Electrical analogies

Exercise session

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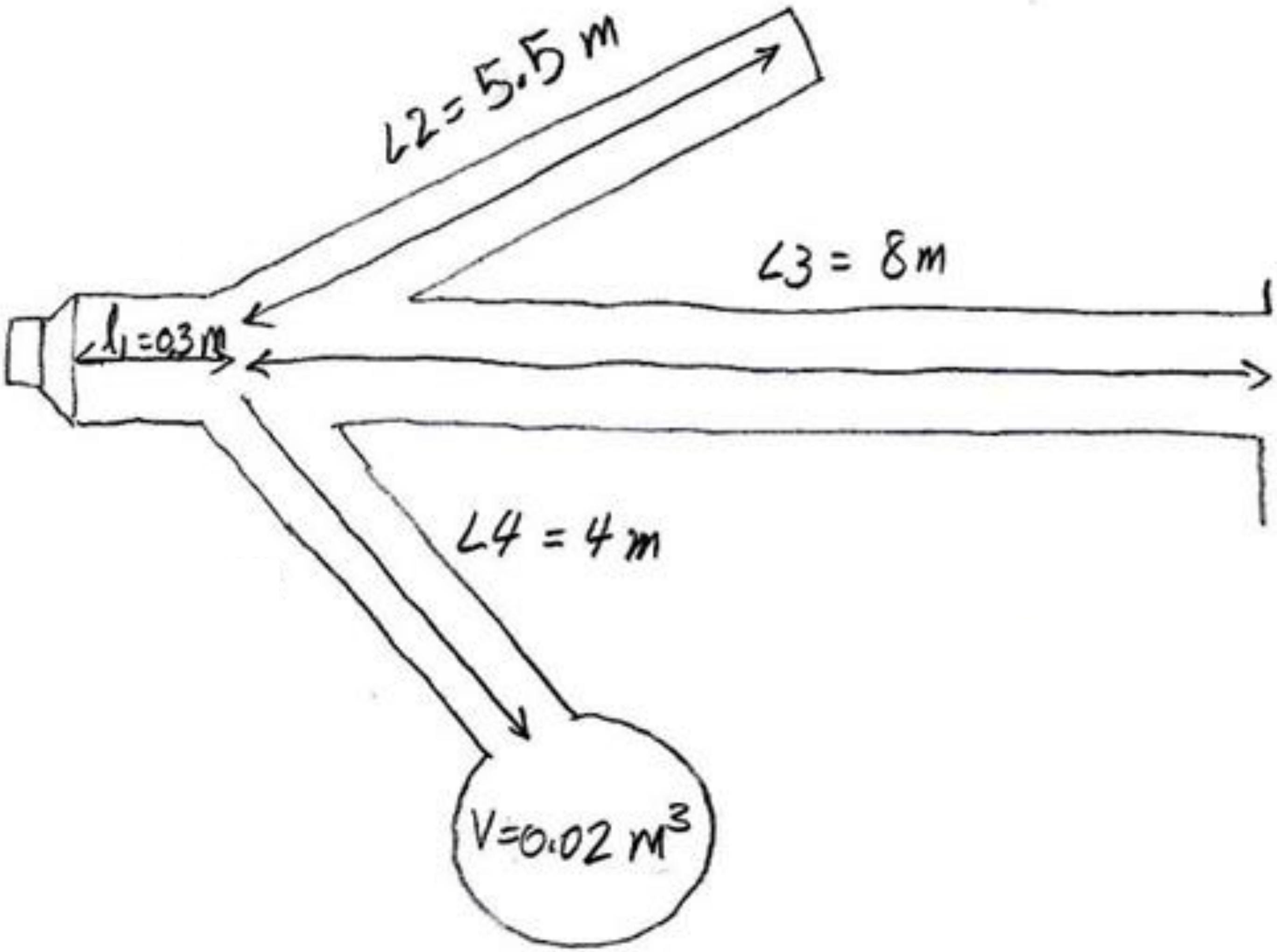
Acoustics and electroacoustics
27 Mars 2018

Question 1:



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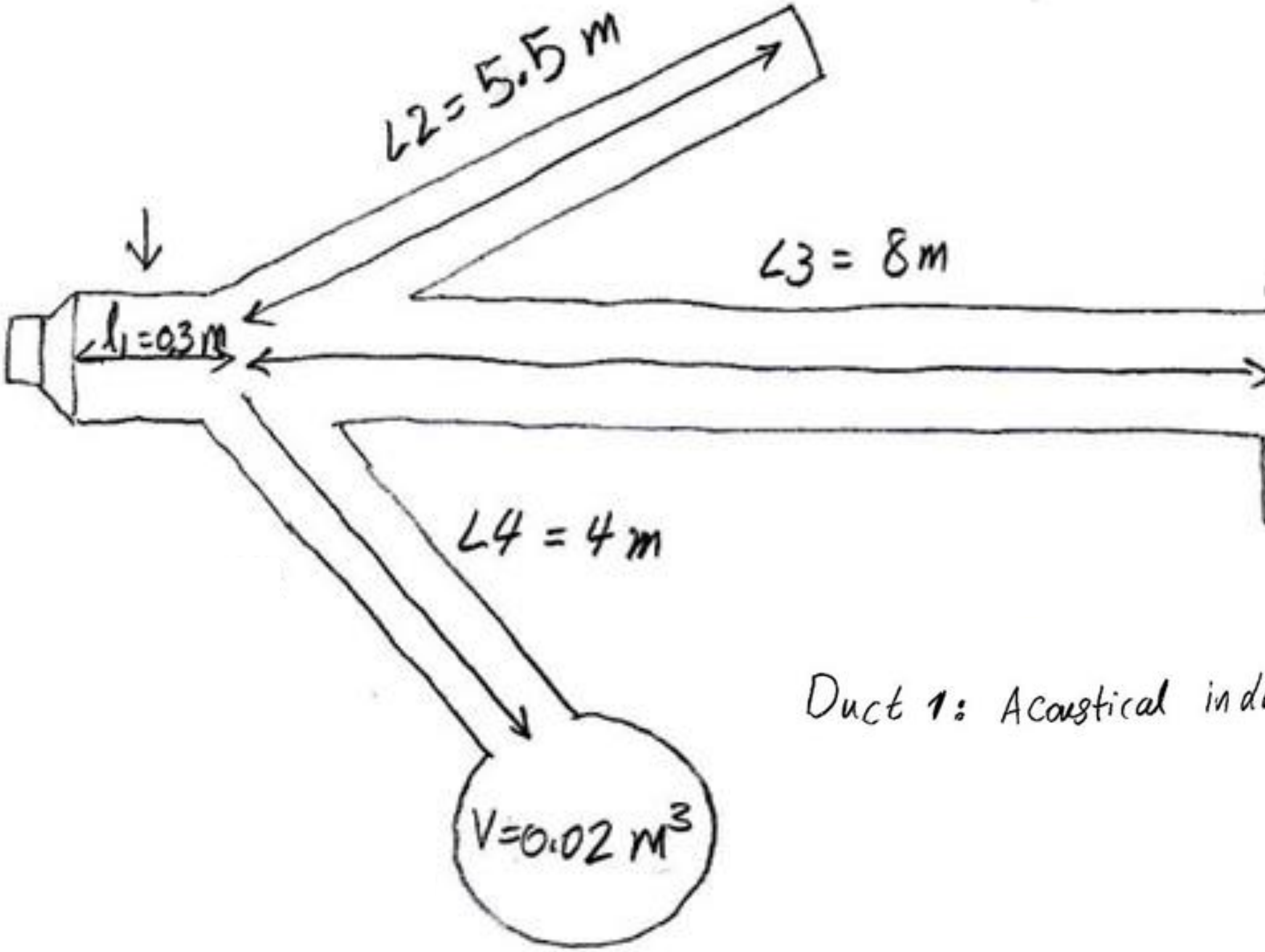

