ELEN0037
Microelectronics
Tutorials

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Tutorial 1: MOSFET Operation and Modelling

Device Model Summary (Constants)

$q = 1.602 \times 10^{-19}\,C$

$k = 1.38 \times 10^{-23}\,JK^{-1}$

$n_i = 1.1 \times 10^{16}\,carriers/m^3$ @ $T = 300\,K$

n_i doubles for every 11°C increase in temperature

$n \times p = n_i^2$

$\varepsilon_0 = 8.854 \times 10^{-12} Fm^{-1}$

$K_{ox} \cong 3.9$

$K_s \cong 11.8$

Device Model Summary (Diode)

Diode equations (Forward-Biased):

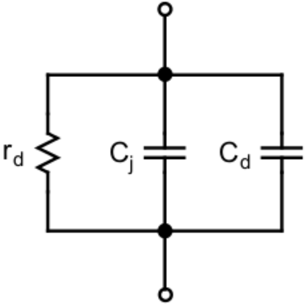
$$I_D = I_S \exp\left(\frac{V_D}{V_T}\right)$$
$$I_S = A_D q n_i \left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D}\right)$$
$$V_T = \frac{kT}{q} \cong 26\, mV\ @\, 300K$$

Diode equations (Reverse-Biased):

$$Q = 2C_{j0}\Phi_0\sqrt{1+\frac{V_R}{\Phi_0}}$$
$$C_j = \frac{C_{j0}}{\sqrt{1+\frac{V_R}{\Phi_0}}}$$
$$C_{j0} = \sqrt{\frac{qK_s\varepsilon_0}{2\Phi_0}\frac{N_A N_D}{N_A+N_D}}$$
$$C_{j0} = \sqrt{\frac{qK_s\varepsilon_0}{2\Phi_0}N_D} \text{ if } N_A \gg N_D$$
$$\Phi_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

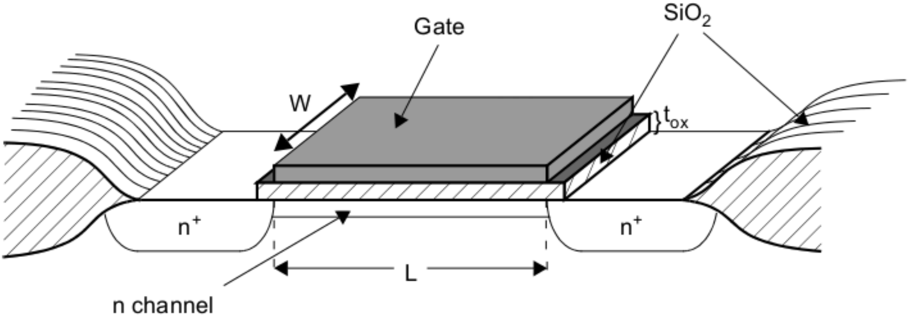
Device Model Summary (Diode)

Small-Signal Model of Forward-Biased Diode:



$r_d = \frac{V_T}{I_D}$
$C_T = C_d + C_j$
$C_d = \tau_t \frac{I_D}{V_T}$
$C_j \cong 2C_{j0}$

Device Model Summary (MOSFET)



Device Model Summary (MOSFET)

The following equations are for n-channel MOST. For p-channel MOST, put negative signs in front of all voltages. Also, the short-channel effects are not taken into account ($L < 2L_{min}$).

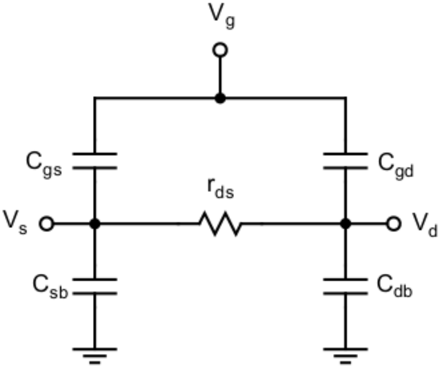
Triode region ($V_{GS} > V_{tn}$, $V_{DS} \leq V_{eff}$):

$$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right) \left[\left(V_{GS} - V_{tn}\right) V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$V_{eff} = V_{GS} - V_{tn}$$
$$V_{tn} = V_{tn-0} + \gamma \left(\sqrt{V_{SB} + 2\Phi_F} - \sqrt{2\Phi_F}\right)$$
$$\Phi_F = V_T \ln\left(\frac{N_A}{n_i}\right)$$
$$\gamma = \frac{\sqrt{2qK_s\varepsilon_0 N_A}}{C_{ox}}$$
$$C_{ox} = \frac{K_{ox}\varepsilon_0}{t_{ox}}$$

