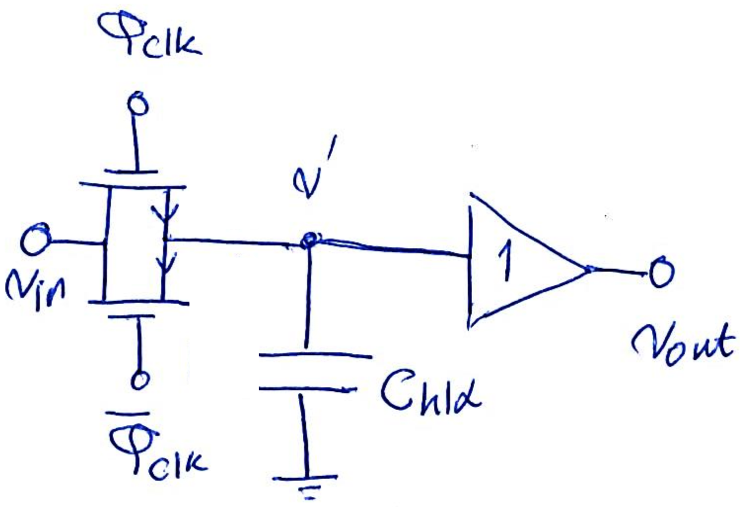
Exercise 1



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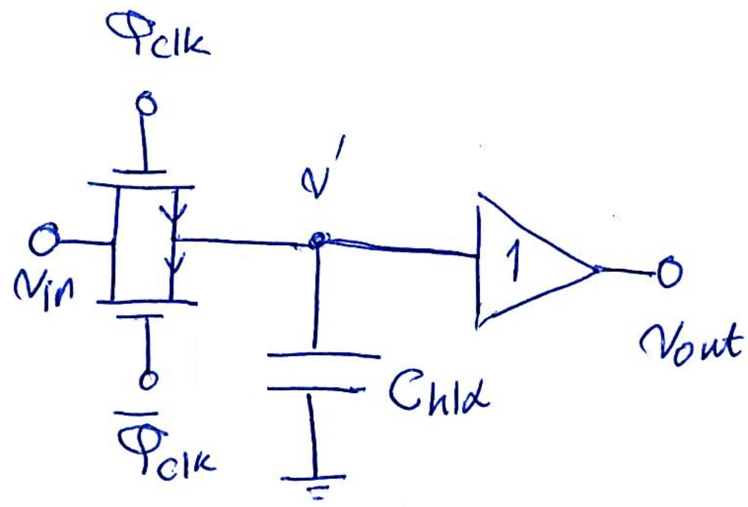
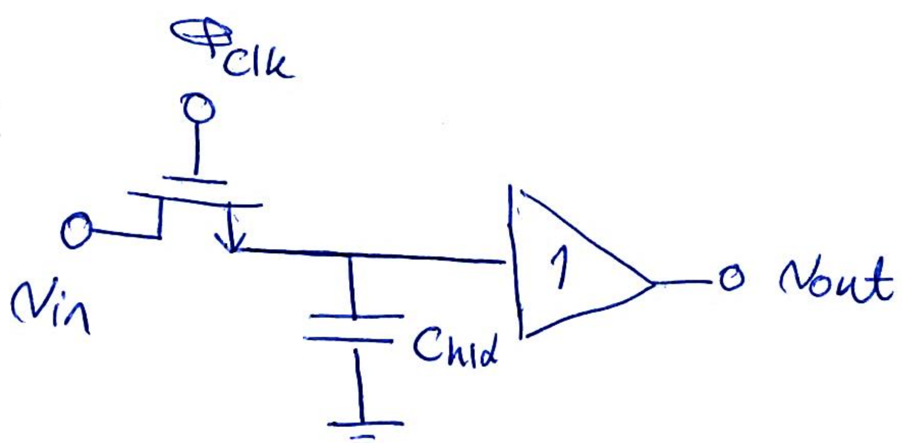


φ_{clk}
v'
V_{in}
1
V_{out}
C_{hld}
$\overline{\varphi}_{clk}$



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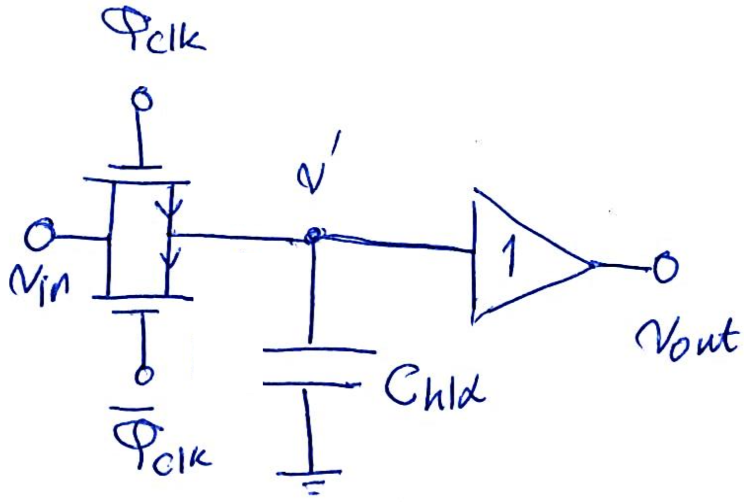

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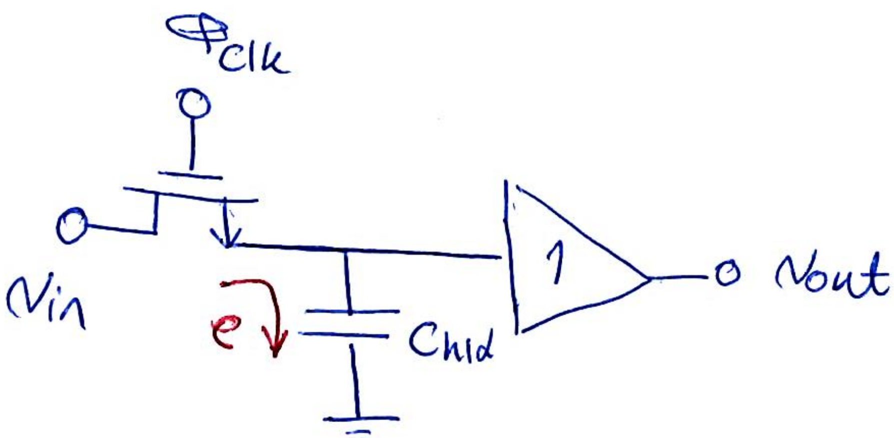

φ_{clk}
V_{in}
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V_{out}
e
C_{hld}

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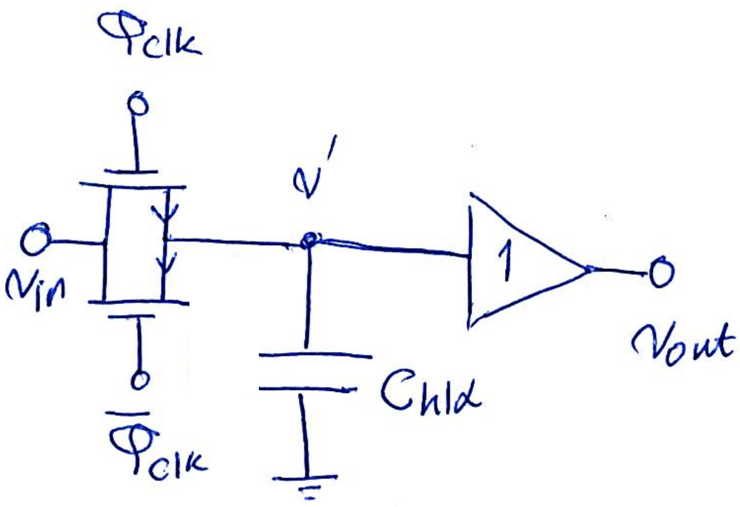

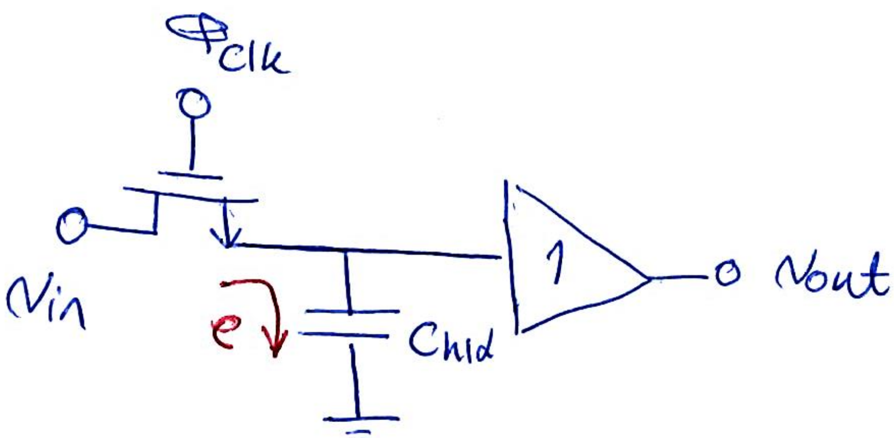

Problem:

Charge injection

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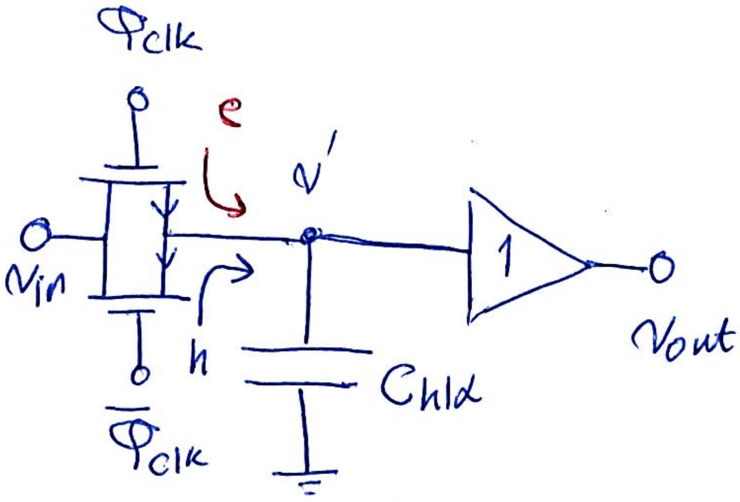

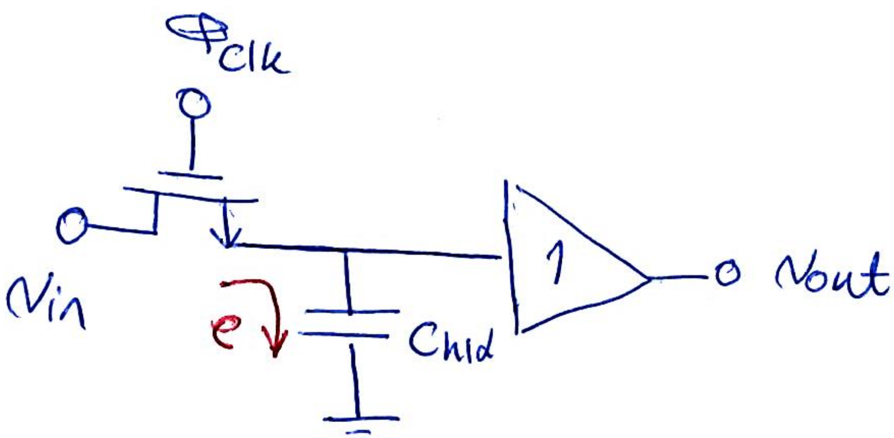

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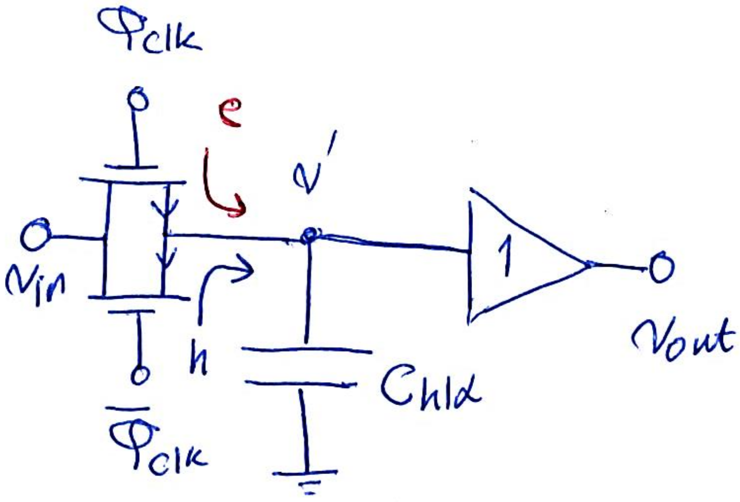
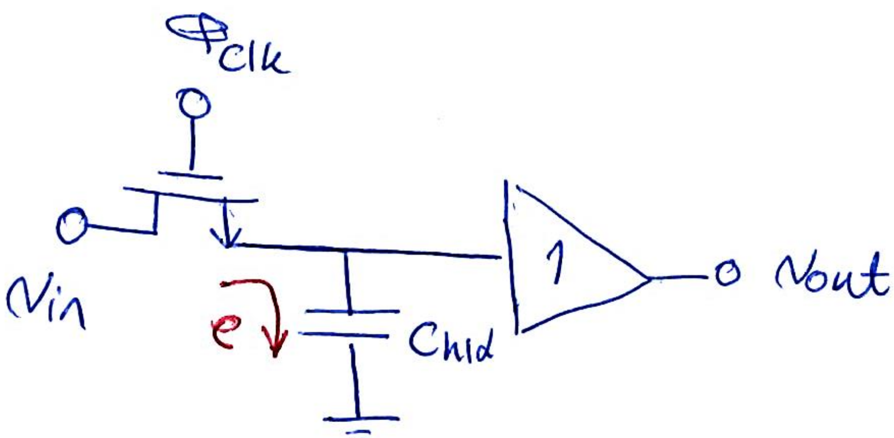


Solution:

Charge injection ↓

accuracy ↑



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$$\varphi_{clk} = V_{in} + 0.8\ V$$

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$|\Delta\varphi|$ is maximized at $V_{in} = -1$ for which, $|\Delta\varphi|_{max} = 2.1\ V$

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$\Delta t_{max} = ?$

We can find the answer using the proportion method.

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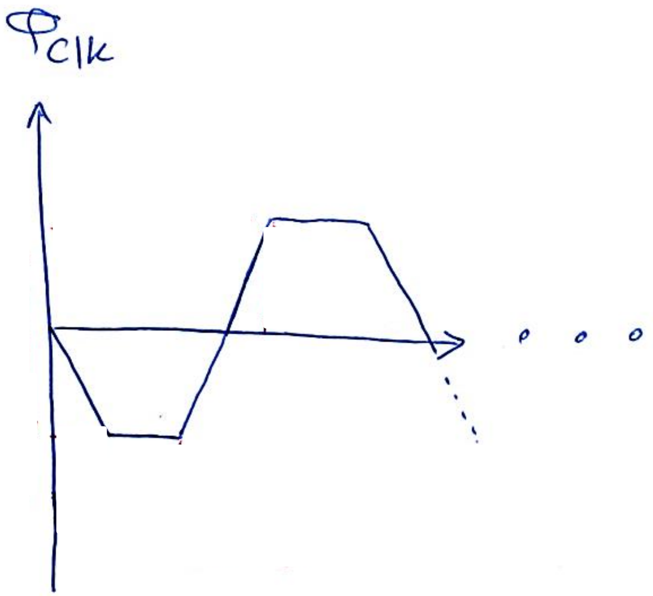

Volt

time

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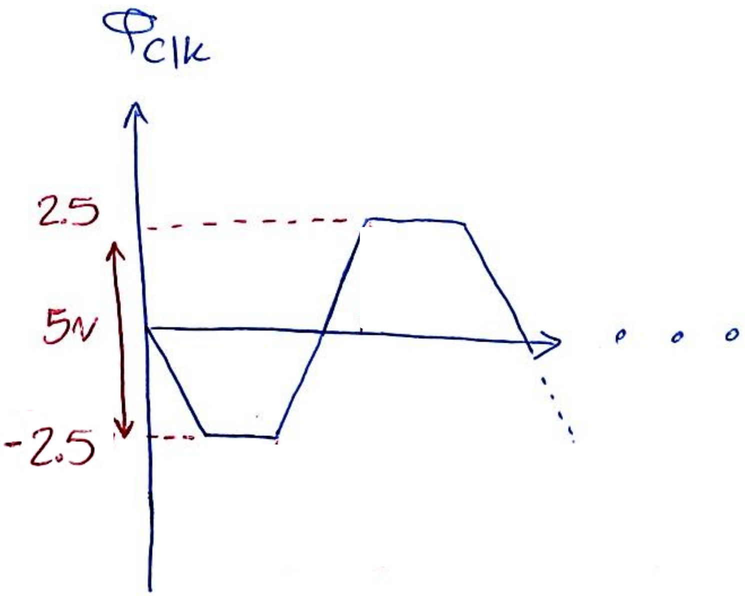

$\frac{\text{Volt}}{5}$ $\quad$ $\frac{\text{time}}{}$

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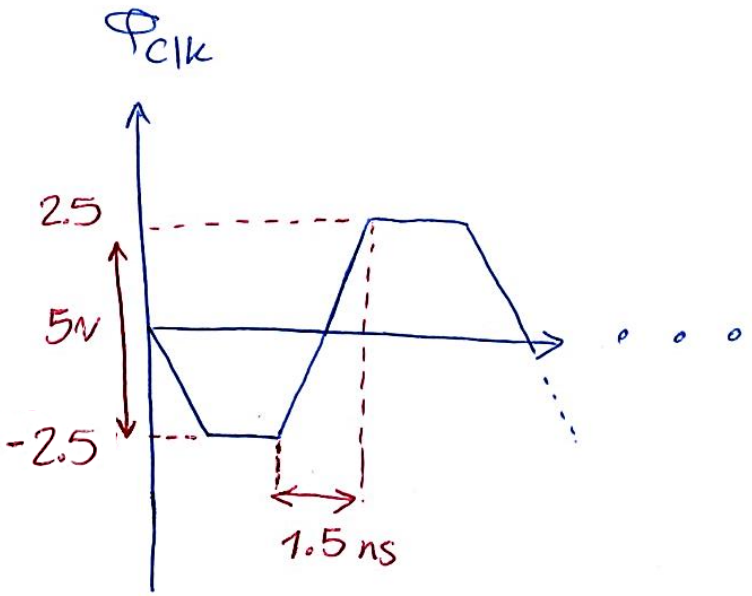

$$\frac{\text{Volt}}{5} \qquad \frac{\text{time}}{1.5\ ns}$$

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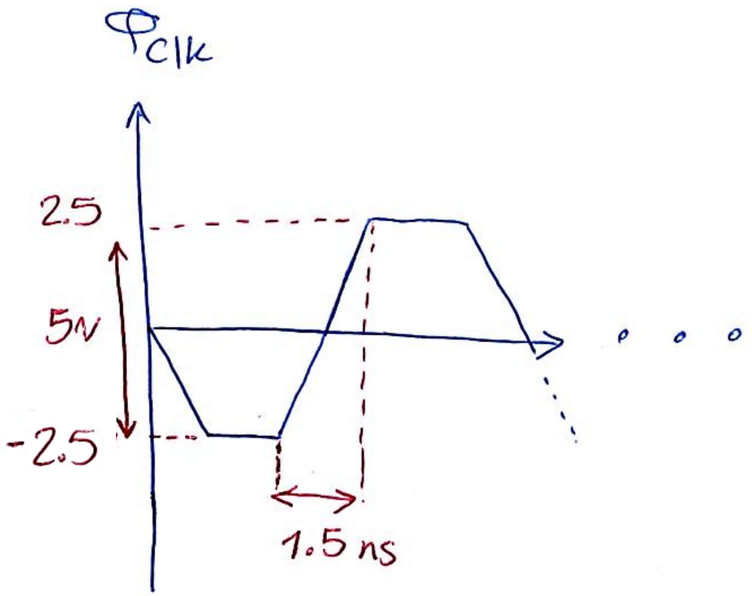

