Review: z-transform

z-transform of a sequence $x[n]$:

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}$$

Review: Z-transform

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What is z ?

Review: z-transform

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What is z? z represent any Complex number.

Review: Z-transform

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Review: z-transform

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* The set of values "z" which,

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Region of Convergence ( ).

Review: z-transform

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Review: z-transform

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Review: Z-transform

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Review: Z-transform

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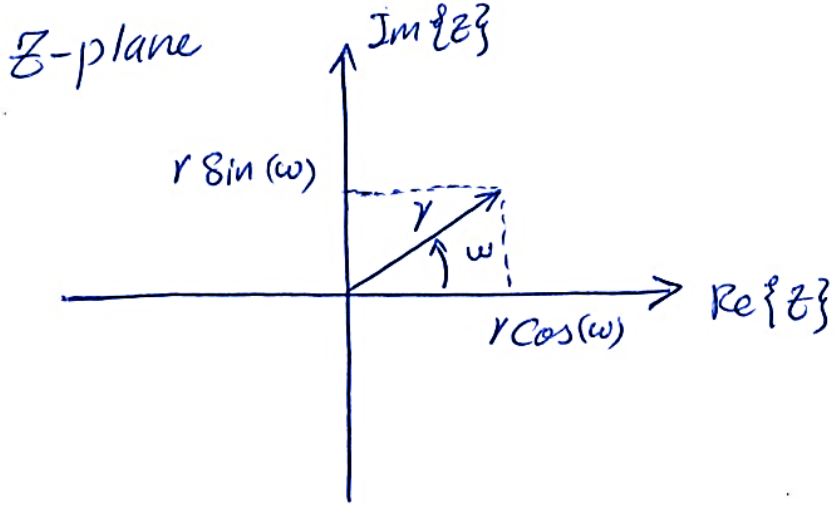
* The set of values of "Z" for which, $X(Z)$ is infinite called,

Review: z-transform

* The set of values of "z" for which, $X(z) = 0$ are called, <u>zeros</u> of $X(z)$.

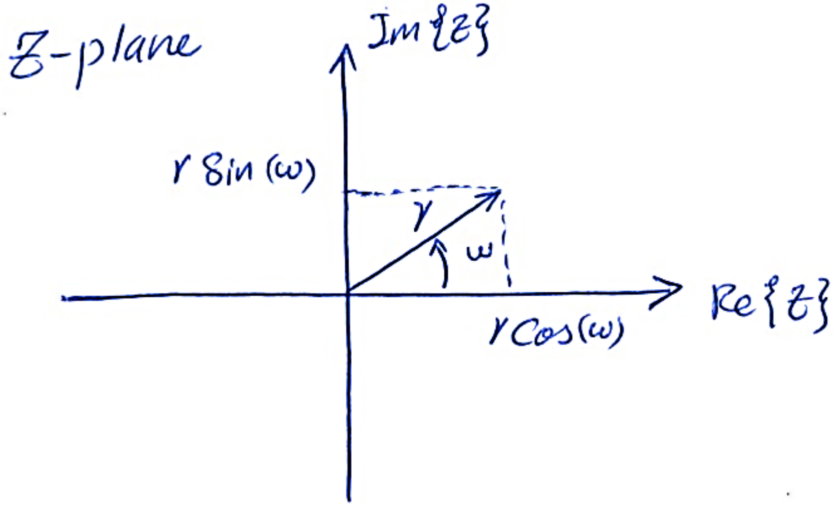
* The set of values of "z" for which, $X(z)$ is infinite called, <u>poles</u> of $X(z)$.

