Sabine theorem

Exercise session

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1a) Minimum of absorbing material

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length 50 m

Width 20 m

height 8 m

V = room volume = $50 \times 20 \times 8 = 8000\ m^3$

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S_{abs} : unknown

$$a \geq \frac{0.163\,V}{RT_{max}}$$

$$\alpha_c \delta_c + \alpha_f \delta_f + \alpha_w \delta_w + \delta_{abs}(\alpha_{abs} - \alpha_w) \geq \frac{0.163\,V}{RT_{max}}$$

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$$\underbrace{\alpha_c \delta_c + \alpha_f \delta_f + \alpha_w \delta_w}_{a'} + \delta_{abs} \underbrace{(\alpha_{abs} - \alpha_w)}_{>0\ \forall freq} \geq \frac{0.163\, V}{R T_{max}}$$

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$$\Rightarrow \quad \underbrace{\delta_{abs} \geq \left(\frac{0.163\, V}{R T_{max}} - a' \right) / (\alpha_{abs} - \alpha_w)}_{\text{minimum surface of absorbing material.}}$$

@ 250 Hz : $S_{abs} \geq 150\ m^2$

1 kHz : $S_{abs} \geq 360\ m^2$

4 kHz : $S_{abs} \geq 878\ m^2$

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this one is the solution that is valid for other frequencies.

1b) L_p as a function of the distance

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source power
(100 dB)

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PWL ↙ source power (100 dB)

↙ r: distance from source

1b) L_p as a function of the distance

→ directivity coefficent

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$$R' = \frac{a}{1 - a/S_T}$$

S_T = total surface of walls + ceiling + floor = 3120 m^2

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Q → directivity coefficent

PWL → source power (100 dB)

r: distance from source

$$R' = \frac{a}{1 - a/S_T}$$ with $a = \sum \alpha_i S_i$

$a = 1563\ m^2$ @ 1kHz

(with $S_{abs} = 878\ m^2$)

S_T = total surface of walls + ceiling + floor $= 3120\ m^2$

⚠ ⚠

$R' = 3133\ m^2$ @ 1 kHz

$\Rightarrow\ 10 \log \frac{4}{R'} = -29\ dB$

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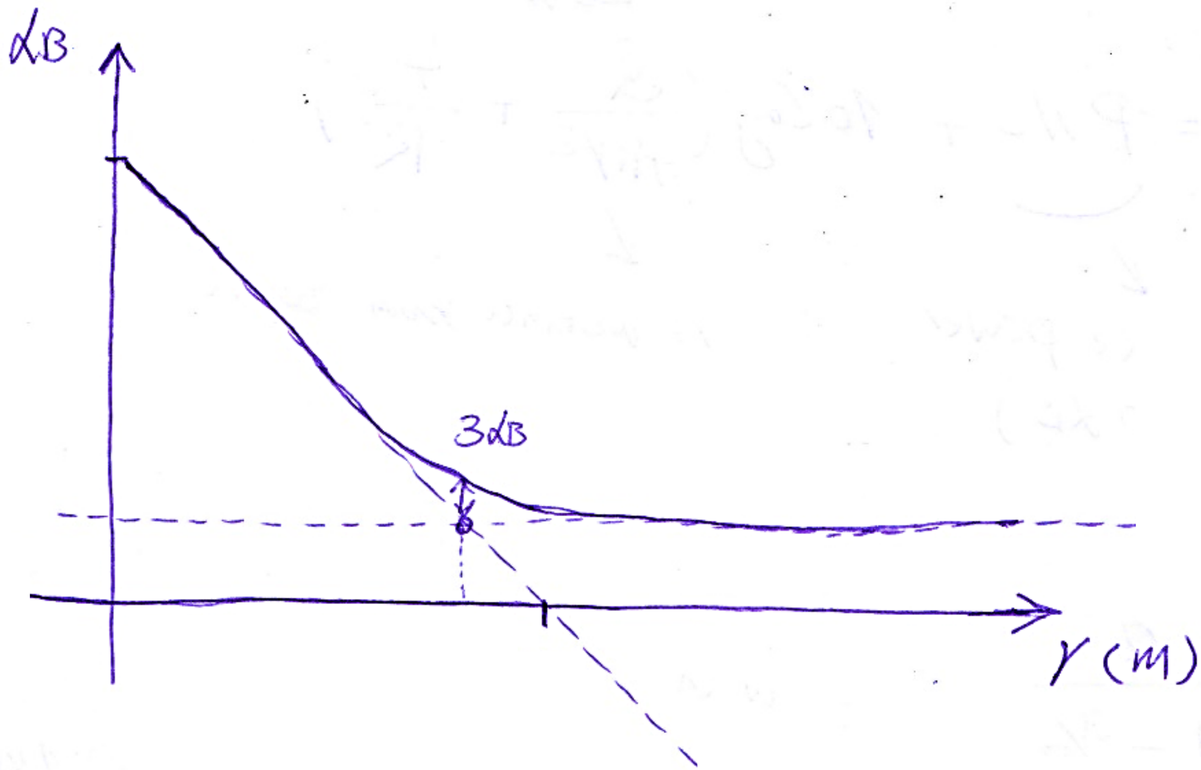

