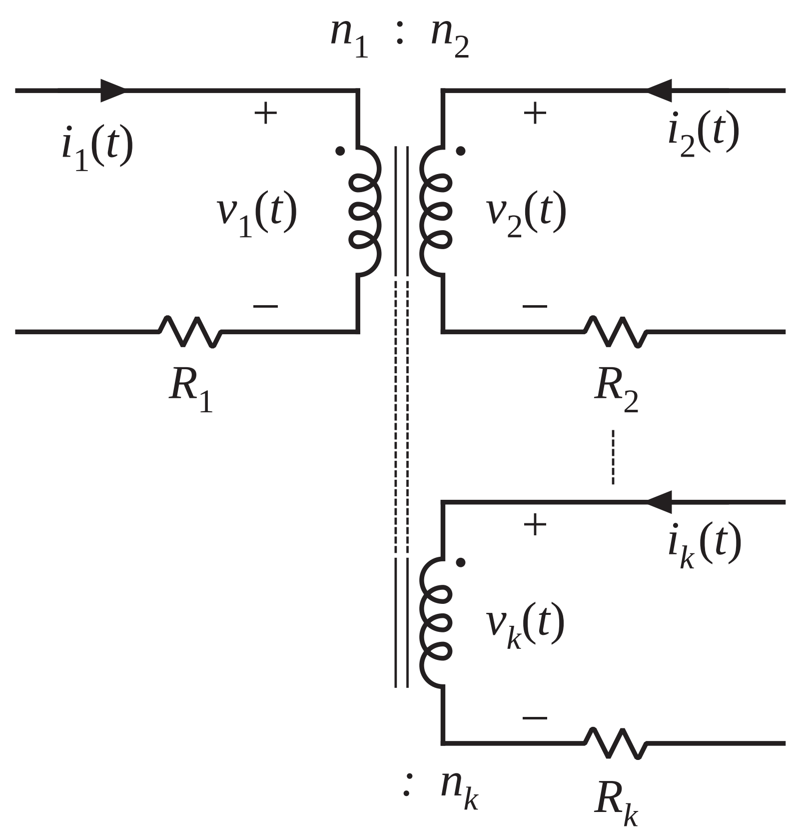
Chapter 15 Transformer Design

Some more advanced design issues, not considered in previous chapter:

- Inclusion of core loss
- Selection of operating flux density to optimize total loss
- Multiple winding design: as in the coupled-inductor case, allocate the available window area among several windings
- A transformer design procedure
- How switching frequency affects transformer size



Chapter 15 Transformer Design

15.1 Transformer design: Basic constraints

15.2 A step-by-step transformer design procedure

15.3 Examples

15.4 AC inductor design

15.5 Summary

15.1 Transformer Design:

Basic Constraints

Core loss

$$P_{fe} = K_{fe}(\Delta B)^{\beta} A_c \ell_m$$

Typical value of β for ferrite materials: 2.6 or 2.7

ΔB is the peak value of the ac component of $B(t)$, *i.e.*, the peak ac flux density

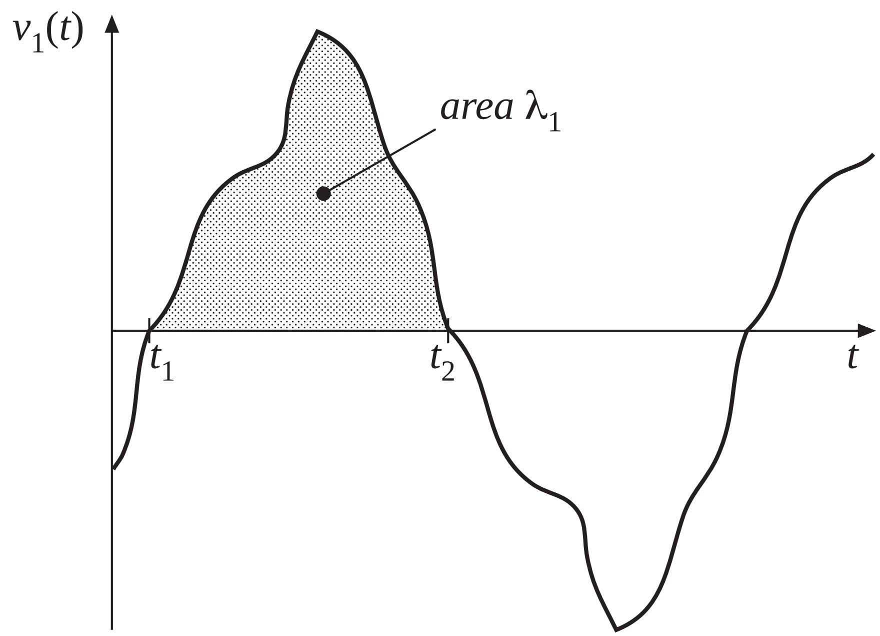
So increasing ΔB causes core loss to increase rapidly