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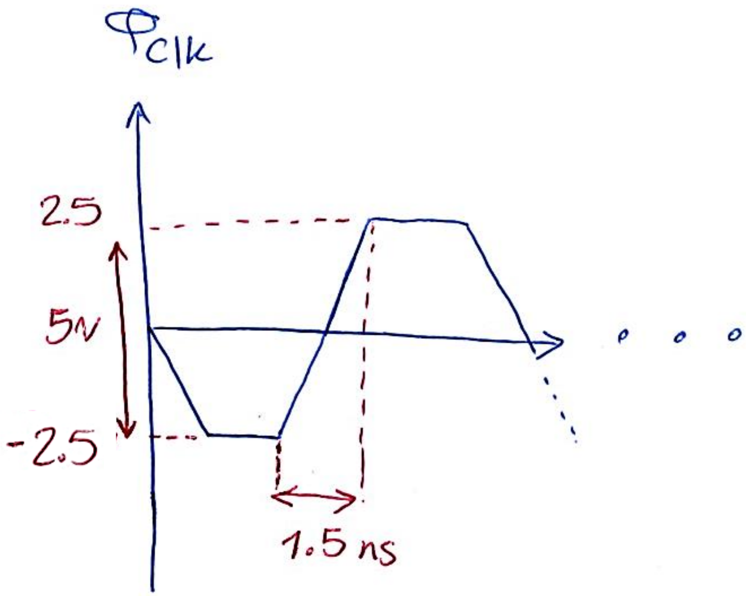
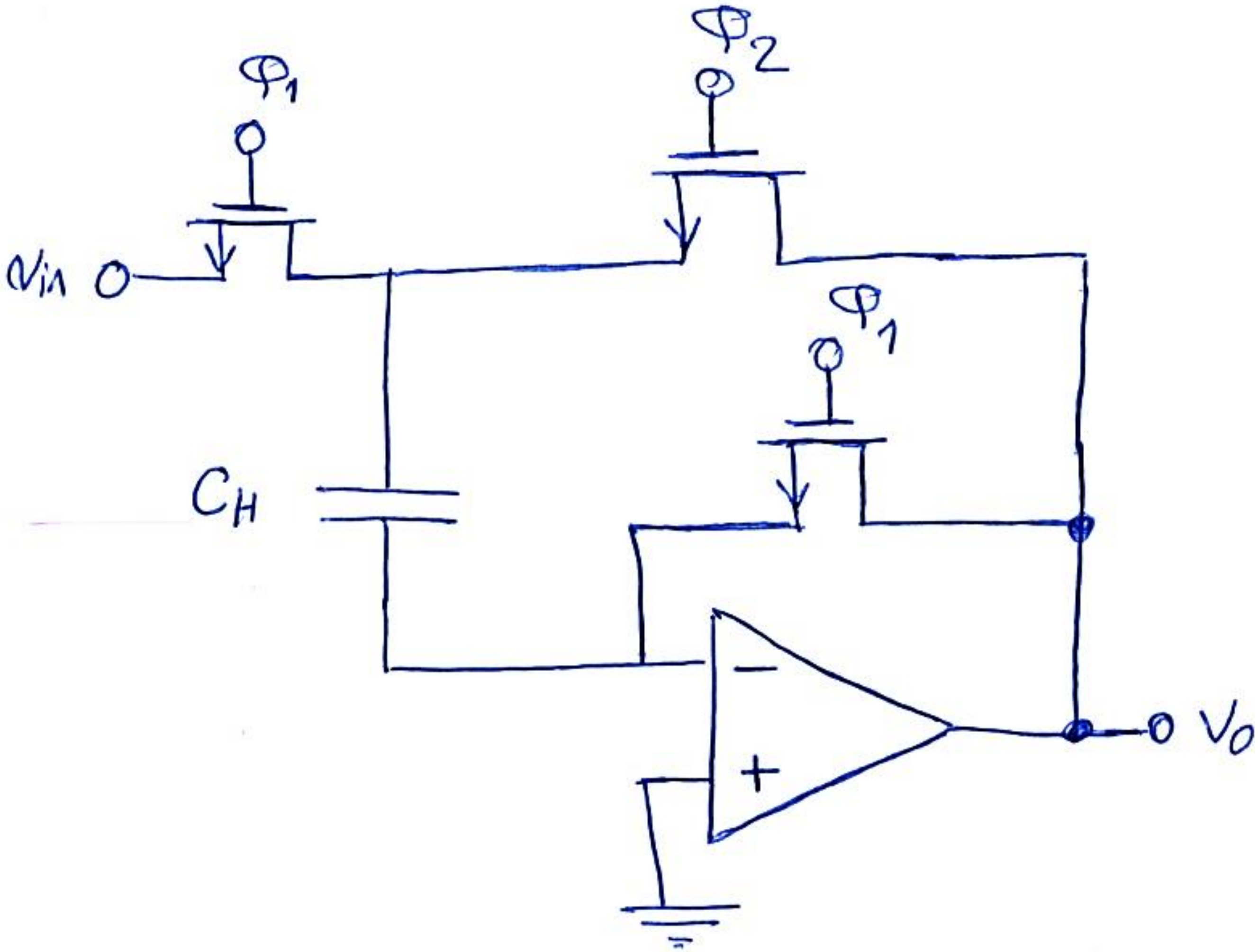


Volt	time
5	1.5 ns
2.1	Δt_{max}

$$\Delta t_{max} = \frac{2.1}{5} \times 1.5\ ns = 0.63\ ns$$

Exercise 2



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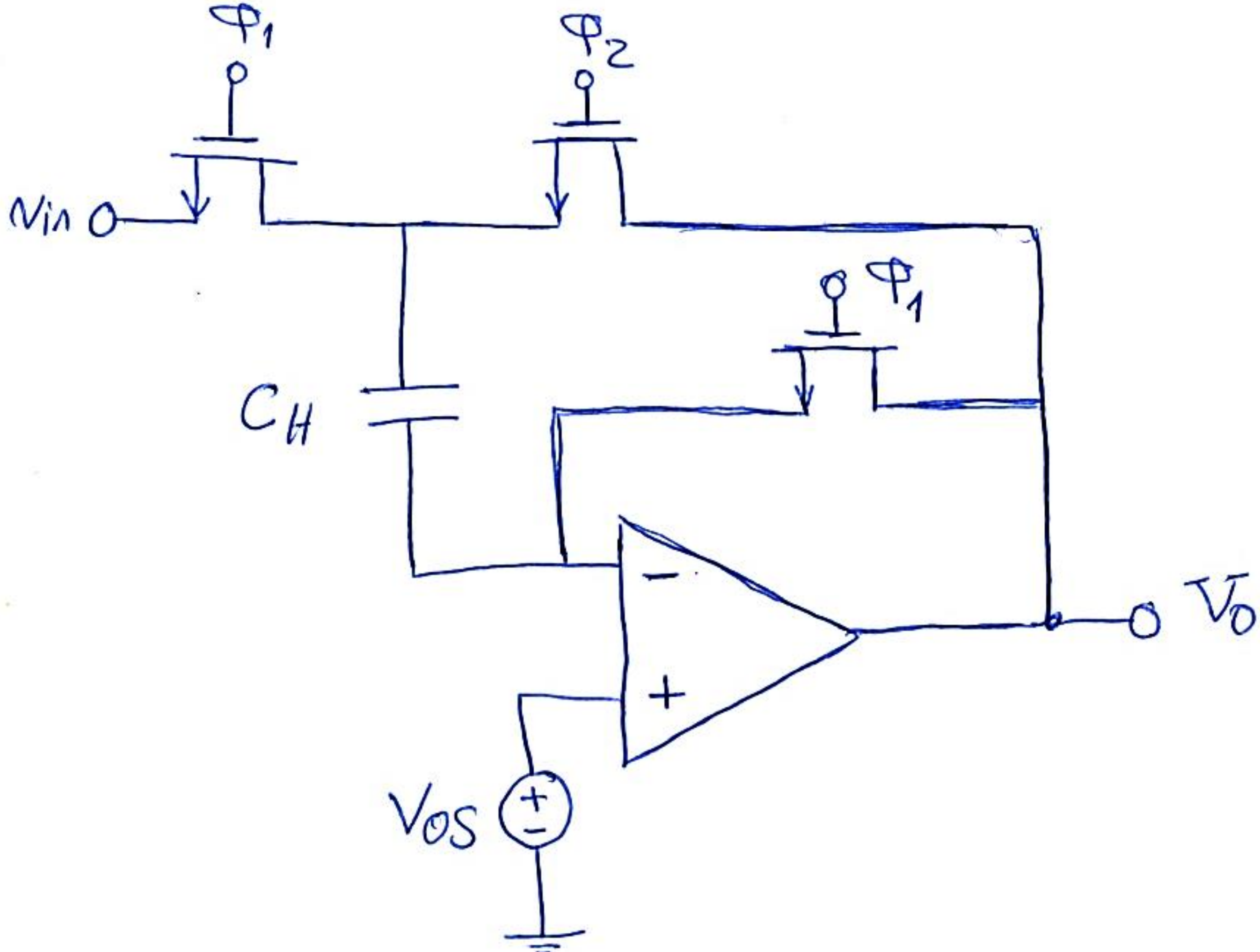