Device Model Summary (MOSFET)

Small-Signal Model, Triode region (for $V_{DS} \ll V_{eff}$):



$$r_{ds} = \frac{\partial V_{DS}}{\partial I_D} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{eff} - V_{DS})} \cong \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)V_{eff}}$$

$$C_{gd} = C_{gs} \cong \frac{1}{2}WLC_{ox} + WL_{ov}C_{ox}$$

$$C_{sb} = C_{db} = \frac{C_{j0}(A_s + WL/2)}{\sqrt{1 + \frac{V_{sb}}{\Phi_0}}}$$

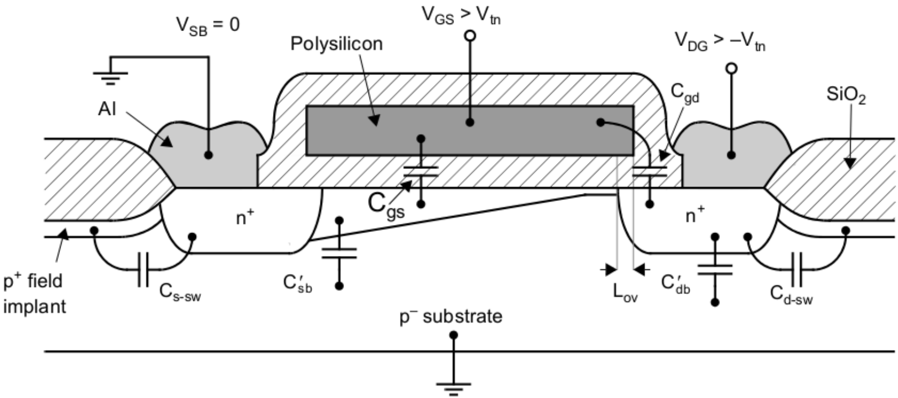
Device Model Summary (MOSFET)

Active (or Pinch-Off) Region ($V_{GS} > V_{tn},\ V_{DS} \geq V_{eff}$):

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{tn})^2\left[1 + \lambda\left(V_{DS} - V_{eff}\right)\right]$$
$$\lambda = \frac{k_{ds}}{2L\sqrt{V_{DS} - V_{eff} + \Phi_0}}$$
$$k_{ds} = \sqrt{\frac{2K_s\varepsilon_0}{qN_A}}$$
$$V_{eff} = V_{GS} - V_{tn} = \sqrt{\frac{2I_D}{\mu_n C_{ox} W/L}}$$
$$V_{tn} = V_{tn-0} + \gamma\left(\sqrt{V_{SB} + 2\Phi_F} - \sqrt{2\Phi_F}\right)$$

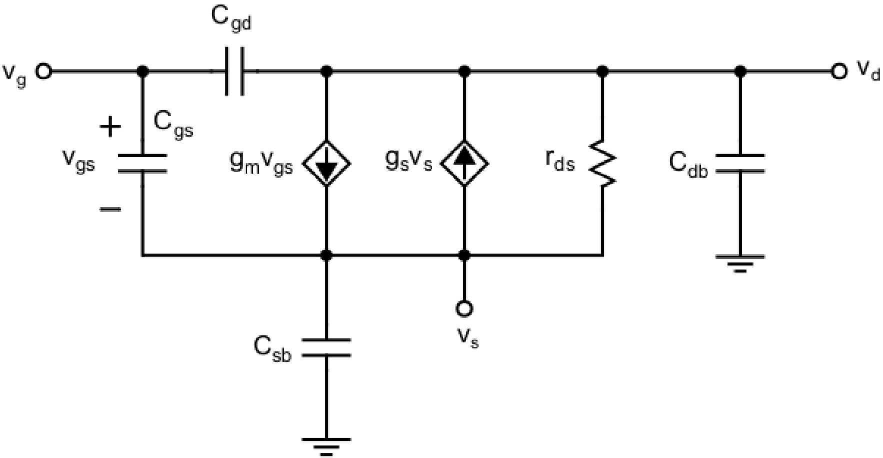
Device Model Summary (MOSFET)

Small-Signal Model, Active region ($V_{GS} > V_{tn}$, $V_{DS} \geq V_{eff}$):



Device Model Summary (MOSFET)

Small-Signal Model, Active region ($V_{GS} > V_{tn}$, $V_{DS} \geq V_{eff}$):



Device Model Summary (MOSFET)

Small-Signal Model, Active region ($V_{GS} > V_{tn}$, $V_{DS} \geq V_{eff}$):