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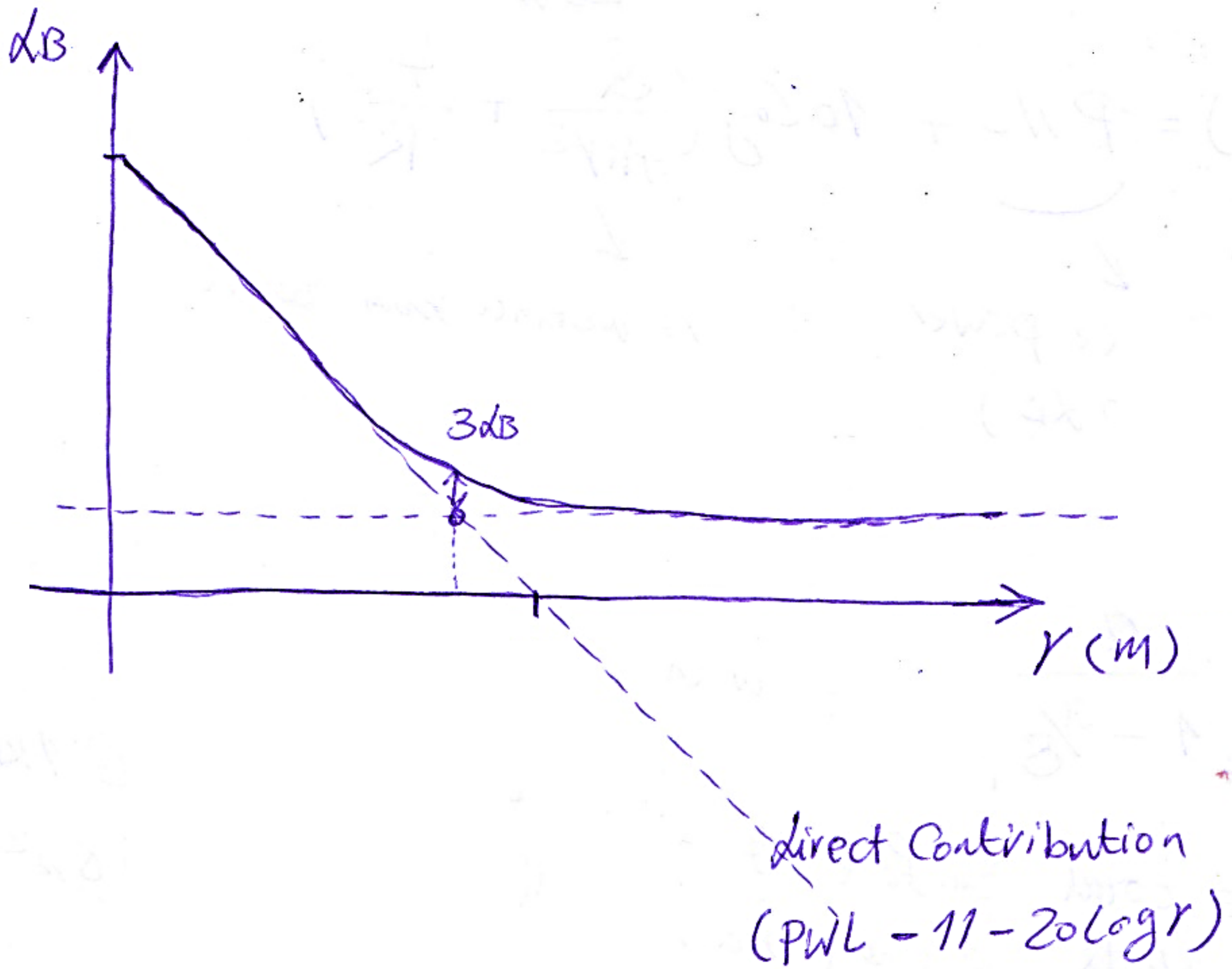