Duct 1: Acoustical inductance

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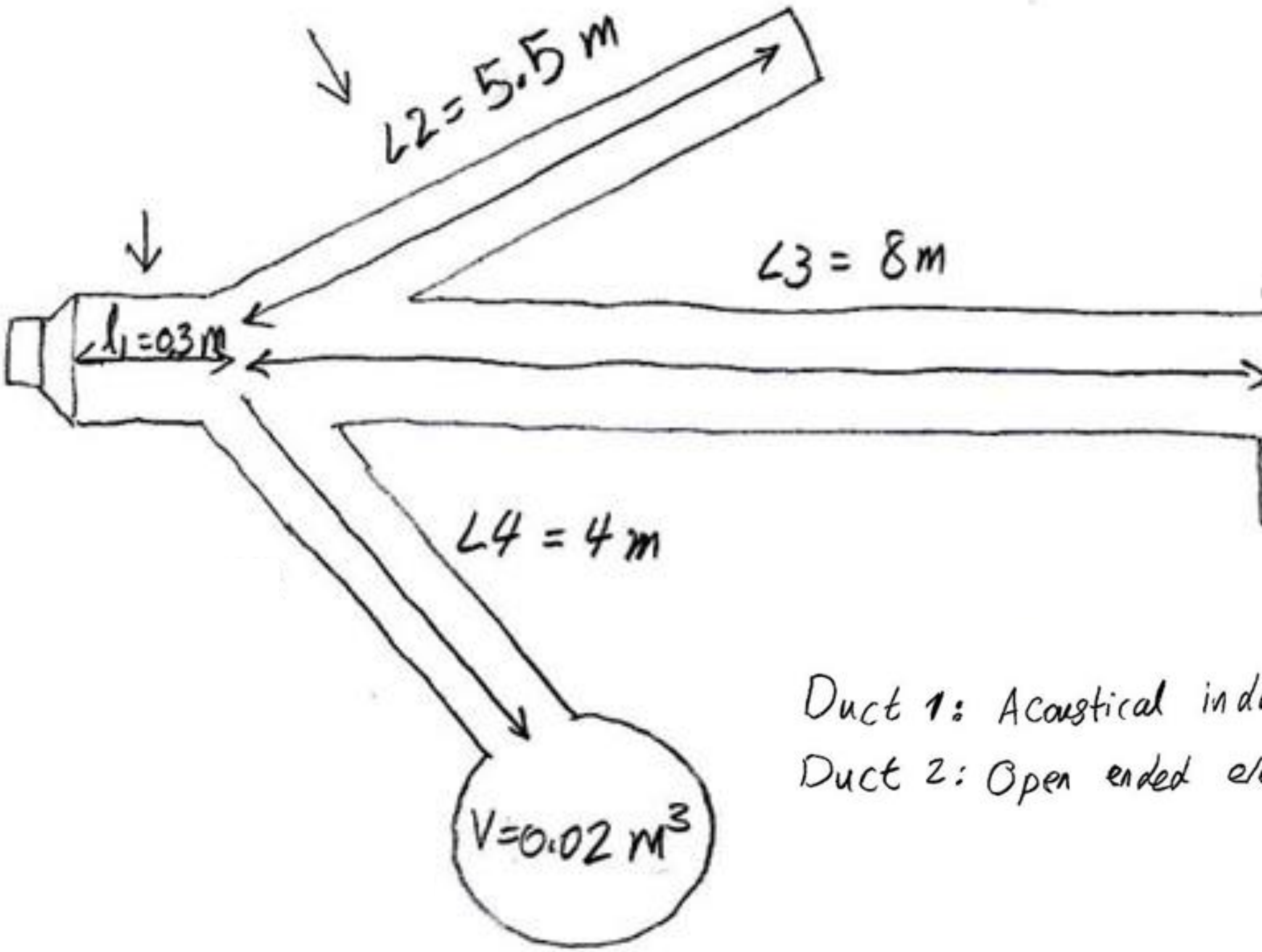

Duct 1: Acoustical inductance

Duct 2: Open ended electrical line

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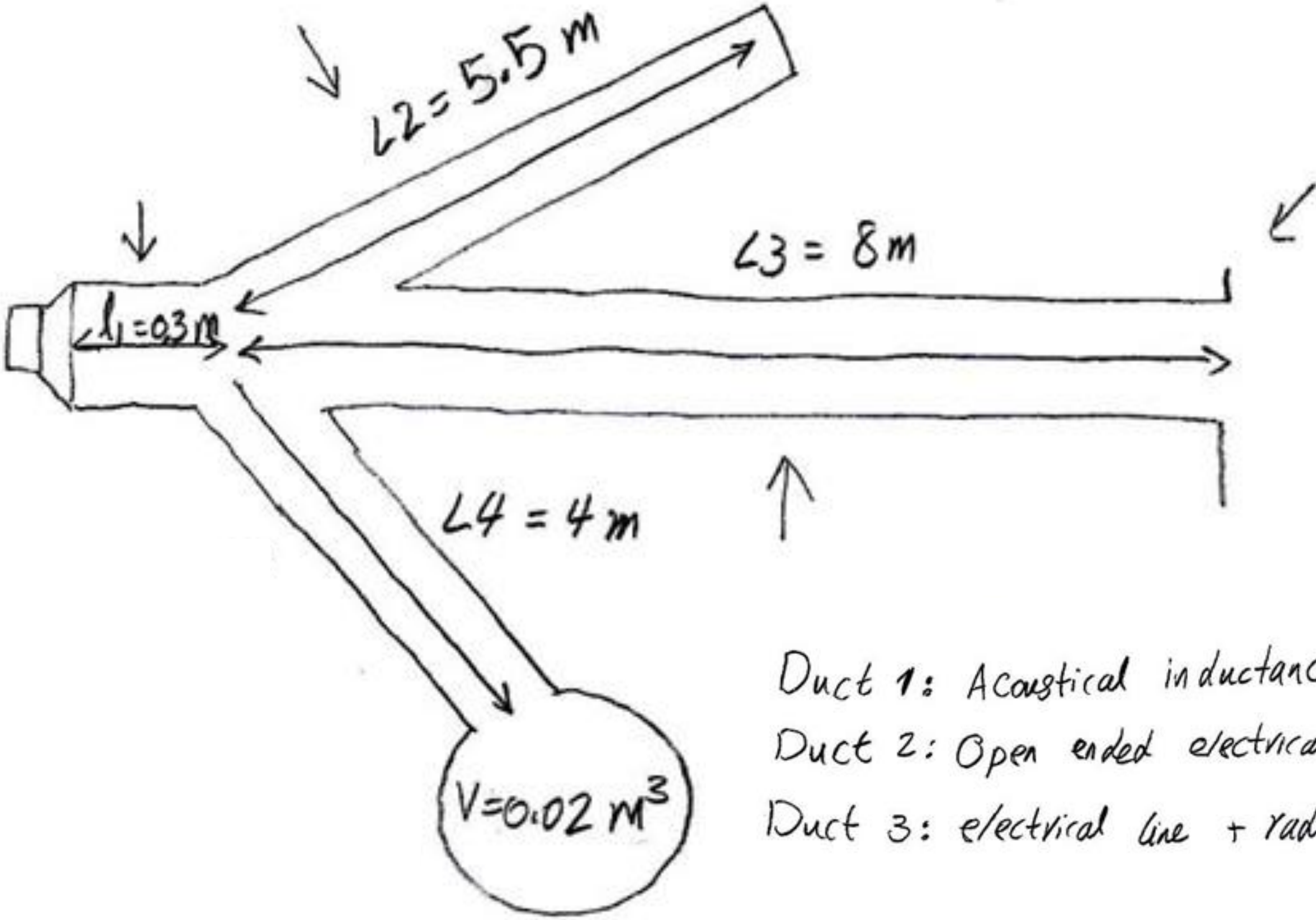

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Duct 2: Open ended electrical line