This is the first constraint

Flux density
Constraint #2

Flux density $B(t)$ is related to the applied winding voltage according to Faraday's Law. Denote the volt-seconds applied to the primary winding during the positive portion of $v_1(t)$ as λ_1:

$$\lambda_1 = \int_{t_1}^{t_2} v_1(t)dt$$



This causes the flux to change from its negative peak to its positive peak. From Faraday's law, the peak value of the ac component of flux density is

$$\Delta B = \frac{\lambda_1}{2n_1A_c}$$

To attain a given flux density, the primary turns should be chosen according to

$$n_1 = \frac{\lambda_1}{2\Delta BA_c}$$

Copper loss
Constraint #3

- Allocate window area between windings in optimum manner, as described in previous section
- Total copper loss is then equal to

$$P_{cu} = \frac{\rho (MLT) n_1^2 I_{tot}^2}{W_A K_u}$$

with

$$I_{tot} = \sum_{j=1}^{k} \frac{n_j}{n_1} I_j$$

Eliminate n_1, using result of previous slide:

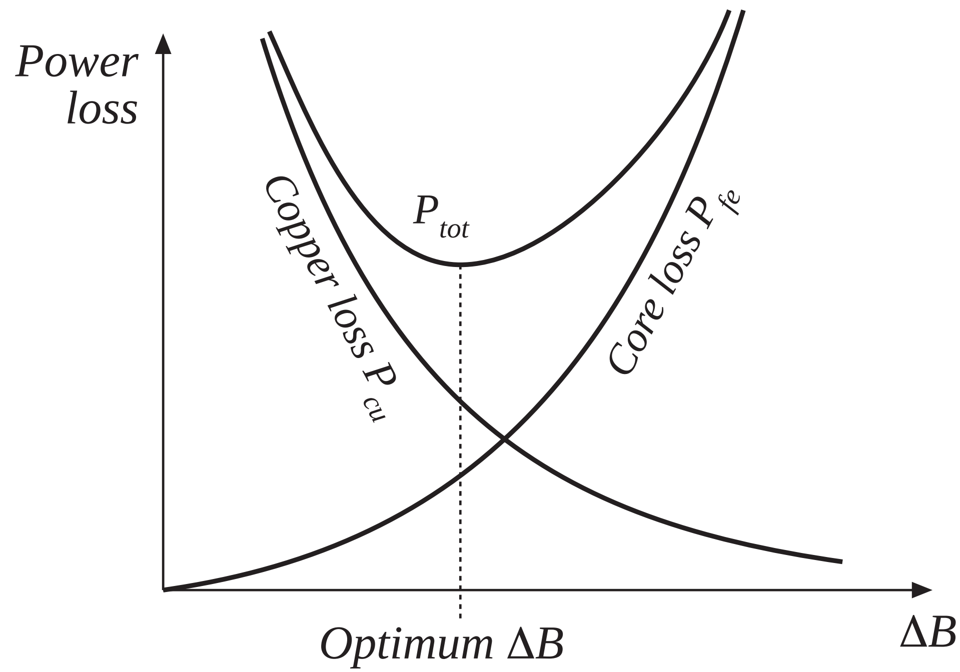
$$P_{cu} = \left(\frac{\rho \lambda_1^2 I_{tot}^2}{4 K_u} \right) \left(\frac{(MLT)}{W_A A_c^2} \right) \left(\frac{1}{\Delta B} \right)^2$$

Note that copper loss decreases rapidly as ΔB is increased

Total power loss

4. $P_{tot} = P_{cu} + P_{fe}$

There is a value of ΔB that minimizes the total power loss



$$P_{tot} = P_{fe} + P_{cu}$$

$$P_{fe} = K_{fe}(\Delta B)^{\beta} A_c \ell_m$$

$$P_{cu} = \left(\frac{\rho \lambda_1^2 I_{tot}^2}{4K_u}\right)\left(\frac{(MLT)}{W_A A_c^2}\right)\left(\frac{1}{\Delta B}\right)^2$$

