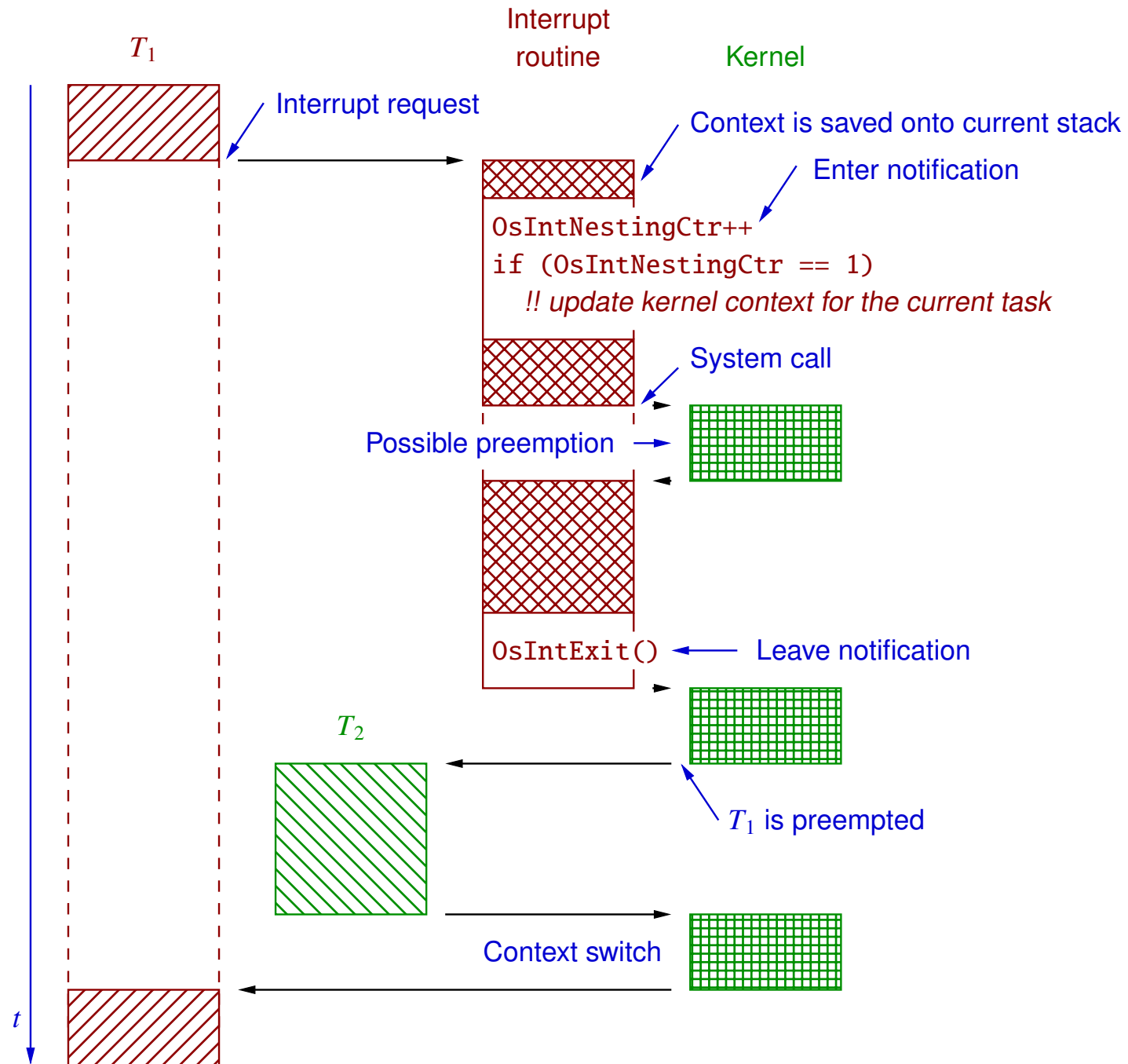


Solution 1: Call **dedicated OS services** at the beginning and the end of interrupt routines, **informing the kernel** that the processor is currently running an interrupt routine. Between those calls, **preemption** of the current task is inhibited.

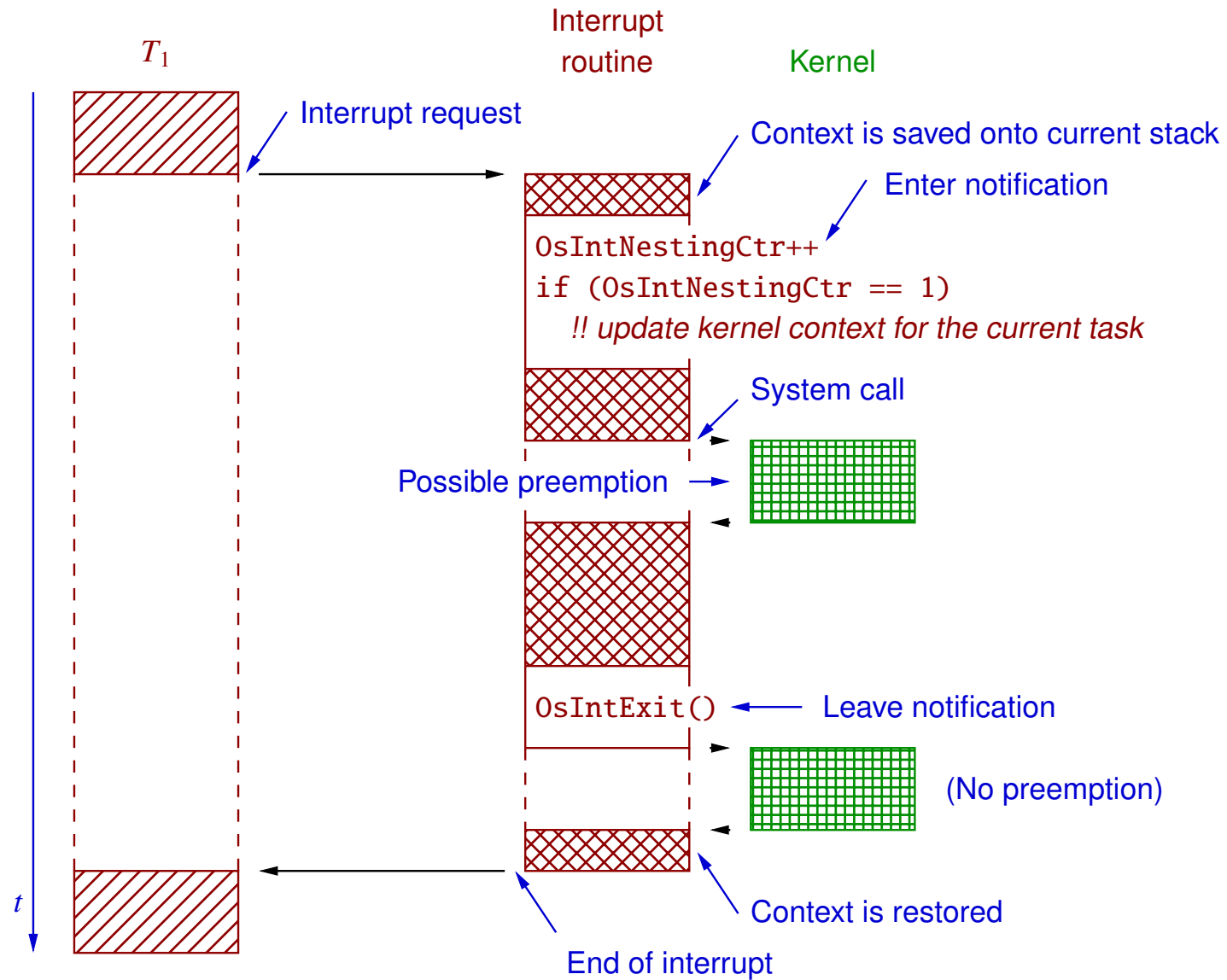
Notes:

- This approach increases **interrupt latency**.
- In the case of multiple interrupt priorities, **nested interrupt routine calls** must be correctly handled.
- At the end of an interrupt routine, **context** has to be switched to the appropriate task.

Example (μ COS-III):



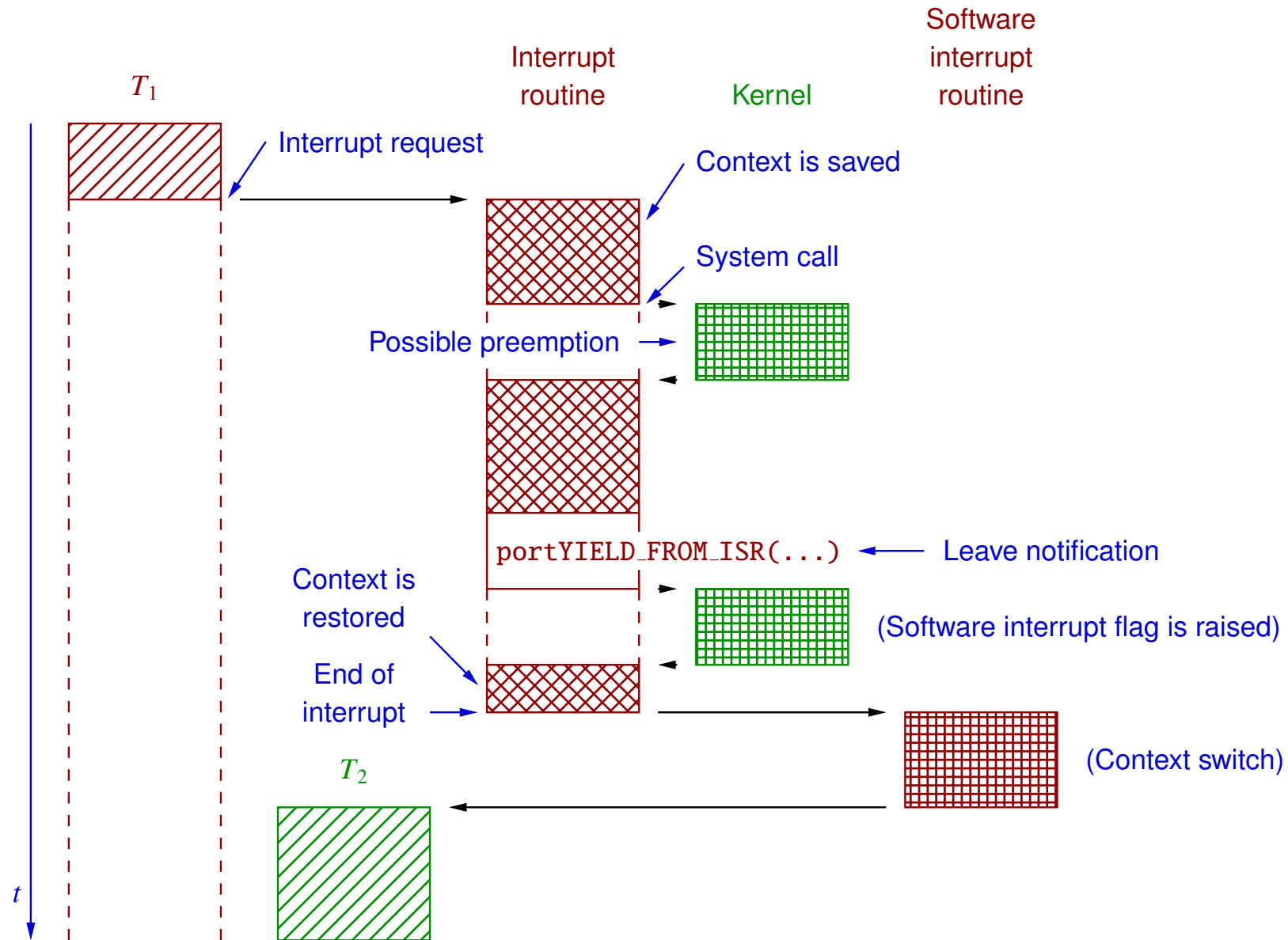
If T_1 is not preempted:



Solution 2: Use **alternate versions** of OS services in interrupt routines, that **do not induce preemption**.

Note: There must exist a mechanism for **switching the current task** immediately after returning from an interrupt routine, if this task must be preempted.

Example (FreeRTOS):



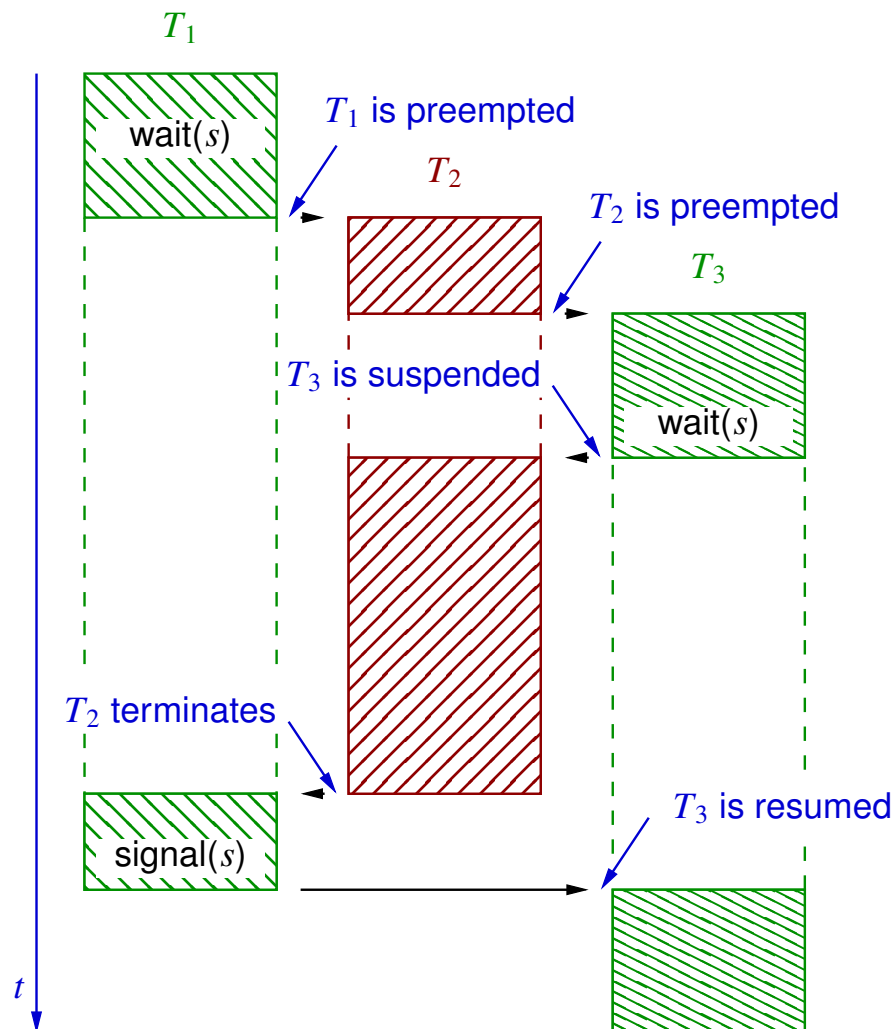
Notes:

- The software interrupt routine implementing the context switch is part of the kernel, and has the **lowest interrupt priority**.
- If the interrupt leave service is **not called**, preemption will only occur the next time that the **scheduler is called**.

