Correlation of signals

MATLAB tutorial series (Part 1.2)

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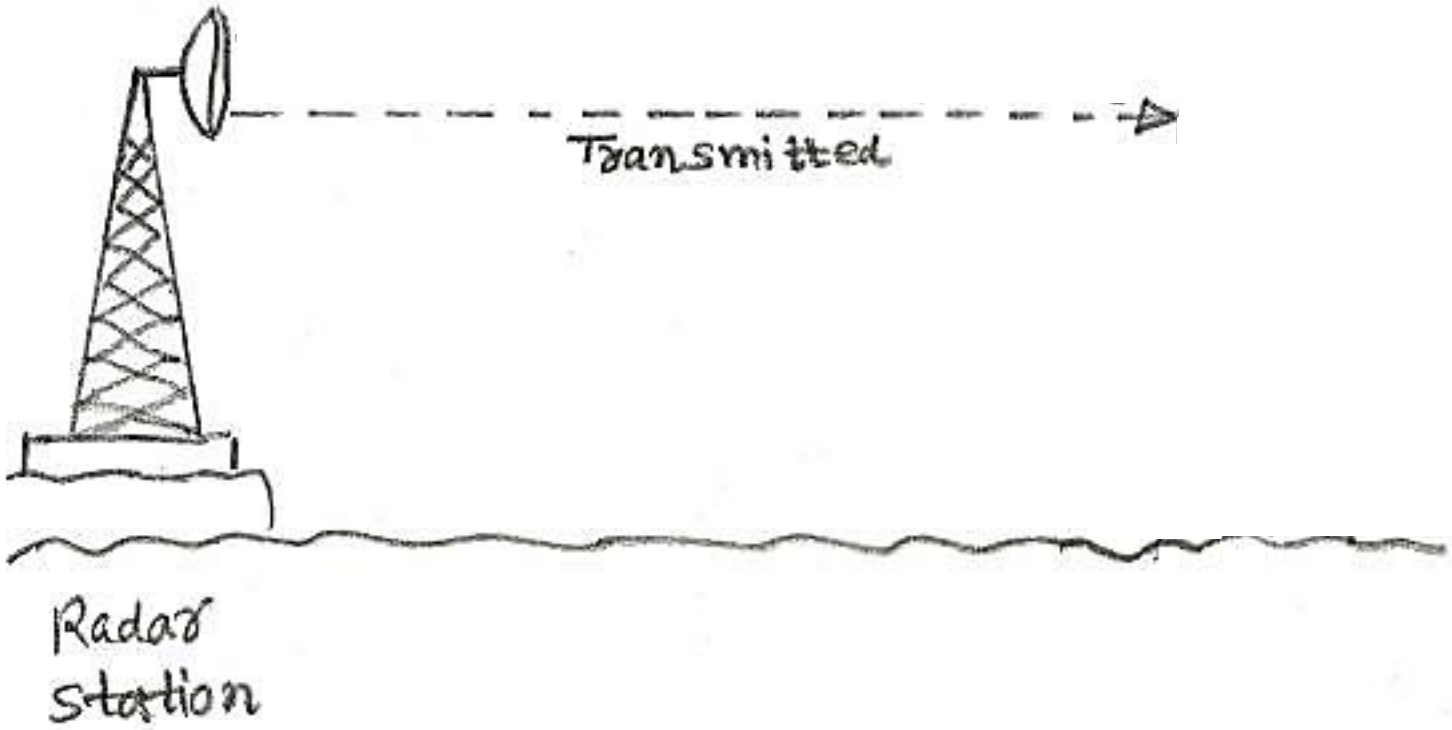
Motivation

- **We wish to measure the similarity between a signal of interest and a reference signal.**



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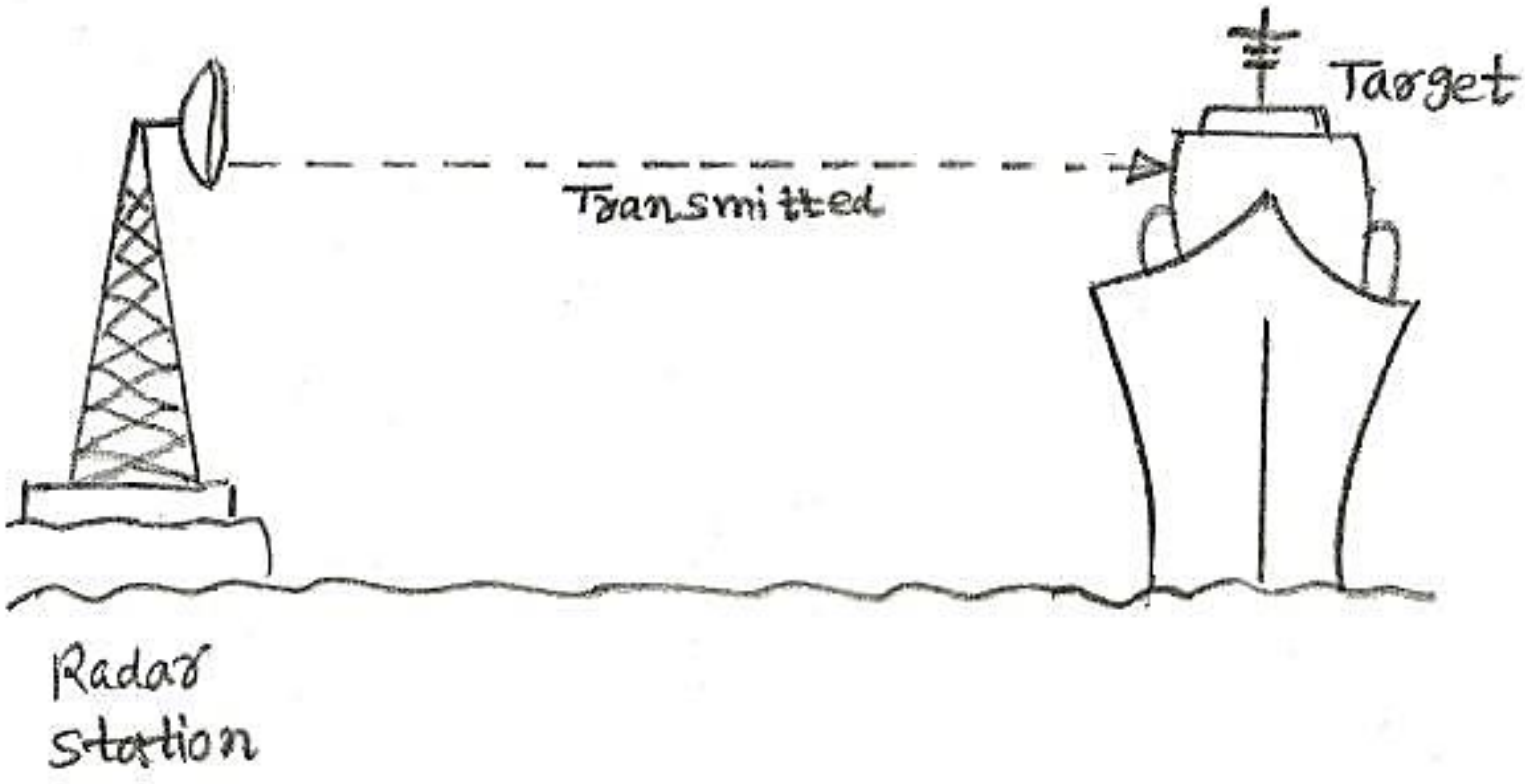