Duct 3: electrical line + radiation impedance

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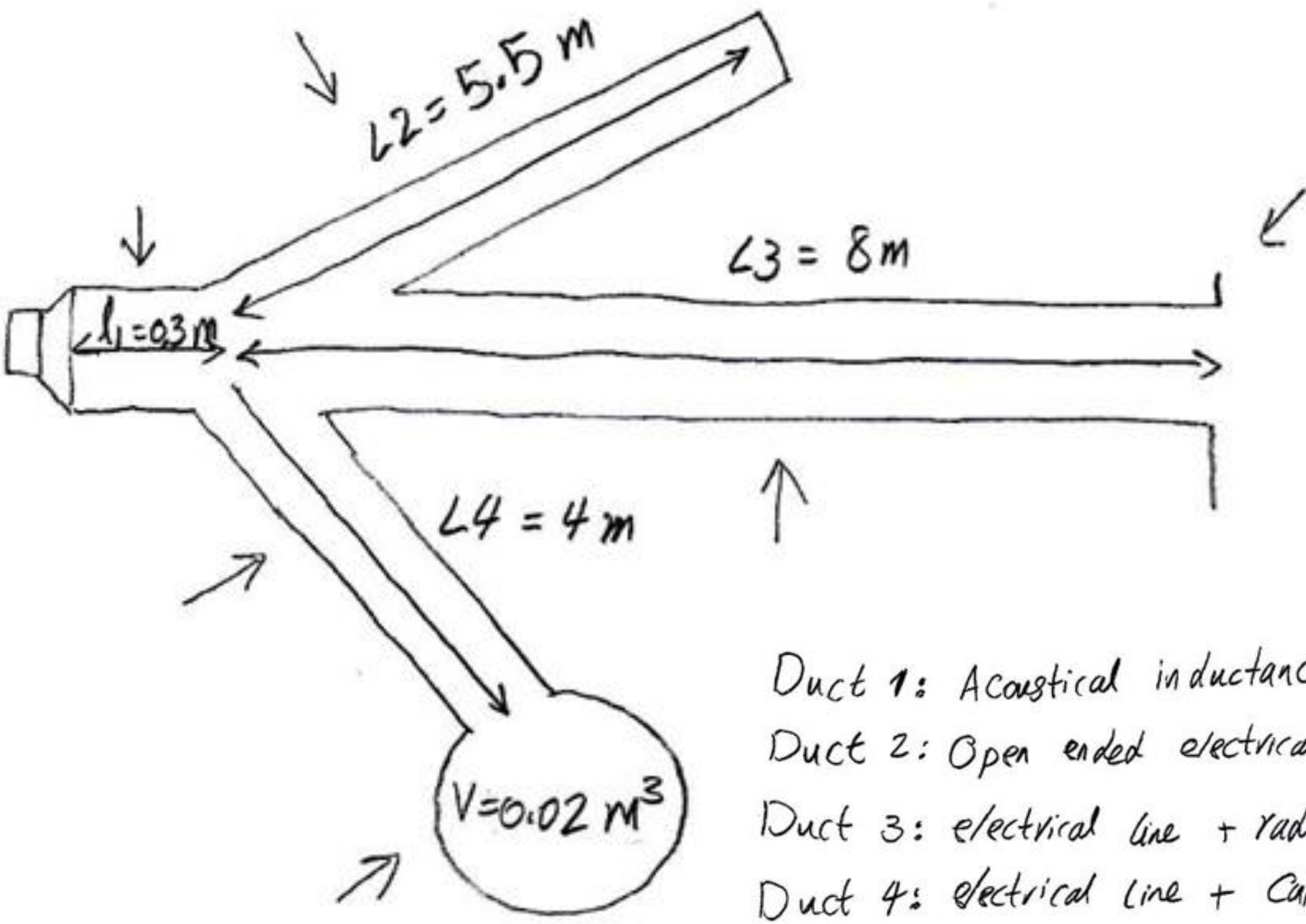

Duct 1: Acoustical inductance

Duct 2: Open ended electrical line

Duct 3: electrical line + radiation impedance

Duct 4: electrical line + Capacitance

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tube L1 can be modeled as an inductance

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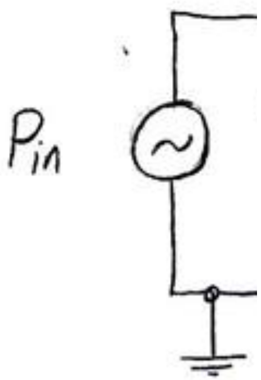
Pin: input power

analog electrical Circuit:



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P_{in}: input power
S: Section of tube 1

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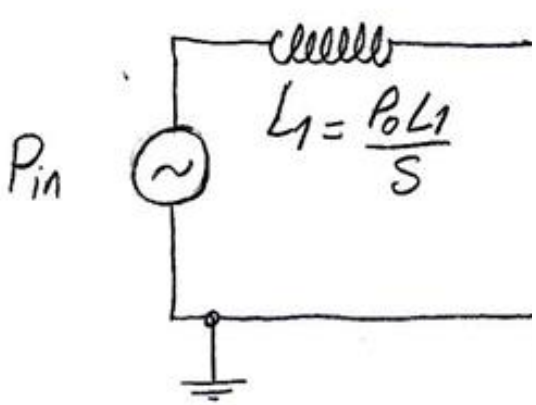

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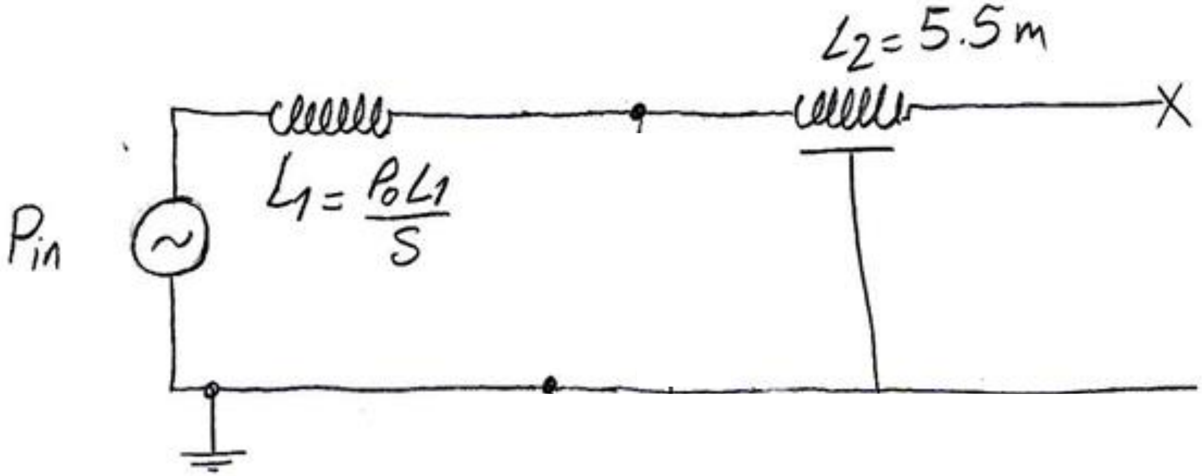
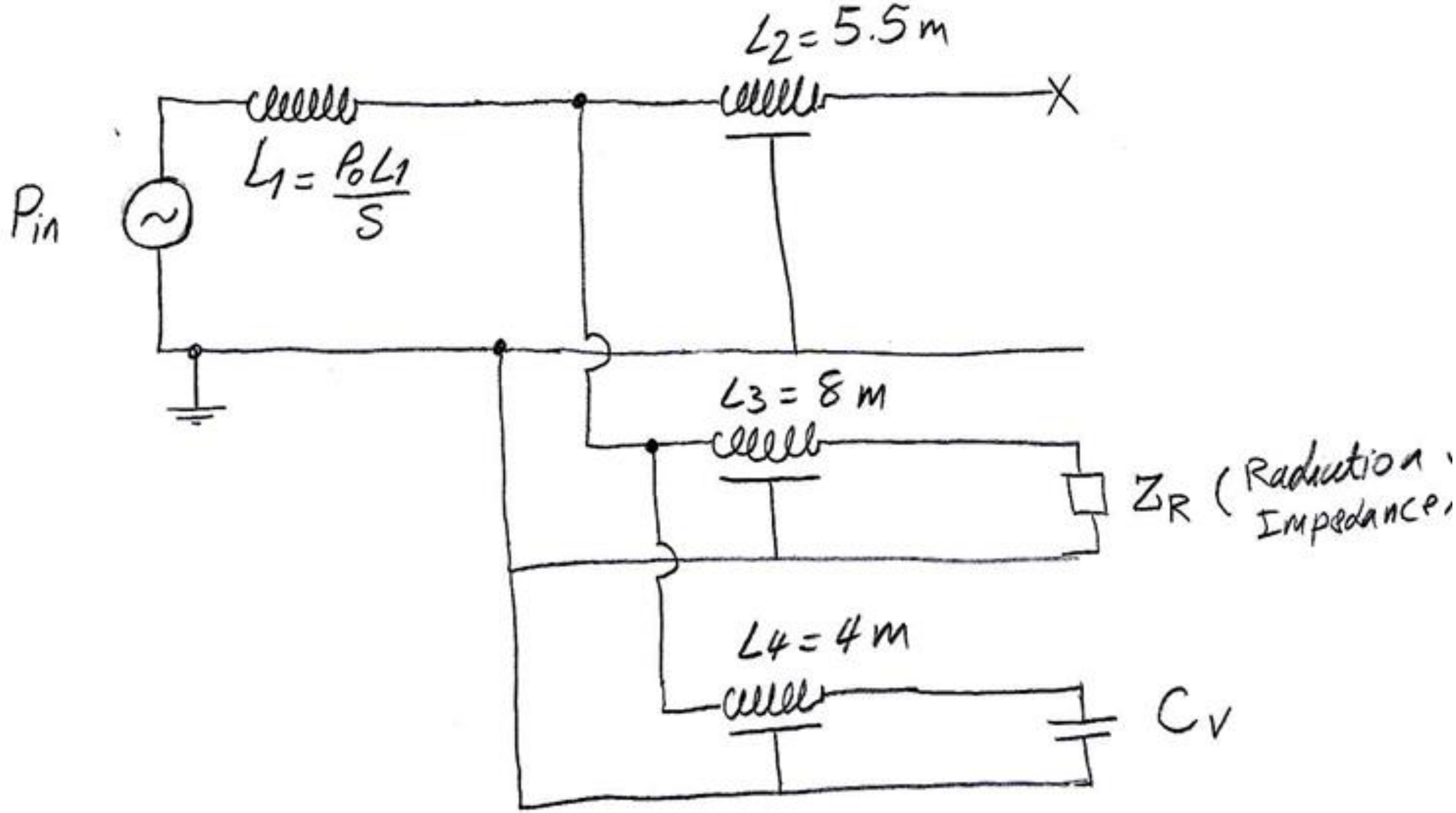