Priority inversion

Priority inversion happens when a task is **blocked** waiting for a resource controlled by another task with a **lower priority**.

Example:



Problem:

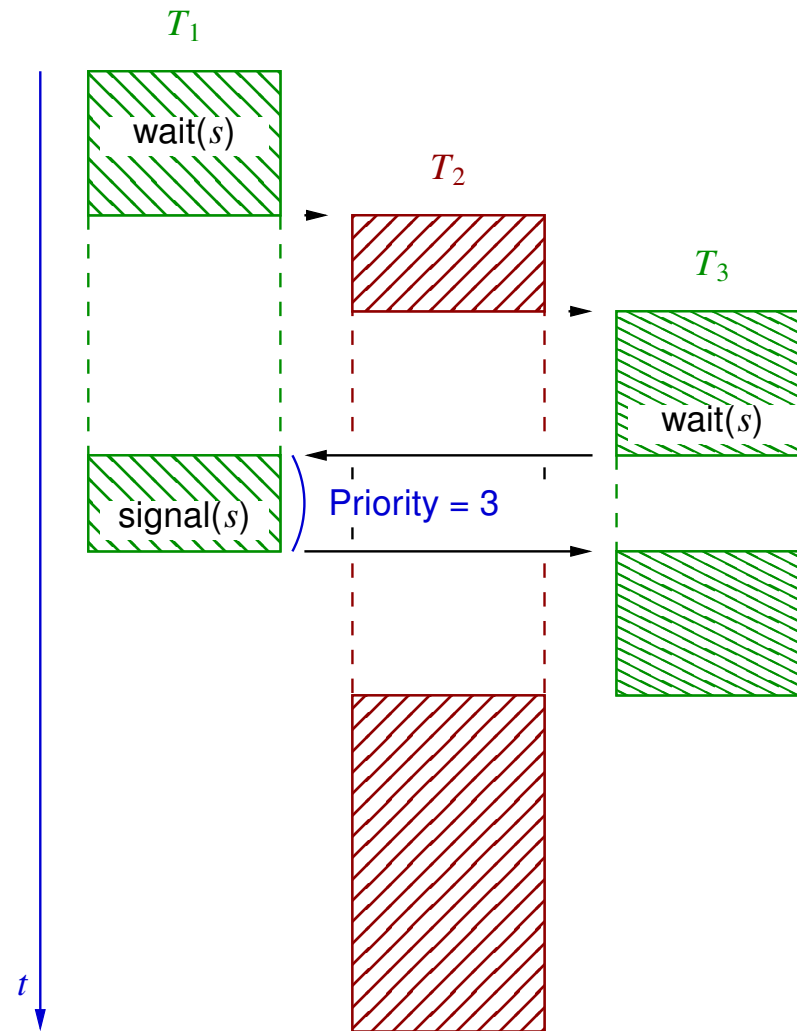
In such a situation, the **effective priority** of T_3 becomes equal to the one of T_1 .

Solution:

The priority of T_1 can be **momentarily increased** (becoming equal to that of T_3) during all the time that T_3 is suspended **waiting for the semaphore acquired by T_1** .

This **priority inheritance** mechanism is **automatically applied** by some operating systems (e.g, FreeRTOS, with a special form of semaphores called **mutexes**).

Illustration:



Time-oriented services

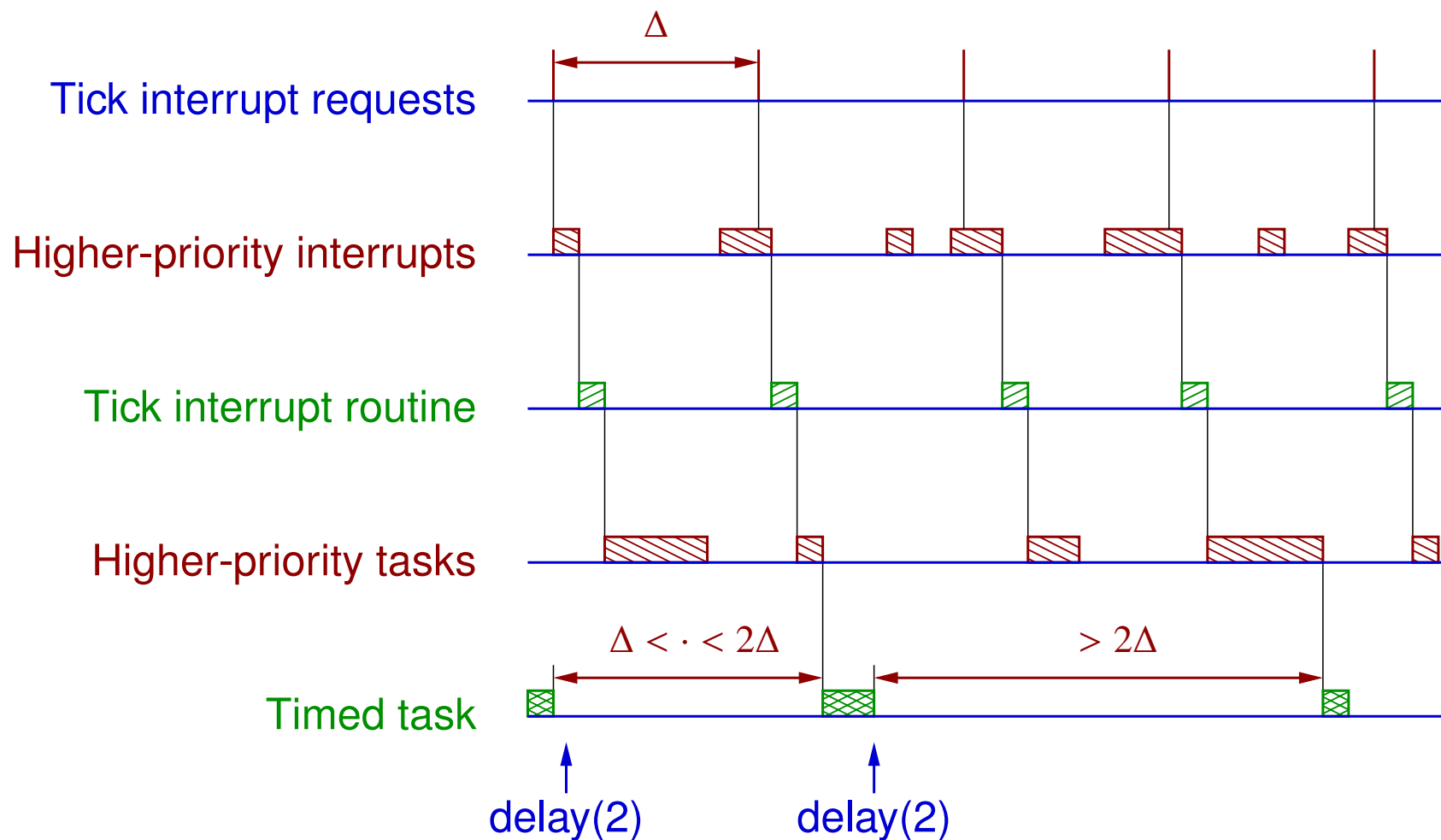
The real-time operating systems offer **timed services**, for instance for **suspending a task** for a predefined amount of time, or specifying a **timeout** for services that can block the current task.

Principles:

- A dedicated component triggers **periodic requests** for an interrupt that
 - has a **low priority**, and
 - is serviced by a routine implemented by the **kernel**. This routine
 - * updates the **state** of the tasks that need to be woken up (or delegates this operation to a kernel task),
 - * manages **time slicing**, and
 - * invokes the **scheduler**, possibly triggering a context switch.
- The delay during which a task is suspended is expressed in the **number of occurrences** (**ticks**) of this interrupt request signal.

Note: The **precision** is limited. Asking to suspend the task during k ticks only ensures that the suspension delay is greater than or equal to the interval between $k - 1$ invocations of the tick interrupt routine.

Example:



Chapter 6

Real-time operating systems: Implementation

Overview of the main difficulties

- The implementation of the OS should remain **generic**, even though some operations (e.g., context switching) are highly architecture-dependent.
- The following operations need to be **efficient**:
 - Identifying the process with the **highest priority** in a list, in order to make it active, or to unblock it following an operation on a synchronization or communication object.
(Ideally, this should have a maximum execution time that is **independent from the number of tasks** managed by the operating system.)
 - Unblocking processes suspended for a **given delay**.
 - Performing a **context switch**.
- There must exist a mechanism for **invoking the scheduler** immediately upon exiting interrupt routines, if preemption is needed.
- For some applications, the real-time operating system has to **share the processor** with another operating system.

Keeping the OS implementation generic

Note: For the illustrations in this chapter, we consider **FreeRTOS**, compiled with **GCC** on a **ARM Cortex M4F** architecture (e.g., STM32F4 MCUs).

Principles:

- The main part of the RTOS code is **architecture and compiler-agnostic**.
- A small subset of platform-dependent functions is provided in specific **ports**.

Illustration:

```
FreeRTOS-Kernel/list.c
                    queue.c
                    tasks.c
                    timers.c
                    portable/GCC/ARM_CM4F/port.c
                                portmacro.h
                    include/
```

Task control blocks

A **Task Control Block (TCB)** is a data structure that represents a **process** inside the kernel memory. This structure contains:

- The current **priority** of the task.

Note: When the task priorities are **fixed and unique**, they can also be used as **process identifiers**.

- The **context of the task**, i.e., the state of the processor saved the last time that the task became non-running.

This data consists in either

- values for all the **processor registers**, or
 - only the **stack pointer(s)**, with the other registers saved on the process stack.
- Pointers linking the TCB to the **global data structures** of the kernel (list of ready processes, lists of processes blocked on synchronization or communication objects, list of delayed processes, ...).

- The **current state** of the process (ready, active, blocked, or interrupted), if it cannot be determined from other data.
- Data needed for managing **priority inversion**, if implemented.
- Values of **thread-local** variables (e.g., errno).
- **Auxiliary data**: statistics, debugging aids (e.g., process name), safety features (e.g., for stack overflow and underflow detection), ...

Global data structures of the kernel

The global information managed by the kernel essentially contains:

- Sets of task control blocks corresponding to
 - the ready (or active) processes,
 - the processes suspended for a given delay.

Those sets are organized as doubly-linked lists, in order to be able to manipulate them in constant time.

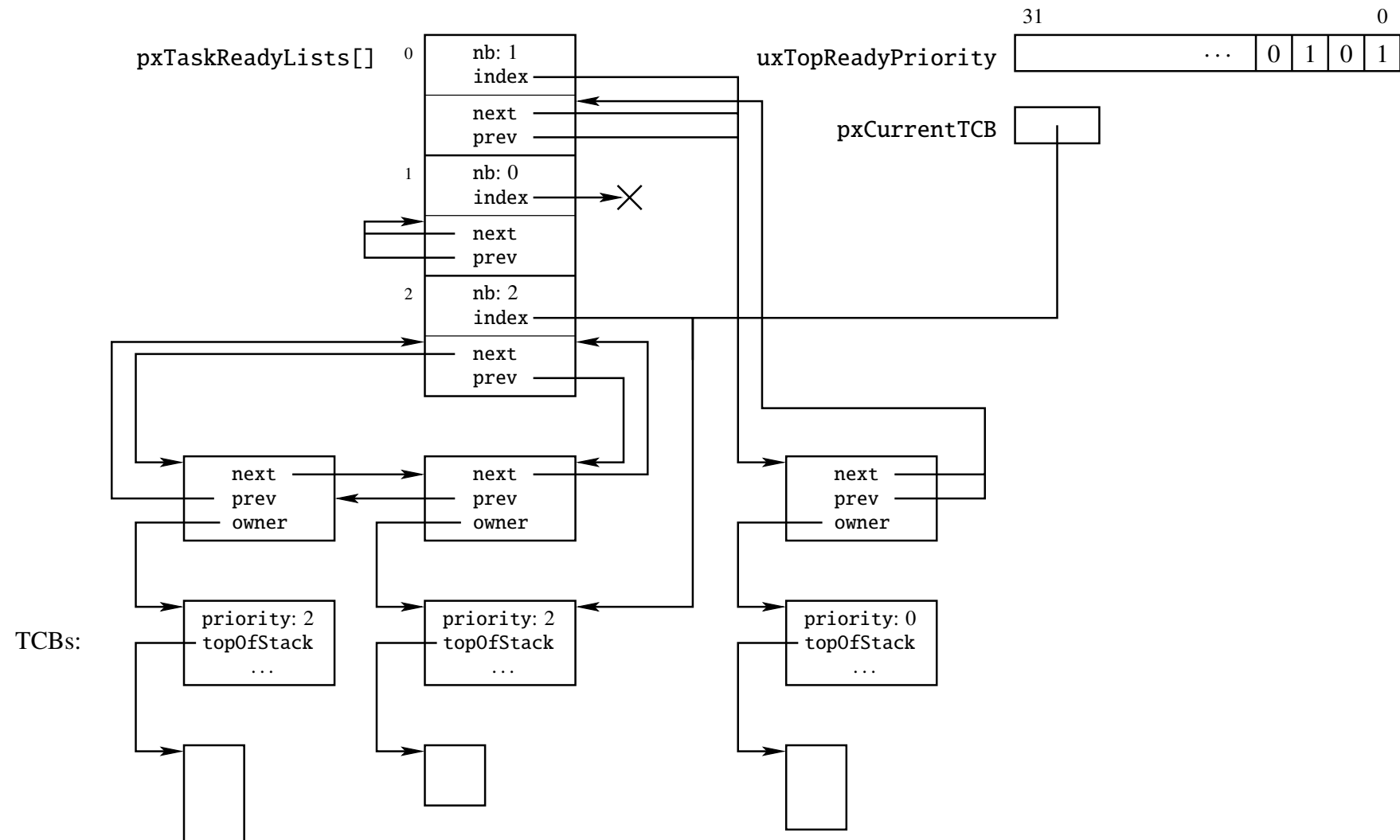
- Optionally, a structure for identifying efficiently the ready process with the highest priority.
- An index for accessing directly a task control block from its corresponding process identifier (if those identifiers do not take the form of pointers to TCBs).
- Data structures representing the state of synchronization and communication objects.