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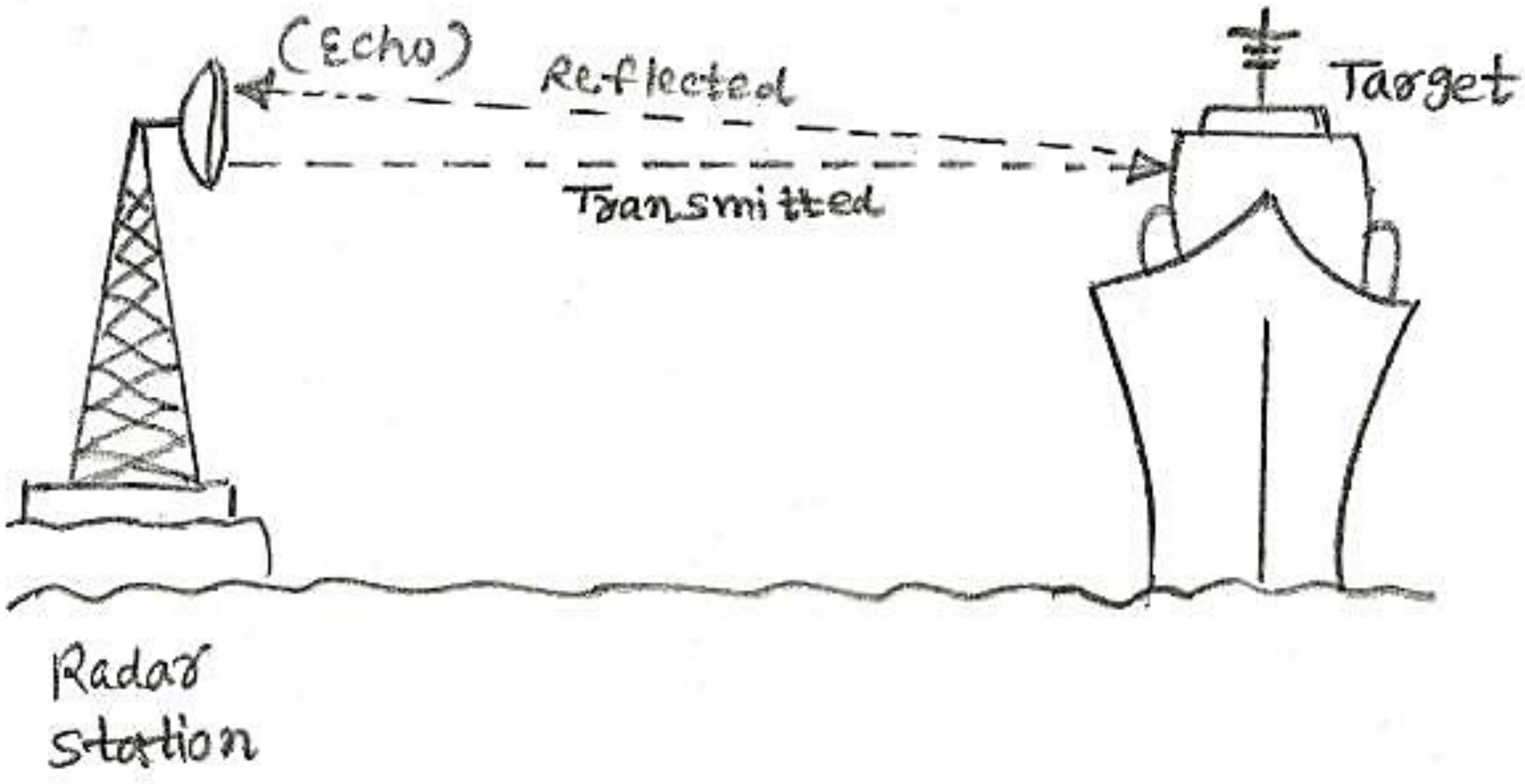

Correlation signal

Correlation signal (main formula):

$$r_{xy}[l] = \sum_{n=-\infty}^{\infty} x[n] \times y[n-l] \qquad -\infty < l < \infty$$

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Normalized correlation:

$$-1 \leq \rho_{xy}[l] \triangleq \frac{r_{xy}[l]}{\sqrt{E_x}\sqrt{E_y}} \leq 1$$

How does it work?!

Example 1:

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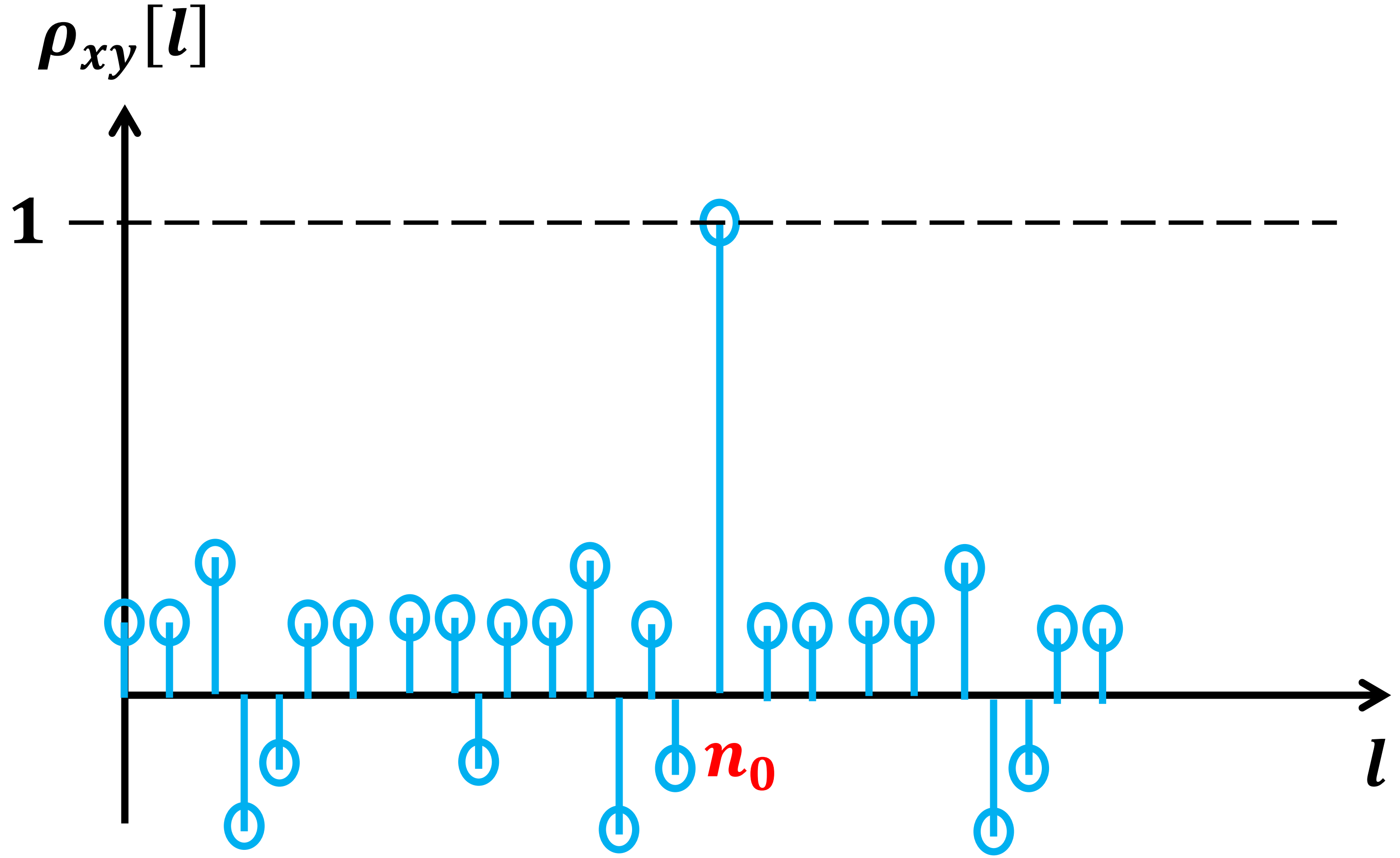
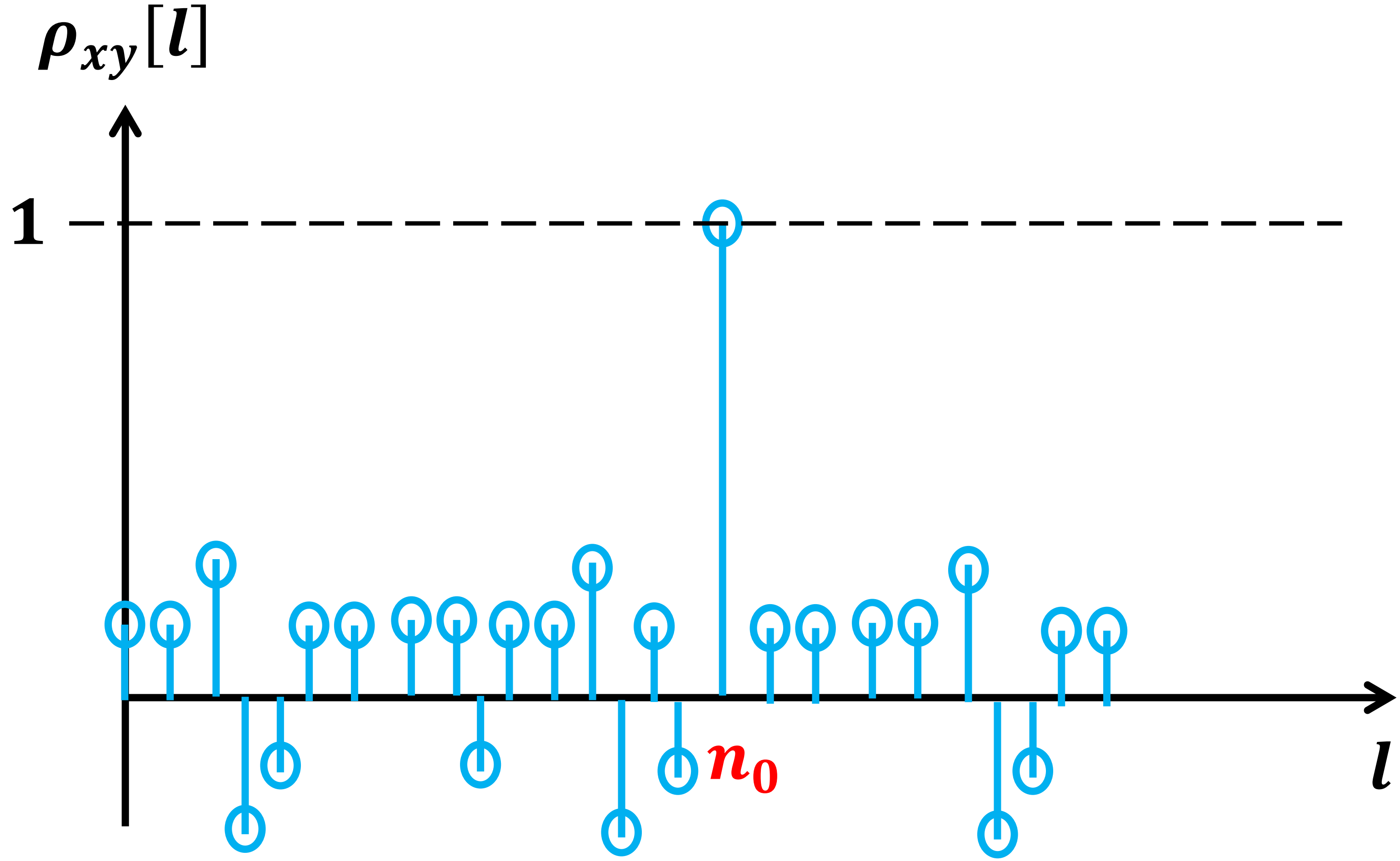


How does it work?!