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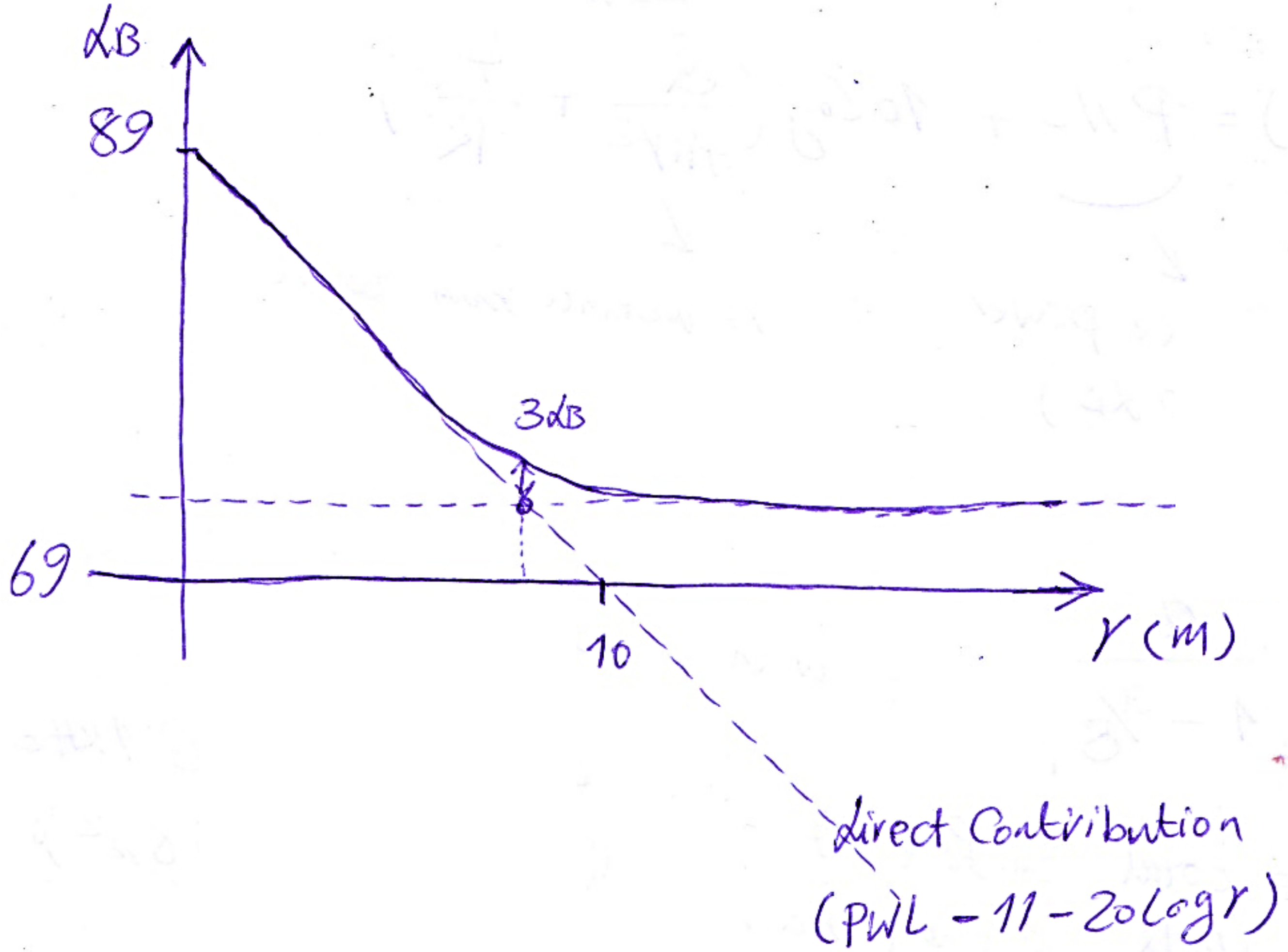