Review: z-transform



$$z = \underline{r}\, e^{j\omega}$$

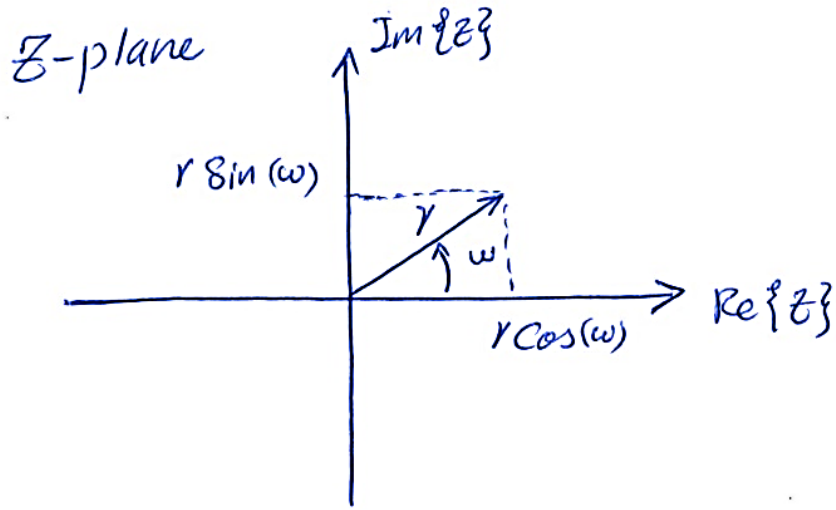
Review: z-transform



$z = \underline{r} e^{j\omega}$

$\Downarrow$ set $r = 1$

Review: z-transform



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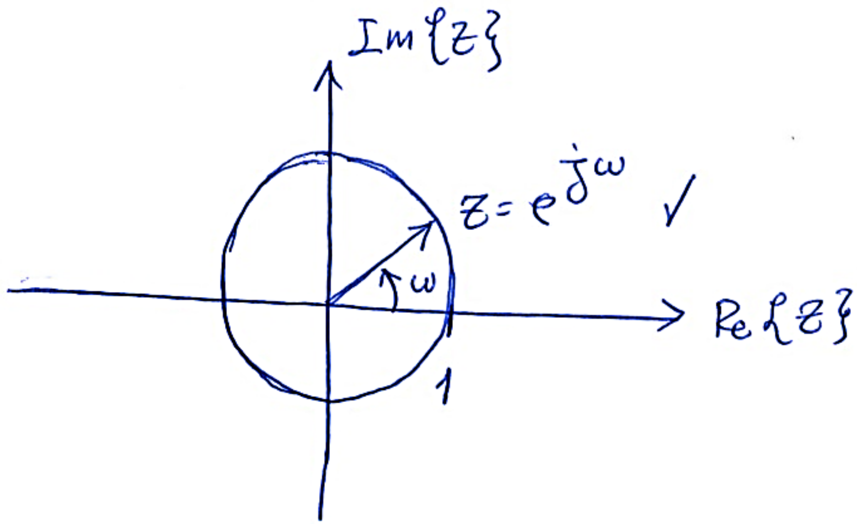

<u>unit circle</u> ✓

Review: z-transform

Stability:

* A system is stable iff all poles, located inside the unit circle!

Review: z-transform

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Example:

$$h[n] = b\, a^n u[n]$$

a, b real.

Review: z-transform

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$$h[n] = b\,a^n u[n] \overset{ZT}{\Longleftrightarrow} H(z) = \frac{b}{1 - a z^{-1}}$$

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Review: z-transform

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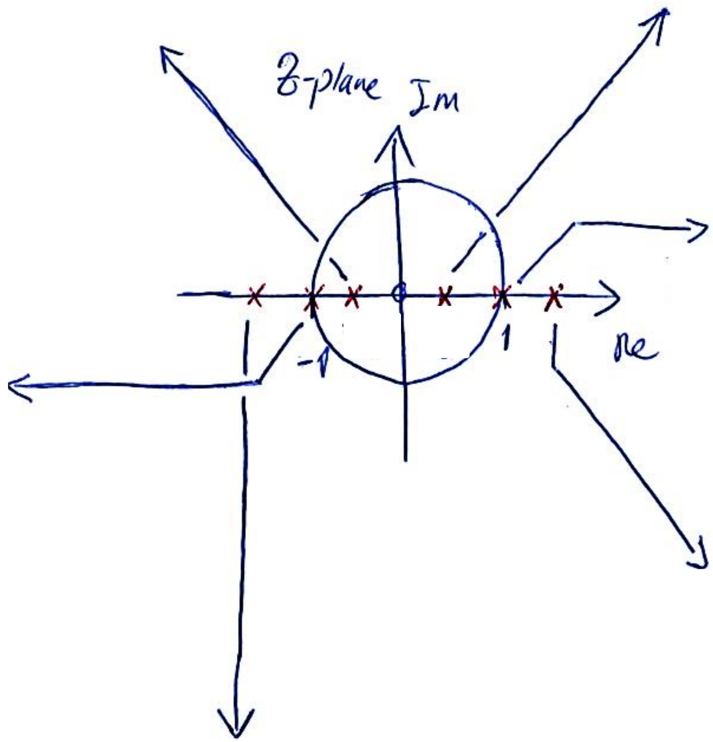
Example:

$$h[n] = b\, a^n u[n] \xleftrightarrow{ZT} H(z) = \frac{b}{1 - a z^{-1}}$$

a, b real.

Zero at $z = 0$ and pole at $p = a$.

Review: z-transform



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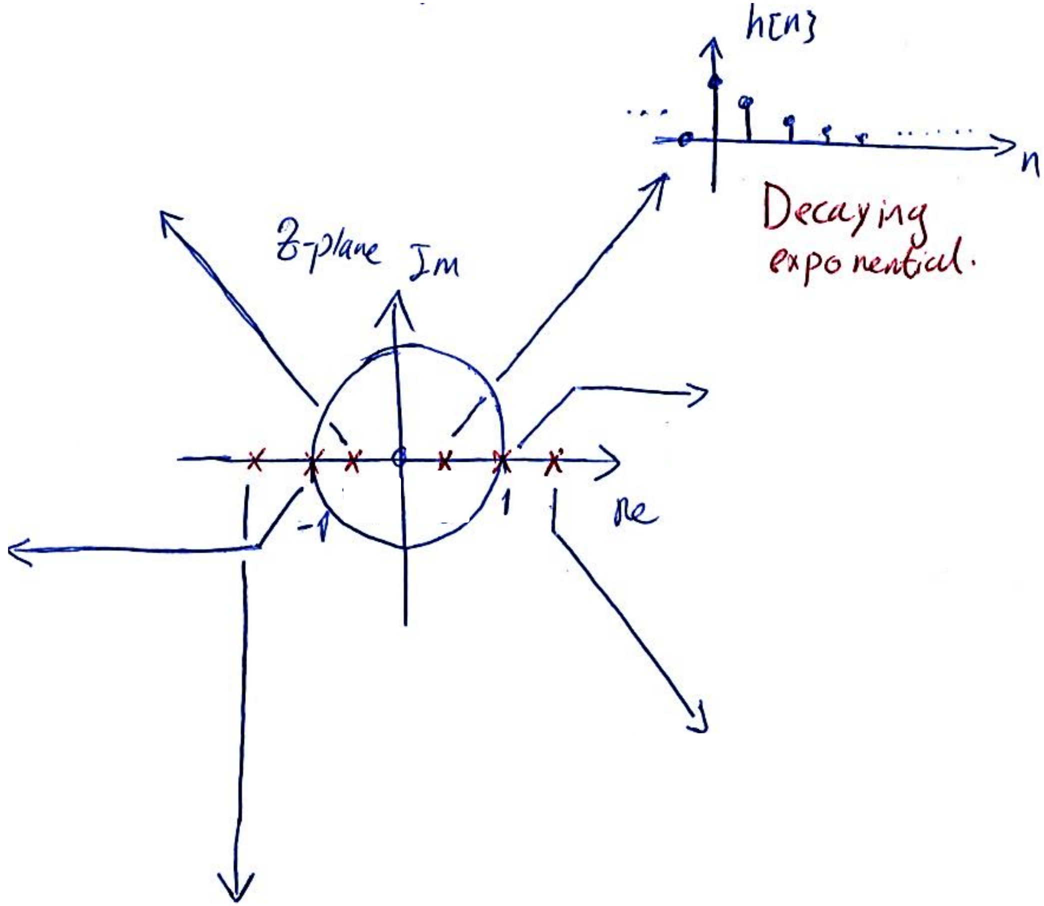