Example 1:

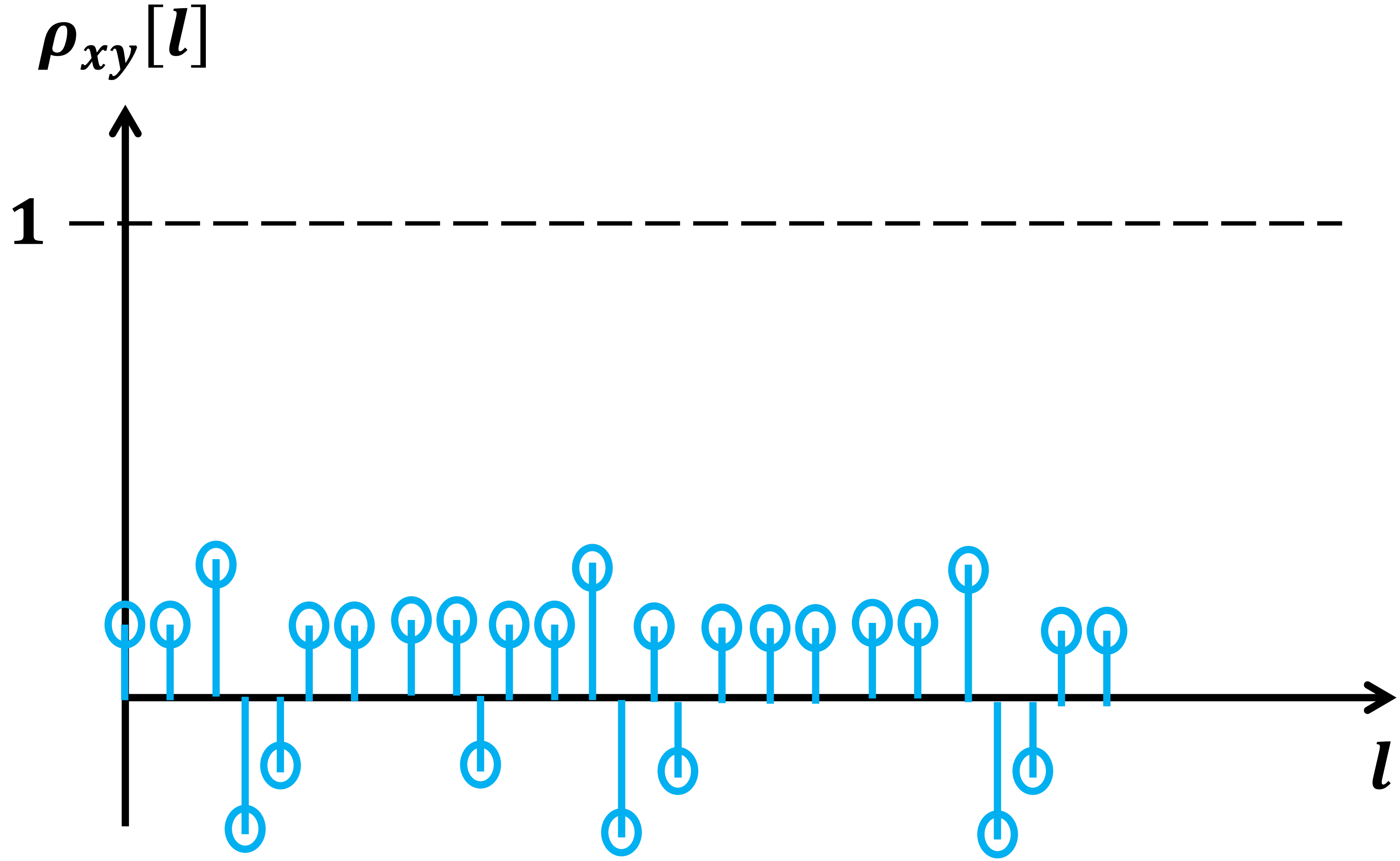
let $x[n] = cy[n - n_0],\ c > 0 \rightarrow \rho_{xy}[n_0] = 1$.



How does it work?!

Example 2:

let $x[n]$ and $y[n]$ be two uncorrelated signal.



Autocorrelation

Autocorrelation is the correlation of signal with itself.

$$r_x[l] = x[l] * x[-l]$$

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Wiener-Khintchine theorem:

$$r_x[l] = x[l] * x[-l] \xleftrightarrow{\text{DTFT}} R_x(\omega) = \left|X(e^{j\omega})\right|^2$$

Numerical computation of correlation signal

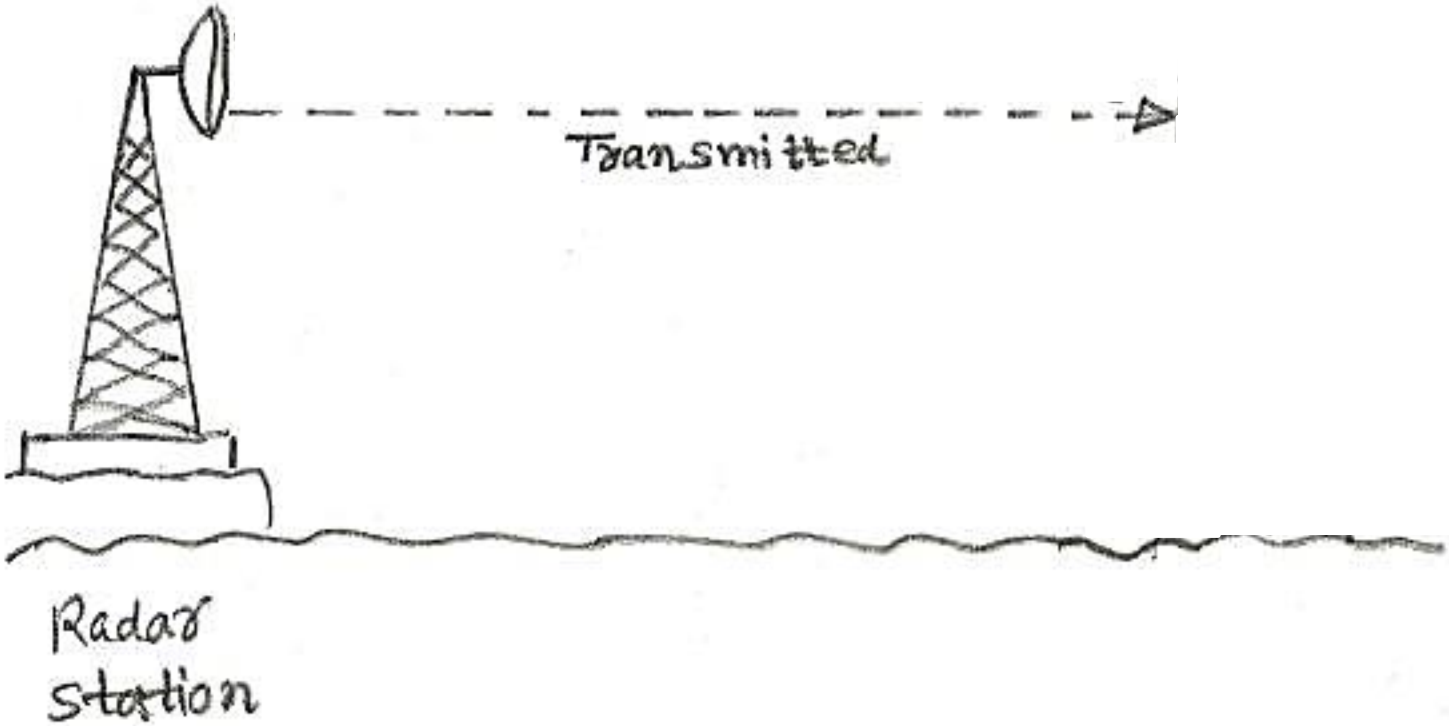
Cross correlation:

$$r_{xy}[l] = \sum_{n=-\infty}^{\infty} x[n] \times y[n-l] \qquad -\infty < l < \infty$$