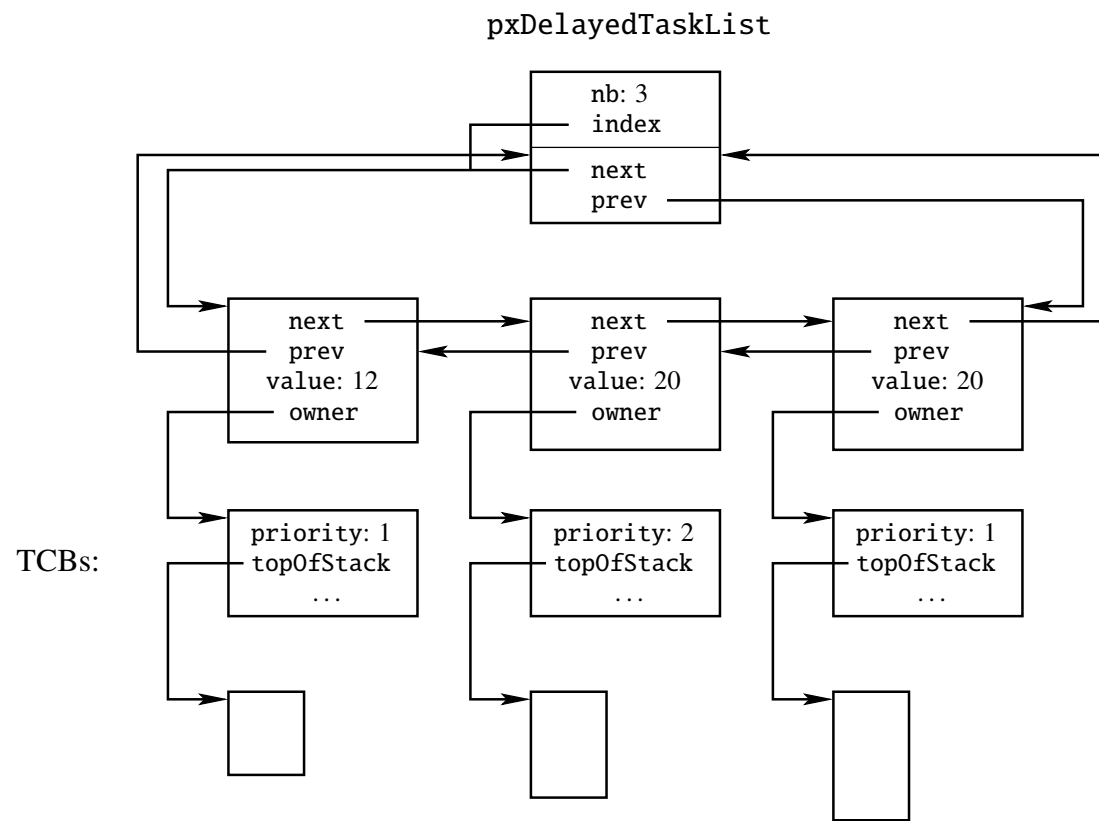
Example (FreeRTOS):

- The **maximum number of process priorities** is a compile-time configuration parameter of the kernel (`configMAX_PRIORITIES`).
- The set of **ready processes** is represented by
 - an array `pxReadyTaskLists[]` of **circular doubly linked lists** of pointers to TCBs, indexed by process priorities.

Each such list contains a **cursor** used for managing time slicing, since FreeRTOS allows several processes to **share the same priority**.
 - an optional **bitfield** `uxTopReadyPriority` for speeding up the search for the non-empty list of processes with the highest priority.
 - A pointer `pxCurrentTCB` to the TCB of the **currently active** task.
- The set of processes **suspended for some delay** is represented by a list `pxDelayedTaskList` of pointers to TCBs, in which the processes are sorted by increasing deadline, then by decreasing priority.

Illustration





Note: Identifying quickly the ready process with the **highest priority** is achieved by exploiting **specific processor instructions** (e.g., *Count Leading Zeros*, CLZ).

The scheduler

The scheduler is implemented by a **kernel function** called after each operation that can potentially **influence the state of processes**:

- **Creating or destroying** a task, or modifying a **task priority**.
- Performing an operation on a **synchronization or communication object**.
- **Suspending** a task for a given delay.
- Servicing the **tick interrupt**.

This function must be kept **simple and efficient**, and only performs the following operations:

1. Checking whether the scheduler is **allowed to preempt tasks**.
2. Identifying the ready task with the **highest priority**.
3. Performing a **context switch** in order to assign the processor to this task.

Note: The possibility of **enabling or disabling** preemption is offered because

- preempting the current task should be prevented inside **interrupt routines** (cf. Chapter 5);
- it provides a simple mechanism for **manipulating atomically** shared variables or communication objects (*preemption locks*). However, this mechanism
 - **increases the latency** of the tasks (by the duration of the **longest interval** during which preemption is disabled), and
 - affects **all the tasks** of the system (not only those that need to be coordinated).

Context switching

The main issue for implementing context switching is to be able to **save and restore** all processor registers.

Some processors **automatically perform** (totally or in part) this operation during interrupts:

- **When an interrupt routine is called:** The **current value of the registers** (or some of them) is saved on the **runtime stack** of the interrupted task.
- **When an interrupt routine returns:** The values extracted from the **current stack** are loaded into the processor registers.

A simple solution (when allowed by the processor architecture) consists of implementing the scheduler **inside an interrupt routine**.

If this routine has the **lowest** possible interrupt priority, then this has the advantage of preventing context switches from occurring in (regular) interrupt routines.

Illustration (FreeRTOS, ARM Cortex M4F)

Principles:

- The scheduler is implemented in the **PendSV** interrupt routine, which has the lowest priority.
- A call for **preemption** thus amounts to raising the corresponding interrupt event flag.
- The processor **automatically pushes** some processor registers (r0-r3, r12, r14, ...) on the stack upon servicing an interrupt.

PendSV interrupt routine:

1. Push **remaining registers** (r4-r11, FPU registers, ...) on the current stack.
2. Store the current **stack pointer** into `pxCurrentTCB -> pxTopOfStack`.
3. Temporary raise the **interrupt enable level** in order to prevent reentrancy.
4. Call the function `vxTaskSwitchContext()`, that updates `pxCurrentTCB` with a pointer to the TCB of the task that must become **active**.

5. Reset the **interrupt enable level** to its lowest value.
6. Set the current **stack pointer** to `pxCurrentTCB -> pxTopOfStack`.
7. Pop the **register values** (r4-r11, FPU registers, ...) from the current stack.
8. Execute a **return from interrupt** instruction.

Task creation and deletion

Creating or deleting a process essentially amounts to updating the data structures managed by the kernel.

Task creation:

1. Allocate (statically or dynamically) a new TCB, and fill it with user-supplied parameters (priority, process name, ...).
2. Allocate a stack for the new process.
3. Create an initial context, in which
 - the program counter is equal to the entry point of the new task,
 - the stack pointer points to the appropriate location, and
 - the other registers are either left uninitialized, or set to 0.
4. Insert the new TCB into the list of ready tasks.
5. Call the scheduler.

Note: A **parameter** can generally be passed to a newly created task, in order to make it possible for **several tasks sharing the same code** to behave differently.

Task deletion:

1. Remove the TCB of the task from its **containing list** (ready processes, processes suspended during a given delay, or waiting list of a synchronization or communication object).
2. If needed, **unallocate** the process stack and the TCB itself.
3. Call the **scheduler**.

Idle task(s)

Some operating systems systematically create one or many **internal tasks**, with a lower priority than all other processes.

There are **many advantages** to this approach:

- The scheduler can be **more efficient**, since it does not have to check whether there exists at least one ready task.
- Such tasks make it possible to measure the **processor utilization**, e.g., by repeatedly incrementing a **counter** that is queried at periodic intervals.
- An idle task can put the processor and some peripherals in a mode that minimizes **energy consumption**.

Example (FreeRTOS): One idle task that either

- runs an **infinite loop**, or
- puts the processor in **deep sleep mode**, and suppresses unneeded tick interrupts (**tickless** idle mode).

Time management

Quantitative time management is performed by the tick interrupt routine:

1. Increment a counter aimed at measuring elapsed time.
2. Compute the set of blocked tasks that must become ready again.

Note: This includes tasks blocked on a timeout.

3. Update the state of tasks involved in time slicing.
4. Call the scheduler.

Illustration (FreeRTOS, ARM Cortex M4F):

- The priority of the tick interrupt is immediately above the one of PendSV.
- The tick interrupt routine processes the TCBs at the head of the list of delayed tasks.

The processing time is linear in the number of tasks to be unblocked.

- Time slicing is implemented by periodically **rotating** the list of ready tasks at the highest priority.
- This routine also signals a **dedicated task** (*Timer/Daemon* task), the purpose of which is to manage **software timers**.

Communication and synchronization objects

In the kernel memory, a **communication or synchronization object** is represented by a structure containing:

- The **type** of the object (semaphore, message queue, ...).
- Data representing the **state of the object** (e.g., for a semaphore: an integer counter).
- One or several lists of **blocked tasks**, waiting on specific operations.

In the case of a **priority-based** selection policy, such a list can be represented by a **doubly-linked list** of TCBs, sorted in **decreasing priority** order.

If necessary, the kernel also maintains a **table of allocated objects**.

Finally, the TCB of each task **waiting for an object** contains a pointer to the structure managing this object.

Note: Implementing kernel services that **update the state of objects** does not require specific instructions such as **test and set**, since it is sufficient to **disable interrupts** during non-atomic operations.

Combining a real-time and a non-real-time operating systems

It is possible to combine in a single application a **real-time operating system (RTOS)** with **another operating system (host OS)**. There are two possibilities:

- The operations of the host operating system are **suspended** when the real-time OS is started, and **get the processor back** when the RTOS terminates (e.g., μ COS-III).
- The host operating system is seen as **special task** that has a **lower priority** than all the tasks managed by the real-time OS (e.g., RTAI).

For implementing this approach, it is necessary to ensure that the host OS **can never disable interrupts** managed by the kernel or the real-time tasks.

Chapter 7

Periodic Tasks Scheduling

Periodic tasks

We consider a **simplified programming environment** satisfying the following hypotheses:

- The **number of tasks** to be executed is fixed.
- Each task is characterized by a distinct and constant **priority**.
- The **execution requests** for each task occur **periodically**, i.e., with a constant delay between two successive requests.

In particular, the timing of execution requests for a task **cannot depend** on operations performed by other tasks.

- The **execution time** of each task is constant.

- The following **real-time constraint** must be satisfied:

Each execution of a task must finish before or at the same time as the next request for executing this task.

- **Context switches** are instantaneous and preemptive.

Critical instants and critical zones

In addition to its priority, each task τ_i is characterized by

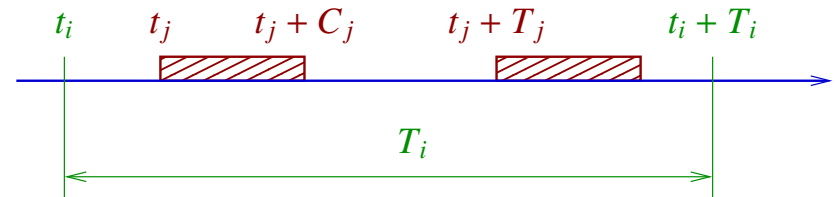
- its period T_i , and
- its execution time (for each period) C_i .

Definitions:

- The response time of an execution request for τ_i is the delay between this request and the end of the corresponding execution of this task.
- A critical instant for the task τ_i is an occurrence of an execution request for τ_i that leads to the largest possible response time for this task.
- A critical zone for τ_i is an interval of duration T_i that starts at a critical instant (for τ_i).

Theorem 1: A **critical instant** for τ_i occurs when an execution request for this task coincides with requests for executing **all the tasks that have a higher priority** than τ_i .

Proof: Assume that an execution request for τ_i occurs at $t = t_i$, and that an execution request for a **higher-priority task** τ_j is received at $t = t_j$.



Advancing the request for τ_j from t_j **to** t_i can **never decrease** the response time of τ_i .

(Indeed, for each instruction I of τ_i , advancing τ_j by **one instruction** either leaves unchanged the execution time of I , or **postpones it**.)

The same reasoning can be applied to **all the tasks** that have a higher priority than τ_i .

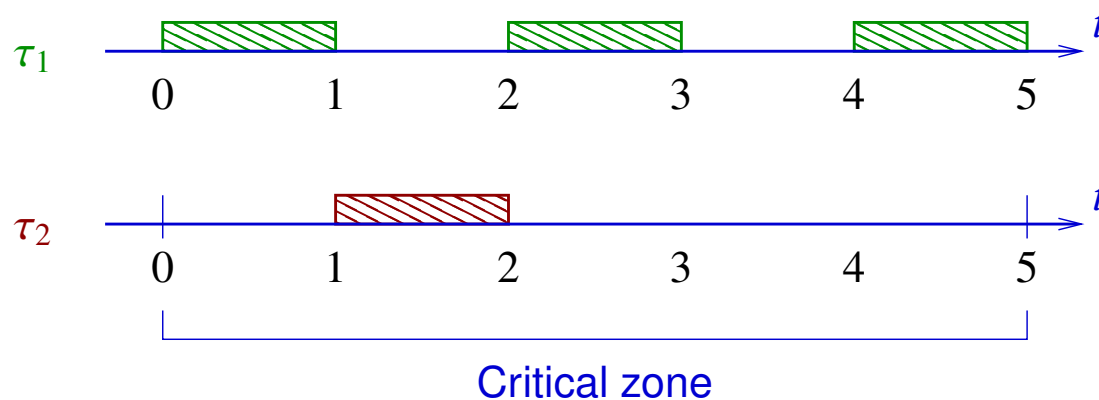
Schedulable tasks

Definition: A set of tasks is **schedulable** (with respect to a given assignment of priorities) if the **response time** of each task τ_i is always **less than or equal to its period T_i** .