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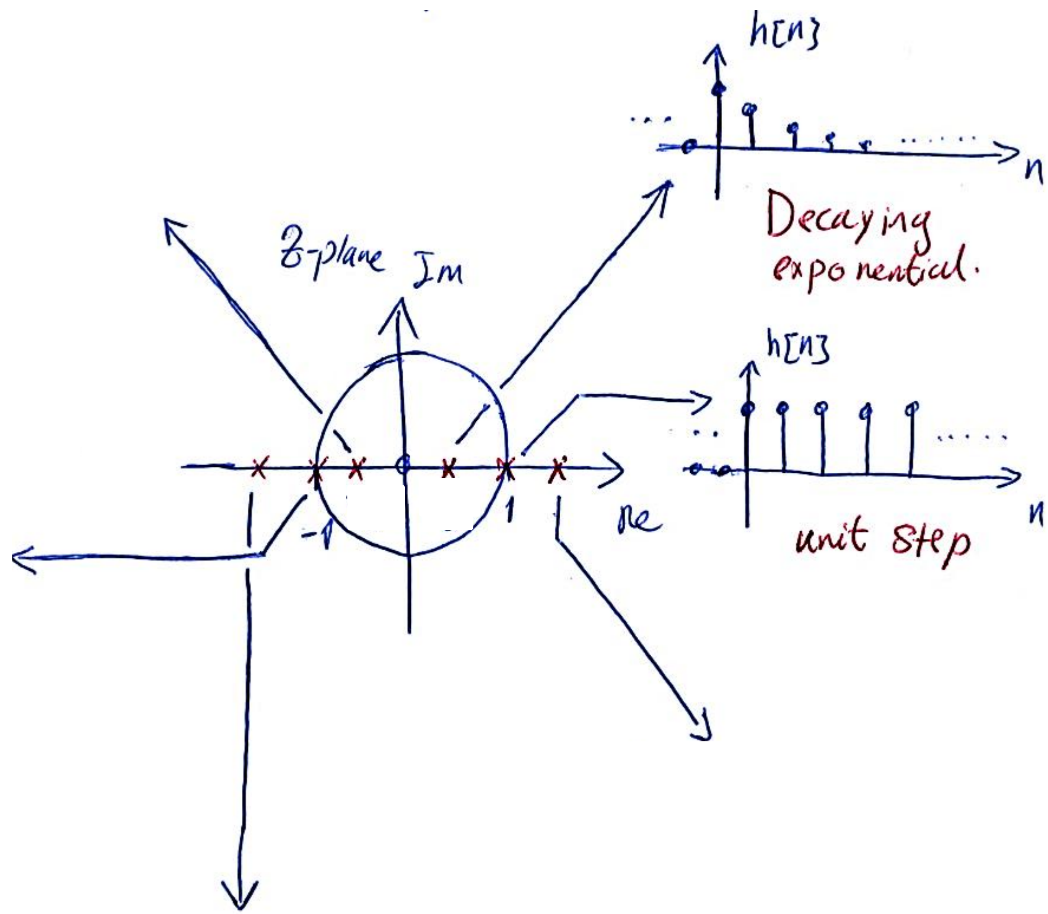