Question 1:

P_{in}: input power
S: Section of tube 1
Z_R: radiation impedance of a piston in an infinit rigid baffle
C_V: acoustic Capacitance of special volume V

analog electrical Circuit:



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<u>tube L_3</u>: $kL_3 = 4\pi \Rightarrow Z_{in} = Z_{load} = Z_R$

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tube $L3$: $kL_3 = 4\pi \Rightarrow Z_{in} = Z_{load} = Z_R$

tube $L2$: $kL_2 = 2\pi\left(\frac{5.5}{4}\right) = \frac{11\pi}{4}$

↳ input impedance $= -j\frac{\rho_0 c}{S}\underbrace{\cot g(kL_2)}_{-1}$

$$= j\frac{\rho_0 c}{S}$$

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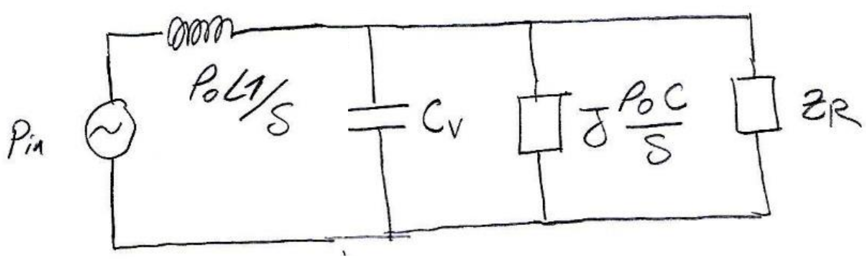
New analog Circuit:



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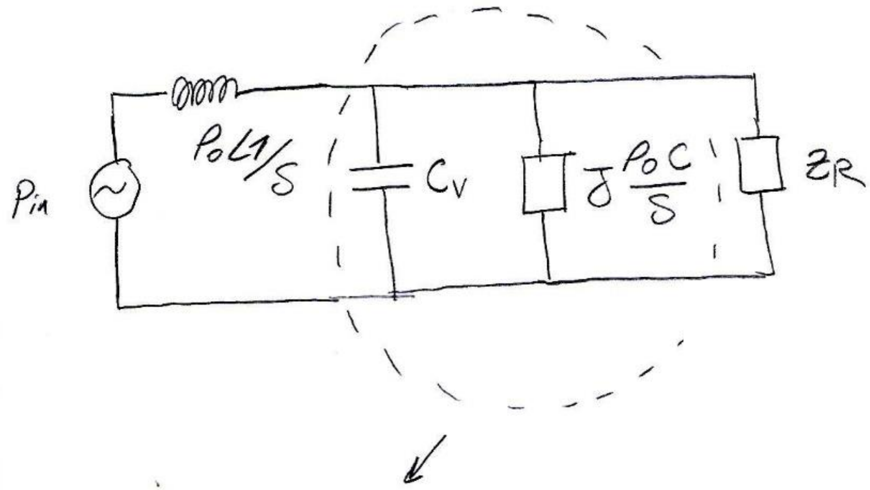

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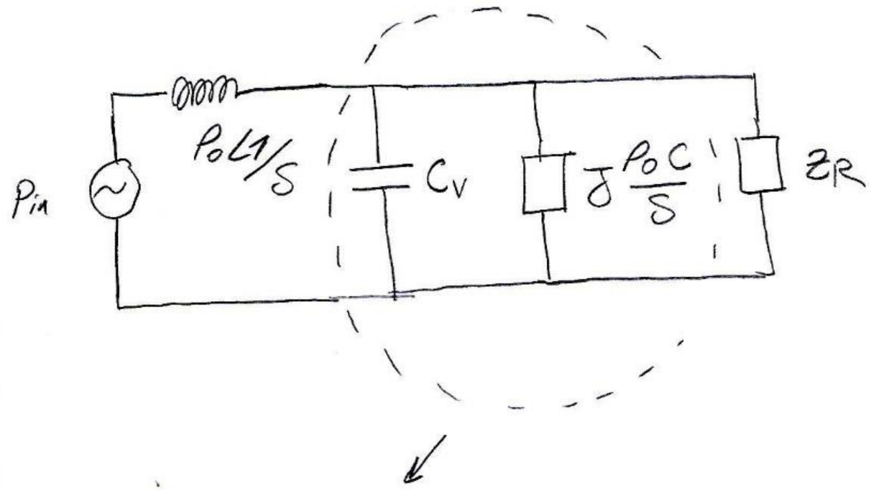

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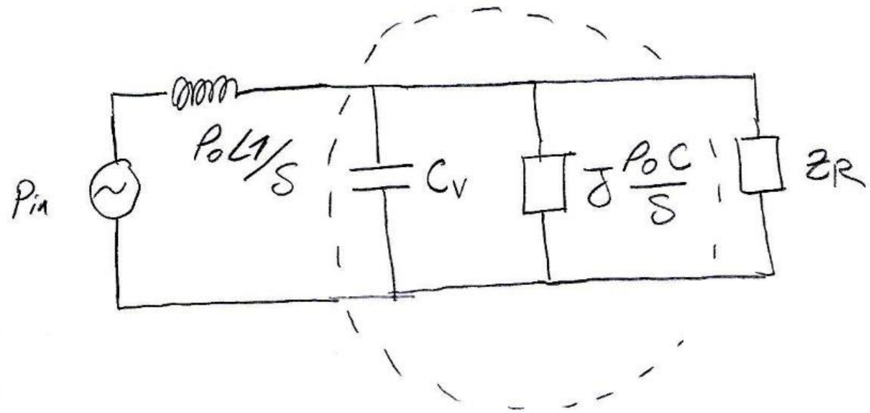


$$\text{impedance} = \frac{1}{\text{admittance}}$$

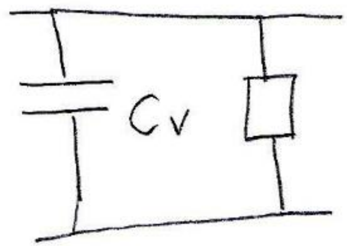
$$\text{admittance} = j\omega C_V + S/j\rho_0 c$$

$$= j(2\pi\, 85)\, V/\rho_0 c^2 - \frac{j\pi(0.1)}{\rho_0 c}$$

$$= \frac{j\pi}{\rho_0 c}\left(\frac{(170)(0.02)}{340} - 0.01\right) = 0$$

Question 1:

So,



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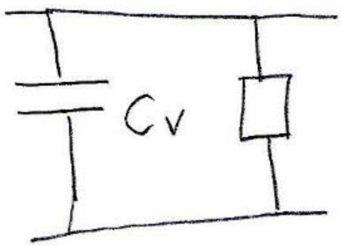

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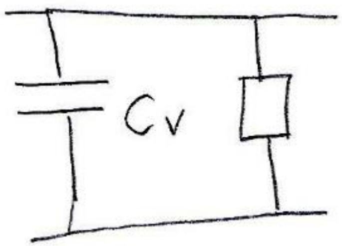

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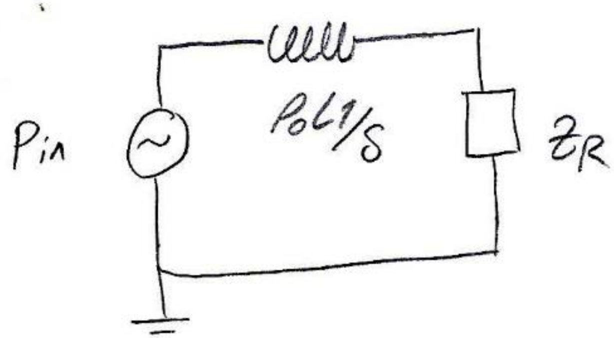

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$$kR = \frac{2\pi}{\lambda}(0.1) = \frac{2\pi}{4}(0.1) \leq 0.2 \text{ ok!}$$

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$$= j\omega \frac{(1.2)(0.3)}{\pi(0.1)^2} + \underbrace{\frac{(1.2)}{2\pi(340)}\omega^2 + j\omega\frac{8(1.2)}{3\pi^2(0.1)}}_{Z_R}$$

With $\omega = 2\pi(85) = 534.1$ rad/s

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$$\Rightarrow Z_{in} = 160.2 + j\,7852$$

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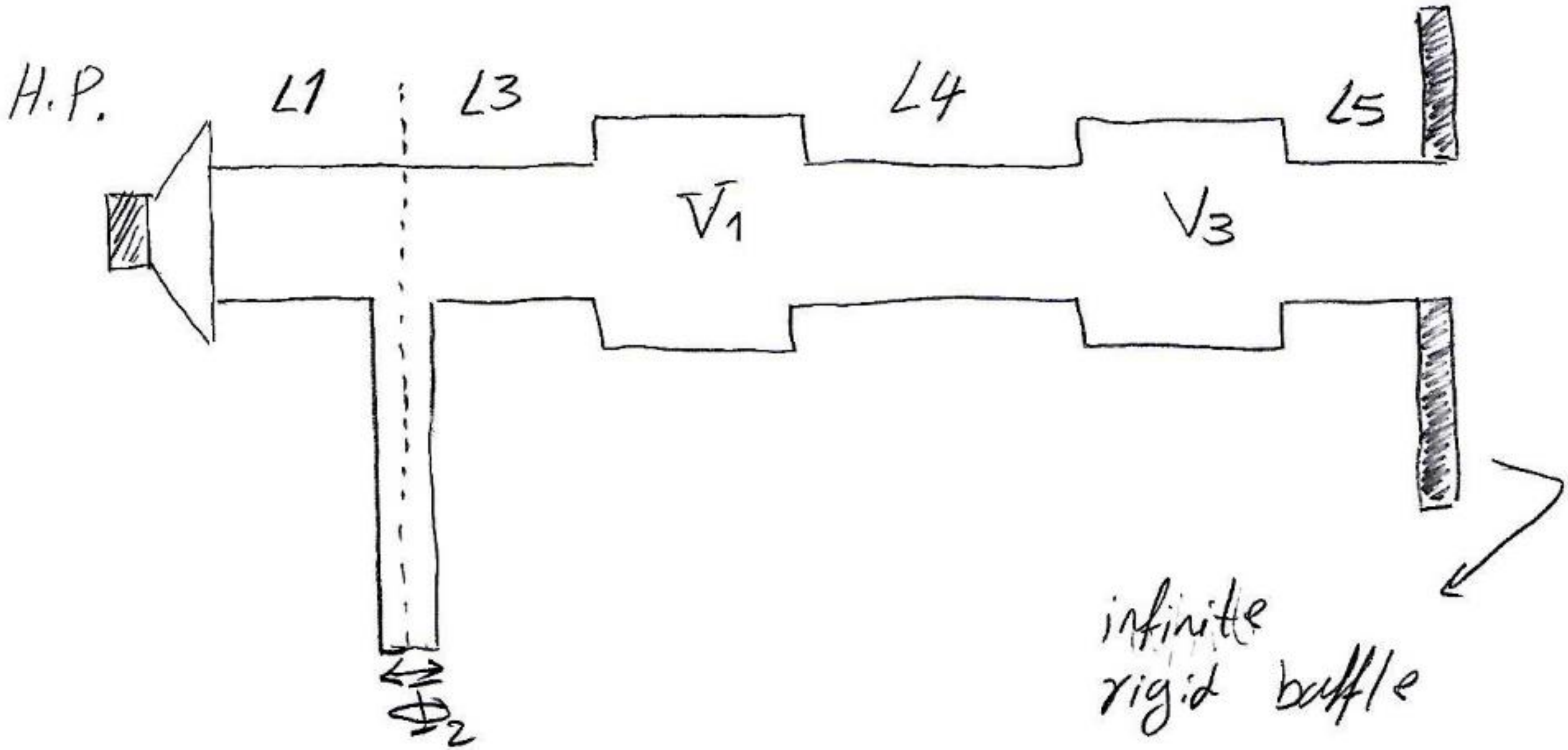
$$= |P_{in,rms}|^2 \times \frac{160.2}{|160.2 + j7852|^2}$$

Question 2 //



Question 2//

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Check 1/: plane wave if $f \leq 0.586\ c/\Phi$

((tube, Φ = diameter of section))

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$\Phi_1 = 10$ cm $\Rightarrow f \leq 2016$ Hz

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) ok @ 500 H[...] / 1000 H[...]