5. Find optimum flux density ΔB

Given that

$$P_{tot} = P_{fe} + P_{cu}$$

Then, at the ΔB that minimizes P_{tot}, we can write

$$\frac{dP_{tot}}{d(\Delta B)} = \frac{dP_{fe}}{d(\Delta B)} + \frac{dP_{cu}}{d(\Delta B)} = 0$$

Note: optimum does not necessarily occur where $P_{fe} = P_{cu}$. Rather, it occurs where

$$\frac{dP_{fe}}{d(\Delta B)} = -\frac{dP_{cu}}{d(\Delta B)}$$

Take derivatives of core and copper loss

$$P_{fe} = K_{fe} (\Delta B)^{\beta} A_c \ell_m \qquad P_{cu} = \left(\frac{\rho \lambda_1^2 I_{tot}^2}{4 K_u} \right) \left(\frac{(MLT)}{W_A A_c^2} \right) \left(\frac{1}{\Delta B} \right)^2$$

$$\frac{dP_{fe}}{d(\Delta B)} = \beta K_{fe} (\Delta B)^{(\beta - 1)} A_c \ell_m \qquad \frac{dP_{cu}}{d(\Delta B)} = -2 \left(\frac{\rho \lambda_1^2 I_{tot}^2}{4 K_u} \right) \left(\frac{(MLT)}{W_A A_c^2} \right) (\Delta B)^{-3}$$

Now, substitute into $\frac{dP_{fe}}{d(\Delta B)} = - \frac{dP_{cu}}{d(\Delta B)}$ and solve for ΔB:

$$\Delta B = \left[\frac{\rho \lambda_1^2 I_{tot}^2}{2 K_u} \frac{(MLT)}{W_A A_c^3 \ell_m} \frac{1}{\beta K_{fe}} \right]^{\left(\frac{1}{\beta + 2} \right)}$$

Optimum ΔB for a given core and application

Total loss

Substitute optimum ΔB into expressions for P_{cu} and P_{fe}. The total loss is:

$$P_{tot} = \left[A_c \ell_m K_{fe}\right]^{\left(\frac{2}{\beta+2}\right)} \left[\frac{\rho \lambda_1^2 I_{tot}^2}{4K_u} \frac{(MLT)}{W_A A_c^2}\right]^{\left(\frac{\beta}{\beta+2}\right)} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]$$

Rearrange as follows:

$$\frac{W_A \left(A_c\right)^{\left(2(\beta-1)/\beta\right)}}{(MLT)\ell_m^{\left(2/\beta\right)}} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]^{-\left(\frac{\beta+2}{\beta}\right)} = \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{\left(2/\beta\right)}}{4K_u \left(P_{tot}\right)^{\left(\left(\beta+2\right)/\beta\right)}}$$

Left side: terms depend on core geometry

Right side: terms depend on specifications of the application

The core geometrical constant K_{gfe}

Define
$$K_{gfe} = \frac{W_A (A_c)^{(2(\beta-1)/\beta)}}{(MLT)\ell_m^{(2/\beta)}} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)} \right]^{-\left(\frac{\beta+2}{\beta}\right)}$$

Design procedure: select a core that satisfies

$$K_{gfe} \geq \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4 K_u (P_{tot})^{((\beta+2)/\beta)}}$$

Appendix D lists the values of K_{gfe} for common ferrite cores

K_{gfe} is similar to the K_g geometrical constant used in Chapter 14:

- K_g is used when B_{max} is specified
- K_{gfe} is used when ΔB is to be chosen to minimize total loss

15.2 Step-by-step transformer design procedure

The following quantities are specified, using the units noted:

Wire effective resistivity	ρ	(Ω-cm)
Total rms winding current, ref to pri	I_{tot}	(A)
Desired turns ratios	n_2/n_1, n_3/n_1, etc.	
Applied pri volt-sec	λ_1	(V-sec)
Allowed total power dissipation	P_{tot}	(W)
Winding fill factor	K_u	
Core loss exponent	β	
Core loss coefficient	K_{fe}	(W/cm^3T^β)

Other quantities and their dimensions:

Core cross-sectional area	A_c	(cm^2)
Core window area	W_A	(cm^2)
Mean length per turn	MLT	(cm)
Magnetic path length	ℓ_e	(cm)
Wire areas	A_{w1}, …	(cm^2)
Peak ac flux density	ΔB	(T)

Procedure
1. Determine core size

$$K_{gfe} \ge \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4 K_u \left(P_{tot}\right)^{\left((\beta+2)/\beta\right)}} 10^8$$

Select a core from Appendix D that satisfies this inequality.

