Anisotropy preserving interpolation of diffusion tensors

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The growing importance of statistical studies of Diffusion Tensor Images (DTI) requires the development of a processing framework that accounts for the non-scalar and nonlinear nature of diffusion tensors. This motivation led a number of authors to consider a Riemannian framework for DTI processing because a Riemannian structure on the data space is sufficient to redefine most processing operations. As a prominent example, the Log-Euclidean metric proposed in [1] has emerged as a popular tool because it accounts for the tensor nature of DTI data at a computational cost that remains competitive with respect to standard tools. A limitation of the Log-Euclidean metric is its tendency to degrade the anisotropy of tensors through the standard operations of processing. Because anisotropy is the core information that motivates tensor imaging, the present paper proposes a novel metric that is anisotropy preserving while retaining the desirable properties of the Log-Euclidean metric. The properties of the proposed metric are illustrated on the basic operation of interpolating between diffusion tensors.

1 Methods

The proposed Riemannian geometry is rooted in the spectral decomposition of the tensors, which models any positive definite matrix as a diagonal positive scaling in a basis appropriately rotated from the canonical basis. This parametrization suggests a metric that weighs separately the rotation and the scaling using the geometries of both groups. This basic idea appears in early work on DTI [2] but we introduce additional features to make it practical and relevant for DTI processing. First, we use the invariant metrics of each group in order to recover the invariance properties of the Log-Euclidean metric by scaling and rotation. Second, the anisotropy of the tensor is used to balance the contributions of rotation and scaling: by making rotations of isotropic tensors cheaper, one obtains a geometric framework that inherently accounts for the uncertainty about the measured directions of diffusion. Third, we use quaternions for all computations involving rotations. The computational saving is significant and analog to the one obtained when working with the Log-Euclidean metric as opposed to the affine invariant metric.

2 Results

The benefits of the proposed framework are illustrated on the basic operation of interpolation. Figure 1 compares the interpolation of two anisotropic tensors within both the Log-Euclidean and the proposed frameworks. The Log-Euclidean interpolation causes a large decrease of the relative anisotropy (RA), resulting in degraded anisotropy information for the average tensor, while the proposed framework overcomes this limitation because the RA evolves monotonically between the two tensors. Thanks to the use of quaternions, the computational cost of generating these results is comparable in both frameworks.

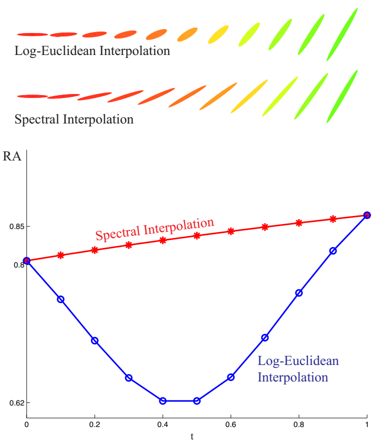

Figure 1: (Top) Interpolation between two tensors with the Log-Euclidean and the spectral framework. (Bottom): Evolution of the relative anisotropy through the interpolation with the Log-Euclidean and the spectral framework.

References

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