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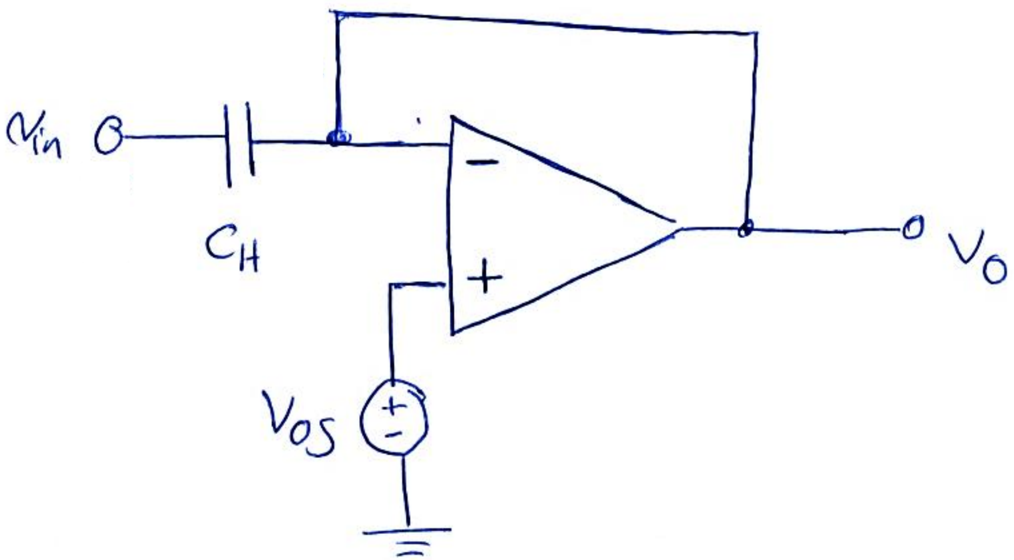
When φ_1 is on:



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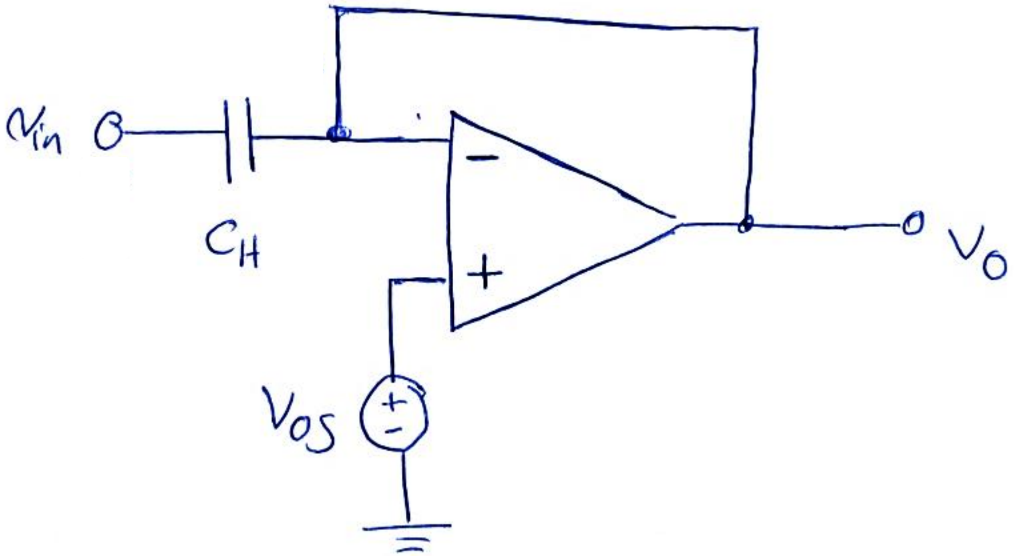

for any op-Amp:

$$A(V_+ - V_-) = V_{out}$$

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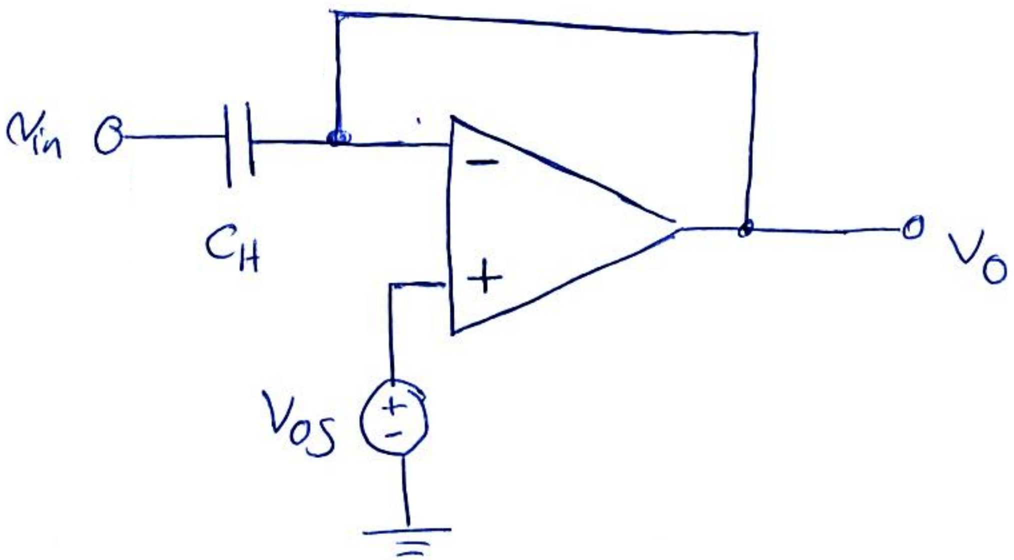

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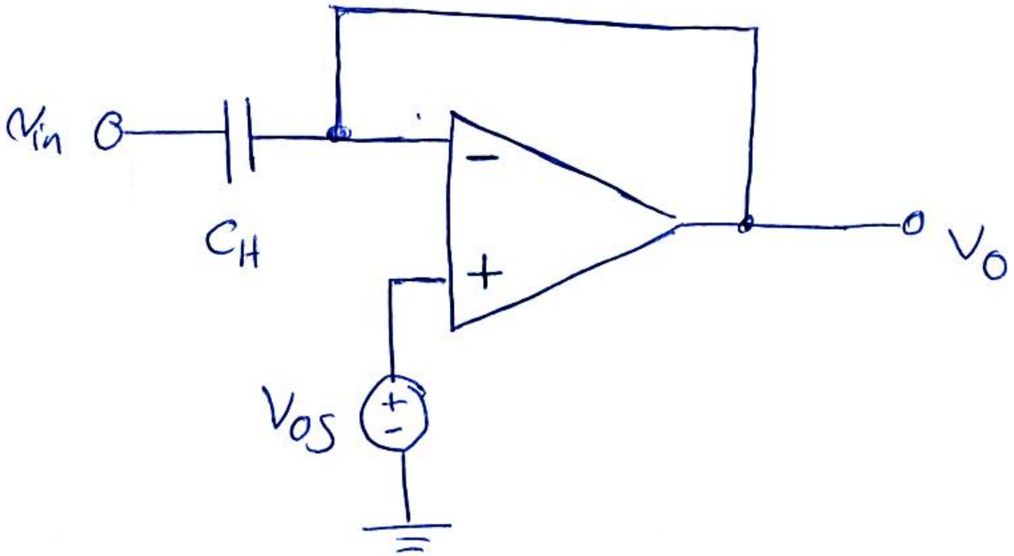

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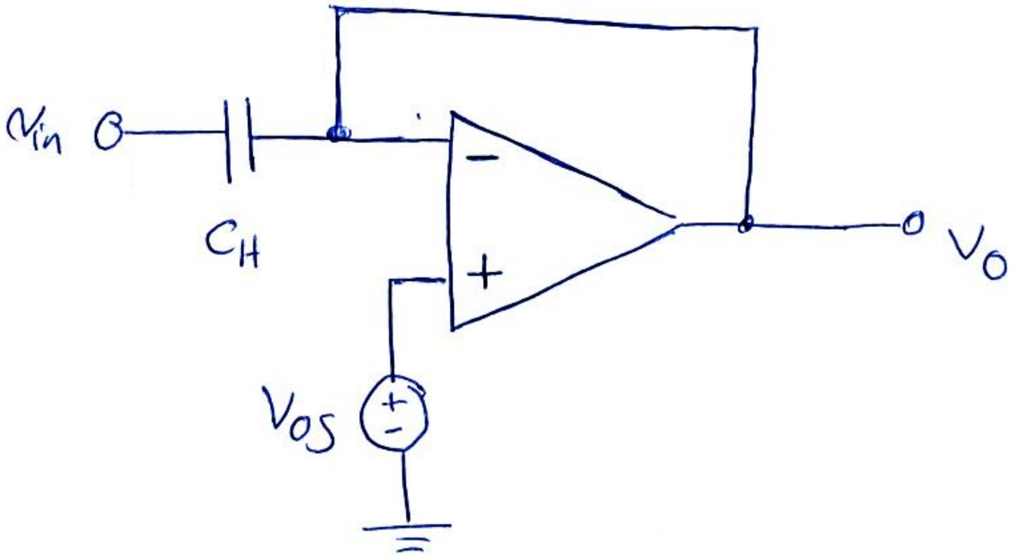

So, during sample mode, voltage accross C_H is " $V_{in} - \frac{A}{A+1} V_{os}$ ".