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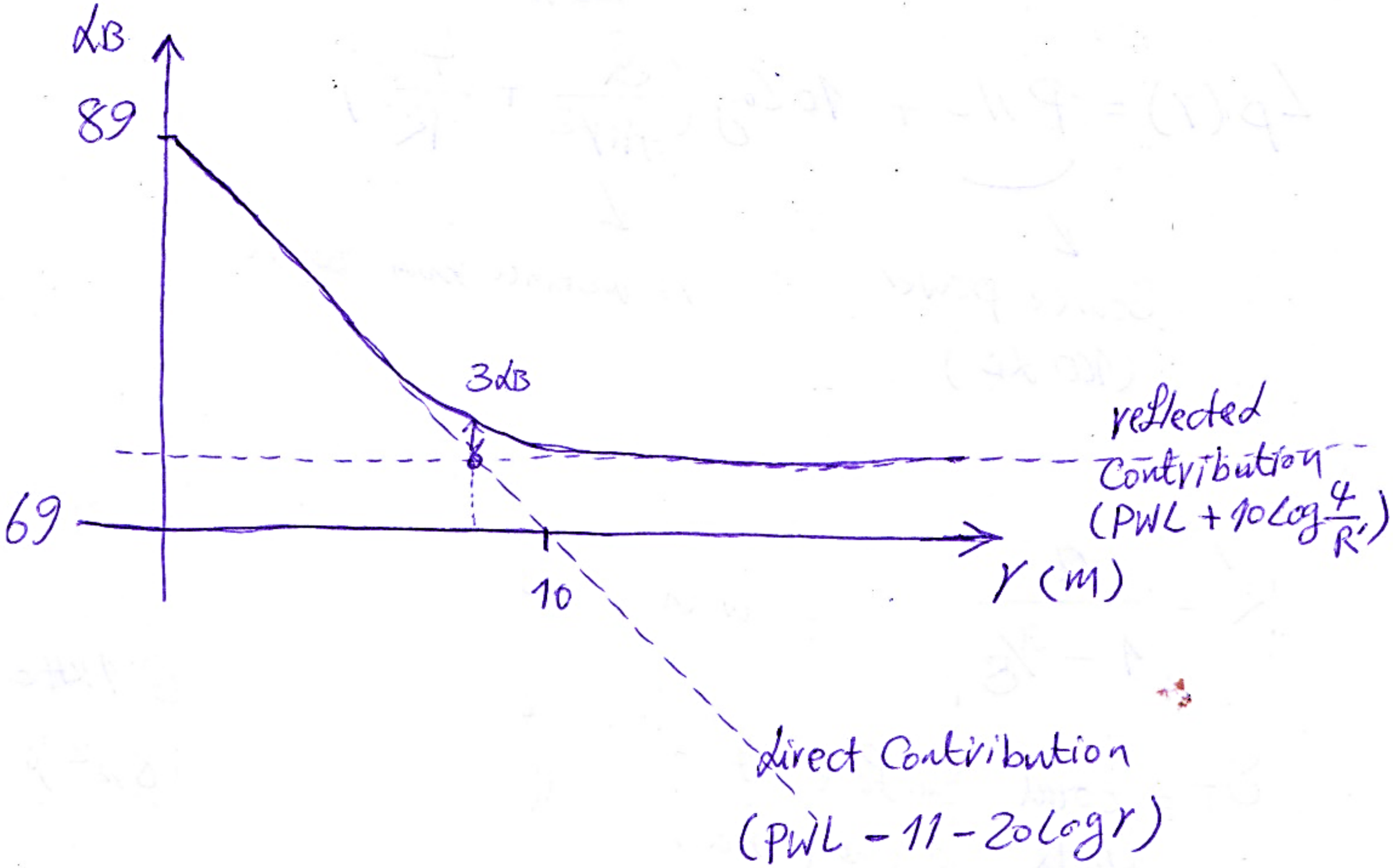