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Check 2//: Condensed elements (tube) if
length $\leq 0.08\, c/f = 5.5$ cm @ 500 Hz

Question 2//

analog electric Circuit:

HYP. → General Case: tube ≡ electrical line

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HyP. → General Case: tube ≡ electrical line

→ V_1 and V_2 ≡ Capacitances

→ Loud Speaker ≡ power generator

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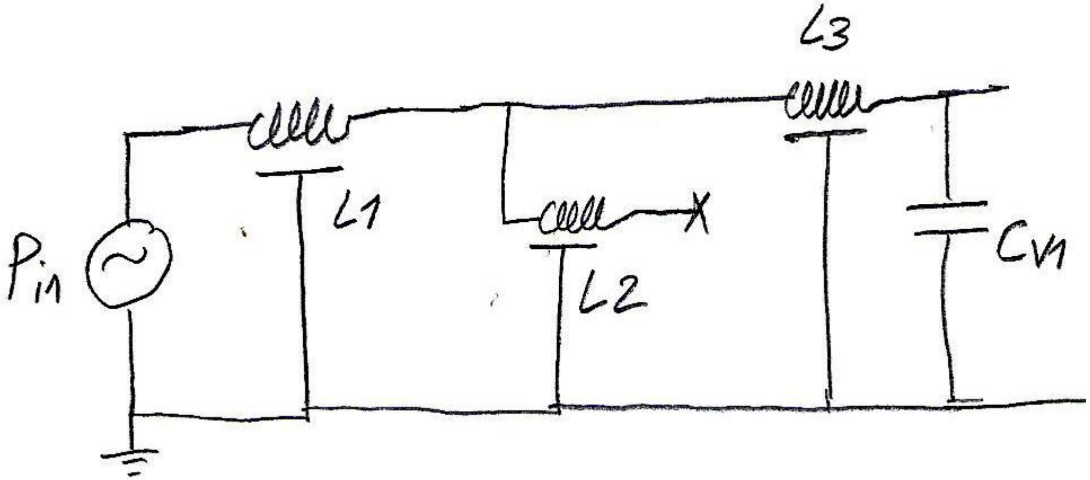

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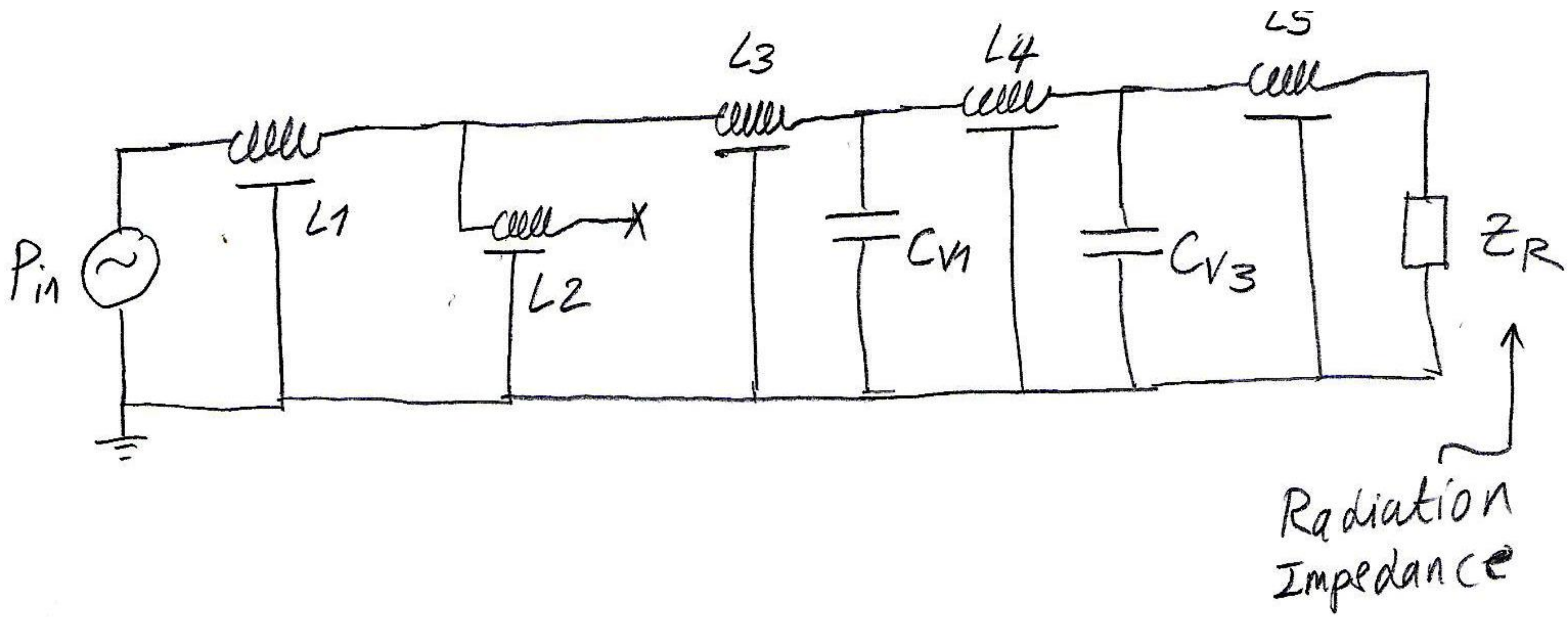

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2.2) Tube L_2: input impedance

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$$Z_{in} = 0 \quad \text{if} \quad kL_2 = \frac{2\pi(500)}{344} L_2 = \pi/2 + m\pi$$

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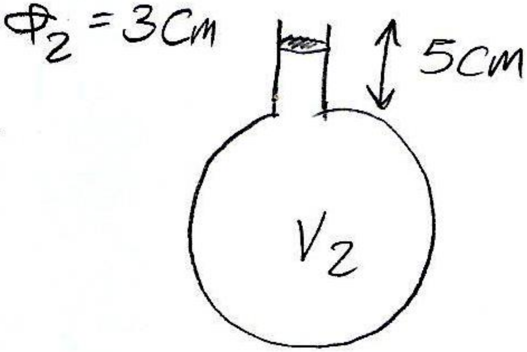
$Z_{in} = 0$ if $kL_2 = \frac{2\pi(500)}{344} L_2 = \pi/2 + m\pi$

minimum value of $L_2 = \frac{344}{2000} = 0.172\,m$

Question 2//

2.3)

Helmholtz resonator



Question 2//

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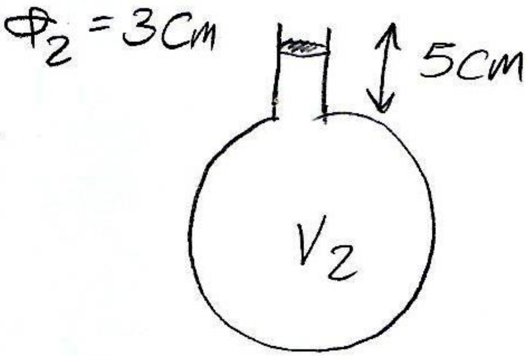

tube 5 cm ⇒ Condensed element @ 500 Hz

Question 2//

in the analog Circuit, Line L_2 replace by

$M = \frac{P_0 (0.0}{S}$

C_{V_2}

$$Z_{in} = j\omega M + \frac{1}{j\omega C_{V_2}} = 0 \quad \text{if} \ \ldots$$

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$$\Rightarrow V_2 = \frac{1}{[2\pi(500)]^2} \cdot \frac{\pi(0.015)^2}{(0.05)} C^2$$

$$= 1.70 \times 10^{-4} m^3 = 0.17 \text{ liters}$$

Question 2//

2.4)

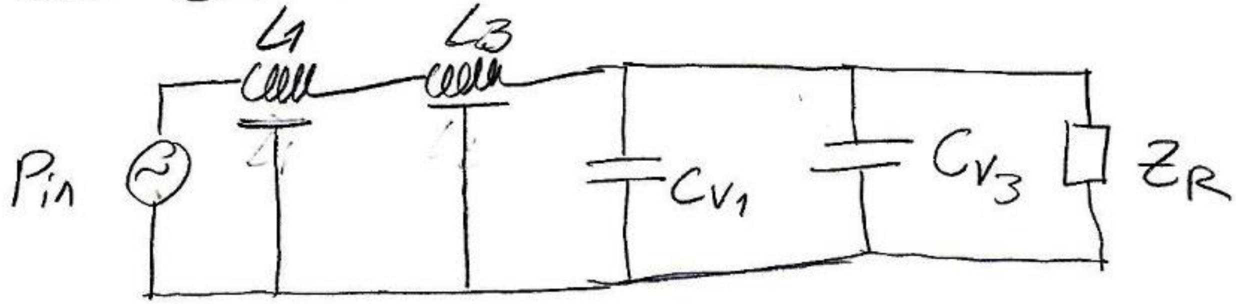
if $kL_2 = \pi/2$ @ 500 Hz $\Rightarrow$