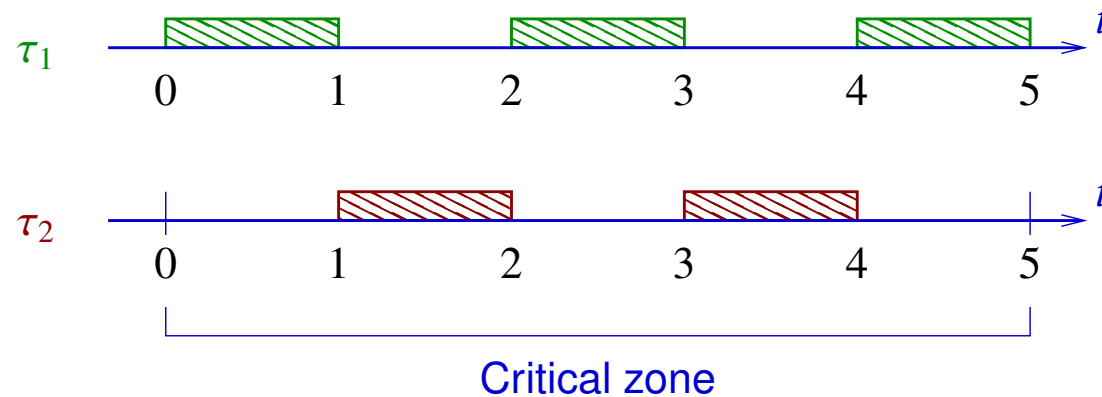
Thanks to Theorem 1, checking whether a given set of tasks is **schedulable** reduces to simulating the **scheduling strategy** in the particular case of **simultaneous execution requests** for all tasks at $t = 0$.

Examples: Consider two tasks τ_1 and τ_2 , with $T_1 = 2$, $T_2 = 5$, $C_1 = 1$ and $C_2 = 1$.

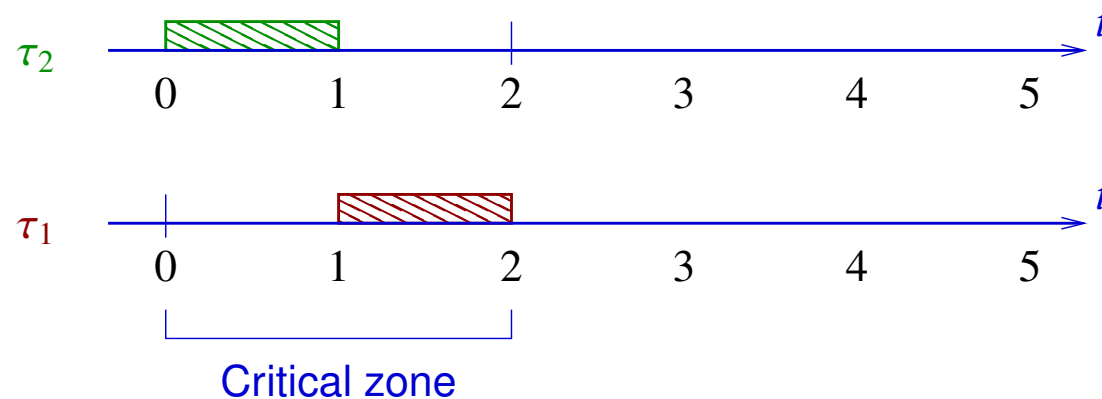
- If τ_1 has a higher priority than τ_2 .



The tasks **are schedulable**, and remain schedulable even if the execution time of τ_2 is increased by one time unit ($C_2 = 2$):



- If τ_2 has a higher priority than τ_1 .



The tasks **are schedulable**.

Note: In this case, the execution time of τ_1 and τ_2 **cannot be increased** anymore.

Rate-Monotonic Scheduling

In the previous example, the best strategy was to assign the **highest priority** to the task that has the **smallest period**.

Definition: Given a set of tasks $\tau_1, \tau_2, \dots, \tau_n$ with respective periods $T_1, T_2, \dots, T_n$, the **Rate-Monotonic Scheduling (RMS)** strategy consists in assigning distinct priorities $P_1, P_2, \dots, P_n$ to the tasks, such that for all i, j :

$$T_i < T_j \Rightarrow P_i > P_j.$$

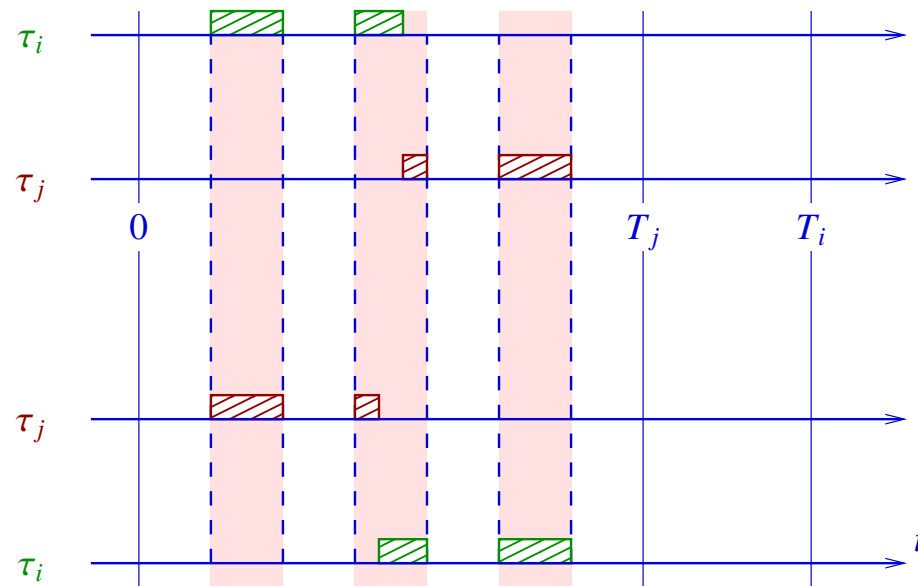
The following result establishes that the RMS strategy is **optimal**:

Theorem 2: If a set of tasks is **schedulable** with respect to some priorities assignment, then it is **schedulable as well** with respect to priorities defined by the RMS strategy.

Proof: Consider a set of tasks $\tau_1, \tau_2, \dots, \tau_n$ for which there exists a priorities assignment $P_1, P_2, \dots, P_n$ that makes them schedulable.

Let τ_i and τ_j two tasks with adjacent priorities P_i and P_j , such that $P_i > P_j$.

If $T_i > T_j$, then the priorities of τ_i and τ_j can be swapped:



The resulting set of tasks remains schedulable.

By performing repeatedly this operation, one eventually obtains a priorities assignment corresponding to the RMS strategy.

The processor load factor

Consider a set of tasks $\tau_1, \tau_2, \dots, \tau_n$ with respective periods and execution times $T_1, T_2, \dots, T_n$ and $C_1, C_2, \dots, C_n$.

The **processor load factor** U corresponding to this set of tasks represents the **relative amount of CPU time** needed for executing them:

$$U = \sum_{i=1}^n \frac{C_i}{T_i}.$$

Definition: A set of tasks **fully uses** the processor if

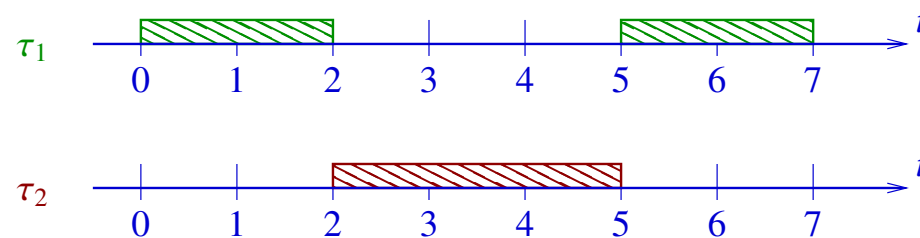
- this set of tasks is **schedulable**, and
- **any increase** of the execution time of a task (and hence of the **processor load factor**) yields a set of tasks that is **not schedulable** anymore.

Notes:

- Thanks to Theorem 2, checking whether a set of tasks is **schedulable or not** can be done by assigning **RMS priorities** to those tasks.
- A set of tasks that has a processor load factor **less than 1** is **not necessarily schedulable**:

Example:

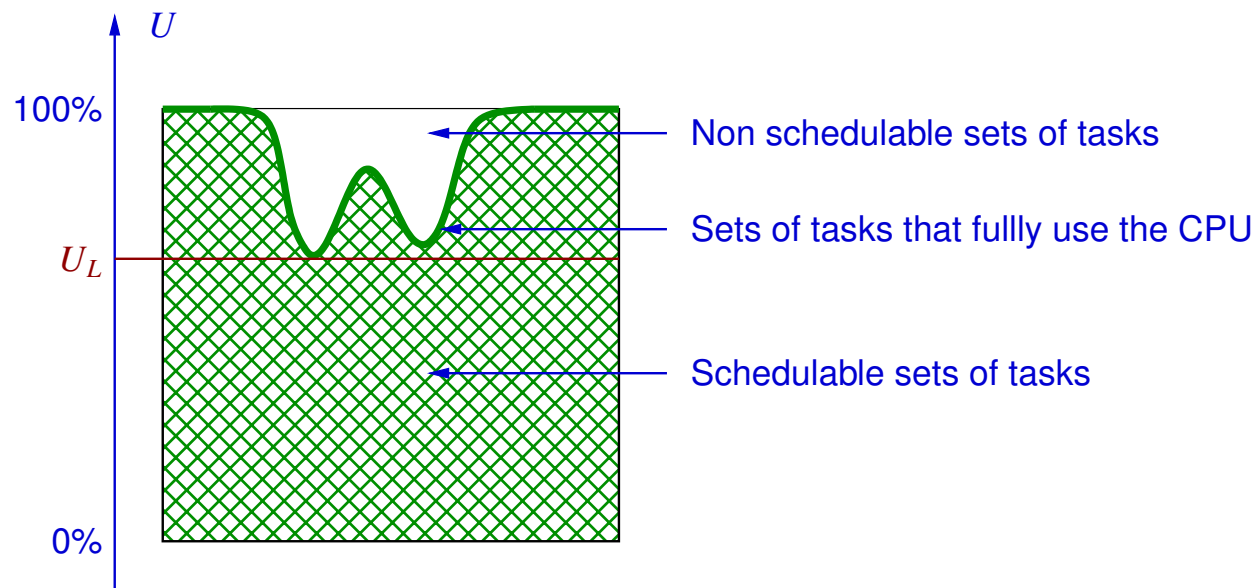
$$\left. \begin{array}{l} \tau_1 : T_1 = 5, C_1 = 2 \\ \tau_2 : T_2 = 7, C_2 = 4 \end{array} \right\} U = \frac{2}{5} + \frac{4}{7} \approx 97\%$$



Classifying sets of tasks

The **set of sets of tasks** can be partitioned into three classes:

- The **non schedulable** sets of tasks.
- The sets of tasks that **fully use** the processor.
- The schedulable sets of tasks that **do not fully use** the processor.



The best lower bound U_L on the processor load factor of the sets of tasks that fully use the processor is such that:

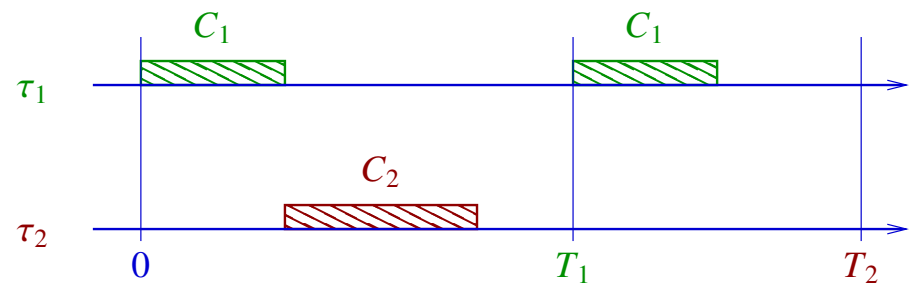
- If the processor load factor of a set of tasks is less than or equal to U_L , then this set of tasks is schedulable (regardless of the periods and execution times of the tasks!).
- If the processor load factor of a set of tasks is greater than U_L , then this set of tasks may or may not be schedulable, depending on the details of the tasks.

U_L : Case of two tasks

Let τ_1 and τ_2 be two tasks with respective periods and execution times T_1, T_2 and C_1, C_2 . We assume $T_1 < T_2$. According to the RMS strategy, we assign a **higher priority** to τ_1 .

During a **critical zone** of τ_2 , the number of execution requests for τ_1 is equal to $\left\lceil \frac{T_2}{T_1} \right\rceil$.

- If all the executions of τ_1 in the interval $[0, T_2]$ terminate earlier than or at $t = T_2$.