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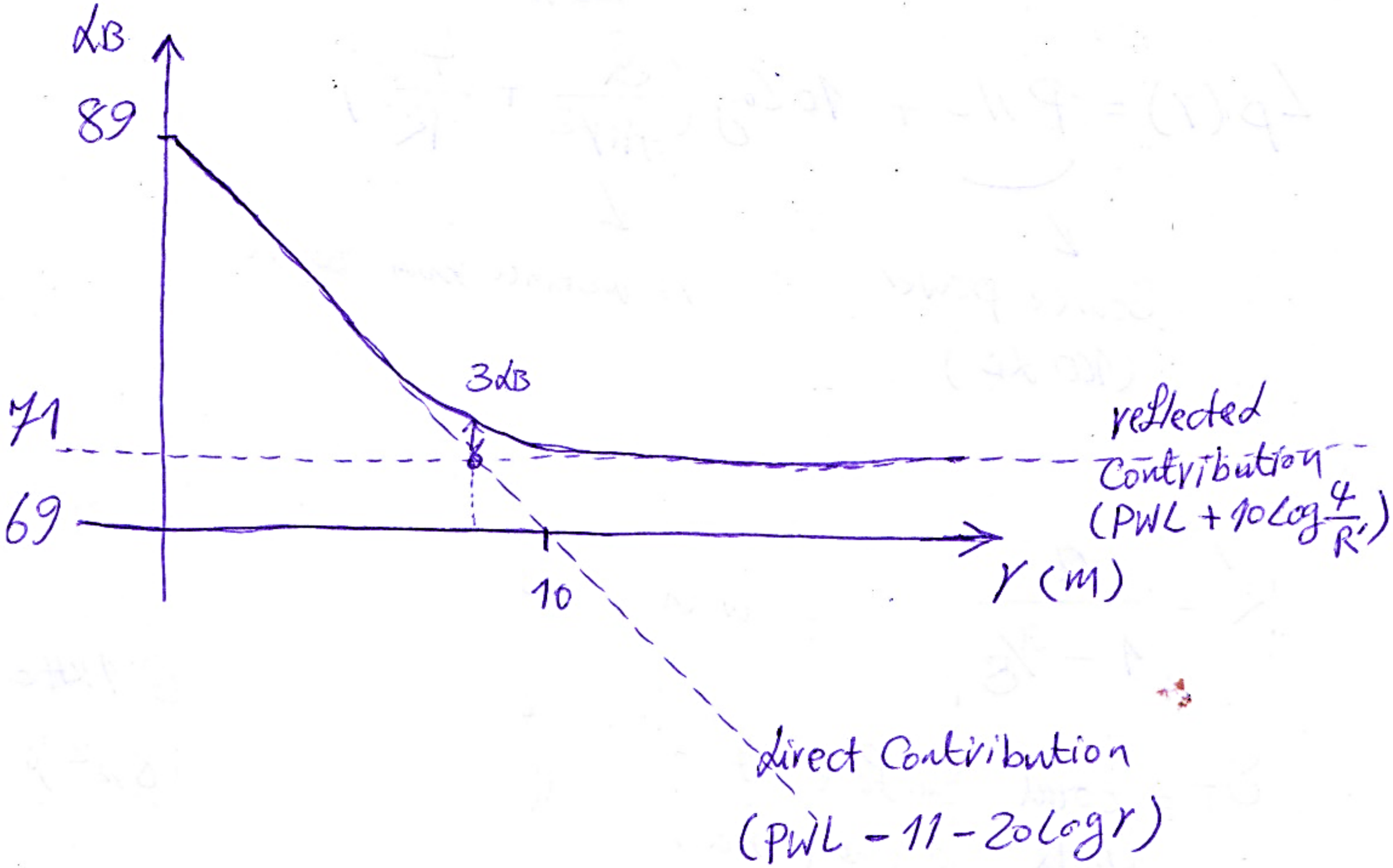