MATLAB function:

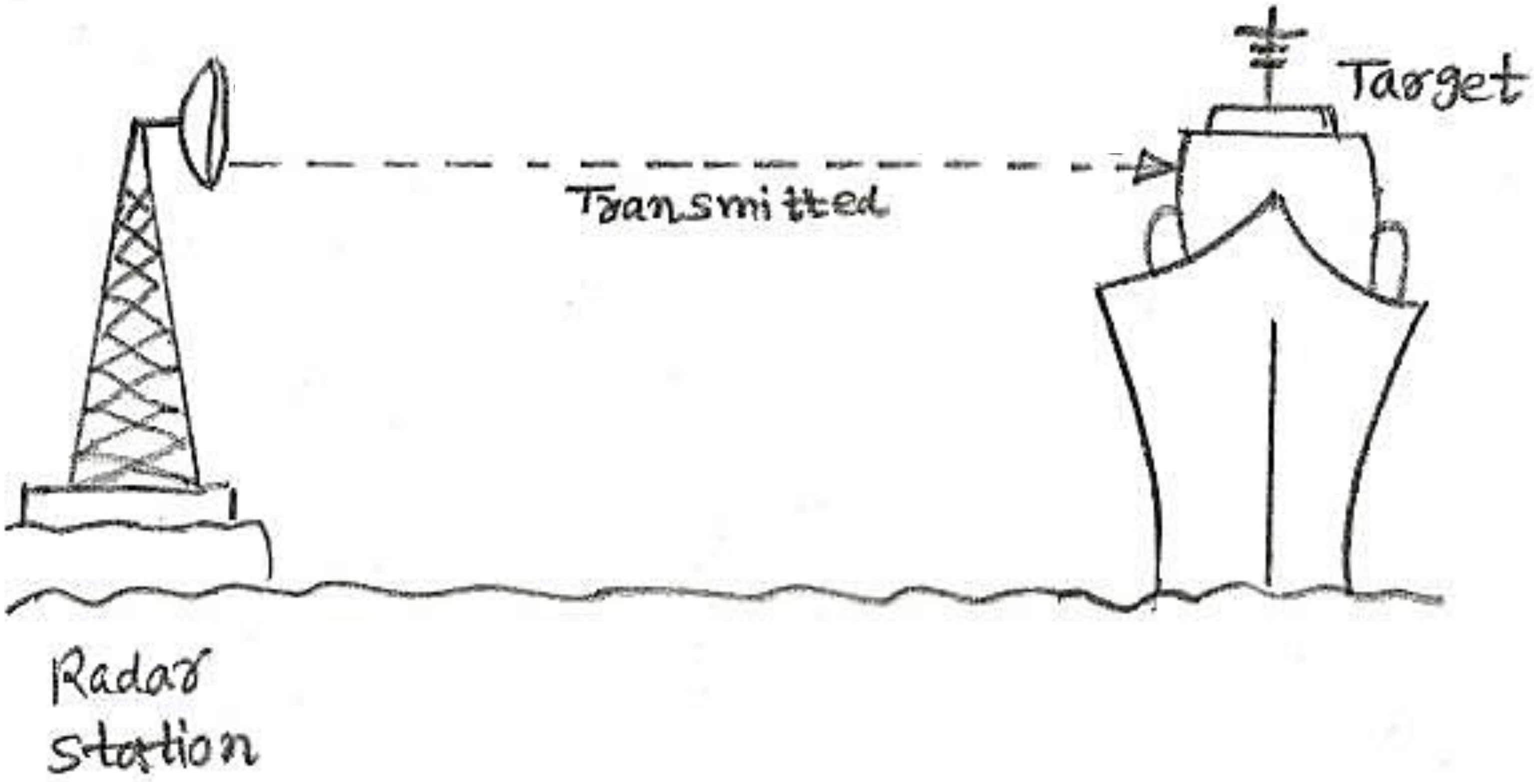
```
[rxy,Lag]=xcorr(x,y)  % returns cross-correlation

rxy=conv(x,flipud(y)) % alternative method
```