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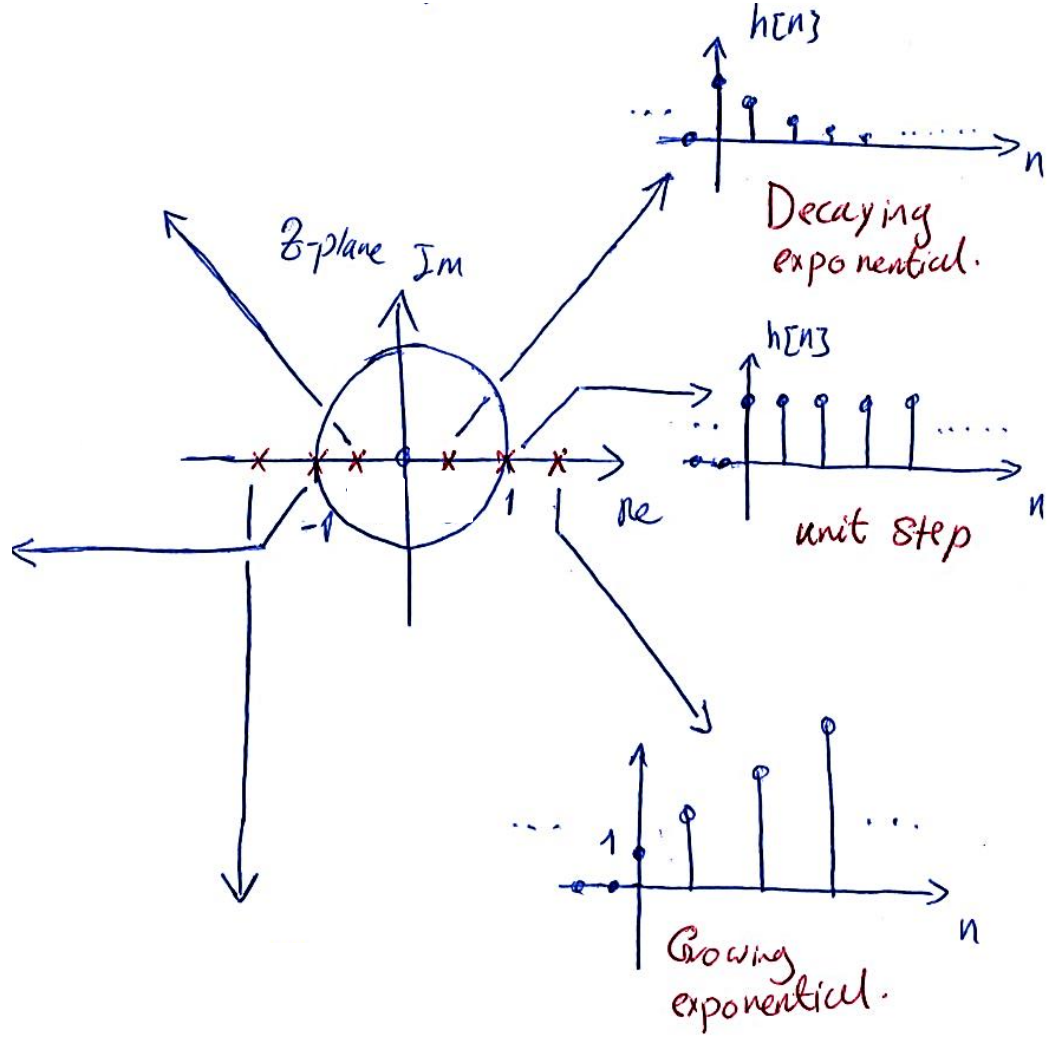

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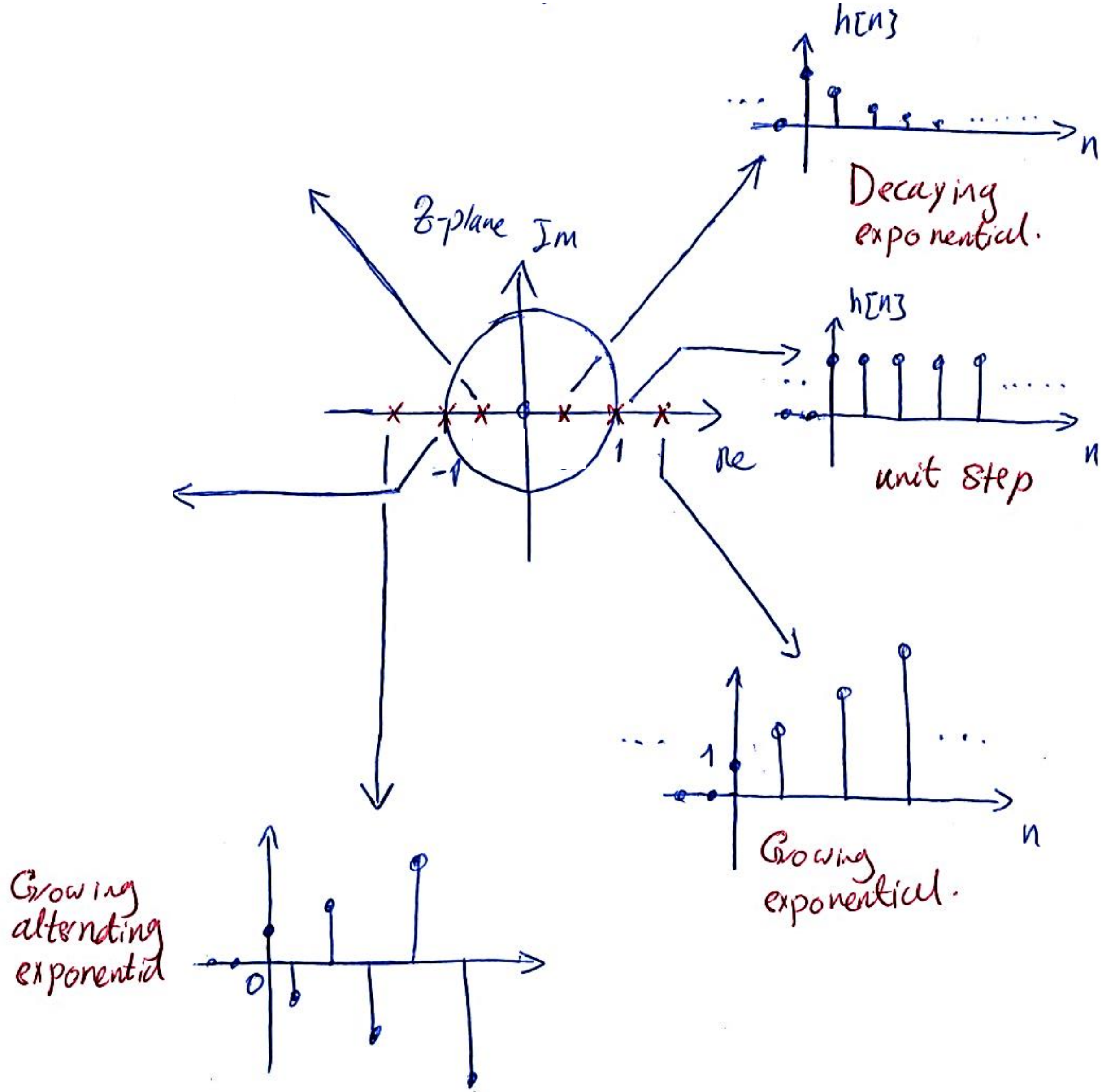