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)V_{eff} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D} = \frac{2I_D}{V_{eff}}$$

$$g_s = \frac{\partial I_D}{\partial V_{SB}} = \frac{\gamma g_m}{2\sqrt{V_{SB}+2\Phi_F}}$$

$$r_{ds} = \frac{\partial V_{DS}}{\partial I_D} \cong \frac{1}{\lambda I_D}$$

$$\lambda = \frac{k_{ds}}{2L\sqrt{V_{DS}-V_{eff}+\Phi_0}}$$

$$k_{ds} = \sqrt{\frac{2K_s\varepsilon_0}{qN_A}}$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$$

$$C_{gd} = WL_{ov}C_{ox}$$

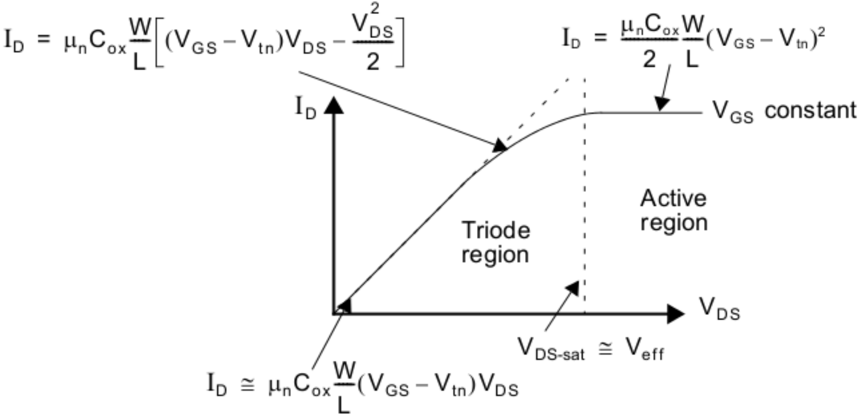
$$C_{sb} = (A_s + WL)\,C_{js} + P_s C_{j-sw}$$

$$C_{js} = \frac{C_{j0}}{\sqrt{1+\frac{V_{sb}}{\Phi_0}}}$$

$$C_{db} = A_d C_{jd} + P_d C_{j-sw}$$

$$C_{jd} = \frac{C_{j0}}{\sqrt{1+\frac{V_{db}}{\Phi_0}}}$$

Device Model Summary (MOSFET)



$$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS} - V_{tn})V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS} - V_{tn})^2$$

V_{GS} constant

I_D

Triode region

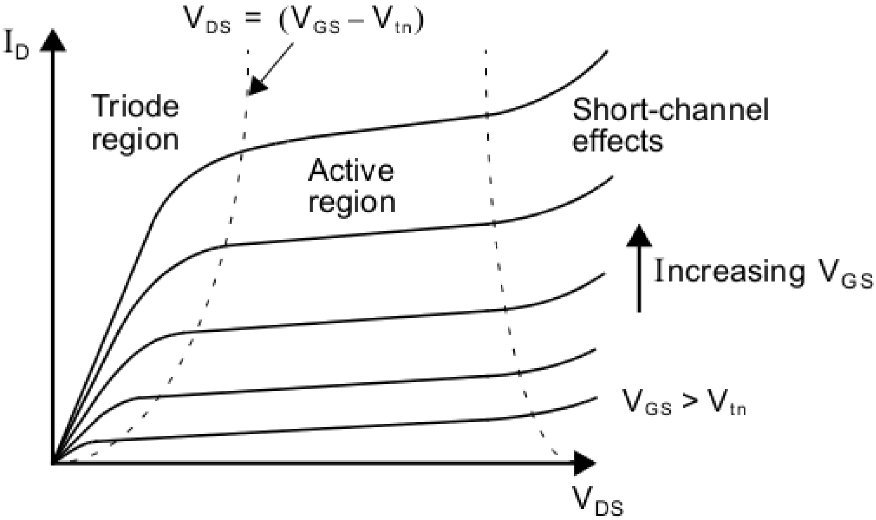
Active region

V_{DS}

$V_{DS\text{-sat}} \cong V_{eff}$

$$I_D \cong \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{tn})V_{DS}$$

Device Model Summary (MOSFET)



Device Model Summary (MOSFET)