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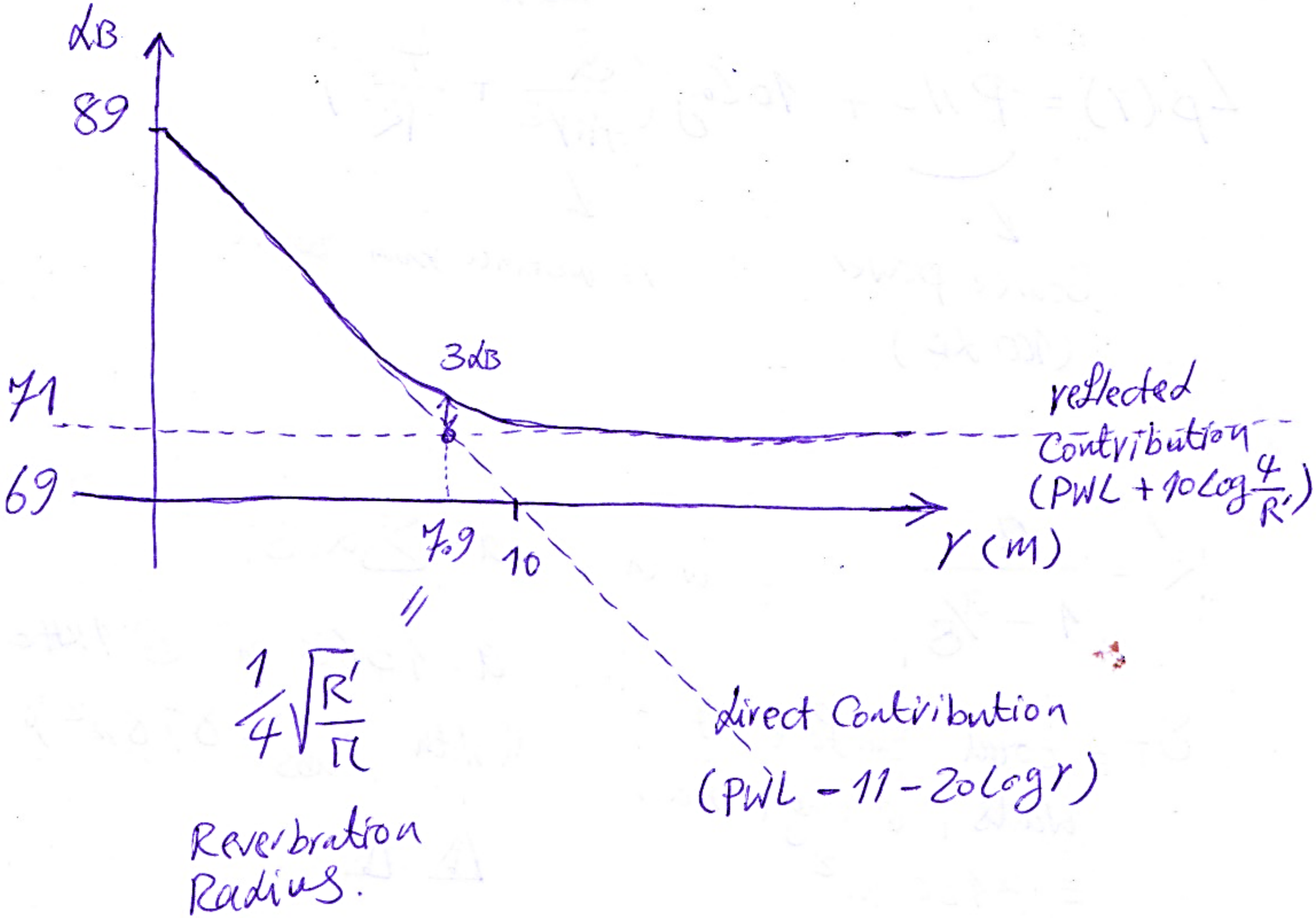