The following condition is satisfied:

$$C_1 \leq T_2 - T_1 \left\lceil \frac{T_2}{T_1} \right\rceil.$$

For a given value of C_1 , the largest possible value of C_2 is given by

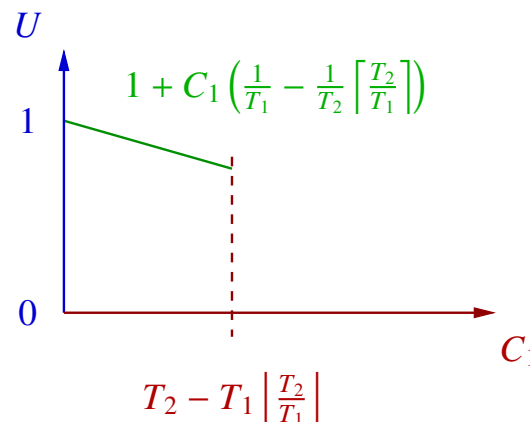
$$C_2 = T_2 - C_1 \left\lceil \frac{T_2}{T_1} \right\rceil.$$

It follows that the highest possible processor load factor is equal to

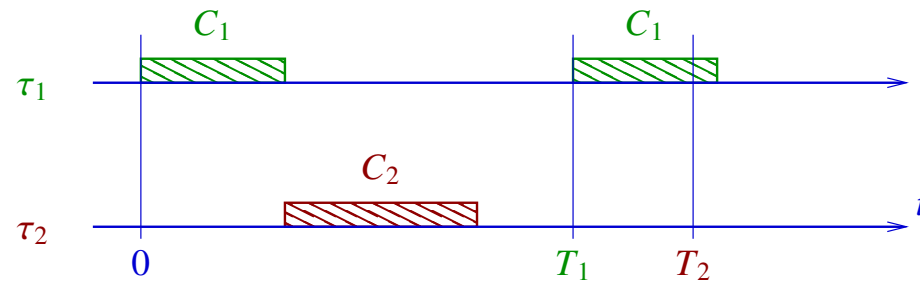
$$\begin{aligned} U &= \frac{C_1}{T_1} + \frac{C_2}{T_2} \\ &= 1 + C_1 \left(\frac{1}{T_1} - \frac{1}{T_2} \left\lceil \frac{T_2}{T_1} \right\rceil \right). \end{aligned}$$

Note that we have $\frac{1}{T_1} - \frac{1}{T_2} \left\lceil \frac{T_2}{T_1} \right\rceil \leq 0$.

Therefore, for given values of T_1 and T_2 , the maximum processor load factor decreases with C_1 .



- If an execution of τ_1 is still unfinished at $t = T_2$.



The following condition is satisfied:

$$C_1 > T_2 - T_1 \left\lfloor \frac{T_2}{T_1} \right\rfloor.$$

For a **given value of C_1** , the **largest possible value of C_2** is given by

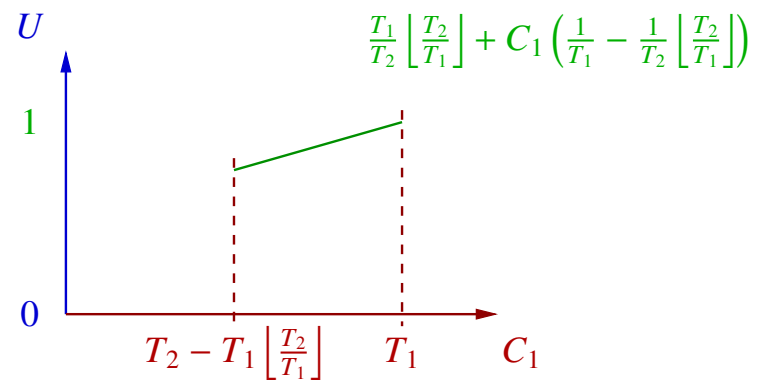
$$C_2 = (T_1 - C_1) \left\lfloor \frac{T_2}{T_1} \right\rfloor.$$

Hence, the **highest possible processor load factor** is equal to

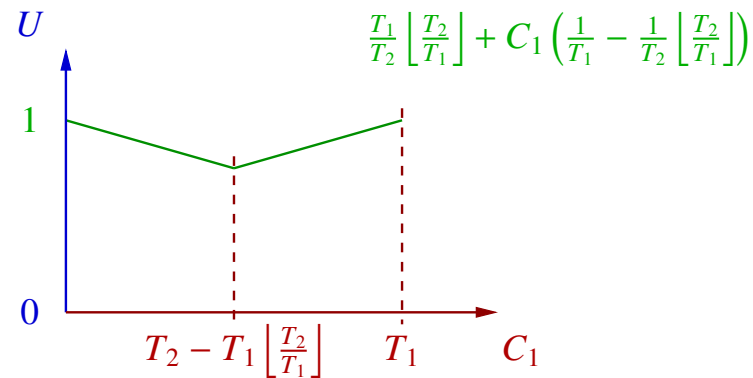
$$U = \frac{T_1}{T_2} \left\lfloor \frac{T_2}{T_1} \right\rfloor + C_1 \left(\frac{1}{T_1} - \frac{1}{T_2} \left\lfloor \frac{T_2}{T_1} \right\rfloor \right).$$

For given values of T_1 and T_2 , this expression increases with C_1 , since

$$\frac{1}{T_1} - \frac{1}{T_2} \left\lfloor \frac{T_2}{T_1} \right\rfloor \geq 0.$$



Summary:



The **smallest value of U** corresponds to the **boundary between the two cases**, where we have

$$C_1 = T_2 - T_1 \left\lfloor \frac{T_2}{T_1} \right\rfloor.$$

By **introducing this value** in the expression of U , one obtains

$$\begin{aligned} U &= \frac{T_1}{T_2} \left\lfloor \frac{T_2}{T_1} \right\rfloor + \left(T_2 - T_1 \left\lfloor \frac{T_2}{T_1} \right\rfloor \right) \left(\frac{1}{T_1} - \frac{1}{T_2} \left\lfloor \frac{T_2}{T_1} \right\rfloor \right) \\ &= \frac{T_1}{T_2} \left\lfloor \frac{T_2}{T_1} \right\rfloor + \frac{T_2}{T_1} - 2 \left\lfloor \frac{T_2}{T_1} \right\rfloor + \frac{T_1}{T_2} \left\lfloor \frac{T_2}{T_1} \right\rfloor^2. \end{aligned}$$

Let us define $I = \left\lfloor \frac{T_2}{T_1} \right\rfloor$ and $f = \frac{T_2}{T_1} - \left\lfloor \frac{T_2}{T_1} \right\rfloor$.

The previous expression becomes

$$\begin{aligned} U &= \frac{I}{I+f} + (I+f) - 2I + \frac{I^2}{I+f} \\ &= 1 - f \frac{1-f}{I+f}. \end{aligned}$$

The **smallest possible value of U** is obtained with $I = 1$. We then have

$$U = 1 - f \frac{1-f}{1+f},$$

and

$$\frac{dU}{df} = \frac{f^2 + 2f - 1}{(1+f)^2}.$$

The **best lower bound U_L** on U is thus obtained with $I = 1$ and $f = -1 + \sqrt{2}$:

$$U_L = 1 - (\sqrt{2} - 1) \left(\frac{2 - \sqrt{2}}{\sqrt{2}} \right) = 2(\sqrt{2} - 1) \approx 0.83.$$

Case of two tasks: Conclusions

Theorem 3: If a set of two periodic tasks has a processor load factor that is **less than or equal to $2(\sqrt{2} - 1)$** , then this set of tasks is **schedulable**.

Notes:

- This **sufficient criterion** is **independent** from the periods and execution times of the tasks.
- In the particular case where **T_2 is an integer multiple of T_1** , one has **$f = 0$** , hence

$$U_L = 1.$$

All pairs of tasks satisfying this condition (and such that $\frac{C_1}{T_1} + \frac{C_2}{T_2} \leq 1$!) are thus **schedulable**.

U_L : Case of n tasks

The goal is now to compute the value of U_L

- for a **given number** n of tasks, and
- for **any number** of tasks.

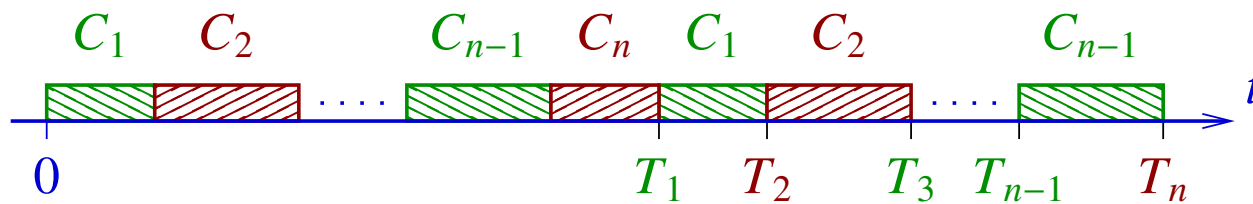
The first step is to establish an intermediate result:

Lemma 1: Let $\tau_1, \tau_2, \dots, \tau_n$ be periodic tasks with the respective periods and execution times $T_1, T_2, \dots, T_n$ and $C_1, C_2, \dots, C_n$, such that

- This set of tasks **fully uses the processor**,
- $0 < T_1 < T_2 < \dots < T_{n-1} < T_n < 2T_1$,
- The processor load factor of this set of tasks is **minimum** among all sets of tasks that fully use the processor.

In this case, one has

$$\begin{aligned}
 C_1 &= T_2 - T_1, \\
 C_2 &= T_3 - T_2, \\
 &\vdots \\
 C_{n-1} &= T_n - T_{n-1}, \\
 C_n &= T_n - 2(C_1 + C_2 + \cdots C_{n-1}) \\
 &= 2T_1 - T_n.
 \end{aligned}$$

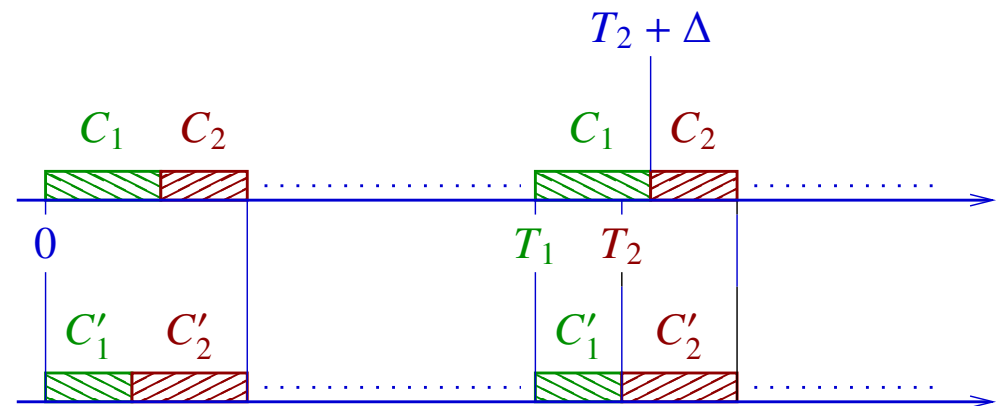


Proof: By contradiction, let us show that we must have $C_1 = T_2 - T_1$.

- If $C_1 = T_2 - T_1 + \Delta$, with $\Delta > 0$.

We modify the **execution time of tasks** in the following way:

$$\begin{aligned} C'_1 &= C_1 - \Delta, \\ C'_2 &= C_2 + \Delta, \\ C'_3 &= C_3, \\ &\vdots \\ C'_{n-1} &= C_{n-1}, \\ C'_n &= C_n. \end{aligned}$$



After the modification, the new set of tasks still **fully uses the processor**. However, the **processor load factor** now becomes

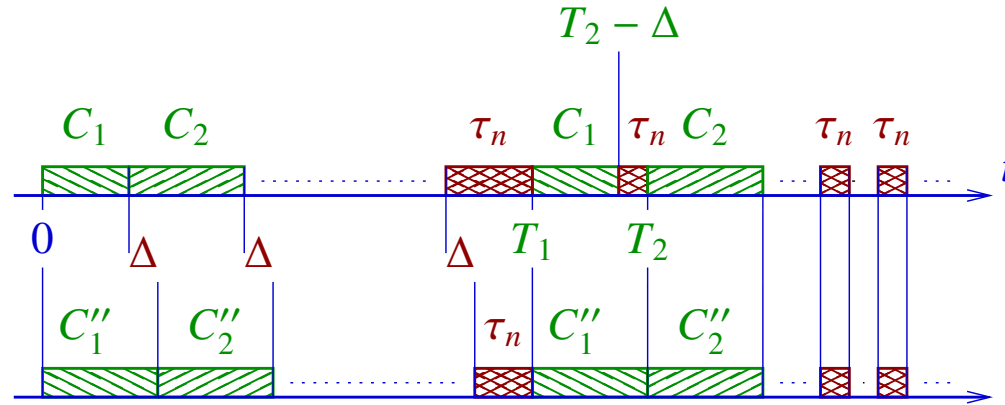
$$U' = U - \frac{\Delta}{T_1} + \frac{\Delta}{T_2} < U,$$

which contradicts the hypothesis that **U is minimum**.

- If $C_1 = T_2 - T_1 - \Delta$, with $\Delta > 0$.

We now modify the **execution time of tasks** as follows:

$$\begin{aligned} C_1'' &= C_1 + \Delta, \\ C_2'' &= C_2, \\ C_3'' &= C_3, \\ &\vdots \\ C_{n-1}'' &= C_{n-1}, \\ C_n'' &= C_n - 2\Delta. \end{aligned}$$



The resulting set of tasks **fully uses the processor**. The **processor load factor** becomes

$$U' = U + \frac{\Delta}{T_1} - \frac{2\Delta}{T_n}.$$

Since we have by hypothesis $T_n < 2T_1$, this property contradicts $U' < U$.