It may be possible to reduce the core size by choosing a core material that has lower loss, i.e., lower K_{fe}.

2. Evaluate peak ac flux density

$$\Delta B = \left[10^8 \; \frac{\rho \lambda_1^2 I_{tot}^2}{2K_u} \; \frac{(MLT)}{W_A A_c^3 \ell_m} \; \frac{1}{\beta K_{fe}} \right]^{\left(\frac{1}{\beta + 2}\right)}$$

At this point, one should check whether the saturation flux density is exceeded. If the core operates with a flux dc bias B_{dc}, then $\Delta B + B_{dc}$ should be less than the saturation flux density B_{sat}.

If the core will saturate, then there are two choices:

- Specify ΔB using the K_g method of Chapter 14, or
- Choose a core material having greater core loss, then repeat steps 1 and 2

3. and 4. Evaluate turns

Primary turns:

$$n_1 = \frac{\lambda_1}{2\Delta B A_c} \; 10^4$$

Choose secondary turns according to desired turns ratios:

$$n_2 = n_1 \left(\frac{n_2}{n_1} \right)$$

$$n_3 = n_1 \left(\frac{n_3}{n_1} \right)$$

$$\vdots$$

5. and 6. Choose wire sizes

Fraction of window area assigned to each winding:

$$\alpha_1 = \frac{n_1 I_1}{n_1 I_{tot}}$$

$$\alpha_2 = \frac{n_2 I_2}{n_1 I_{tot}}$$

$$\vdots$$

$$\alpha_k = \frac{n_k I_k}{n_1 I_{tot}}$$

Choose wire sizes according to:

$$A_{w1} \leq \frac{\alpha_1 K_u W_A}{n_1}$$

$$A_{w2} \leq \frac{\alpha_2 K_u W_A}{n_2}$$

$$\vdots$$

Check: computed transformer model

Predicted magnetizing inductance, referred to primary:

$$L_M = \frac{\mu n_1^2 A_c}{\ell_m}$$

Peak magnetizing current:

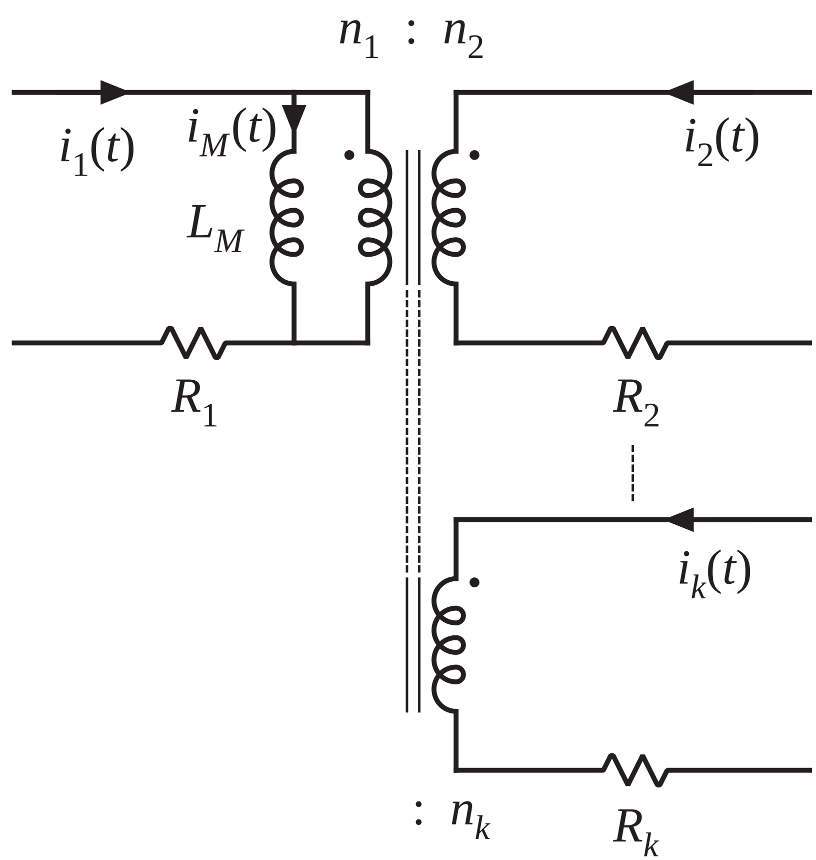
$$i_{M,\,pk} = \frac{\lambda_1}{2L_M}$$

Predicted winding resistances:

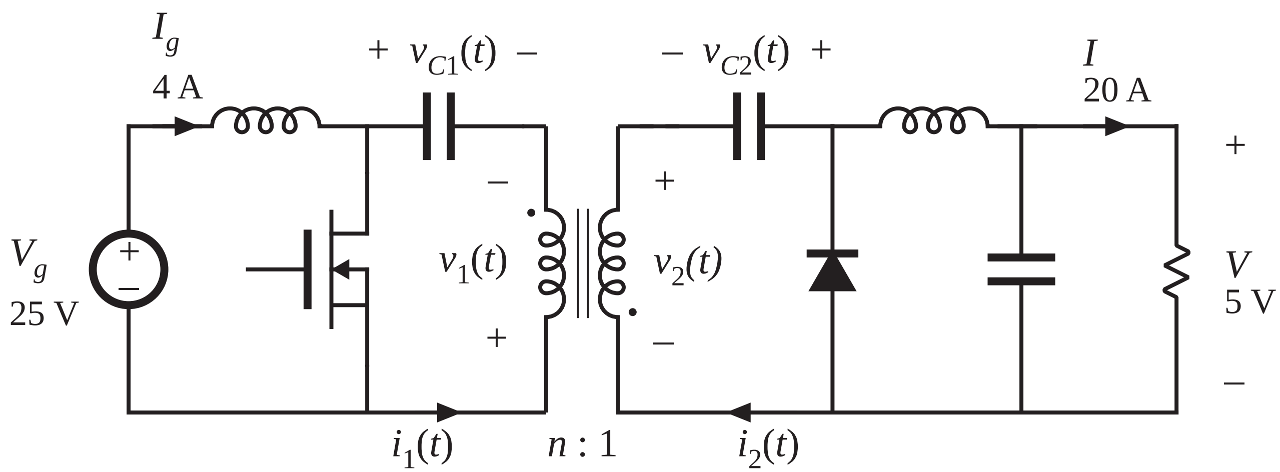
$$R_1 = \frac{\rho n_1 (MLT)}{A_{w1}}$$

$$R_2 = \frac{\rho n_2 (MLT)}{A_{w2}}$$

$$\vdots$$



15.4.1 Example 1: Single-output isolated Cuk converter



100 W | $f_s = 200$ kHz

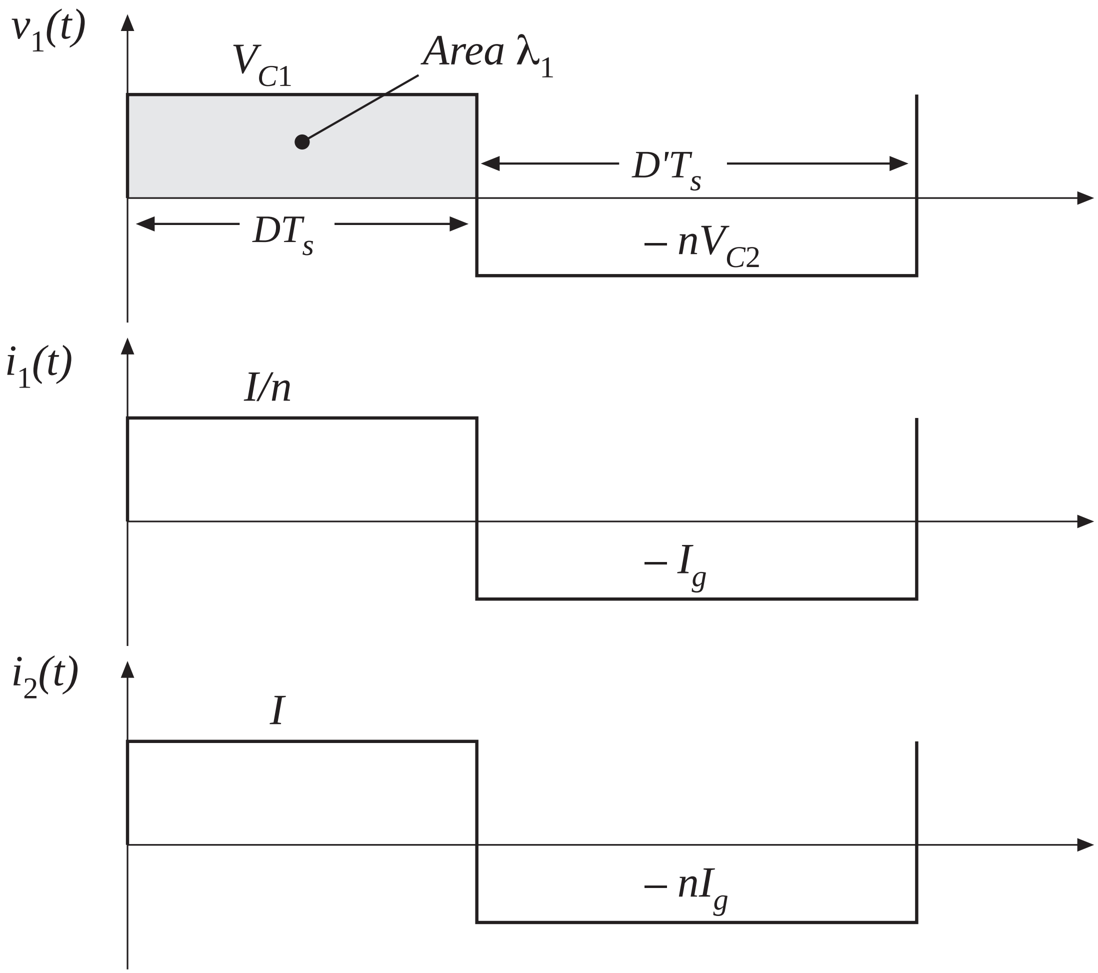
$D = 0.5$ | $n = 5$

$K_u = 0.5$ | Allow $P_{tot} = 0.25$ W

Use a ferrite pot core, with Magnetics Inc. P material. Loss parameters at 200 kHz are

$K_{fe} = 24.7$ | $\beta = 2.6$

Waveforms



Applied primary volt-seconds:

$$\lambda_1 = DT_s V_{c1} = (0.5)\,(5\ \mu\text{sec})\,(25\ \text{V}) = 62.5\ \text{V–}\mu\text{sec}$$

Applied primary rms current:

$$I_1 = \sqrt{D\left(\frac{I}{n}\right)^2 + D'\left(I_g\right)^2} = 4\ \text{A}$$

Applied secondary rms current:

$$I_2 = nI_1 = 20\ \text{A}$$

Total rms winding current:

$$I_{tot} = I_1 + \frac{1}{n} I_2 = 8\ \text{A}$$

Choose core size

