Selecting an appropriate metric for the processing of Diffusion Tensor Images

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For a few years, Diffusion Tensor Imaging (DTI) has received much attention. This new method of imaging allows non-invasive quantification of the diffusion of water in the brain. The formalism introduced by Basser [1] enables to assess the diffusion in each voxel (small cubes) of the brain. Conceptually, a diffusion tensor D is assigned to each voxel. The anisotropy of the tensors is then used to numerically reconstruct nervous fibers of the brain. This new development of brain imaging raises new challenges for its processing. Classical image processing algorithms are indeed developed for scalar images. The processing of Diffusion Tensor Images, which can be viewed as symmetric positive definite matrices fields, thus requires the definition of novel algorithms. One common feature of these algorithms is their use of a distance function between images. At a finer level, a distance function between tensors has to be defined. In this work, we propose a novel metric which is particularly appropriate for the processing of DTI.

Choosing a distance between diffusion tensors

The first algorithms which were developed in the context of DTI process each of the 6 components of the tensors as 6 independent scalar. However, Diffusion tensors are not only matrices, they represent some physical quantities. For this reason, the metric underlying the DTI processing should fulfill important criteria:

Invariances Such as in the whole field of images processing, invariances are crucial in DTI. We consider here that invariances by rotation and scaling are crucial to robustness. In other words, for $\Sigma_1, \Sigma_2 \in S^+(3)$, s a scalar and R a rotation matrix, we require that $d(\Sigma_1, \Sigma_2) = d(sR\Sigma_1 R^T, sR\Sigma_2 R^T)$.

Conservation of the anisotropy Since the main goal of DTI is to reconstruct nervous fibers (a technique called tractography), the image processing should minimally affect the (relative) anisotropy of the tensor, which gives essential information about the underlying cerebral tissues.

Computational cost Brain images are usually of dimension $128 \times 128 \times 64$, which means that we have more than 10^6 tensors to process for each subject. This explains why the computational cost is a critical factor which has to be considered when choosing a metric.

Metrics

A fundamental geometric metric studied by Pennec *et al.*[2] in the context of DTI processing is the *affine-invariant metric*. This metric is invariant by any affine transformation. In the DTI literature, the affine-invariant metric is often approximated by the *Log-Euclidean* metric [3], which retains the invariance by rotation and scaling and performs much faster than the affine-invariant framework. However, these metrics can be shown to degrade the anisotropy of the tensors during the processing. Metrics of another type can be defined, which are based on a spectral decomposition of matrices [4]. The separated regularizations of eigenvectors on one hand and eigenvalues on the other hand has the inconvenient of introducing some swelling effect.

In this work, we introduce a novel metric, which is close to the spectral methods but use a coupling between eigenvectors and eigenvalues. We thus avoid the swelling effect and conserve the anisotropy during the processing. In a similar way than the Log-Euclidean metric, which improves the computational cost of the affine-invariant metric, the use of quaternions will improve the cost of our novel framework. We show that the proposed metric offers the combined advantages of being invariant by rotations and scaling, preserving the relative anisotropy and reduced computational cost.

References

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