MOSFET parameters representative of various CMOS technologies

	0.8 μm		0.35 μm		0.18 μm		45 nm	
Technology	**NMOS**	**PMOS**	**NMOS**	**PMOS**	**NMOS**	**PMOS**	**NMOS**	**PMOS**
$\mu C_{ox}\ \left(\mu A/V^2\right)$	92	30	190	55	270	70	280	70
$V_{t0}\ (V)$	0.80	-0.90	0.57	-0.71	0.45	-0.45	0.45	-0.45
$\lambda\ L\ (\mu m/V)$	0.12	0.08	0.16	0.16	0.08	0.08	0.10	0.15
$C_{ox}\ \left(fF/\mu m^2\right)$	1.8	1.8	4.5	4.5	8.5	8.5	25	25
$t_{ox}\ (nm)$	18	18	8	8	5	5	1.2	1.2
n	1.5	1.5	1.8	1.7	1.6	1.7	1.85	1.85
$\theta\ \left(V^{-1}\right)$	0.06	0.135	1.5	1.0	1.7	1.0	2.3	2.0
m	1.0	1.0	1.8	1.8	1.6	2.4	3.0	3.0
$C_{ox}/W = L_{ov}V_{ox}\ (fF/\mu m)$	0.20	0.20	0.20	0.20	0.35	0.35	0.50	0.50
$C_{db}/W \cong C_{sb}/W\ (fF/\mu m)$	0.50	0.80	0.75	1.10	0.50	0.55	0.45	0.50

Device Model Summary (MOSFET)

Default parameters for n-channel MOS transistors:

$T = 300K$ (Room temperature)
$\mu_n C_{ox} = 92\mu A/V^2$
$V_{tn} = 0.8V$
$\gamma = 0.5V^{1/2}$
$r_{ds}(\Omega) = 8000L(\mu m)/I_D(mA)$ in active region
$C_j = 2.4 \times 10^{-4} pF/(\mu m)^2$
$C_{j-sw} = 2.0 \times 10^{-4} pF/\mu m$
$C_{ox} = 1.9 \times 10^{-3} pF/(\mu m)^2$
$C_{gs(\mathrm{overlap})} = C_{gd(\mathrm{overlap})} = 2.0 \times 10^{-4} pF/\mu m$

Device Model Summary (MOSFET)

Default parameters for p-channel MOS transistors:

$T = 300K$ (Room temperature)
$\mu_p C_{ox} = 30\mu A/V^2$
$V_{tp} = -0.9V$
$\gamma = 0.8V^{1/2}$
$r_{ds}(\Omega) = 12000L(\mu m)/I_D(mA)$ in active region
$C_j = 4.5 \times 10^{-4} pF/(\mu m)^2$
$C_{j-sw} = 2.5 \times 10^{-4} pF/\mu m$
$C_{ox} = 1.9 \times 10^{-3} pF/(\mu m)^2$
$C_{gs(\mathrm{overlap})} = C_{gd(\mathrm{overlap})} = 2.0 \times 10^{-4} pF/\mu m$

Exercise 1 (1st/2nd, P1.1)

Estimate the hole and electron concentrations in silicon doped with arsenic at a concentration of $10^{25}\,atoms/m^3$ at a temperature $22°$C above room temperature.[1] Is the resulting material n-type or p-type?

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[1] $n_i = 4.4\,10^{16}\,carriers/m^3$ @ $T = 322\,K$, n-type material

Exercise 2 (1st/2nd, E1.2, P1.2)

A PN junction has $N_A = 10^{25}\, atoms/m^3$ and $N_D = 10^{22}\, atoms/m^3$. What is the built-in junction potential Φ_0?[2] Does the built-in potential increase or decrease when the temperature is increased $11^{\circ}\mathrm{C}$ above room temperature?[3]

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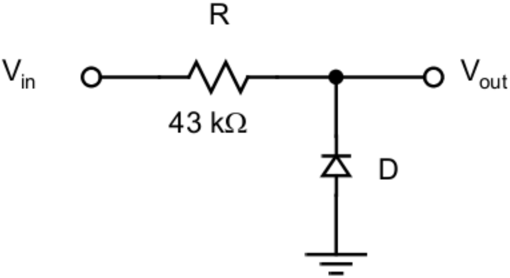
[2] $\Phi_0 = 0.89\ V$

[3] it decreases ($\Phi_0 = 0.88\ V$)

Exercise 3 (1st/2nd, P1.4)

A silicon diode has $\tau_t = 12\,ps$ and $C_{j0} = 15\,fF$. It is reverse-biased by a $43\,k\Omega$ resistor connected between the cathode of the diode and the input signal. Initially the input is $5\,V$, and then at time 0 it changes to $0\,V$. Estimate the time it takes for the output voltage to change from $5\,V$ to $1.5\,V$.[4] Repeat for an input voltage change from $0\,V$ to $5\,V$ and an output voltage change from $0\,V$ to $3.5\,V$.[5]



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[4] $t_{falling} = 0.37\,ns$