$$K_{gfe} \geq \frac{(1.724 \cdot 10^{-6})(62.5 \cdot 10^{-6})^2 (8)^2 (24.7)^{(2/2.6)}}{4\,(0.5)\,(0.25)^{(4.6/2.6)}}\; 10^8$$

$$= 0.00295$$

Pot core data of Appendix D lists 2213 pot core with

$$K_{gfe} = 0.0049$$

Next smaller pot core is not large enough.

Evaluate peak ac flux density

$$\Delta B = \left[10^8 \frac{(1.724 \cdot 10^{-6})(62.5 \cdot 10^{-6})^2 (8)^2}{2\,(0.5)} \frac{(4.42)}{(0.297)(0.635)^3(3.15)} \frac{1}{(2.6)(24.7)} \right]^{(1/4.6)}$$

$$= 0.0858 \text{ Tesla}$$

This is much less than the saturation flux density of approximately 0.35 T. Values of ΔB in the vicinity of 0.1 T are typical for ferrite designs that operate at frequencies in the vicinity of 100 kHz.

Evaluate turns

$$n_1 = 10^4 \frac{(62.5 \cdot 10^{-6})}{2(0.0858)(0.635)}$$
$$= 5.74 \text{ turns}$$

$$n_2 = \frac{n_1}{n} = 1.15 \text{ turns}$$

In practice, we might select

$n_1 = 5$ and $n_2 = 1$

This would lead to a slightly higher flux density and slightly higher loss.