Echo



$$y[n] = x[n]$$

Echo



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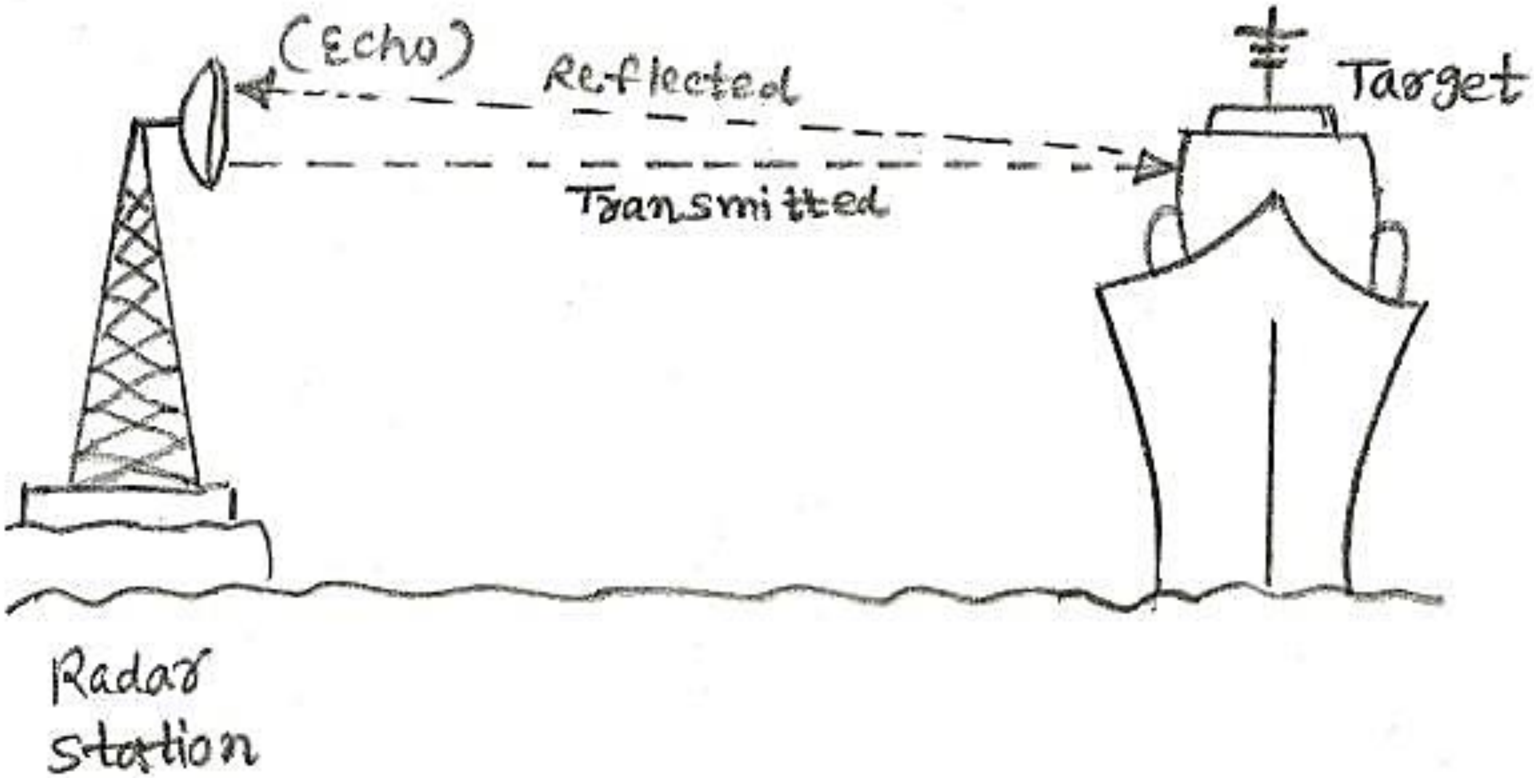
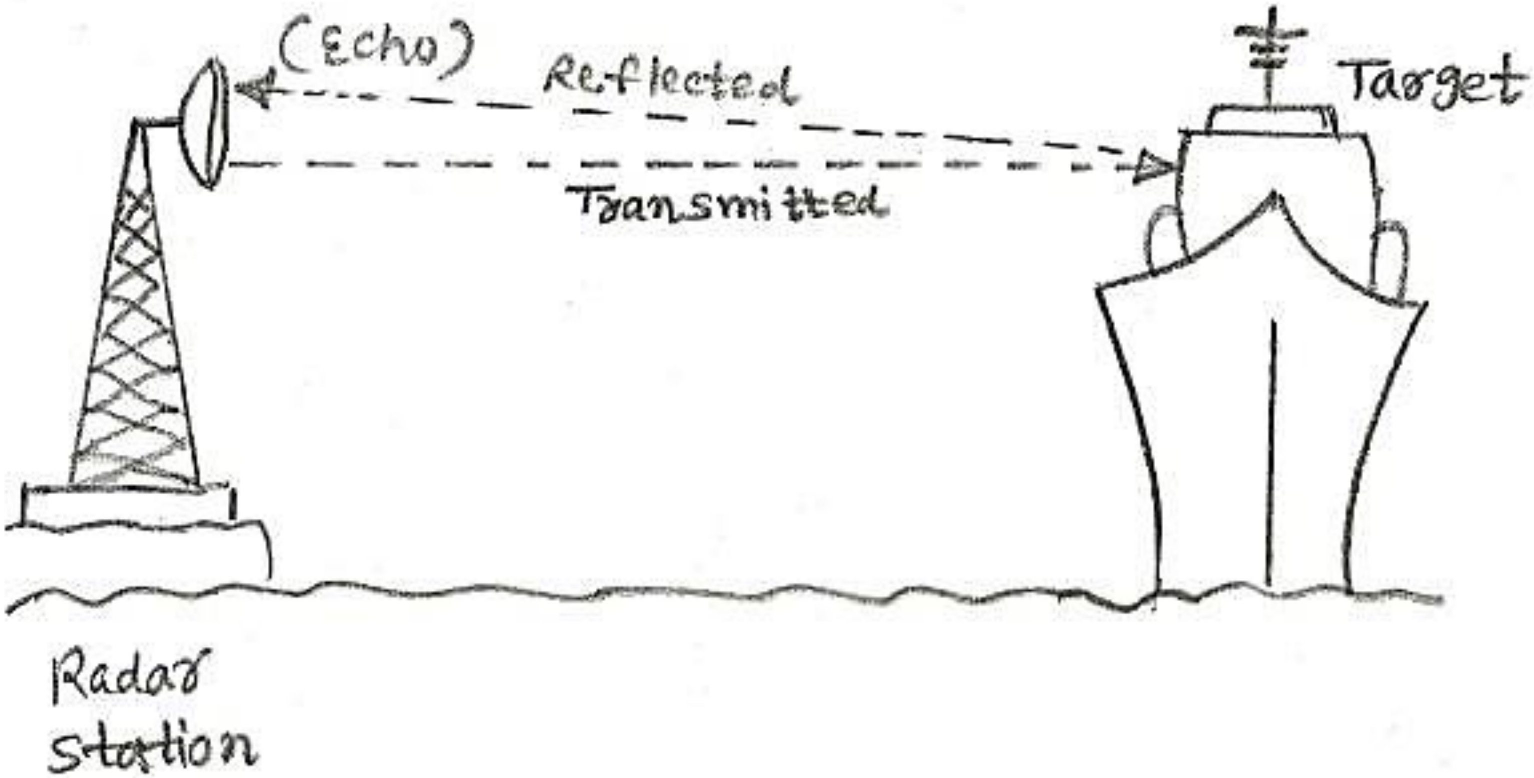