By **similar reasoning**, one obtains successively

$$\begin{aligned} C_2 &= T_3 - T_2, \\ C_3 &= T_4 - T_3, \\ &\vdots \\ C_{n-1} &= T_n - T_{n-1}. \end{aligned}$$

Since the processor is **fully used**, one finally gets

$$C_n = T_n - 2(C_1 + C_2 + \cdots C_{n-1}).$$

Corollary: For each set of tasks that **satisfies the hypotheses of Lemma 1**, the **processor load factor** is equal to

$$\begin{aligned} U &= \frac{T_2 - T_1}{T_1} + \frac{T_3 - T_2}{T_2} + \cdots + \frac{T_n - T_{n-1}}{T_{n-1}} \\ &\quad + \frac{2T_1 - T_n}{T_n} \\ &= \frac{T_2}{T_1} + \frac{T_3}{T_2} + \cdots + \frac{T_n}{T_{n-1}} + 2\frac{T_1}{T_n} - n. \end{aligned}$$

For each $i = 1, 2, \dots, n-1$, let us define $q_i = \frac{T_{i+1}}{T_i}$. We then have

$$U = q_1 + q_2 + \dots + q_{n-1} + \frac{2}{q_1 q_2 \dots q_{n-1}} - n,$$

and thus for each i ,

$$\frac{\partial U}{\partial q_i} = 1 - 2 \frac{q_1 q_2 \dots q_{i-1} q_{i+1} \dots q_{n-1}}{(q_1 q_2 \dots q_{n-1})^2}.$$

The best lower bound U_L of U therefore corresponds to

$$\begin{aligned} \frac{\partial U}{\partial q_i} &= 0 \\ 1 - \frac{1}{q_i} \cdot \frac{2}{q_1 q_2 \dots q_{n-1}} &= 0. \end{aligned}$$

For each i , one has

$$q_i = \frac{2}{q_1 q_2 \cdots q_{n-1}},$$

hence

$$q_1 = q_2 = \cdots = q_{n-1} = 2^{\frac{1}{n}}.$$

By introducing these values in the expression of U , one obtains

$$\begin{aligned} U_L &= (n-1)2^{\frac{1}{n}} + \frac{2}{2^{\frac{n-1}{n}}} - n \\ &= (n-1)2^{\frac{1}{n}} + 2^{\frac{1}{n}} - n \\ &= n(2^{\frac{1}{n}} - 1). \end{aligned}$$

We thus have the following result:

Theorem 4: If the periods $T_1, T_2, \dots, T_n$ of a set of n tasks are such that

$$0 < T_1 < T_2 < \cdots < T_{n-1} < T_n < 2T_1,$$

with a processor load factor that is less than or equal to $n(2^{\frac{1}{n}} - 1)$, then this set of tasks is schedulable.

In the hypotheses of Theorem 4, the **constraint over the task periods** is actually not necessary:

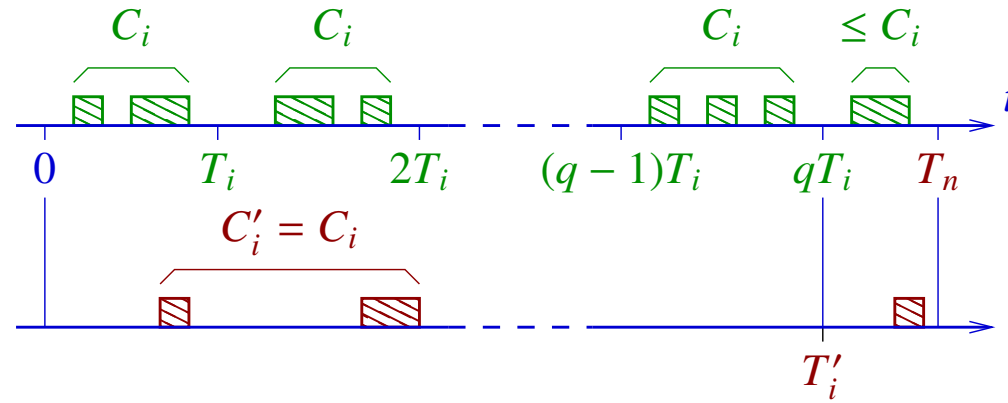
Theorem 5: If a set of n periodic tasks has a processor load factor that is **less than or equal to $n(2^{\frac{1}{n}} - 1)$** , then this set of tasks is **schedulable**.

Proof: Let $\tau_1, \tau_2, \dots, \tau_n$ be tasks with respective periods and execution times $T_1, T_2, \dots, T_n$ and $C_1, C_2, \dots, C_n$. We assume that this set of tasks **fully uses the processor**.

If there exists $i \in \{1, 2, \dots, n-1\}$ such that $2T_i \leq T_n$, then we define $q = \left\lfloor \frac{T_n}{T_i} \right\rfloor$ and $r = T_n - qT_i$ (we thus have $q > 1$ and $r \geq 0$).

We modify the set of tasks in the following way:

- We replace τ_i by τ'_i with the period $T'_i = qT_i$ and the execution time $C'_i = C_i$.
- We replace τ_n by τ'_n , with the period $T'_n = T_n$ and an execution time C'_n chosen so as to **fully use the processor**.



In the **critical zone of τ_n** , the difference between the execution times needed by τ_i and τ'_i is **at most equal to $(q-1)C_i$** . Therefore, one has

$$C'_n - C_n \leq (q-1)C_i.$$

After modifying the set of tasks, the **processor load factor U'** becomes equal to

$$U' \leq U + \frac{C'_i}{T'_i} - \frac{C_i}{T_i} + \frac{(q-1)C_i}{T_n}$$

where U is the processor load factor of the **initial set of tasks**.

One then obtains

$$U' \leq U + C_i \left(\frac{1}{qT_i} - \frac{1}{T_i} + \frac{q-1}{T_n} \right).$$

Since we have $qT_i \leq T_n$, this leads to

$$\begin{aligned} \frac{1}{qT_i} - \frac{1}{T_i} + \frac{q-1}{T_n} &\leq \frac{1}{qT_i} - \frac{1}{T_i} + \frac{q-1}{qT_i} \\ &\leq 0. \end{aligned}$$

As a consequence, we have $U' \leq U$. This implies that our modification of the set of tasks **did not increase** the processor load factor.

By **repeatedly performing** such a modification, one eventually obtains a set of tasks to which **Theorem 4** can be applied.

The limit processor load factor

The value of U_L decreases with the number n of tasks. Indeed,

$$\begin{aligned}\frac{dU_L}{dn} &= \left(1 - \frac{\ln 2}{n}\right) 2^{\frac{1}{n}} - 1 \\ &= (1 - x)e^x - 1,\end{aligned}$$

by defining $x = \frac{\ln 2}{n}$. Let us show that we have

$$(1 - x)e^x < 1$$

for all $x > 0$ (which implies $\frac{dU_L}{dn} < 0$ for all $n > 0$).

For all $x > 0$, we have $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, hence

$$\begin{aligned}(1 - x)e^x &= (1 - x) + (1 - x)x + (1 - x)\frac{x^2}{2!} + (1 - x)\frac{x^3}{3!} + \dots \\ &= 1 - \left(1 - \frac{1}{2!}\right)x^2 - \left(\frac{1}{2!} - \frac{1}{3!}\right)x^3 - \left(\frac{1}{3!} - \frac{1}{4!}\right)x^4 + \dots \\ &< 1.\end{aligned}$$

For an **asymptotically large** number of tasks, we obtain

$$\begin{aligned}\lim_{n \rightarrow \infty} U_L(n) &= \lim_{n \rightarrow \infty} n(2^{\frac{1}{n}} - 1) \\ &= \lim_{n \rightarrow \infty} \frac{2^{\frac{1}{n}} - 1}{\frac{1}{n}} \\ &= \lim_{n \rightarrow \infty} \frac{\frac{\ln 2}{n^2} 2^{\frac{1}{n}}}{\frac{1}{n^2}} \\ &= \ln 2 \\ &\approx 0.69\end{aligned}$$

In summary, we have the following result:

Theorem 6: If a set of periodic tasks has a processor load factor that is less than or equal to $\ln 2$, then this set of tasks is schedulable.

Conclusion: The following algorithm can be used for checking efficiently whether a set of n periodic tasks with a processor load factor equal to U is schedulable or not:

1. If $U > 100\%$, then the set of tasks is not schedulable;
2. If $U \leq 69\%$, then the set of tasks is schedulable;
3. If $U \leq n(2^{\frac{1}{n}} - 1)$, then the set of tasks is schedulable;
4. Otherwise, one performs an exact scheduling simulation, based on a RMS priorities assignment.

Notes

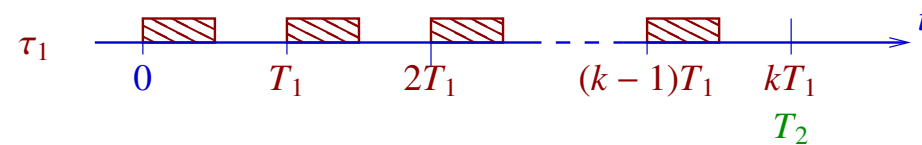
- In situations where $U \leq 69\%$ for the periodic tasks, the processor does not have to remain unused during 31% of the time! One can instead run low-priority tasks that are not bound by real-time constraints.
- For some specific class of sets of tasks, one can obtain $U_L = 100\%$, which guarantees that every set of tasks for which $U \leq 100\%$ is schedulable.

Example: Let $\tau_1, \tau_2, \dots, \tau_n$ be a set of tasks with respective periods and execution times $T_1, T_2, \dots, T_n$ and $C_1, C_2, \dots, C_n$, such that

- $0 < T_1 \leq T_2 \leq \dots \leq T_n$,
- $\forall i, j : i < j \Rightarrow T_j$ is an integer multiple of T_i ,
- $U = \sum_{i=1}^n \frac{C_i}{T_i} \leq 1$.

Let us show that this set of tasks is **schedulable**.

The **critical zone of τ_2** contains $\frac{T_2}{T_1}$ **complete executions** of τ_1 :



Similarly, for each $j \in \{2, 3, \dots, n\}$, the **critical zone of τ_j** contains

$$\frac{T_j}{T_1} \text{ complete executions of } \tau_1,$$

$$\vdots$$

$$\frac{T_j}{T_{j-1}} \text{ complete executions of } \tau_{j-1}.$$

The condition that must be satisfied in order to finish the execution of τ_j **before the end of its critical zone** is thus

$$C_j \leq T_j - \frac{T_j}{T_1}C_1 - \frac{T_j}{T_2}C_2 - \dots - \frac{T_j}{T_{j-1}}C_{j-1}$$

After simplification, this condition becomes

$$\frac{C_1}{T_1} + \frac{C_2}{T_2} + \cdots + \frac{C_j}{T_j} \leq 1,$$

which immediately follows from the hypothesis $U \leq 1$.

Chapter 8

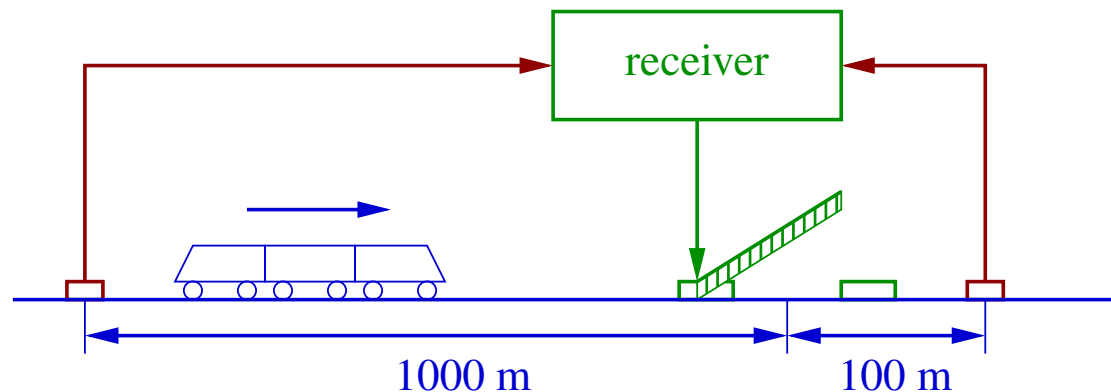
Complex timed systems

Introduction

In order to analyze the **properties** of a complex system, it is not always sufficient to study the **individual behavior** of its components.

Example: An embedded system controlling a **railroad crossing** is composed of the following elements:

- Two **sensors** located on the tracks 1000 meters before and 100 meters after the crossing, aimed at detecting (respectively) that a train **approaches** or **has passed** the crossing.
- A **receiver** that processes the signals emitted by the sensors, and **sends orders** to open or close the gate.



The following information is known:

- The **speed** of the approaching trains is **between 48 and 52 m/s**. Then, after reaching the **first sensor**, their speed is reduced to a value between 40 and 52 m/s.
- After it **receives a signal** from a sensor, the receiver waits for **at most 5 seconds** before **sending an order** to close or to open the gate. During this delay, the receiver **ignores** incoming signals.
- The gate is **closed** (resp. **open**) when its angle is equal to **0** (resp. **90**) deg. The gate is able to move at the rate of **20 deg/s**.
- Two **successive trains** are always separated by **at least 1600 m**.

Question: Is the gate **always closed** when a train passes the crossing?

Modeling a system

In order to analyze the properties of a system, the first step consists in building a **model**, i.e., an **abstract representation** of the system that **describes its relevant properties** without any ambiguity.

For embedded applications, the **modeling formalism** must be able to express

- operations on **integer variables** (used as counters, sequence numbers, identifiers, ...), as well as on **real variables** (for representing positions, speeds, delays, ...).
- **discrete state transitions** (e.g., incrementing a counter) as well as **continuous evolution laws** (e.g., constant-speed movement).
- **composition** of elementary systems into a more complex entity.

Hybrid systems

Hybrid systems are a modeling formalism that meets those requirements.

Syntax:

A hybrid system is composed of:

- a finite number p of **processes** $P_1, P_2, \dots, P_p$,
- a finite number n of **variables** $x_1, x_2, \dots, x_n$, grouped together into a **vector** $\vec{x} \in \mathbb{R}^n$,
- a finite set L of **synchronization labels**.

Each **process** P_i is represented by a **graph** (V_i, E_i) , where

- V_i est a finite set of **control locations**,
- $E_i \subseteq V_i \times V_i$ is a finite set of **transitions**.

Each control location $v \in V_i$ is associated with:

- An activity $dif(v)$, expressed as a conjunction of linear constraints over the variables $x_1, x_2, \dots, x_n$ and their first temporal derivative $\dot{x}_1, \dot{x}_2, \dots, \dot{x}_n$.
- An invariant $inv(v)$, expressed as a conjunction of linear constraints over the variables $x_1, x_2, \dots, x_n$.

Each transition $e \in E_i$ is associated with:

- A guard $guard(e)$, that represents a condition that must be satisfied in order to enable this transition.
- An action $act(e)$, composed of constraints that specify how the values of the variables are modified when this transition is followed.