Determine wire sizes

Fraction of window area allocated to each winding:

$$\alpha_1 = \frac{\left(4\ \mathrm{A}\right)}{\left(8\ \mathrm{A}\right)} = 0.5$$

$$\alpha_2 = \frac{\left(\frac{1}{5}\right)\left(20\ \mathrm{A}\right)}{\left(8\ \mathrm{A}\right)} = 0.5$$

(Since, in this example, the ratio of winding rms currents is equal to the turns ratio, equal areas are allocated to each winding)

Wire areas:

From wire table, Appendix D:

$$A_{w1} = \frac{(0.5)(0.5)(0.297)}{(5)} = 14.8\cdot 10^{-3}\ \mathrm{cm}^2$$

AWG #16

$$A_{w2} = \frac{(0.5)(0.5)(0.297)}{(1)} = 74.2\cdot 10^{-3}\ \mathrm{cm}^2$$

AWG #9

Wire sizes: discussion

Primary

5 turns #16 AWG

Secondary

1 turn #9 AWG

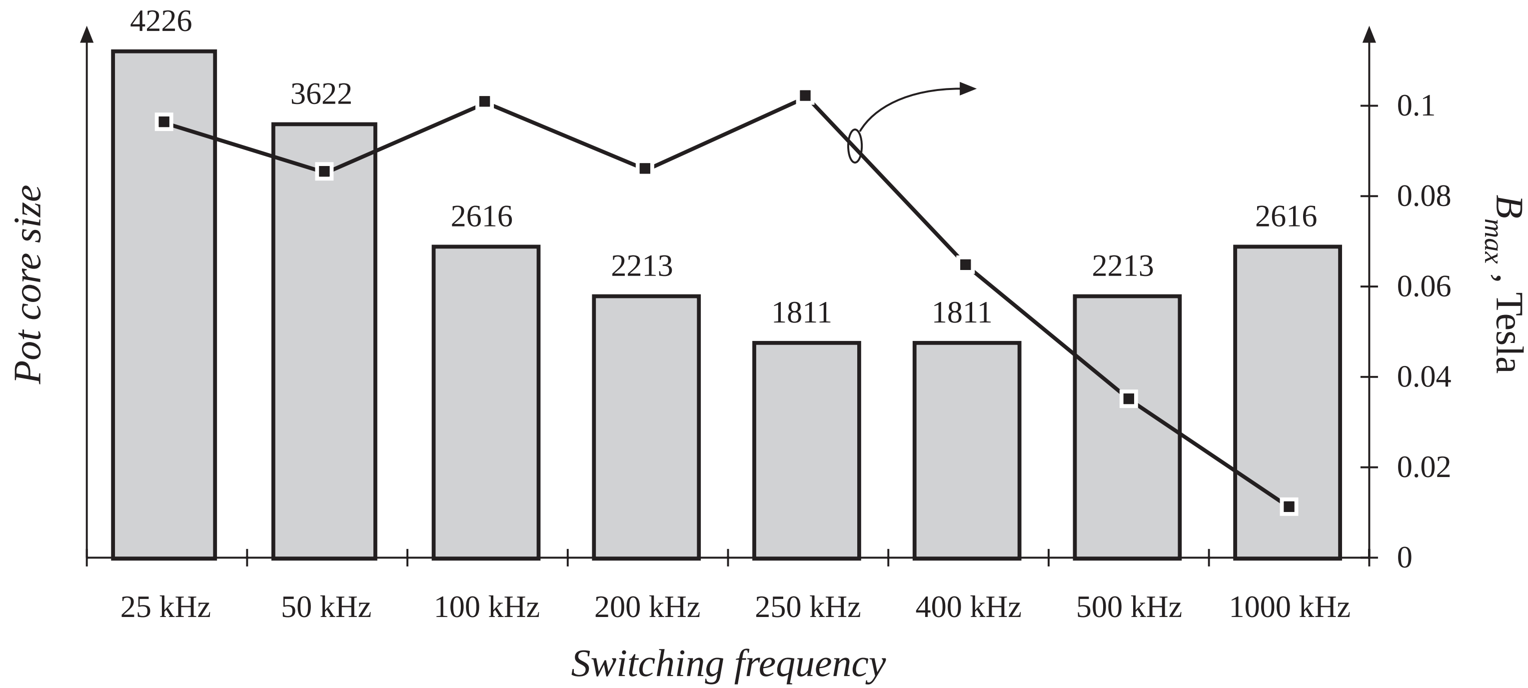
- Very large conductors!
- One turn of #9 AWG is not a practical solution