$$y[n] = x[n] + a\,x[n-D]$$

Echo



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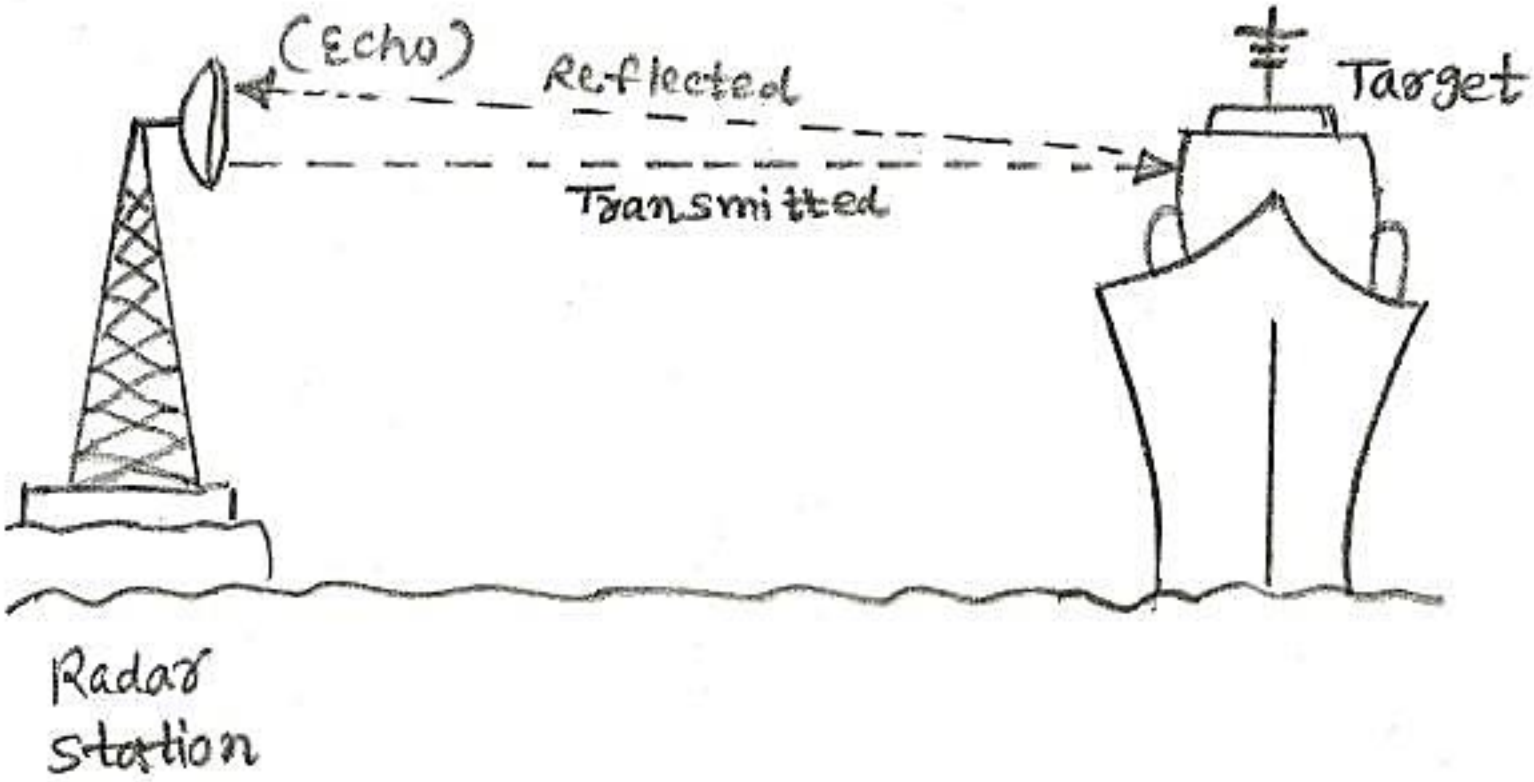
Attenuation **Distance**