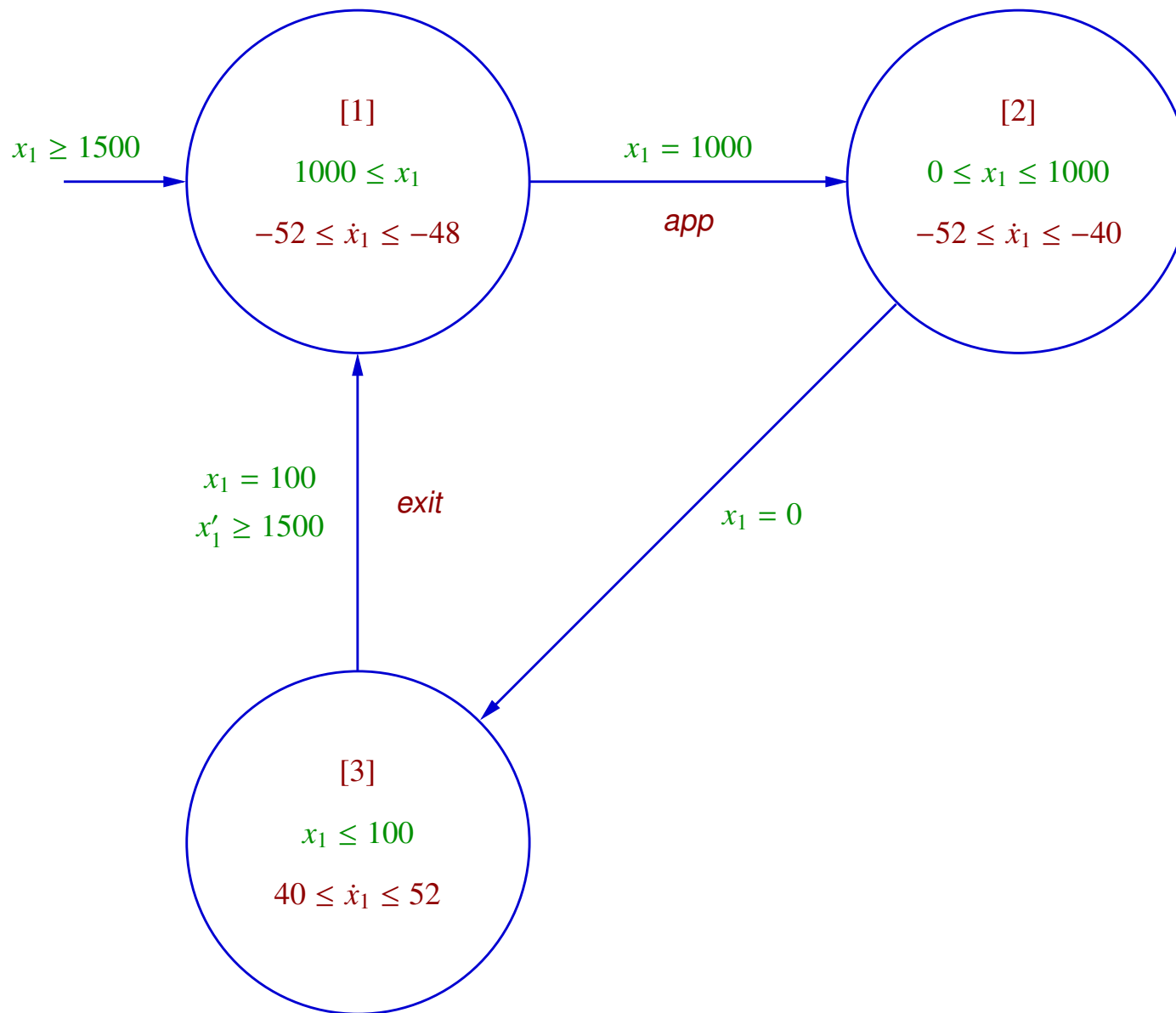
In practice, the guard and the action can be combined into a conjunction of constraints over the values of the variables before $(x_1, x_2, x_3, \dots)$ and after $(x'_1, x'_2, x'_3, \dots)$ following the transition.

- An optional **label** $\text{sync}(e) \in L$ that makes it possible to **synchronize this transition** with one or many transitions belonging to other processes.

Finally, one defines an **initial control location** for each process, and assigns a set of possible **initial values** for each variable, specified as a conjunction of linear constraints.

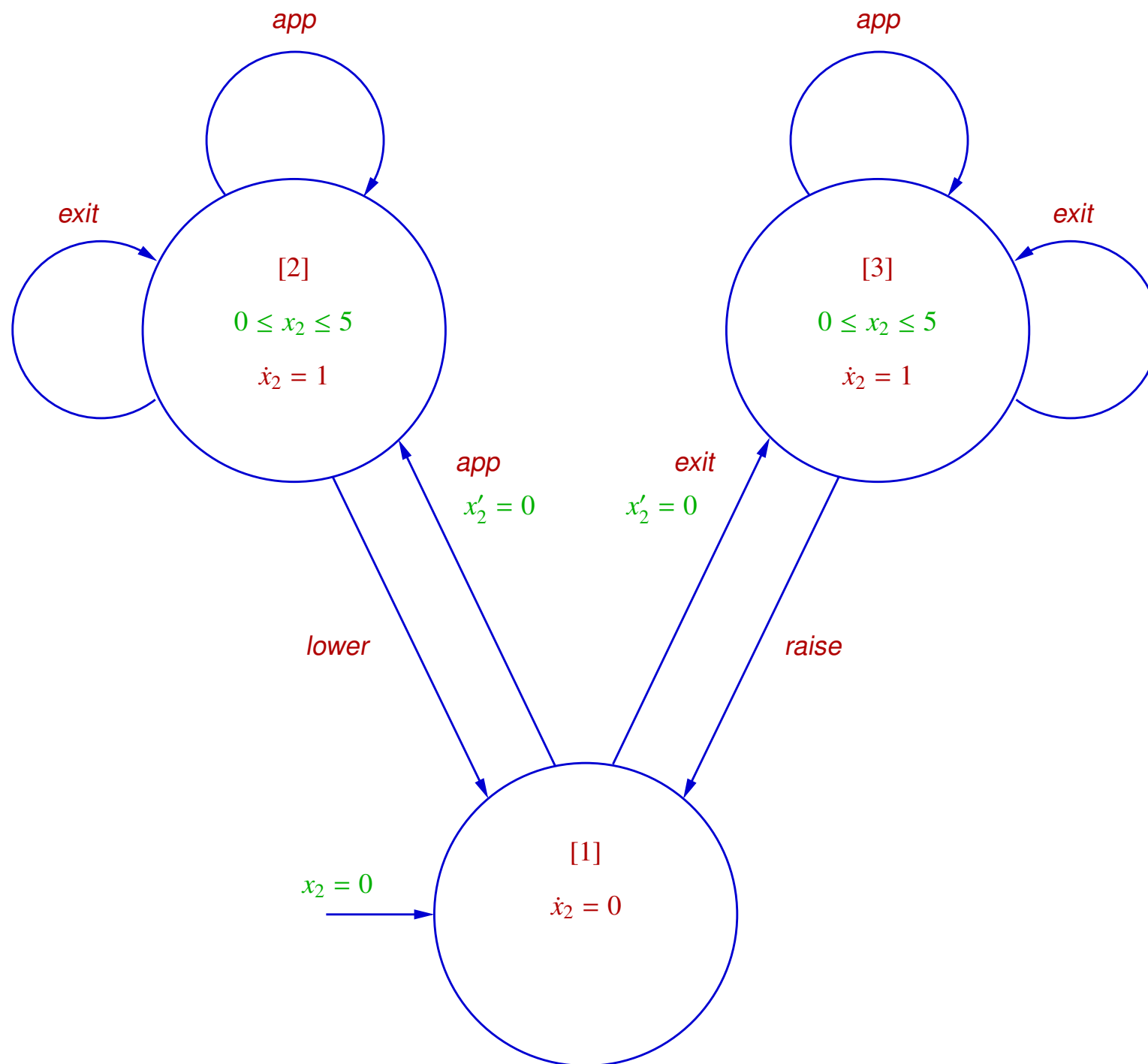
Example: Process modeling the behavior of a train and the two sensors.

- The **distance** between the train and the crossing is represented by a **variable** x_1 .
- The **signals** emitted by the sensors are modeled by two **synchronization labels** *app* and *exit*.



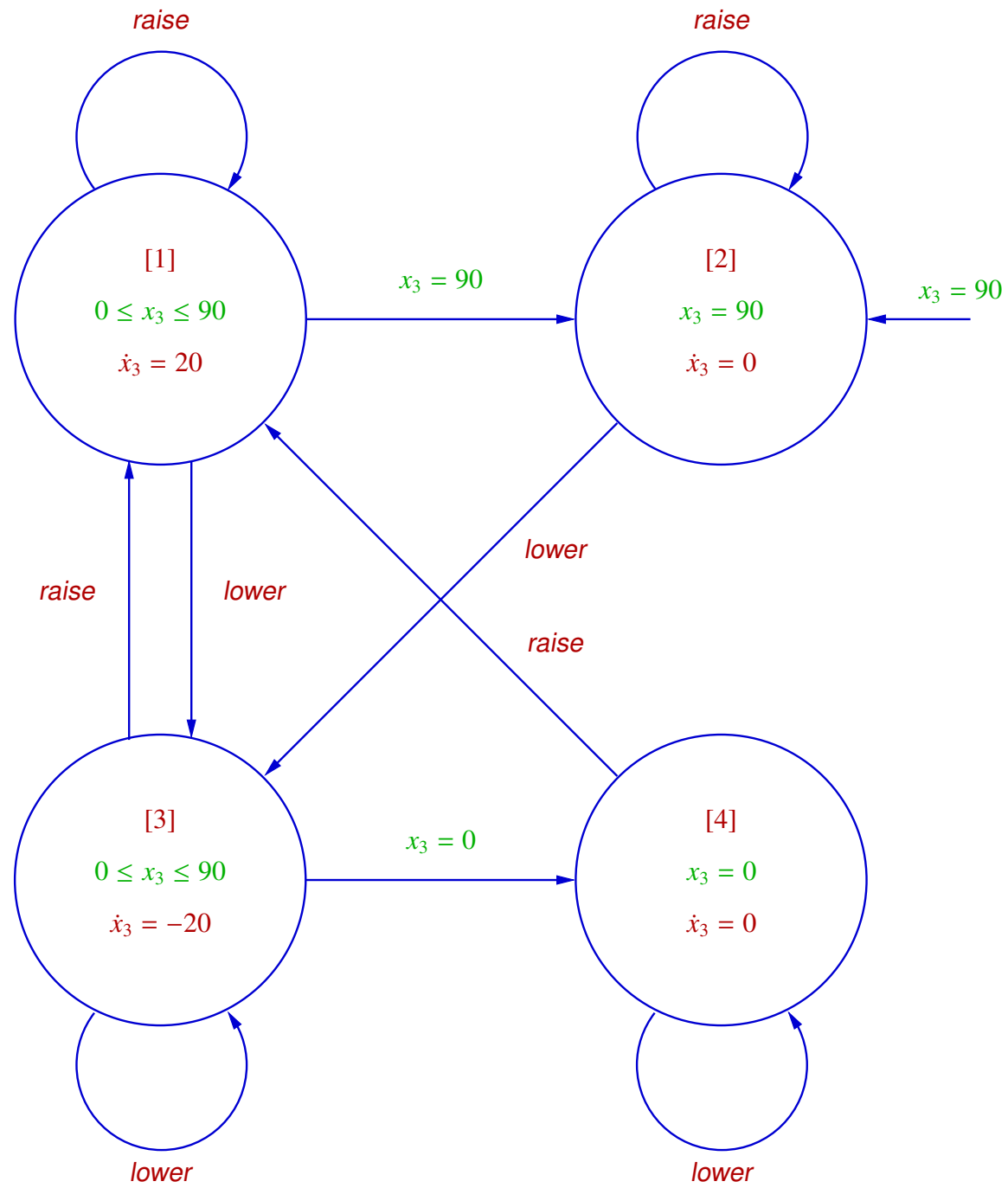
Process modeling the receiver:

- The **delay** between receiving a sensor signal and sending an order to the gate is represented by a **variable** x_2 .
- The **labels** *raise* and *lower* model the **orders sent to the gate**.



Process modeling the **gate**:

- The **variable** x_3 represents the **angular position of the gate**.
- The **labels** *raise* and *lower* correspond to the **orders received**.



Semantics:

At any given time, the current **state** of a hybrid system is characterized by

- a **control location** for each process, and
- a **value** for each variable.

The state of a system can **evolve** in two ways:

- By letting **time elapse** (*time steps*). The control locations of processes stay unchanged, and the **values of the variables** evolve according to the **invariants** and **activities** associated to these locations.
- By **following transitions** (*transition steps*). One can either
 - follow a single **unlabeled transition**, or
 - follow a **pair of transitions** (more generally, a **maximal set** of at least two transitions) belonging to different processes and sharing the **same synchronization label**.

In **both cases**, a transition can only be followed provided that its **guard is satisfied** by the current variable values.

When a transition is followed, the variable values are **modified** according to the **action** associated to the transition. The **invariant** of the destination location must be satisfied by the new variable values (otherwise, the transition **cannot be followed**).

A state s_2 is **reachable from** a state s_1 if there exists a **finite sequence** of time steps and transition steps that **lead from s_1 to s_2** .

A state s is **reachable** if it is reachable from an **initial state**.

Example: The state $([2], [2], [2], 800, 4, 90)$ of the railroad crossing controller model corresponds to

- the control location $[2]$ for each process.
- the respective values 800 , 4 and 90 for the variables x_1 , x_2 and x_3 .

This state is **reachable**. Indeed, one has

$$\begin{aligned} & ([1], [1], [2], 1500, 0, 90) \\ & \xRightarrow{10} ([1], [1], [2], 1000, 0, 90) \\ & \xrightarrow{app} ([2], [2], [2], 1000, 0, 90) \\ & \xRightarrow{4} ([2], [2], [2], 800, 4, 90), \end{aligned}$$

where

- “ $\xRightarrow{\lambda}$ ” denotes a **time step** with a delay equal to λ ,
- “ $\xrightarrow{\ell}$ ” corresponds to following a **pair of transitions** sharing the synchronization label ℓ .

Executions of a hybrid system

An **execution** of a hybrid system is an **infinite sequence** $s_1, s_2, s_3, \dots$ of **states** such that:

- s_1 is an **initial state** of the system.
- For each i , the state s_{i+1} is **reachable** from the state s_i in a **time** $\delta_i \geq 0$.