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$$r_{rev} = \frac{1}{4}\sqrt{\frac{R'}{\pi}} \qquad R' = \frac{a}{1 - a/S_T}$$

$a = \sum_i \alpha_i S_i$ depends on freq.

S_T = total surface in the room (m^2).

Data:

$$RT = \frac{0.163V}{a}$$

1000 m^3

↑

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method: $RT \Rightarrow a \Rightarrow R' \Rightarrow \gamma_{rev}$

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f (Hz)	125	250	500	1k	2k	4k
RT (s)	1.5	1.4	1.2	1.0	0.9	0.6

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r_{rev} (m)	1.6	1.7	1.8	2	2.2	3.0

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1.6 → This is the maximum distance to be in direct field at all freq.

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PWL (dB)	80	80	75	70	70	60

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Lp (dB)	64.9	64.6	58.8	52.7	52.1	39.5

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Lp (dBA)	48.9	56.6	55.8	52.7	58.1	40.5

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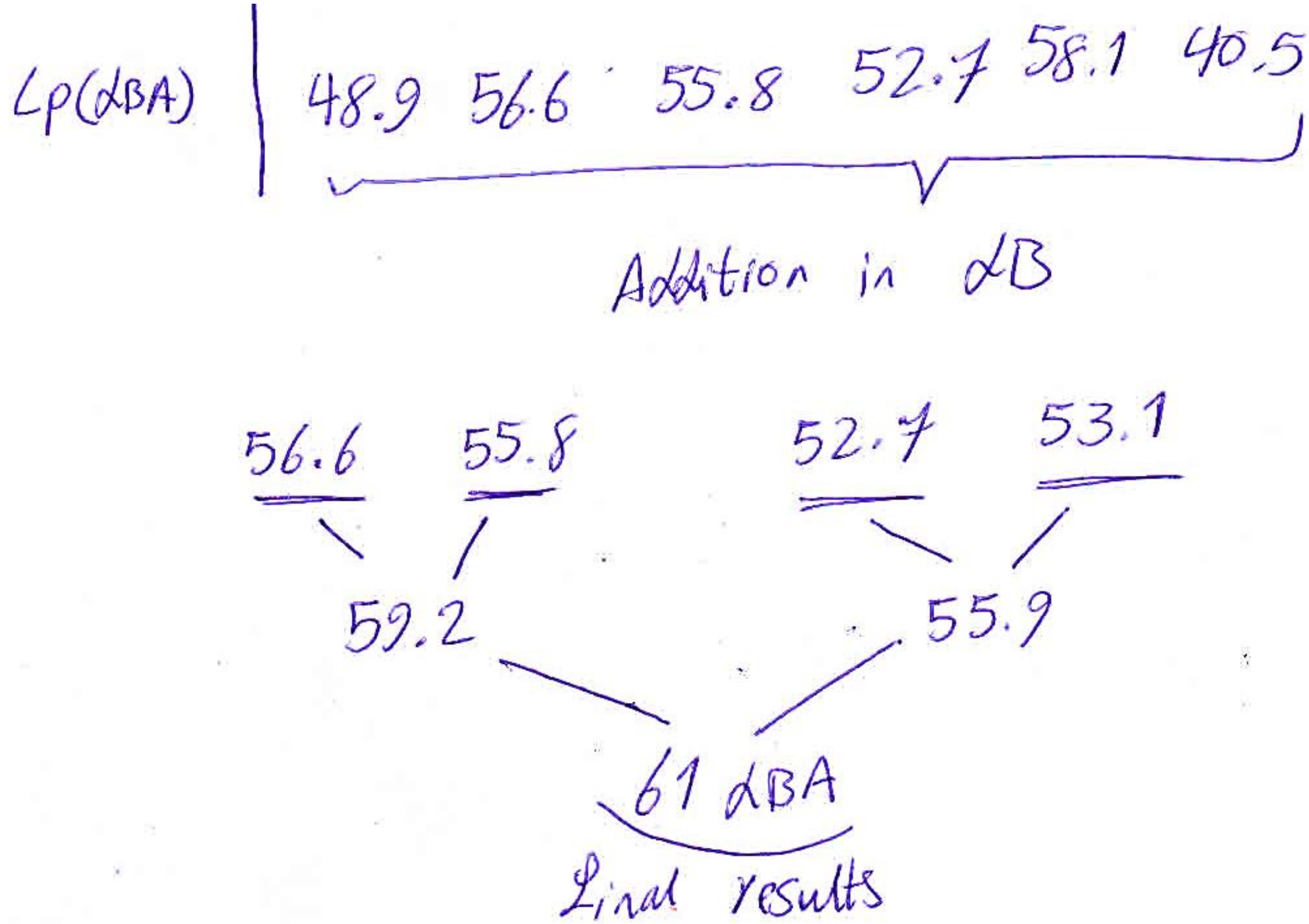
Addition in dB