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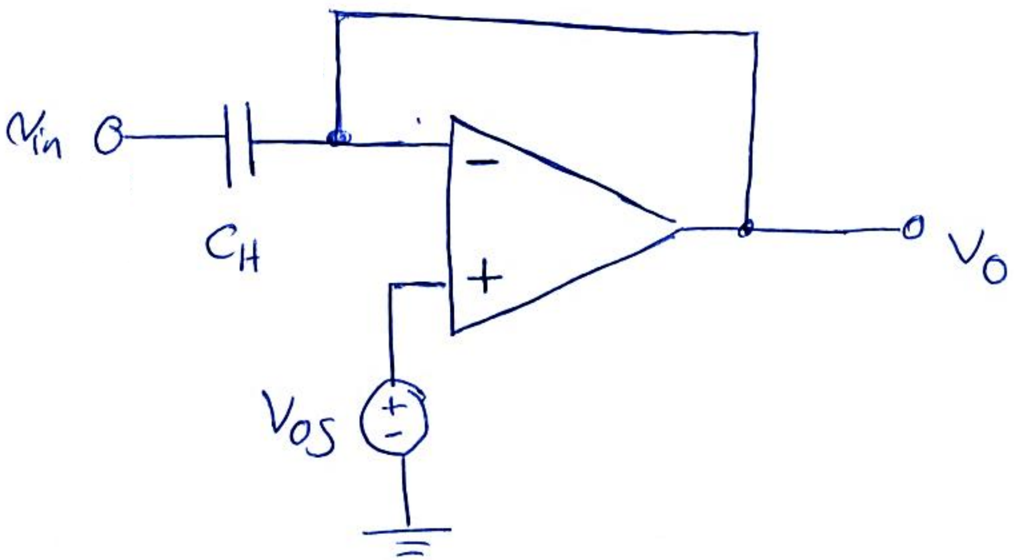

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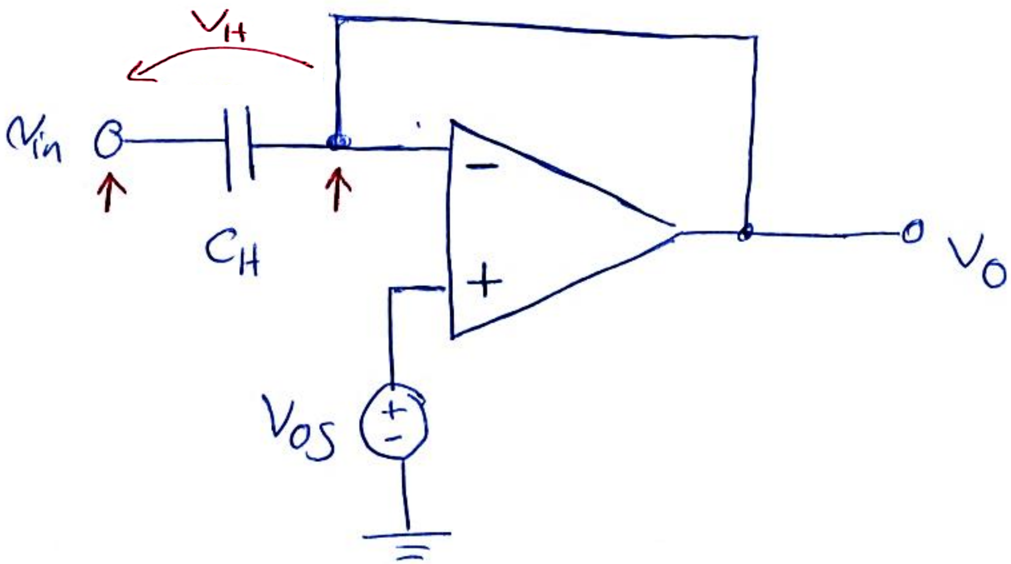

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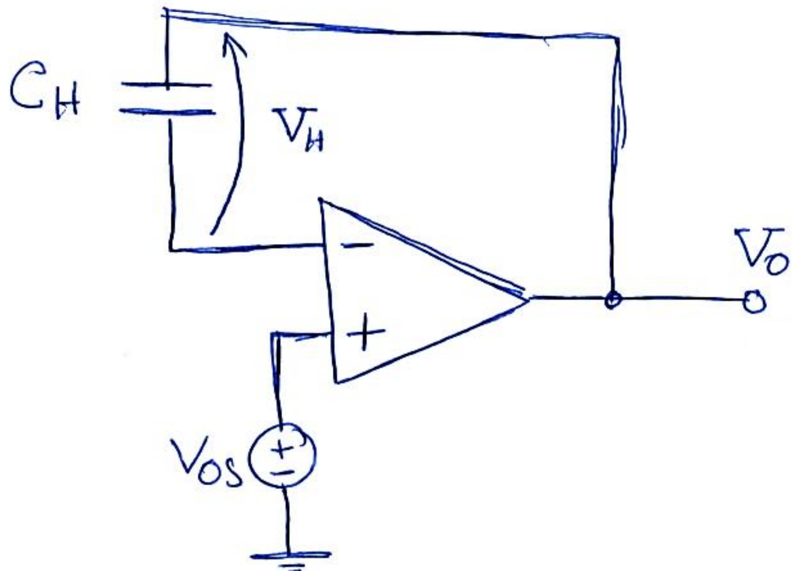

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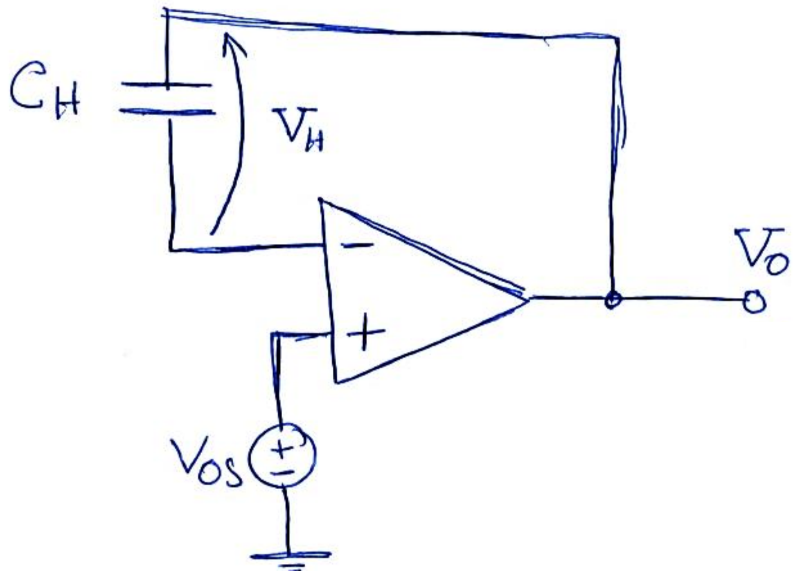

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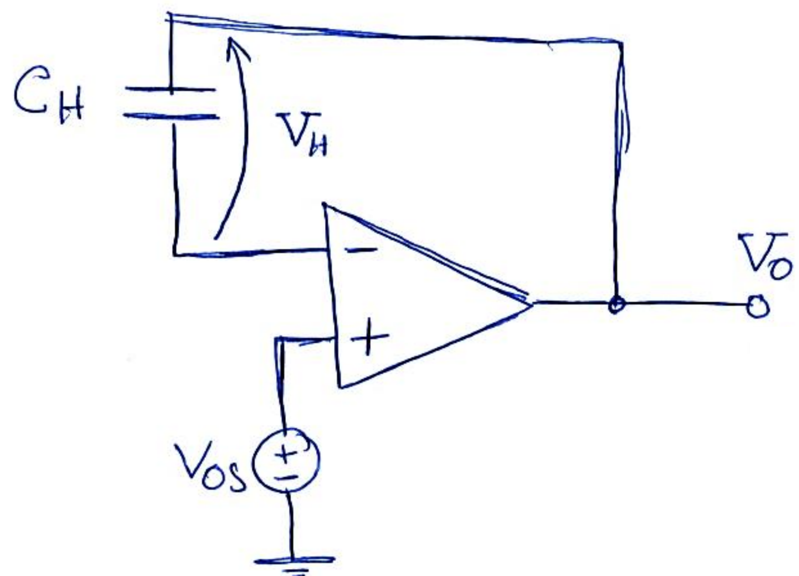