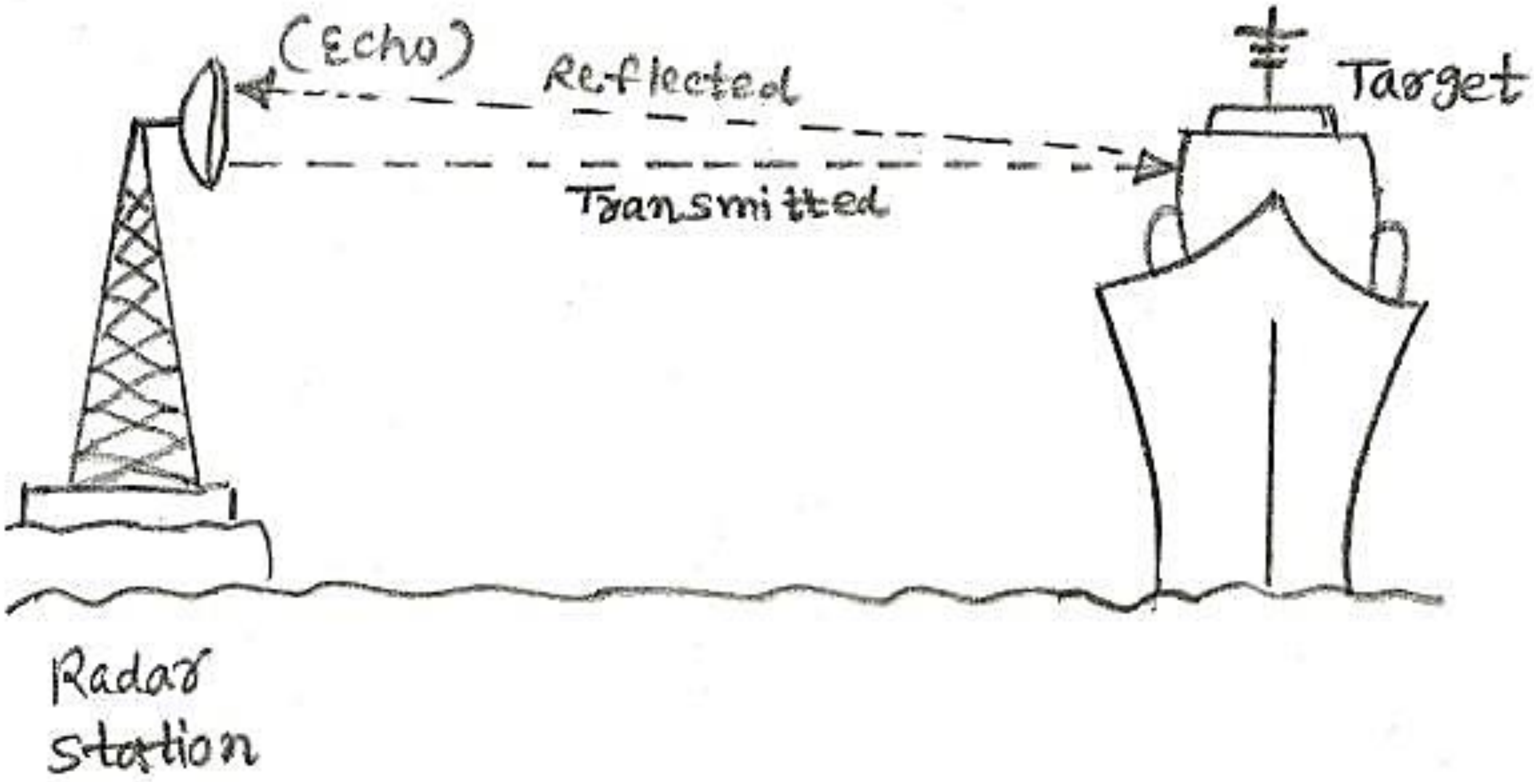
Echo



$$y[n] = x[n] + a\, y[n - D] \rightarrow$$

$$Y[z] = X[z] + a\, X[z]\, z^{-D}$$

Echo

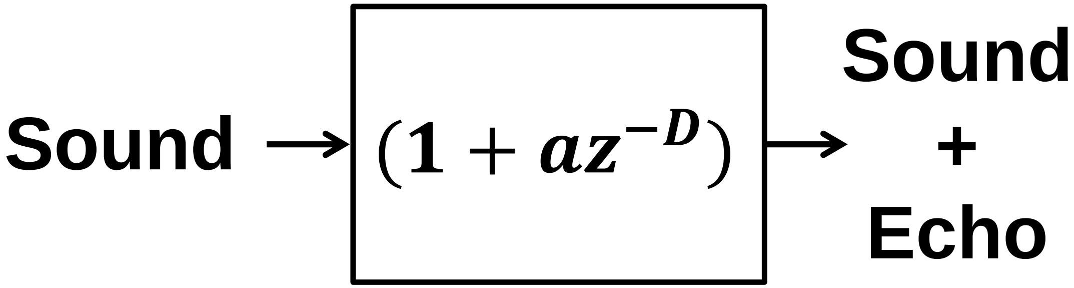


$$\boldsymbol{y[n] = x[n] + a\, y[n - D]} \rightarrow$$

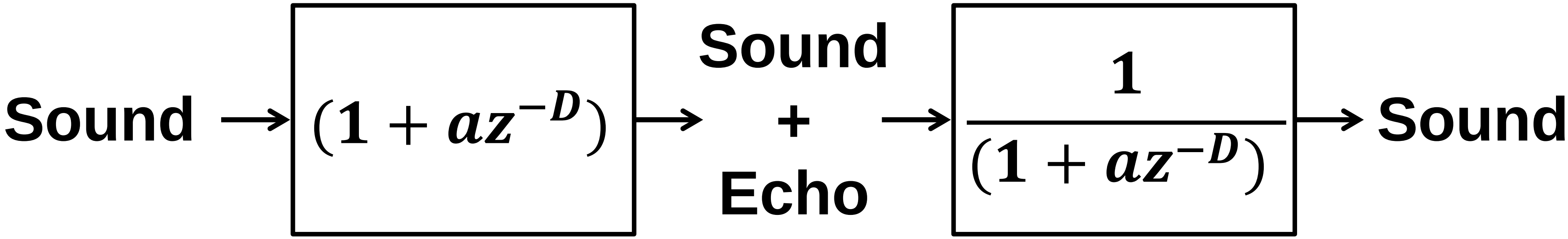
$$\boldsymbol{Y[z] = X[z] + a\, X[z]\, z^{-D}} \rightarrow$$

Echo filter: $\boldsymbol{H[z] = Y\,[z]/X[z] \; = (1 + az^{-D})}$

Application (sound)



Application (sound)



Echo removed !

Application (sound)

MATLAB functions:

[x,Fs]=audioread('Filename.wav')
% Reads audio file and return sampled signal x (all channels), and sampling frequency Fs.

sound(x,Fs) % play the sound

filter(b,a,x)% filter the signal x using the rational transfer function

Example 1.6: play and plot a sound

```
% read audio file .wav
[x,Fs]=audioread('Atonment.wav');
```

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Example 1.6: play and plot a sound

```
% read audio file .wav
[x,Fs]=audioread('Atonment.wav');
% play the sound
sound(x,Fs)
% plot left or right channel
figure(1)
plot(x(:,1))
% compute autocorrelation sequence
[acorrX,lagX]=xcorr(x(:,1),x(:,1));
% plot autocorrelation function
figure(2)
plot(lagX,acorrX,'LineWidth',2.5)
```

Example 1.7: generate echo

```
% read audio file
[x,Fs]=audioread('Atonment.wav');
% delay in seconds (e.g. 0.3, 0.4, 0.5).
% play with these!
delay=0.3;
%alpha (metal room 0.9)
alpha=0.5;
% delay in samples
d=delay*Fs;
```

Example 1.7: generate echo

```
% echo filter coefficents
b=[1, zeros(1,d-1), +alpha];
a=1;
% generate signal + echo
y=filter(b,a,x);
% play new sound
sound(y,Fs)
% compuet autocorrelation
[acorrY,lagY]=xcorr(y(:,1),y(:,1));
% plot autocorr of echo
% /!\ find delay from autocorrelation signal
plot(lagY,acorrY,'LineWidth',2.5);
```

Example 1.7: remove echo

```
b=1;
a=[1, zeros(1,d-1), +alpha];
y1=filter(b,a,y);
sound(y1,Fs)
% compuet autocorrelation
[acorrY1,lagY1]=xcorr(y1(:,1),y1(:,1));
% plot autocorr of echo
plot(lagY1,acorrY1,'LineWidth',2.5);
```

Example 1.9: generate reverberation

```
% read audio file
[x,Fs]=audioread('Atonment.wav');
```

Example 1.9: generate reverberation

```
% read audio file
[x,Fs]=audioread('Atonment.wav');
% delay in seconds (e.g. 0.3, 0.4, 0.5).
% play with these!
delay=0.2;
```

Example 1.9: generate reverberation

```
% read audio file
[x,Fs]=audioread('Atonment.wav');
% delay in seconds (e.g. 0.3, 0.4, 0.5).
% play with these!
delay=0.2;
%alpha (metal room 0.9)
alpha=0.6;
```

Example 1.9: generate reverberation

```
% read audio file
[x,Fs]=audioread('Atonment.wav');
% delay in seconds (e.g. 0.3, 0.4, 0.5).
% play with these!
delay=0.2;
%alpha (metal room 0.9)
alpha=0.6;
% delay in samples
d=delay*Fs;
```

Example 1.9: generate reverberation

```
% read audio file
[x,Fs]=audioread('Atonment.wav');
% delay in seconds (e.g. 0.3, 0.4, 0.5).
% play with these!
delay=0.2;
%alpha (metal room 0.9)
alpha=0.6;
% delay in samples
d=delay*Fs;
% reverberator
b=1;
a=[1, zeros(1,d-1), -alpha];
```

Example 1.9: generate reverberation

```
% generate signal + reverbration
y=filter(b,a,x);
```

Example 1.9: generate reverberation

```
% generate signal + reverberation
y=filter(b,a,x);
% play new sound
sound(y,Fs)
```

Example 1.9: generate reverberation

```
% generate signal + reverberation
y=filter(b,a,x);
% play new sound
sound(y,Fs)
% compuet correlation
[acorrY,lagY]=xcorr(y(:,1),y(:,1));
% plot autocorr of reverberated signal
% /!\ find delay from autocorrelation signal
plot(lagY,acorrY,'LineWidth',2.5)
```

Useful links

- https://nl.mathworks.com/help/matlab/ref/audioread.html
- https://nl.mathworks.com/help/matlab/ref/filter.html
- https://nl.mathworks.com/help/signal/ref/xcorr.html