$kL_2 = \pi$ @ 1000 Hz

$\Rightarrow$ for L_2, L_4, L_5 , $Z_{in} = Z_{load}$

Question 2//

new circuit



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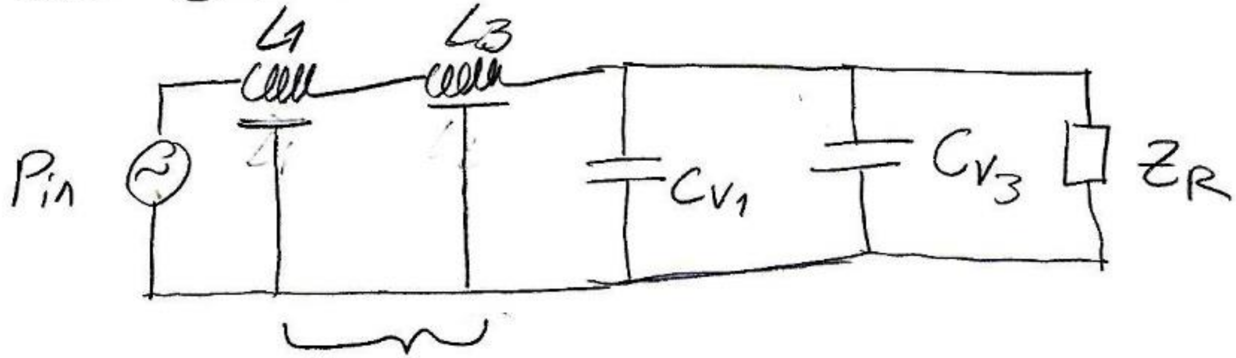

$L_1 + L_3 = L_2 \Rightarrow$ tube of length $L_2 \Rightarrow$

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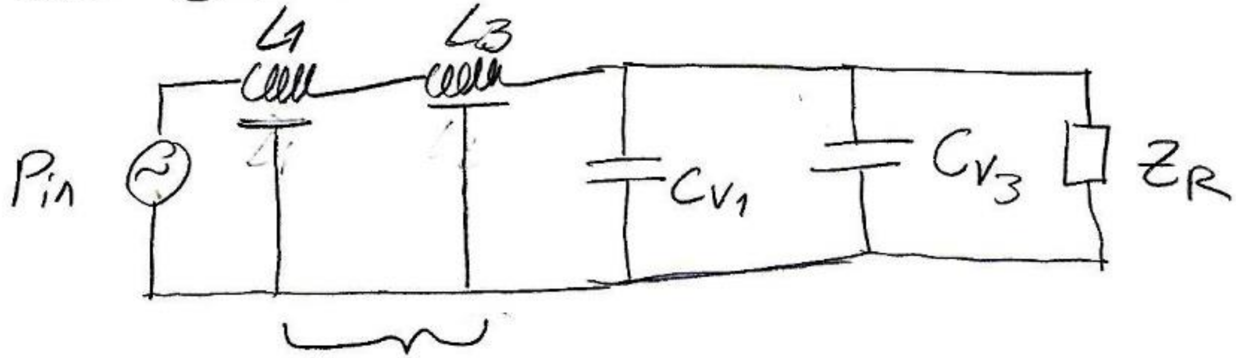

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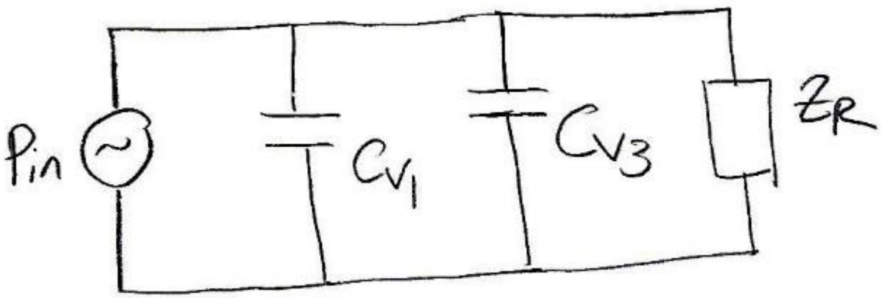

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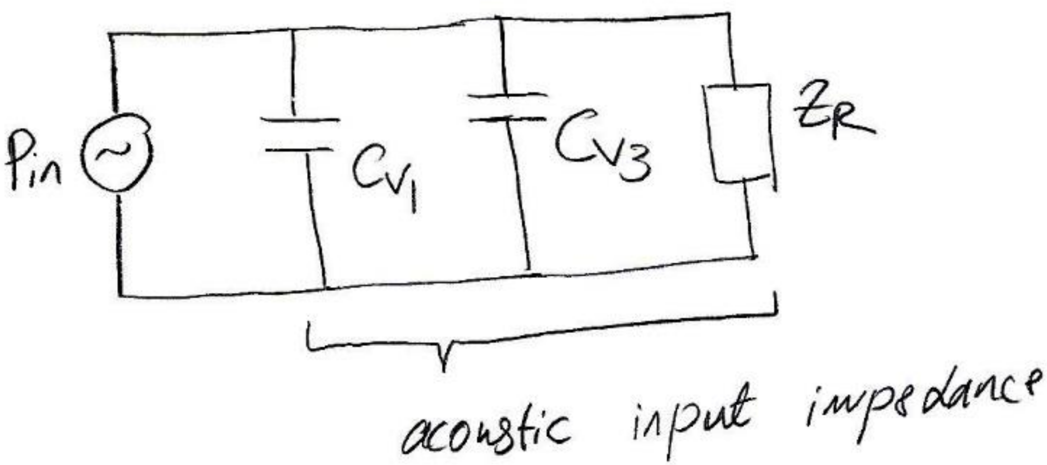

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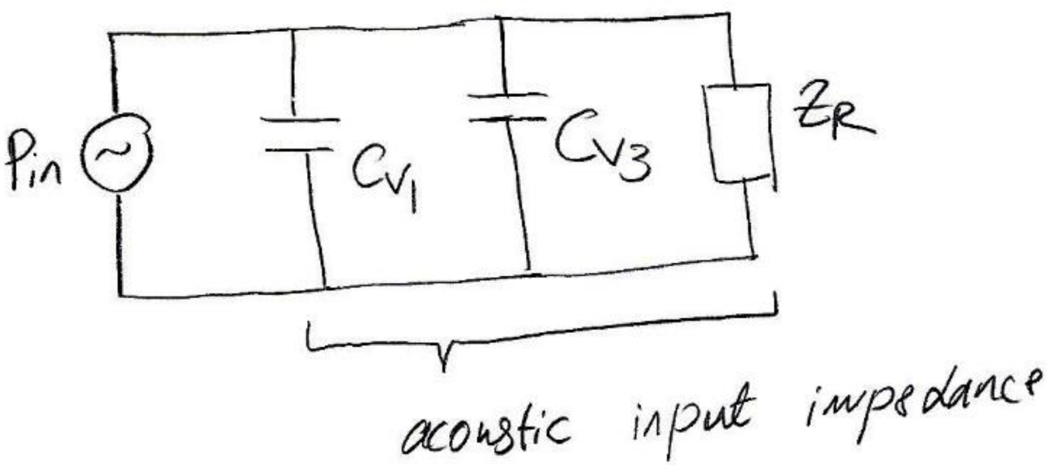

input admitance $= j\omega C_{V_1} + j\omega C_{V_2} + \frac{1}{Z_R}$

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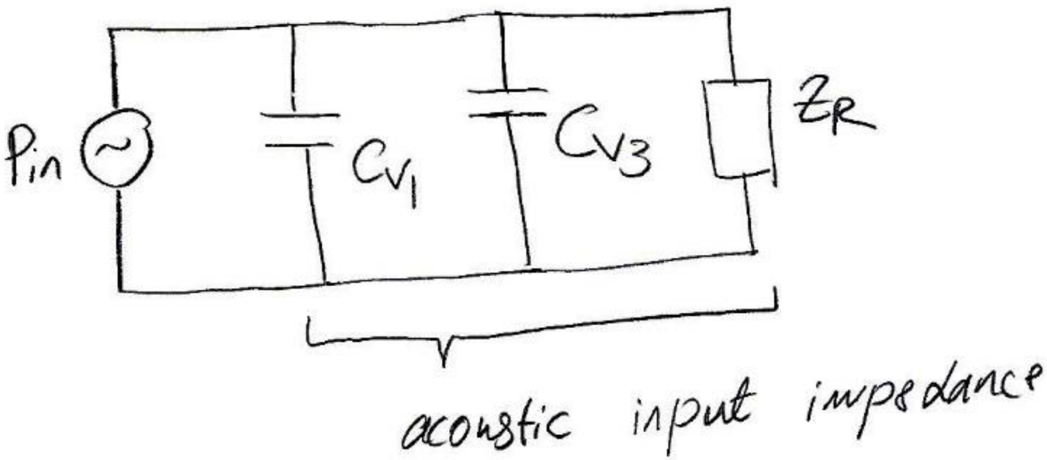

