

Tutorial 10: hybrid systems part 3

December 12, 2025

Exercise 1: January 2025

The hydraulic system driving the rudder of a ship is composed of a pressure pump and a bleed valve, that operate independently from each other. When they are active, the pressure pump increases the pressure in the system at the rate of $3 \cdot 10^6$ Pa/s, and the bleed valve decreases the pressure by $1 \cdot 10^6$ Pa/s. These figures are additive, for instance, when both pump and bleed valve are active, the pressure increases by $2 \cdot 10^6$ Pa/s.

The controller of the pump behaves in the following way: when it detects that the pressure in the system has dropped below $2.8 \cdot 10^7$ Pa, it waits for a random duration between 1 and 5 s, then activates the pump for a random duration between 1 and 3 s, and then gets back to its initial configuration in which it monitors again the pressure. This cycle goes on endlessly.

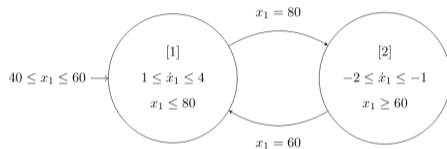
The bleed valve activates whenever it detects that the pressure in the system exceeds $3.5 \cdot 10^7$ Pa. When this happens, the valve remains active for a random duration between 2 and 10 s. It then becomes inactive, and waits for at least 2 s before it is able to become active again.

Initially, the pressure in the system is zero, the pump is active, and the bleed valve is inactive. Note that it is physically impossible for the pressure to become negative during the operation of the hydraulic system.

- (a) Model the behavior of the hydraulic system with a hybrid system.
- (b) Give the first three steps of the state-space exploration of this model.

Exercise 2: January 2024

A thermostat is modeled by the following hybrid system, in which the control locations [1] and [2] of its single process represent (respectively) the heating and non-heating modes of operation.



- (a) Does this hybrid automaton have the Zeno property? (Justify your answer.)
- (b) Compute all the reachable states of this hybrid system.
- (c) Create an additional process for this hybrid system, that moves to a dedicated control location whenever the first process stays continuously in the heating mode of operation for at least 10 time units, and remains in other control locations otherwise. *Note:* You cannot modify the first process, except for adding (if you wish) synchronization labels of your choice to its transitions.