[5] $t_{rising} = 0.48\,ns$

Exercise 4 (1st, P1.7)

Find I_D for an n-channel MOST having doping concentrations of $N_A = 10^{22}\, atoms/m^3$ and $N_D = 10^{25}\, atoms/m^3$, with $W = 50\,\mu m$, $L = 1.5\,\mu m$, $V_{GS} = 1.1\,V$, and $V_{DS} = V_{eff}$.[6] Estimate the new value of I_D if V_{DS} is increased by $0.3\,V$ (we assume λ remains constant).[7]

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[6] $I_D = 138\,\mu A$

[7] $I_D = 143\,\mu A$

Exercise 5 (1st, P1.8)

A MOS transistor in the active region has a drain current of $20\,\mu A$ when $V_{DS} = V_{eff}$. When V_{DS} is increased by $0.5\,V$, I_D increases to $23\,\mu A$. Estimate the output impedance r_{ds}, and the output impedance constant λ.[8]

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[8] $r_{ds} = 167\,k\Omega,\ \lambda = 0.3\,V^{-1}$

Exercise 6 (1st, P1.9)

Derive the low-frequency model parameters (i.e. find g_m, g_s, and r_{ds}) for an n-channel MOST having doping concentrations of $N_A = 10^{22}\, atoms/m^3$ and $N_D = 10^{25}\, atoms/m^3$, with $W = 10\,\mu m$, $L = 1.2\,\mu m$, $V_{GS} = 1.1\,V$, and $V_{DS} = V_{eff}$.[9]

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[9] $r_{ds} = 182\,k\Omega,\ g_m = 230\,\mu A/V,\ g_s = 44\,\mu A/V$

Exercise 7 (1st, P1.10)

Find the capacitances C_{gs}, C_{gd}, C_{sb}, and C_{db} for a MOST having $W = 50\,\mu m$ and $L = 1.2\,\mu m$. Assume that the source and drain junctions extend $4\,\mu m$ beyond the gate, resulting in source and drain areas being $A_s = A_d = 200\,\mu m^2$ and the perimeter of each being $P_s = P_d = 58\,\mu m$.[10]

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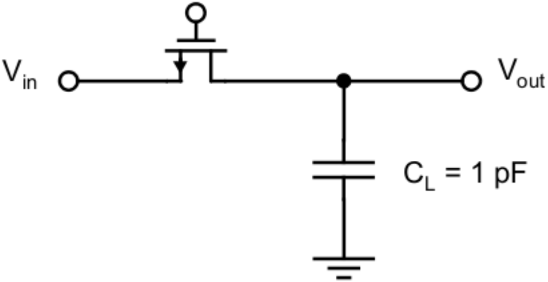
[10] $C_{gs} = 86\,fF$, $C_{gd} = 10\,fF$, $C_{sb} = 74\,fF$, and $C_{db} = 60\,fF$

Exercise 8 (1st, P1.11)

Consider the circuit shown hereafter, where $V_{in} = 1\,V$, $V_G = 5\,V$, $W = 10\,\mu m$ and $L = 0.8\,\mu m$. Taking into account only the channel charge storage, determine the final value of V_{out}, when the transistor is turned off, assuming half the channel charge "goes" to C_L.[11]



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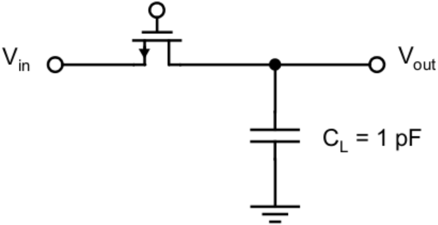
[11] $V_{out} = V_{out}(0) - 0.024 = 1 - 0.024 = 0.976\,V$

Exercise 9 (1st, P1.12, P1.13)

Consider the same circuit as before. The input voltage has a step voltage change at time 0 from $1\,V$ to $1.2\,V$ ($V_G = 5\,V$).

1. Find its $99\,\%$ settling time.[12] You may ignore the body effect and all capacitances except C_L.
2. Repeat the question for V_{in} changing from $3\,V$ to $3.1\,V$.[13]
3. Repeat the same problem, but now take into account the body effect, and assume $N_A = 10^{22}\,atoms/m^3$.[14]



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[12] $t_{settling}(1 \rightarrow 1.2\,V) = 1.25\,ns$

[13] $t_{settling}(3 \rightarrow 3.1\,V) = 3.33\,ns$

[14] $t_{settling}(1 \rightarrow 1.2\,V) = 1.35\,ns$, $t_{settling}(3 \rightarrow 3.1\,V) = 6.1\,ns$