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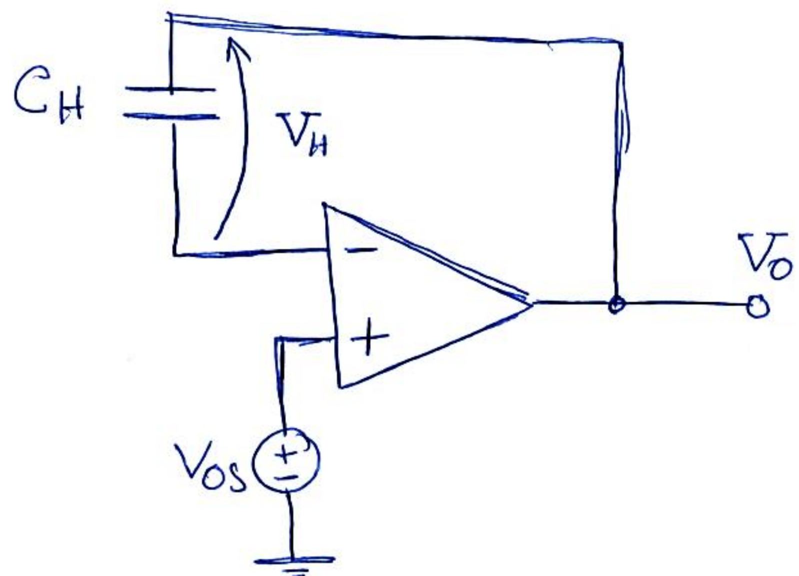

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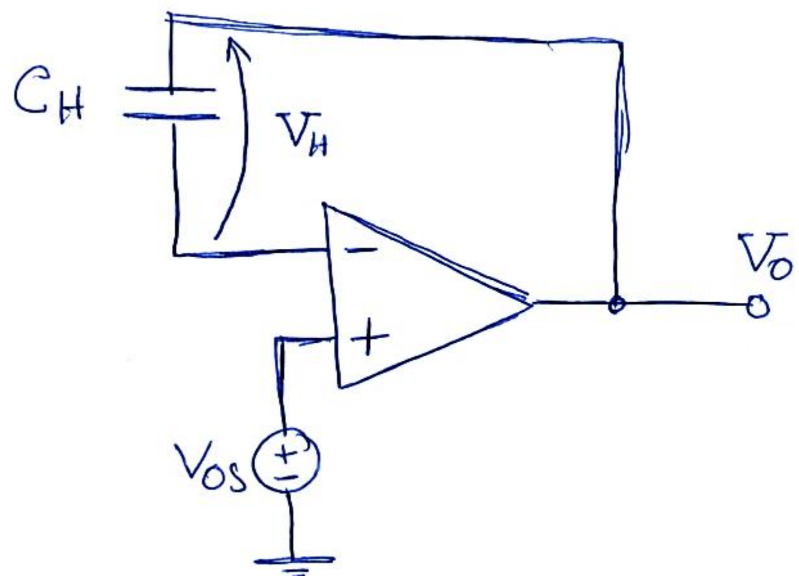

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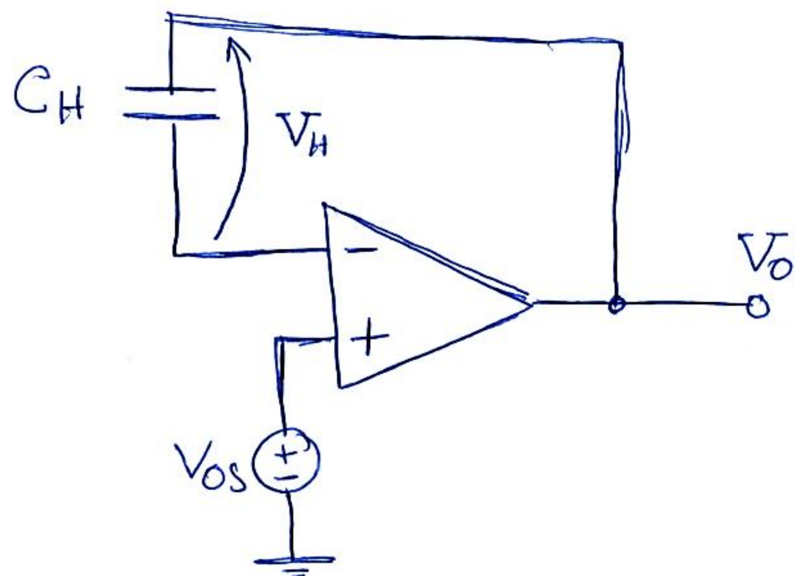

So, I can write,

$$A\left(V_{os} - \left(V_o - \left(V_{in} - \frac{A}{A+1} V_{os}\right)\right)\right) = V_o \Rightarrow$$

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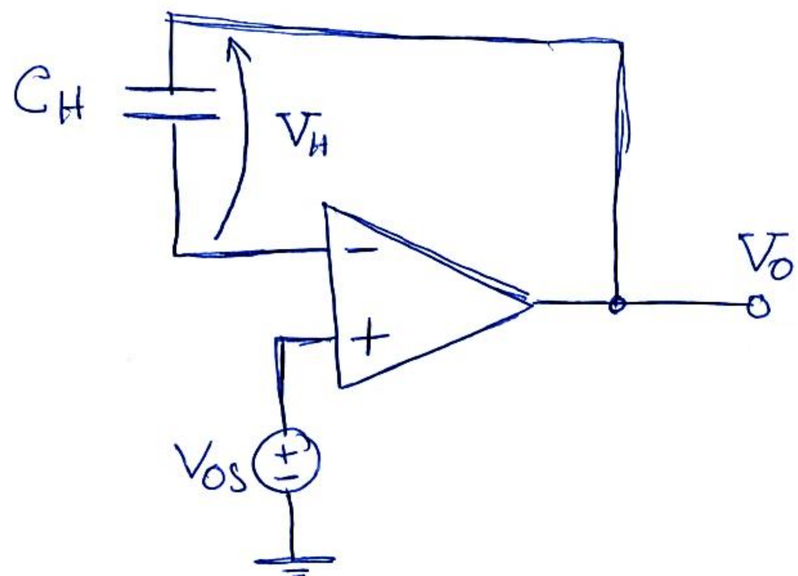

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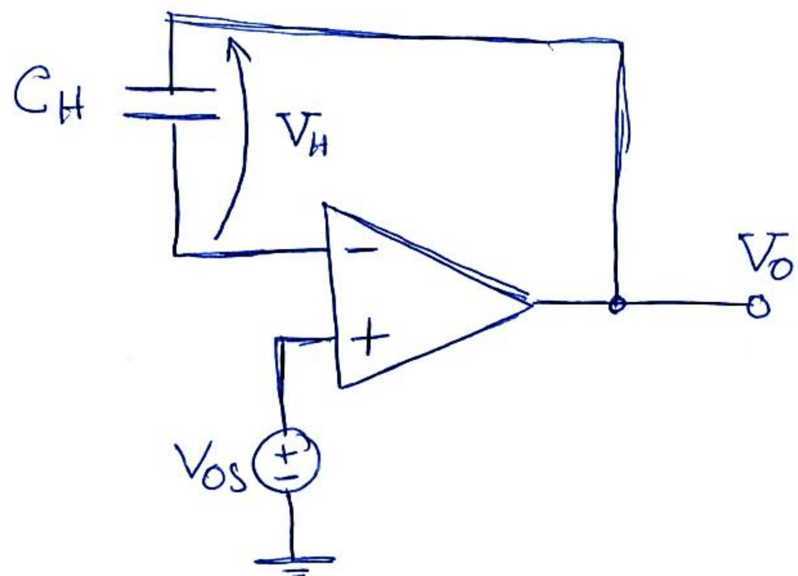

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$$V_o\left(1 + 1/A\right) = V_{in} + \frac{1}{A+1} V_{os}$$

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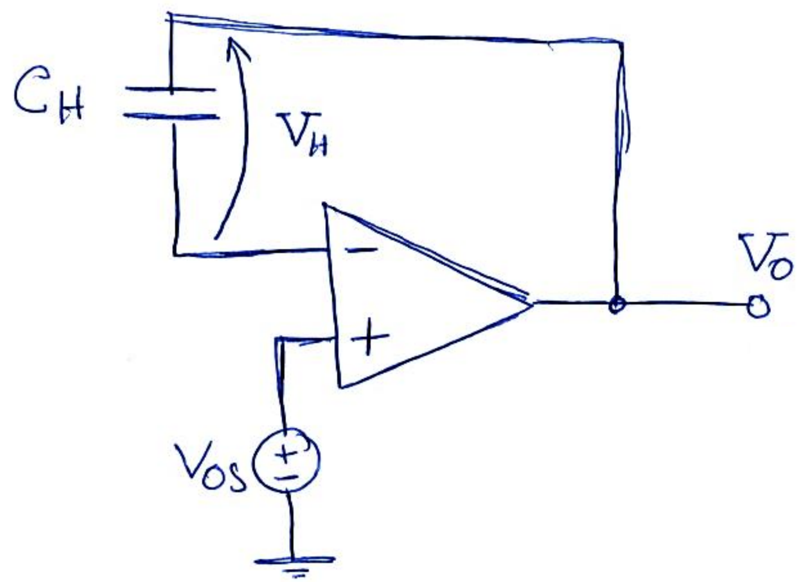

$$V_O = \frac{A}{A+1} V_{in} + \frac{A}{(A+1)^2} V_{OS}$$

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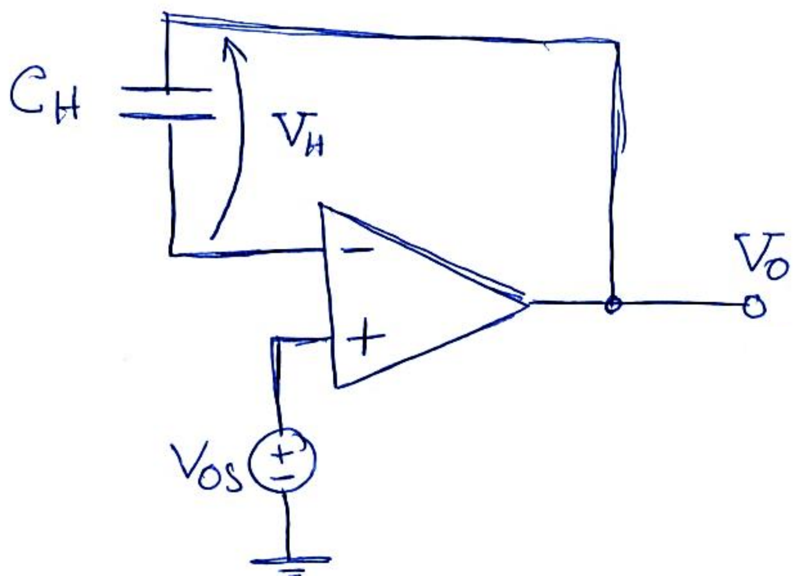

$$V_o = \frac{A}{A+1} V_{in} + \frac{A}{(A+1)^2} V_{os}$$

if $A \to \infty \Rightarrow V_o = 1 \times V_{in} + 0 \times V_{os}$ nice!