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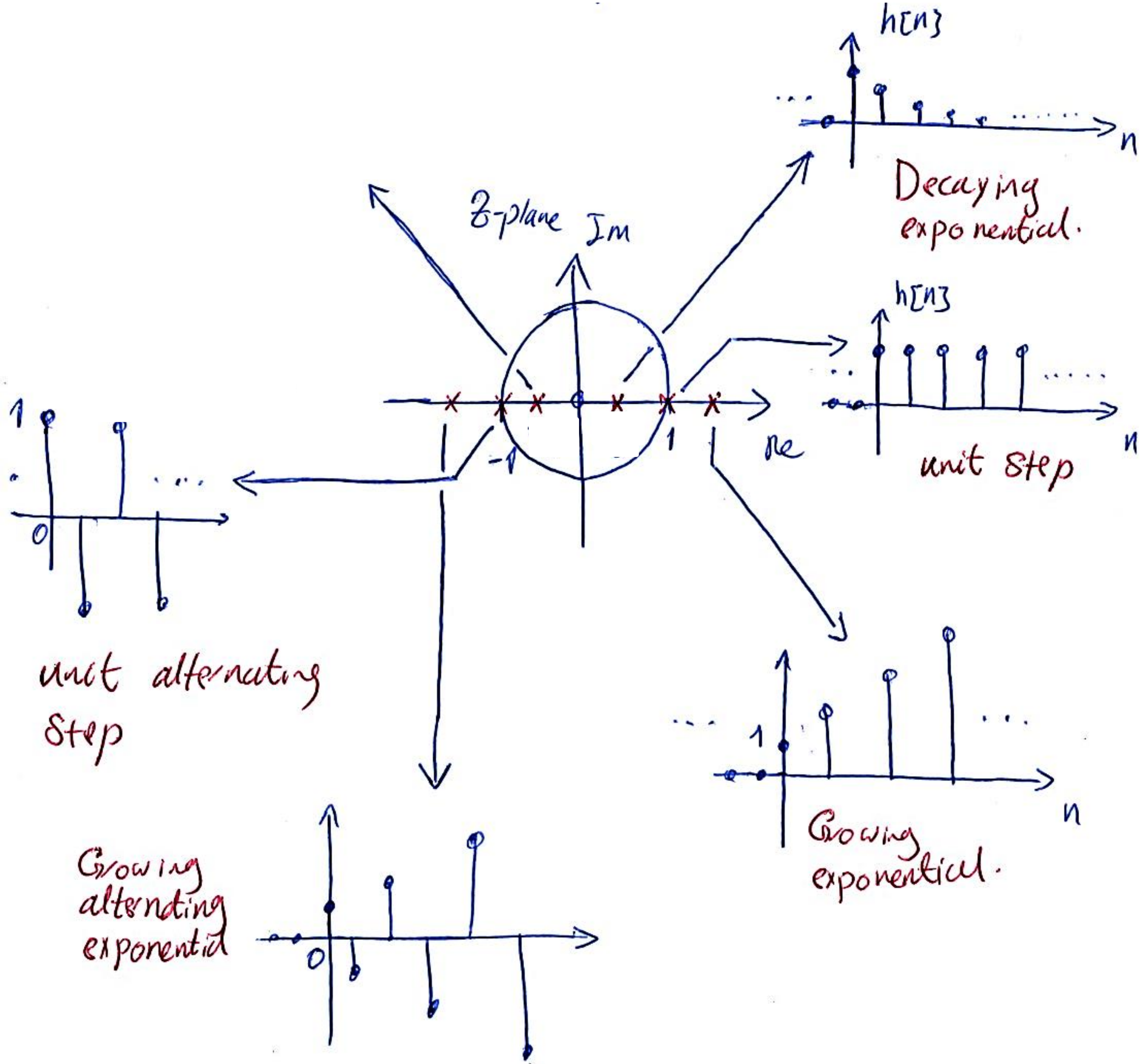