$$\text{input admitance} = j\omega C_{V_1} + j\omega C_{V_2} + \frac{1}{Z_R}$$

$$= j\omega\left(\frac{V_1 + V_3}{\rho_0 c^2}\right) + \frac{1}{Z_R}$$

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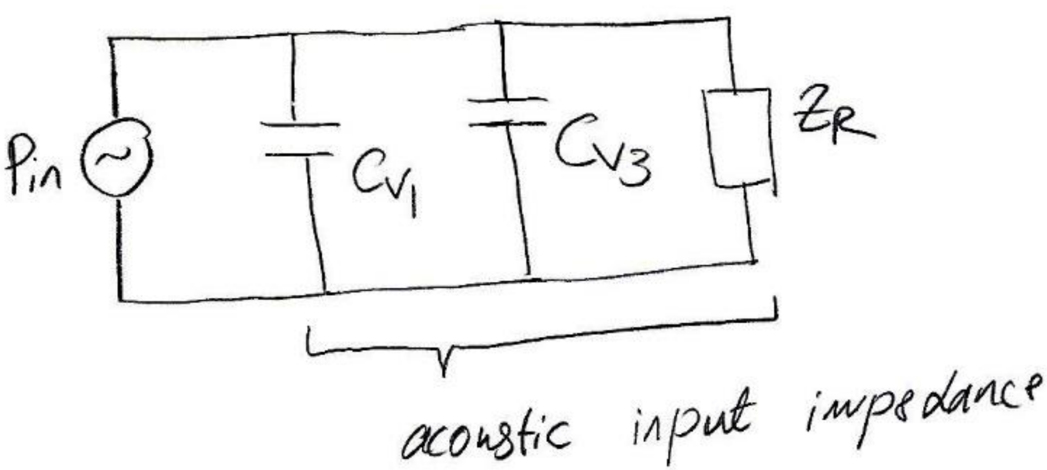

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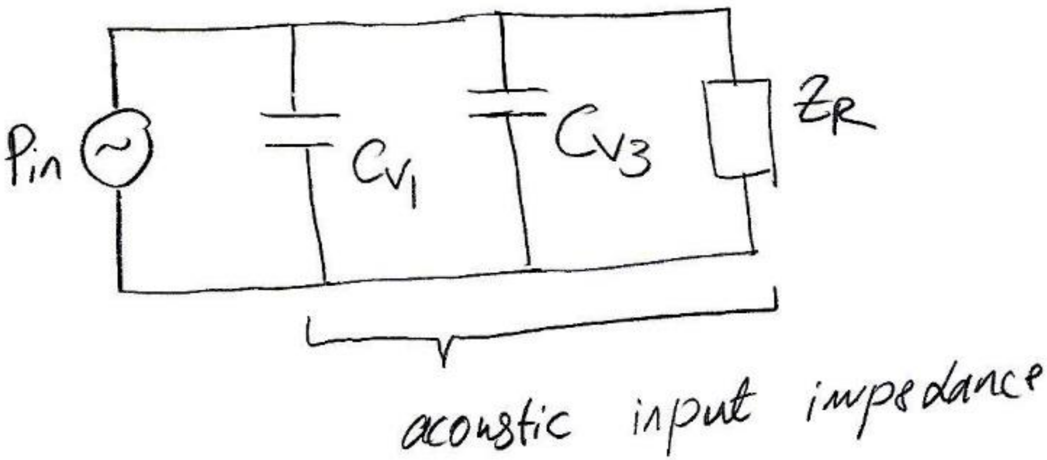

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Simplified formula? $\Rightarrow$ NO

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$\Downarrow$

figure (ZR Piston infinit baffle)

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Simplified formula? ⇒ NO

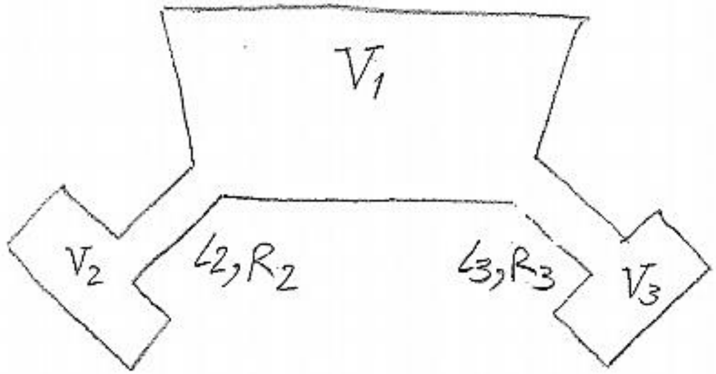
⇓

figure (Z_R Piston infinit baffle)

$$Z_R(kR = 0.91) \cong \frac{\rho_0 c}{S}(0.38 + j\,0.61)$$

$$= 19973 + j\,32062 \ [Pa \cdot S/m^3]$$

Question 5:



"The artificial ear."

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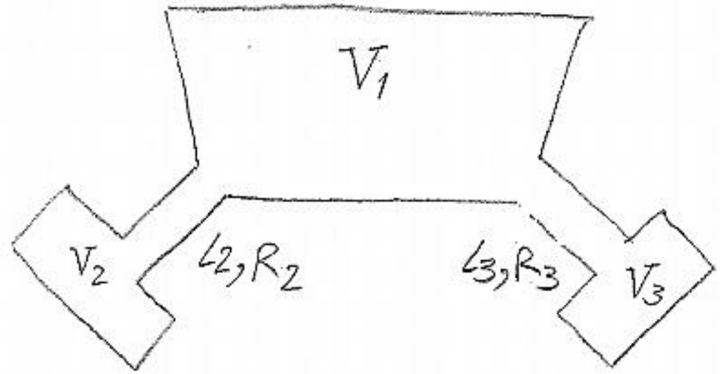

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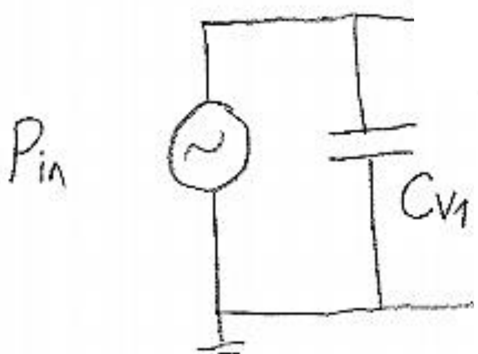

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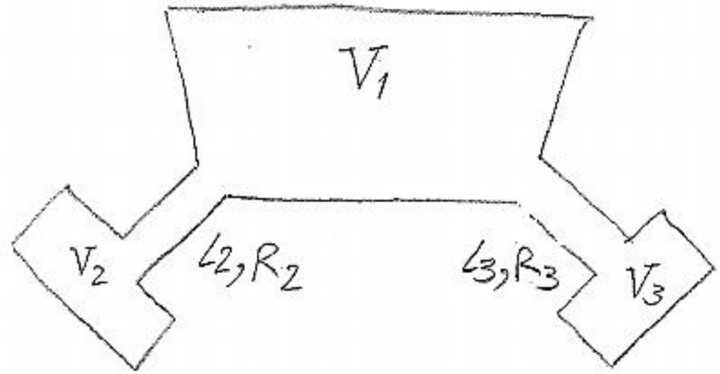

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