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* Let's denote the "end" of Φ_1 by nT.
* Φ_2 is delayed by T rather Φ_1.

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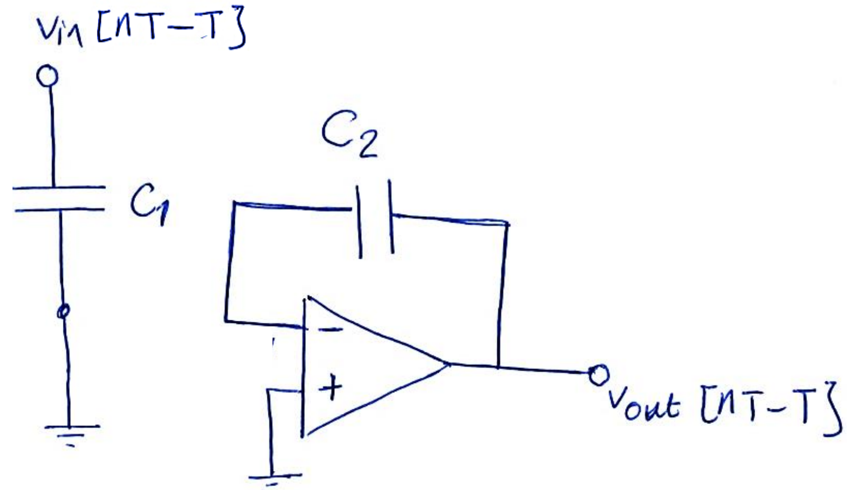

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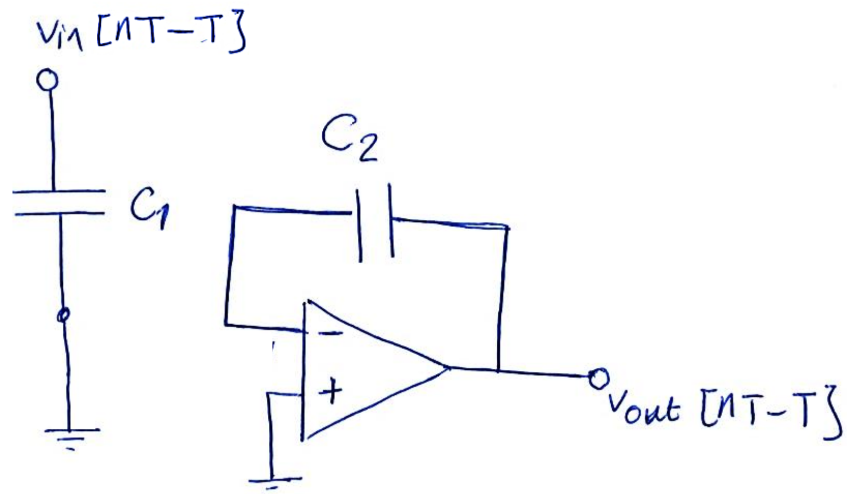

C1 charged by $V_{in}[nT-T]$

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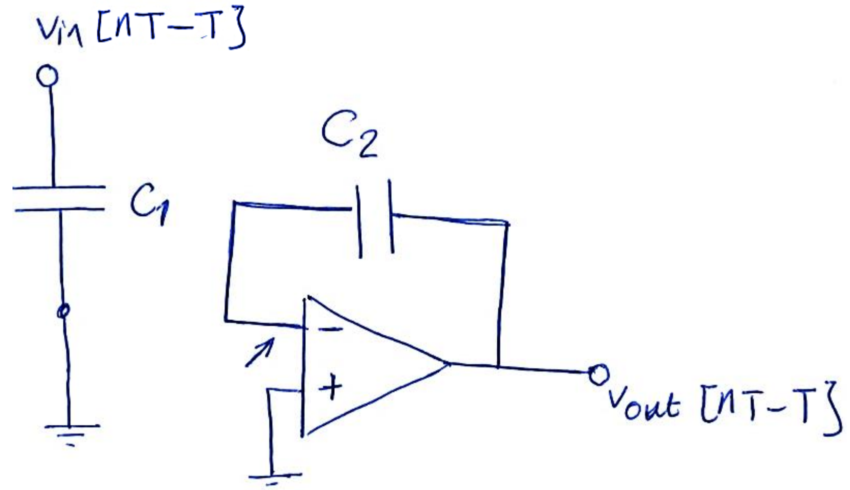


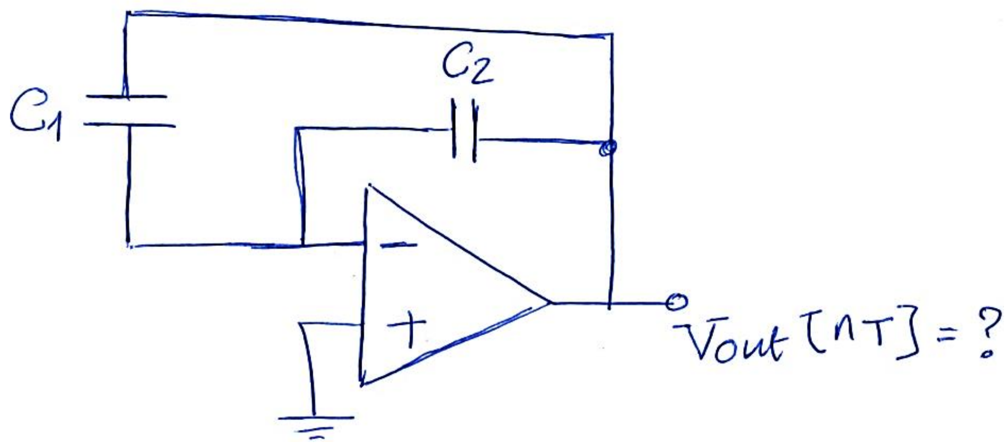
C1 charged by $V_{in}[nT-T]$

C2 charged by $V_{out}[nT-T]$

⚠ Virtual ground.

Exercise 3

@ $t = nT$: φ_2 is on



Exercise 3)

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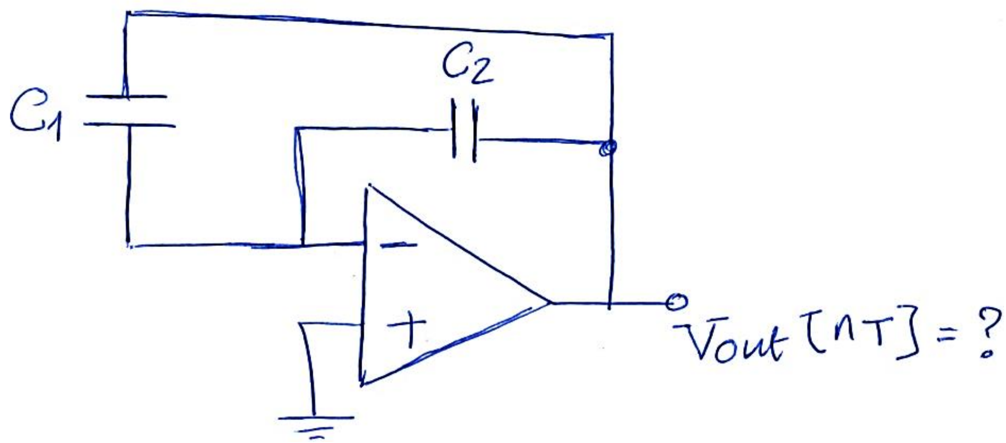
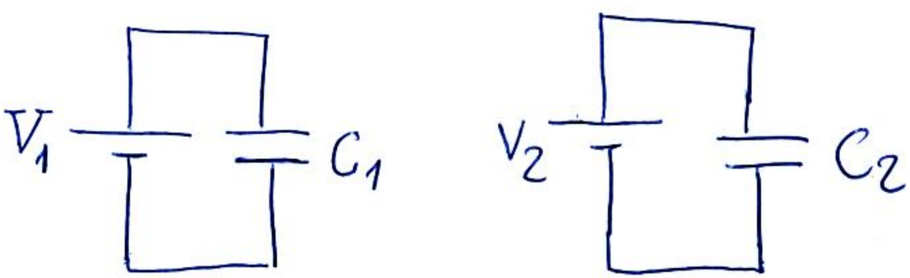


Review:



Exercise 3

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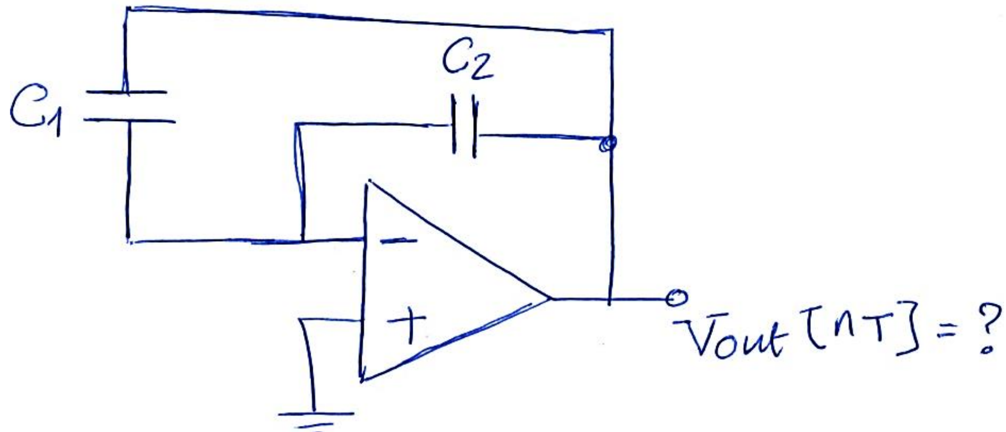

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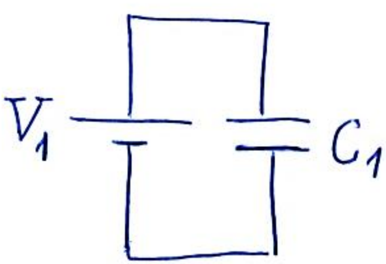

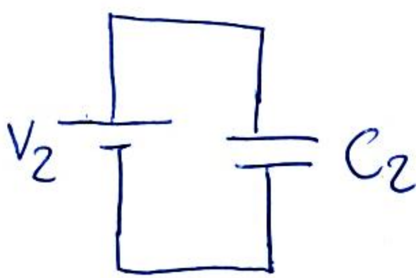

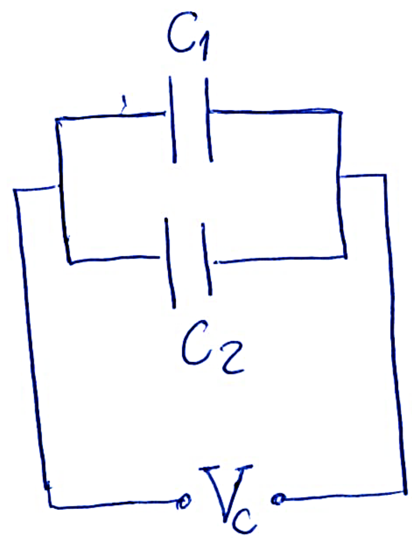

$V_c = ?$

Exercise 3

Charge Conservation:

$$C_1 V_1 + C_2 V_2 = C_1 V_C + C_2 V_C$$

Exercise 3

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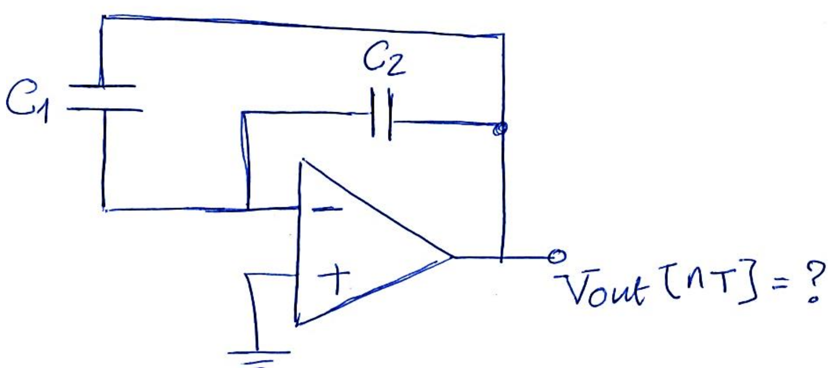
Here is the same:

C_1 is charged by $V_{in}[nT-T]$, is in parallel with C_2 which is charged by $V_{out}[nT-T]$.



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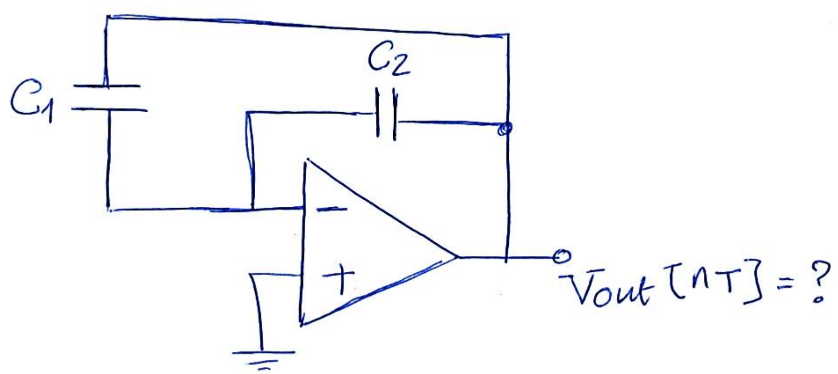

$$V_{out}[nT] = \frac{C_1 V_{in}[nT-T] + C_2 V_{out}[nT-T]}{C_1 + C_2}$$

Exercise 3

Review:

$$x[n] \overset{ZT}{\Longleftrightarrow} X[z]$$

$$x[n-D] \overset{ZT}{\Longleftrightarrow} z^{-D} X[z]$$

$$H(z) \xrightarrow{z=e^{j\omega}} H(e^{j\omega})$$

Exercise 3

We assumed for $\omega T \ll 1$, $z = e^{j\omega T} \simeq 1 + j\omega T$

but why? or how?

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but why? or how?

Euler formula:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

So, $z = e^{j\omega T} = \cos(\omega T) + j\sin(\omega T)$

if $\omega T \ll 1$ we can write,

$$z = e^{j\omega T} \simeq \cos(0) + j\,\omega T = 1 + j\omega T$$

Exercise 3

now, what can I say about z^{-1}:

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so, for the low frq. ($\omega T << 1$):

$$\begin{cases} z = 1 + j\omega T \\ z^{-1} = 1 - j\omega T \end{cases}$$

Exercise 3)

$$V_{out}[nT] = \frac{C_1 V_{in}[nT-T] + C_2 V_{out}[nT-T]}{C_1 + C_2}$$

if we take ZT from both sides,

Exercise 3

$$V_{out}[nT] = \frac{C_1 V_{in}[nT-T] + C_2 V_{out}[nT-T]}{C_1 + C_2}$$

if we take ZT from both sides,

$$(C_1+C_2)V_{out}[z] = C_2 z^{-1} V_o[z] + C_1 z^{-1} V_{in}[z]$$

$$\Rightarrow \frac{V_{out}[z]}{V_{in}[z]} = \frac{z^{-1}}{1 + \frac{C_2}{C_1}(1-z^{-1})}$$

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Substituting:

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Substituting:

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$$\frac{V_o(e^{j\omega T})}{V_{in}(e^{j\omega T})} = \frac{1 - j\omega T}{1 + j\omega C_2/C_1 T}$$

Exercise 3

What is the f_{3dB} cutoff freq. of a LP filter?

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What is the f_{3dB} Cutoff freq. of a LP filter?

The cut off freq. is defined as the freq. where the amplitude of $H(e^{j\omega})$ is $\frac{1}{\sqrt{2}}$ times the DC amplitude (approximately -3dB, half power).

i.e.

$$|H(e^{j\omega_c})| = \frac{1}{\sqrt{2}} \cdot |H(e^{j0})|$$

Exercise 3

$$H(e^{j\omega}) = \frac{1 - j\omega T}{1 + j\omega \frac{C_2}{C_1} T}$$

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$$\frac{\sqrt{1+\omega_c^2 T^2}}{\sqrt{1+\omega_c^2 T^2 (C_2/C_1)^2}} = \frac{1}{\sqrt{2}} \cdot 1 \Rightarrow \quad 1 + \omega_c^2 T^2 (C_2/C_1)^2 = 2 + \omega_c^2 T^2 \Rightarrow$$

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$$(\omega_c^2 T^2)((C_2/C_1)^2 - 1) = 1 .$$

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$$(\omega_c^2 T^2)((C_2/C_1)^2 - 1) = 1 .$$ assume $(C_2/C_1)^2 >> 1 \Rightarrow$

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$$(\omega_c^2 T^2)\left((C_2/C_1)^2 - 1\right) = 1 . \quad \text{assume } (C_2/C_1)^2 >> 1 \Rightarrow$$

$$\omega_c^2 T^2 (C_2/C_1)^2 = 1 \Rightarrow \omega_c = C_1/C_2 \frac{1}{T} \Rightarrow \omega_c = C_1/C_2 f_{clk} \Rightarrow$$

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$$\omega_c^2 T^2 (C_2/C_1)^2 = 1 \Rightarrow \omega_c = \frac{C_1}{C_2}\frac{1}{T} \Rightarrow \omega_c = \frac{C_1}{C_2} f_{clk} \Rightarrow$$

$$2\pi f_{3dB} = \frac{C_1}{C_2} f_{clk} \Rightarrow f_{3dB} = \frac{1}{2\pi}\frac{C_1}{C_2} f_{clk}$$