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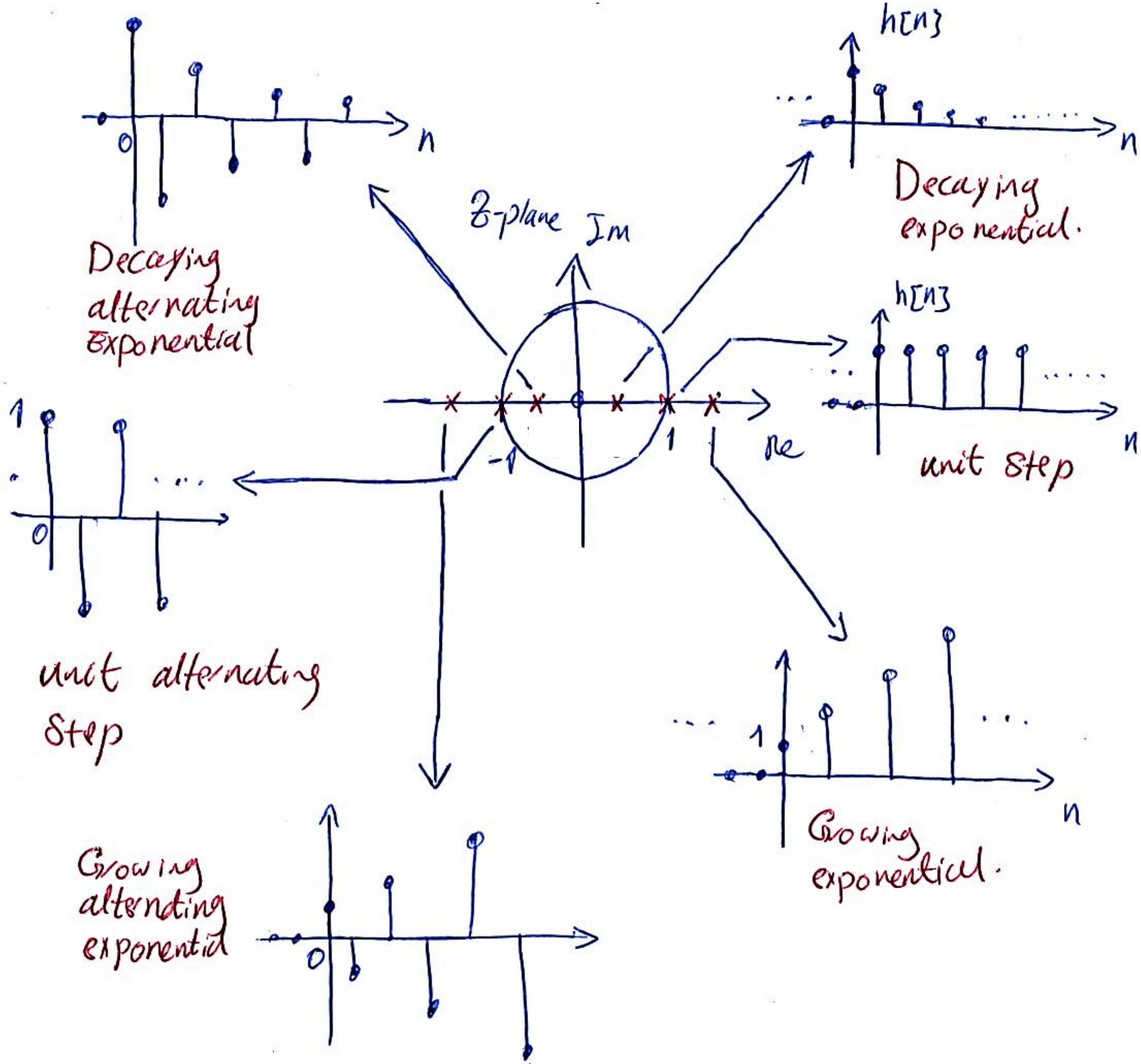
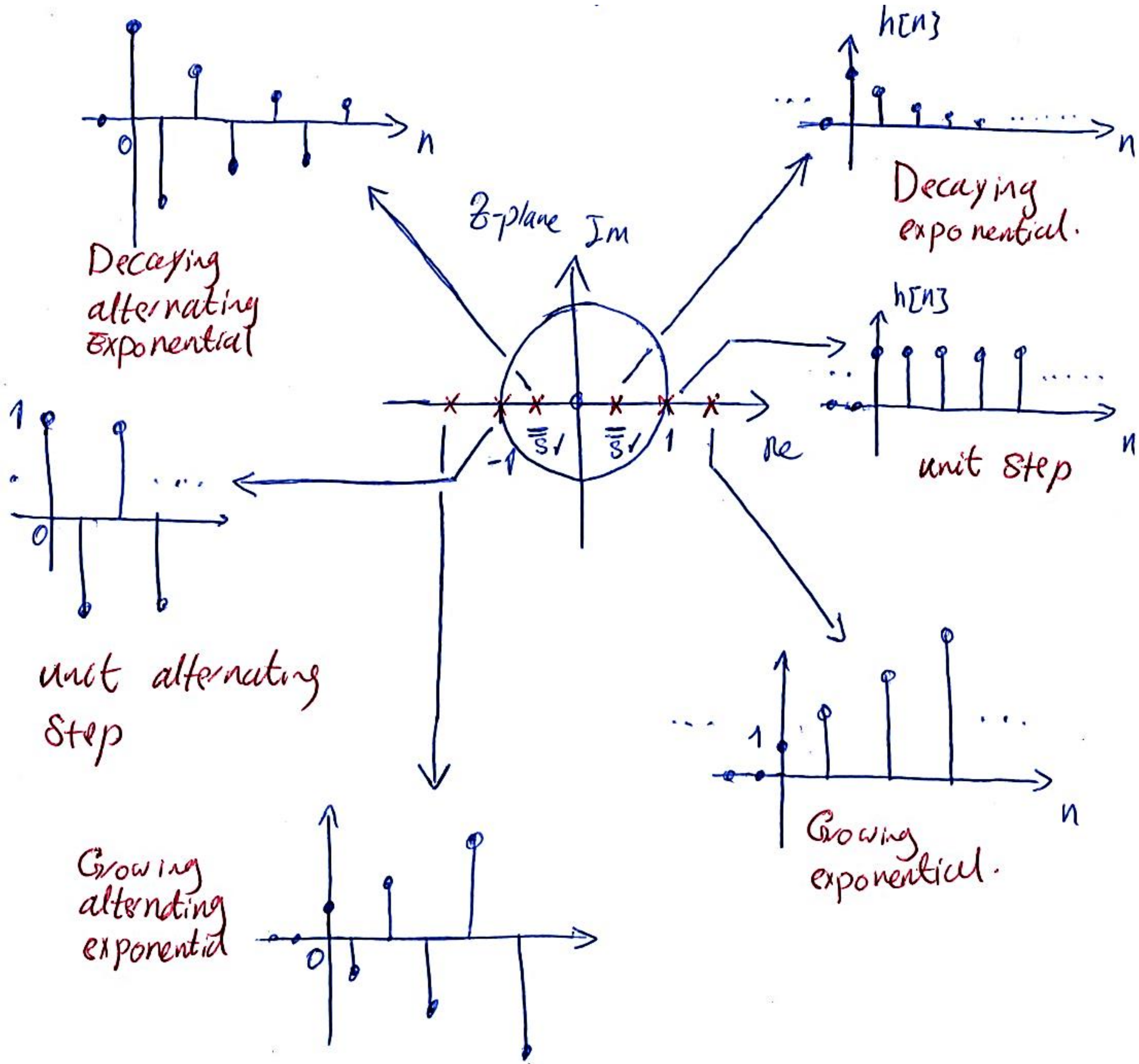