Note: A hybrid system generally admits **several different executions** (*non-determinism*).
Indeed,

- The **time spent at a control location** may not be precisely constrained by the invariant.
- A control location can have **several outgoing transitions** enabled at a given time.

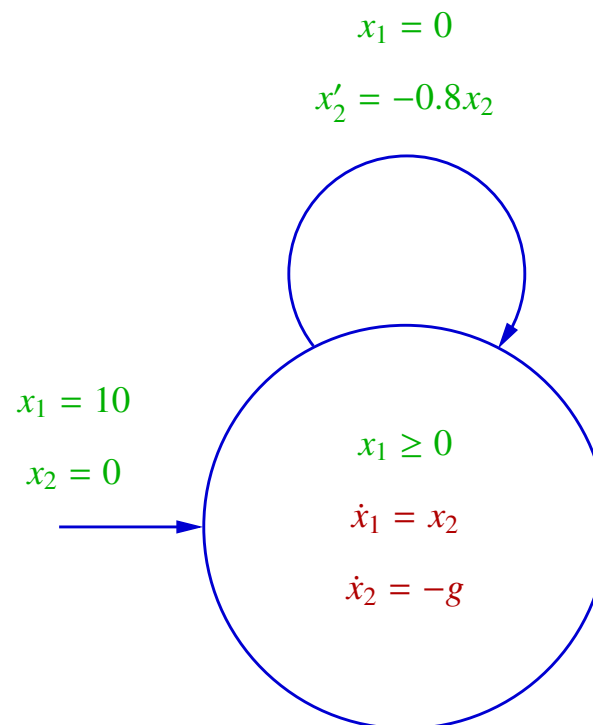
An **execution** $s_1, s_2, s_3, \dots$ beginning at time $t = 0$ is said to be **divergent** if for every $T > 0$, there exists i such that **the state** s_i is reached **later than time** $t = T$.

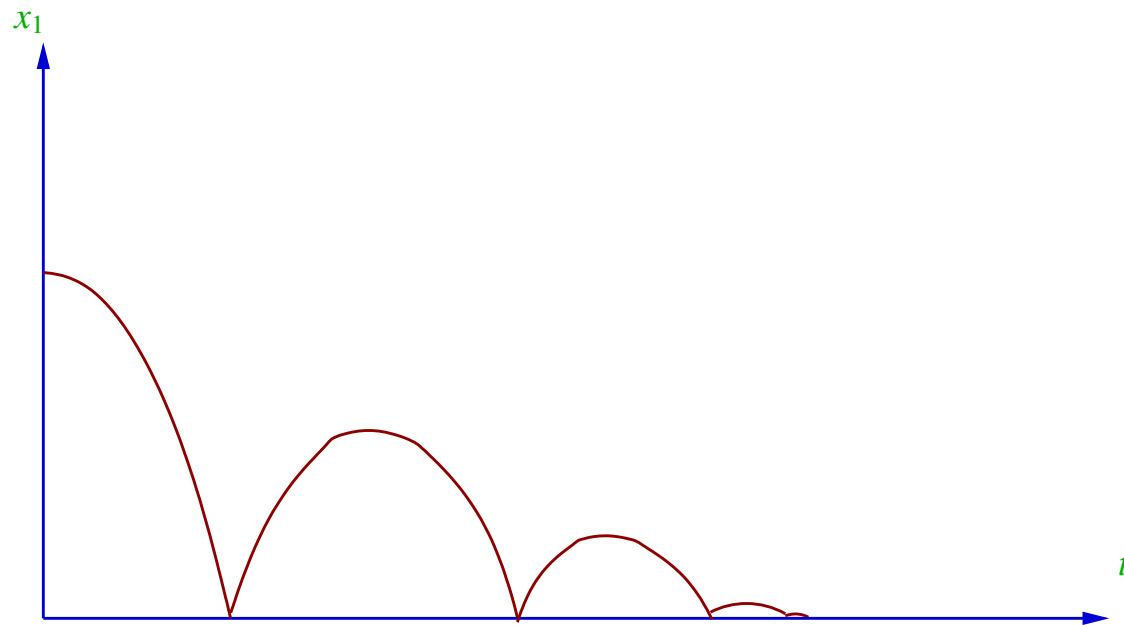
Zeno hybrid systems

A hybrid system is said to have the **Zeno property** if it admits an execution in which at least **one finite prefix is not** a prefix of a **divergent execution**.

In other words, in a Zeno hybrid system, there exists a **reachable state** from which **no execution** is able to **get past** some time bound.

Example: Hybrid system modeling a **bouncing ball**.





Remarks:

- Such models are inconsistent with physical reality and **must be avoided!**
- For some **restricted classes** of hybrid systems, automatic methods have been developed for **transforming any given model** into another one that **does not have the Zeno property**, and admits the same **divergent executions**.

State-space exploration

A large number of **interesting properties** of a hybrid system can be checked by computing its **reachable states**.

This computation can be carried out by building, from every initial state, a **tree** in which each node q represents a **reachable state** $s(q)$, and the children of q correspond to the states that are **reachable from** $s(q)$ by

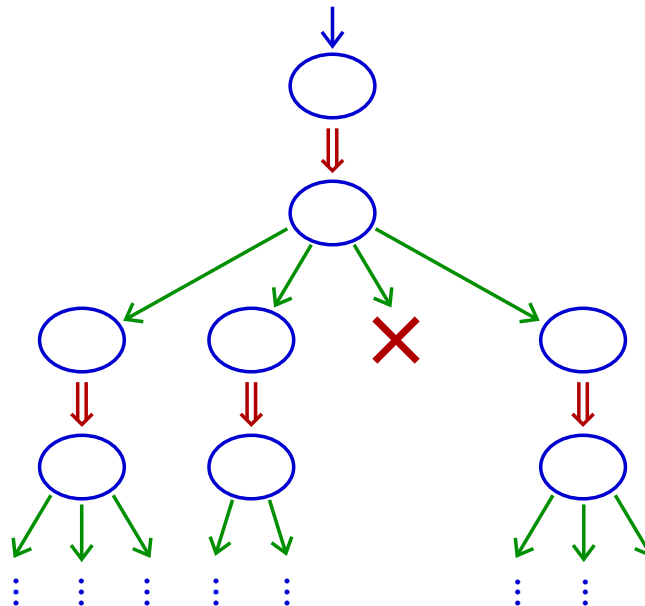
- a **time step**, or
- a **transition step**.

Problems:

- The system may have **infinitely many** initial states.
- The **time spent** at a control location may take an **infinite number** of possible values, which leads to trees of **infinite degree**.
- Since **executions are infinite**, the trees also have an **infinite depth**.

Solutions:

- **Sets of states** sharing the **same control locations** and differing only in the **elapsed time** in those locations can be grouped into **regions**. A tree can be built in which the nodes are associated with **regions** instead of individual states.
- At each exploration step, a first operation **saturates the current region** by **letting time elapse** during **all possible delays**. Then, the **enabled transitions** are individually followed, creating new branches.
- The branches of the exploration tree that only contain **already visited states** can be **pruned**.



Notes:

- Several **exploration strategies** are possible: depth-first search (DFS), breadth-first search (BFS), ...
- For general hybrid systems, the **region tree** can still be **infinite**. It is however possible to define **restricted classes** of models, for which a **finite region tree** can always be computed.

Example: Timed automata are hybrid systems in which

- the **activities** are of the form $\dot{x}_i = 1$,
 - all **invariants, guards and actions** are conjunctions of constraints of the form $x_i \# c$ or $x_i - x_j \# c$, where c is an integer number, and $\# \in \{<, \leq, =, \geq, >\}$.
- Some tools are available for **exploring automatically** the state space of hybrid systems (e.g., *HyTech*, *SpaceEx*, *Hylaa*) or timed automata (e.g., *Uppaal*).

Notes: These tools

- represent and handle regions with the help of **dedicated data structures**, based on logic formulas, convex polyhedra, difference matrices, ...
- are able to check properties that **go beyond** simple reachability.

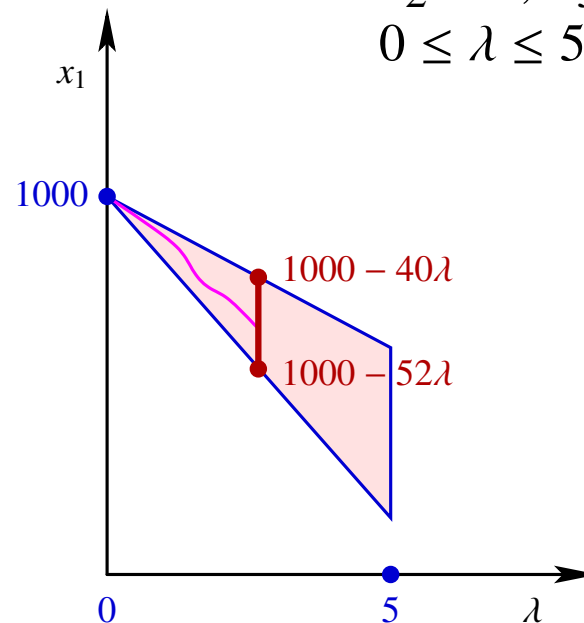
Example: Railroad crossing

$$([1], [1], [2]) : x_1 \geq 1500, x_2 = 0, x_3 = 90.$$

$$\Rightarrow ([1], [1], [2]) : \begin{aligned} x_1 &\geq 1000, \\ x_2 &= 0, x_3 = 90. \end{aligned}$$

$$\xrightarrow{\text{app}} ([2], [2], [2]) : \begin{aligned} x_1 &= 1000, \\ x_2 &= 0, x_3 = 90. \end{aligned}$$

$$\xrightarrow{\leq 5} ([2], [2], [2]) : \begin{aligned} x_1 &\geq 1000 - 52\lambda, \\ x_1 &\leq 1000 - 40\lambda, \\ x_2 &= \lambda, x_3 = 90, \\ 0 &\leq \lambda \leq 5. \end{aligned}$$



lower
 $\longrightarrow$

$$([2], [1], [3]) : \begin{aligned} x_1 &\geq 1000 - 52\lambda, \\ x_1 &\leq 1000 - 40\lambda, \\ x_2 &= \lambda, \quad x_3 = 90, \\ 0 &\leq \lambda \leq 5. \end{aligned}$$

$\leq 9/2$
 $\implies$

$$([2], [1], [3]) : \begin{aligned} x_1 &\geq 1000 - 52(\lambda + \mu), \\ x_1 &\leq 1000 - 40(\lambda + \mu), \\ x_2 &= \lambda, \quad x_3 = 90 - 20\mu, \\ 0 &\leq \lambda \leq 5, \quad 0 \leq \mu \leq 9/2. \end{aligned}$$

$x_3=0$
 $\longrightarrow$

$$([2], [1], [4]) : \begin{aligned} x_1 &\geq 766 - 52\lambda, \\ x_1 &\leq 820 - 40\lambda, \\ x_2 &= \lambda, \quad x_3 = 0, \\ 0 &\leq \lambda \leq 5. \end{aligned}$$

$\implies$

$$([2], [1], [4]) : \begin{aligned} 0 &\leq x_1 \leq 820 - 40\lambda, \\ x_2 &= \lambda, \quad x_3 = 0, \\ 0 &\leq \lambda \leq 5. \end{aligned}$$

$x_1=0$
 $\longrightarrow$

$$([3], [1], [4]) : \begin{aligned} x_1 &= 0, \quad x_2 = \lambda, \\ x_3 &= 0, \quad 0 \leq \lambda \leq 5. \end{aligned}$$

$$\begin{aligned}
&\stackrel{\leq 5/2}{\Rightarrow} ([3], [1], [4]) : \begin{aligned} &0 \leq x_1 \leq 100, \\ &x_2 = \lambda, x_3 = 0, \\ &0 \leq \lambda \leq 5. \end{aligned} \\
&\stackrel{\text{exit}}{\rightarrow} ([1], [3], [4]) : \begin{aligned} &x_1 \geq 1500, x_2 = 0, \\ &x_3 = 0. \end{aligned} \\
&\stackrel{\leq 5}{\Rightarrow} ([1], [3], [4]) : \begin{aligned} &x_1 \geq 1500 - 52\lambda, \\ &x_2 = \lambda, x_3 = 0, \\ &0 \leq \lambda \leq 5. \end{aligned} \\
&\stackrel{\text{raise}}{\rightarrow} ([1], [1], [1]) : \begin{aligned} &x_1 \geq 1500 - 52\lambda, \\ &x_2 = \lambda, x_3 = 0, \\ &0 \leq \lambda \leq 5. \end{aligned} \\
&\stackrel{\leq 9/2}{\Rightarrow} ([1], [1], [1]) : \begin{aligned} &x_1 \geq 1500 - 52(\lambda + \mu), \\ &x_2 = \lambda, x_3 = 20\mu, \\ &0 \leq \lambda \leq 5, 0 \leq \mu \leq 9/2. \end{aligned} \\
&\stackrel{x_3=90}{\rightarrow} ([1], [1], [2]) : \begin{aligned} &x_1 \geq 1266 - 52\lambda, \\ &x_2 = \lambda, x_3 = 90, \\ &0 \leq \lambda \leq 5. \end{aligned}
\end{aligned}$$

$$\begin{aligned} \Rightarrow ([1], [1], [2]) &: x_1 \geq 1000, \\ &x_2 = \lambda, x_3 = 90, \\ &0 \leq \lambda \leq 5. \\ \xrightarrow{\text{app}} ([2], [2], [2]) &: x_1 = 1000, \\ &x_2 = 0, x_3 = 90 \\ &\text{(already obtained).} \end{aligned}$$

Notes:

- In this example, the **regions** correspond to the sets of states obtained after each **time-step operation** (denoted by “ $\Rightarrow$ ”).
- Checking whether the gate is always closed when a train reaches the crossing amounts to verifying that in each reachable region, **$x_1 = 0$ implies $x_3 = 0$** .
- This particular system shows a **very deterministic behavior**: In each reachable state, there is at most **one transition** (or a pair of synchronized transitions) that is enabled.
(This is **generally not the case**!)