Lp(dBA) | 48.9 56.6 55.8 52.7 58.1 40.5

Addition in dB

56.6 55.8 → 59.2

52.7 53.1 → 55.9



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Addition in dB
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55.8
52.7
53.1
59.2
55.9
61 dBA
Final results

Ex 3.

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L_p ↙ in dB; PWL ↓ source power

{2} $$R' = \frac{a}{1 - a/S_T}$$; S_T: total surface of the envelop

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Objective $L_{p,new} = L_{p,old} - 3\,dB$

[1] $\Rightarrow$ $PWL + 10\log\frac{4}{R'_{new}} = PWL + 10\log\frac{4}{R'_{old}} - 3dB$

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$\Rightarrow 10\log\frac{R'_{old}}{R'_{new}} = -3dB$

$$\Rightarrow \boxed{R'_{new} = 2R'_{old}}$$

$$a_{old} = \underbrace{\alpha_f \delta_f}_{\text{floor}} + \underbrace{\alpha_c \delta_c}_{\text{ceiling}} + \underbrace{\alpha_w \delta_w}_{\text{bare walls}}$$

$$= (0.2)(400) + (0.2)(400) + 0.2(320) = 224\ m^2$$

$$\{2\} \Rightarrow R'_{old} = \frac{224}{1 - \frac{224}{1120}} = 80\ m^2$$

$$\{3\} \Rightarrow R'_{new} = 560\ m^2.$$

$$R'_{new} = 560\ m^2 = \frac{a_{new}}{1 - \frac{a_{new}}{8T}}$$

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$$\{3\} \Rightarrow R'_{new} = 560\, m^2$$

$$R'_{new} = 560\, m^2 = \frac{a_{new}}{1 - \frac{a_{new}}{S_T}}$$

$$\Rightarrow a_{new} = \frac{R'_{new}}{1 + \frac{R'_{new}}{S_T}} = 873{,}3\, m^2$$

$$a_{new} = \alpha_f \delta_f + \alpha_c \delta_e + \underbrace{\alpha_w(\delta_w - \delta_{abs})}_{\text{bare wall}} + \underbrace{\delta_{abs} \overbrace{\alpha_{abs}}^{\text{known}}}_{\text{absorbing material}}$$

$$= a_{old} + \delta_{abs}(\alpha_{abs} - \alpha_w)$$

$$373{,}3\ m^2 = 224 + \delta_{abs}(0.6 - 0.2)$$

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$$S_{abs} = 373.3 \ m^2$$

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↓

impossible ⚠

because only $320 \ m^2$ is available on walls!