Review: z-transform



Review: z-transform

Motivation: filter design

Specification of filter:

CT

designe:

DT

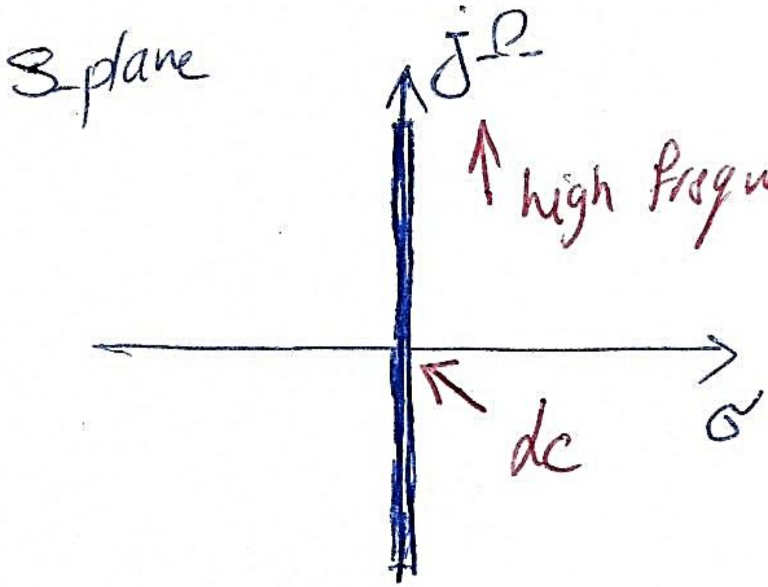
Review: z-transform

Motivation: filter design

Specification of filter:	designe:
CT	DT
$h(S)$	$H(z)$
Laplace domain	z-domain

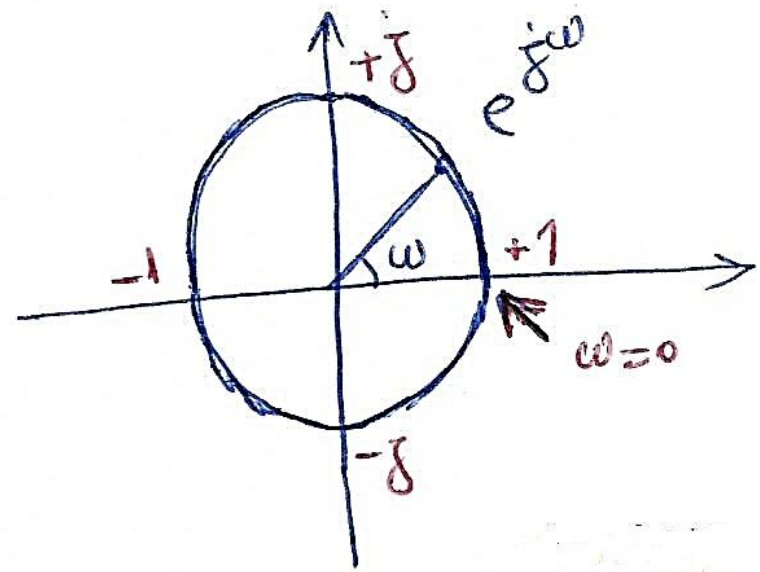
Review: z-transform

s-domain



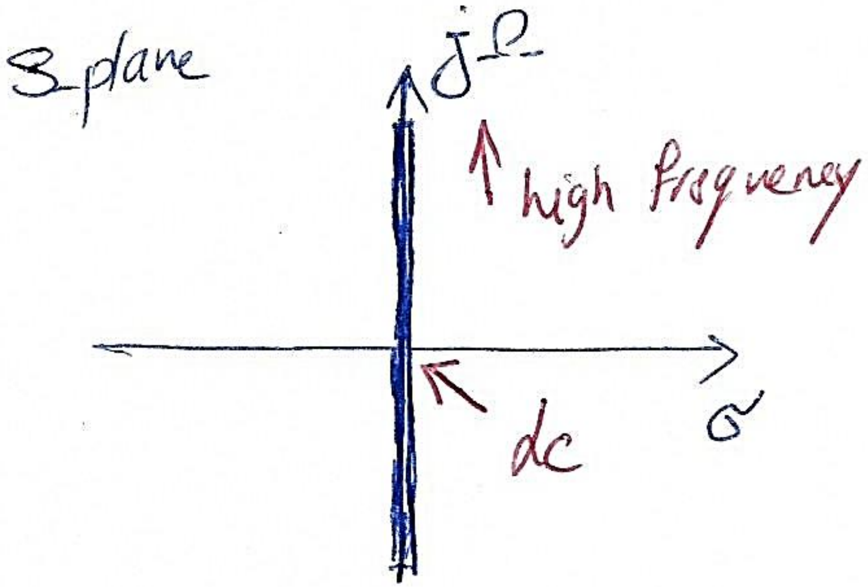
$s = j\Omega$

z-domain



Review: z-transform

s-domain



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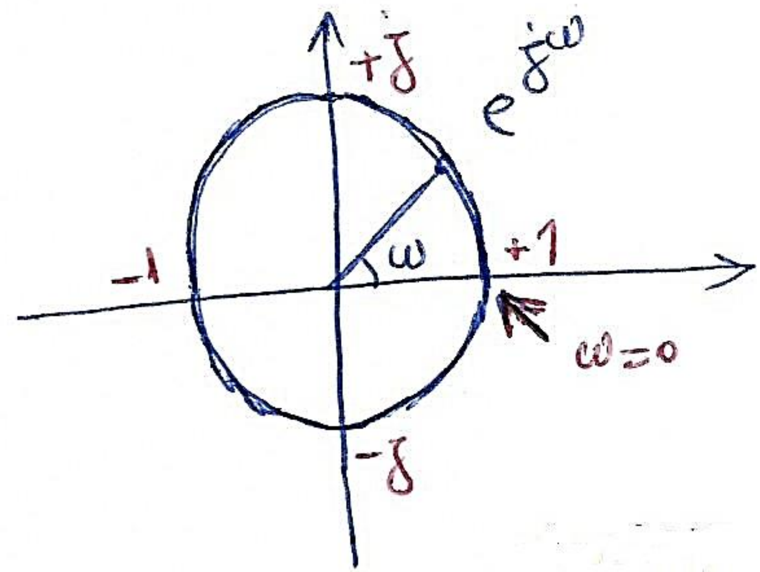

$$\Omega = \tan(\omega/2)$$

page 550 of reference book (13.5.4, bilinear transform)

Bilinear Transform:

$$s = \frac{z-1}{z+1}$$

$$\begin{tikzcd} h(s) \arrow[r, mapsto, bend left] & H(z) \end{tikzcd}$$

page 550 of reference book (13.5.4, bilinear transform)

Bilinear Transform:

$$
\begin{tikzcd}[column sep=huge]
h(s) \arrow[r, mapsto, bend left=50, "s = \frac{z-1}{z+1}"] & H(z) \arrow[l, mapsto, bend left=50, "z = \frac{1+s}{1-s}"]
\end{tikzcd}
$$

Golden relations

page 550 of reference book (13.5.4, bilinear transform)

Golden relations

$$\begin{cases} \omega = 2\pi f / f_S \\ \Omega = \tan(\omega/2) \end{cases}$$

; f_S: clock freq

$$z = e^{j\omega}$$

Golden relations

for first order filters:

$$\delta_p = -\Omega_{3dB}$$

Golden relations

for first order filters:

$$s_p = -\Omega_{3dB}$$

$$z_p = \frac{1+s_p}{1-s_p}$$

Golden relations

for first order filters:

$$s_p = -\Omega_{3dB}$$

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Crazy but true!

Exercise: page 573 of the refrence Book.

Find the system transfer function (i. e. $H(z)$) of a first-order filter such that its 3-dB point is at 10 kHz when clock frequency of 100 kHz is used. It is also desired the filter have zero gain at 50 kHz and the dc gain be unity.

Exercise: page 573 of the reference Book.

Find the system transfer function (i. e. $H(z)$) of a first-order filter such that its 3-dB point is at 10 kHz when clock frequency of 100 kHz is used. It is also desired the filter have zero gain at 50 kHz and the dc gain be unity.

$f_{3\text{-}dB} = 10\ kHz$

$f_s = 100\ kHz$

zero @ 50 kHz

$H(1) = 1$

Exercise: page 573 of the refrence Book.

$$\begin{cases} z = e^{j\omega} \\ \omega = f/f_s \cdot 2\pi \end{cases}$$

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$$\begin{cases} z = e^{j\omega} \\ \omega = f/f_s \cdot 2\pi \end{cases} \Rightarrow z_{zero} = e^{j \cdot 2\pi \cdot 50/100} = e^{j\pi}$$

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$$\Rightarrow \boxed{z_{zero} = -1}$$

Exercise: page 573 of the refrence Book.

$$\omega_{3\text{-dB}} = 2\pi \cdot f_{3\text{-dB}} / f_s = 2\pi \cdot 10/100 = 0.2\pi$$

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$$\Omega_{3dB} = \tan\left(\frac{\omega_{3dB}}{2}\right) = \tan\left(0.2\pi / 2\right) = 0.3249$$

Exercise: page 573 of the refrence Book.

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$$s_p = -\Omega_{3dB} \Rightarrow s_p = -0.3249$$

$$z_p = \frac{1+s_p}{1-s_p} = 0.5095$$

Exercise: page 573 of the refrence Book.

$$H(z) = \frac{k(z+1)}{z - 0.5095}$$

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from $H(1) = 1 \longrightarrow k = 0.24525$

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from $H(1) = 1 \longrightarrow k = 0.24525$

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Exercise: page 573 of the reference Book.

$$H(z) = \frac{k(z+1)}{z - 0.5095}$$

from $H(1) = 1 \longrightarrow k = 0.24525$

$$H(z) = \frac{0.24525\,(z+1)}{z - 0.5095}$$

or equivalently,

$$H(z) = \frac{0.4814z + 0.4814}{1.9627z - 1}$$

Exercise: page 573 of the refrene Book.

from the Course

$$H(z) = -\frac{\left(\frac{C_1 + C_2}{C_A}\right)z - C_1/C_2}{\left(1 + C_3/C_A\right)z - 1}$$