Some alternatives

- Use foil windings
- Use Litz wire or parallel strands of wire

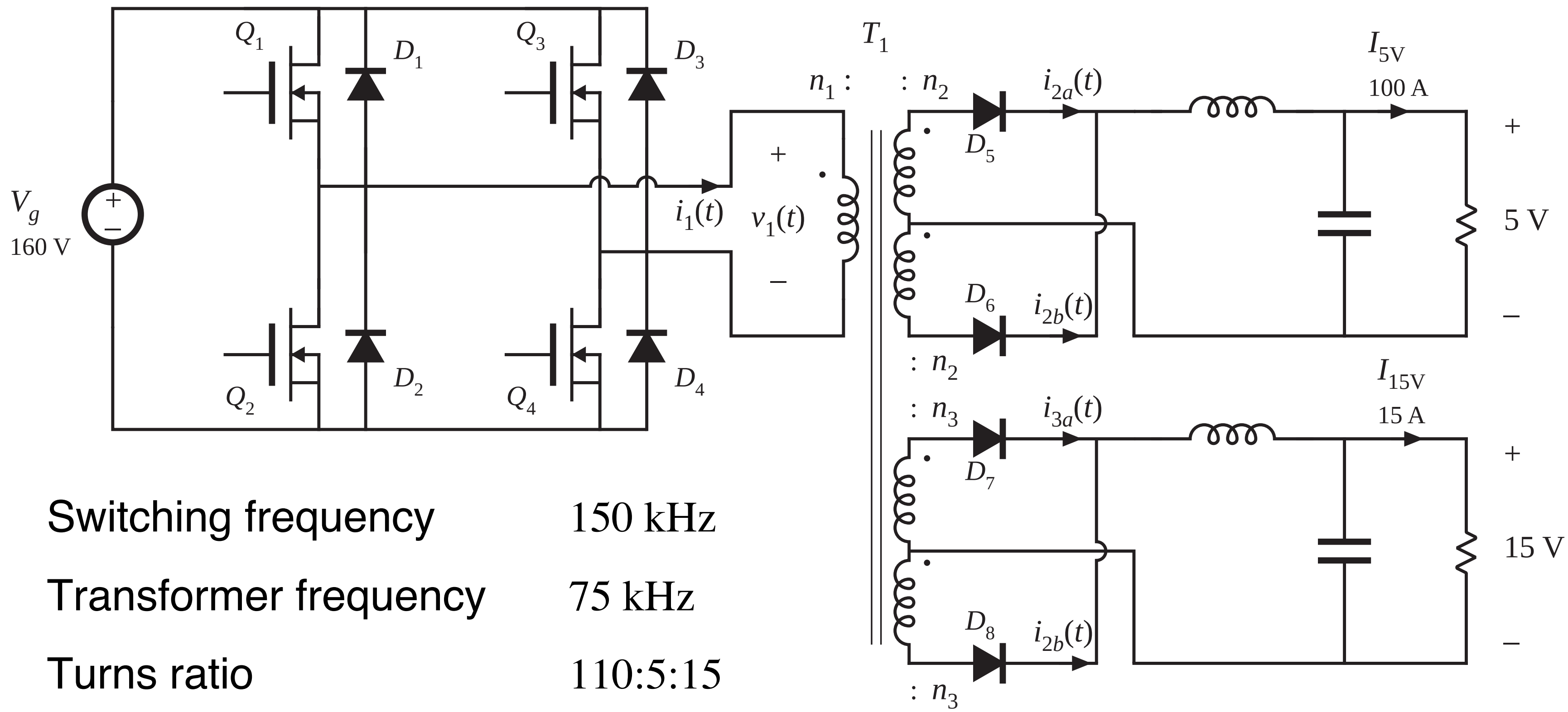
Effect of switching frequency on transformer size

for this P-material Cuk converter example



- As switching frequency is increased from 25 kHz to 250 kHz, core size is dramatically reduced
- As switching frequency is increased from 400 kHz to 1 MHz, core size increases

15.3.2 Example 2
Multiple-Output Full-Bridge Buck Converter



Switching frequency	150 kHz
Transformer frequency	75 kHz
Turns ratio	110:5:15
Optimize transformer at	D = 0.75

Other transformer design details

Use Magnetics, Inc. ferrite P material. Loss parameters at 75 kHz:

$K_{fe} = 7.6\ \mathrm{W/T^{\beta}cm^3}$

$\beta = 2.6$

Use E-E core shape

Assume fill factor of

$K_u = 0.25$ (reduced fill factor accounts for added insulation required in multiple-output off-line application)

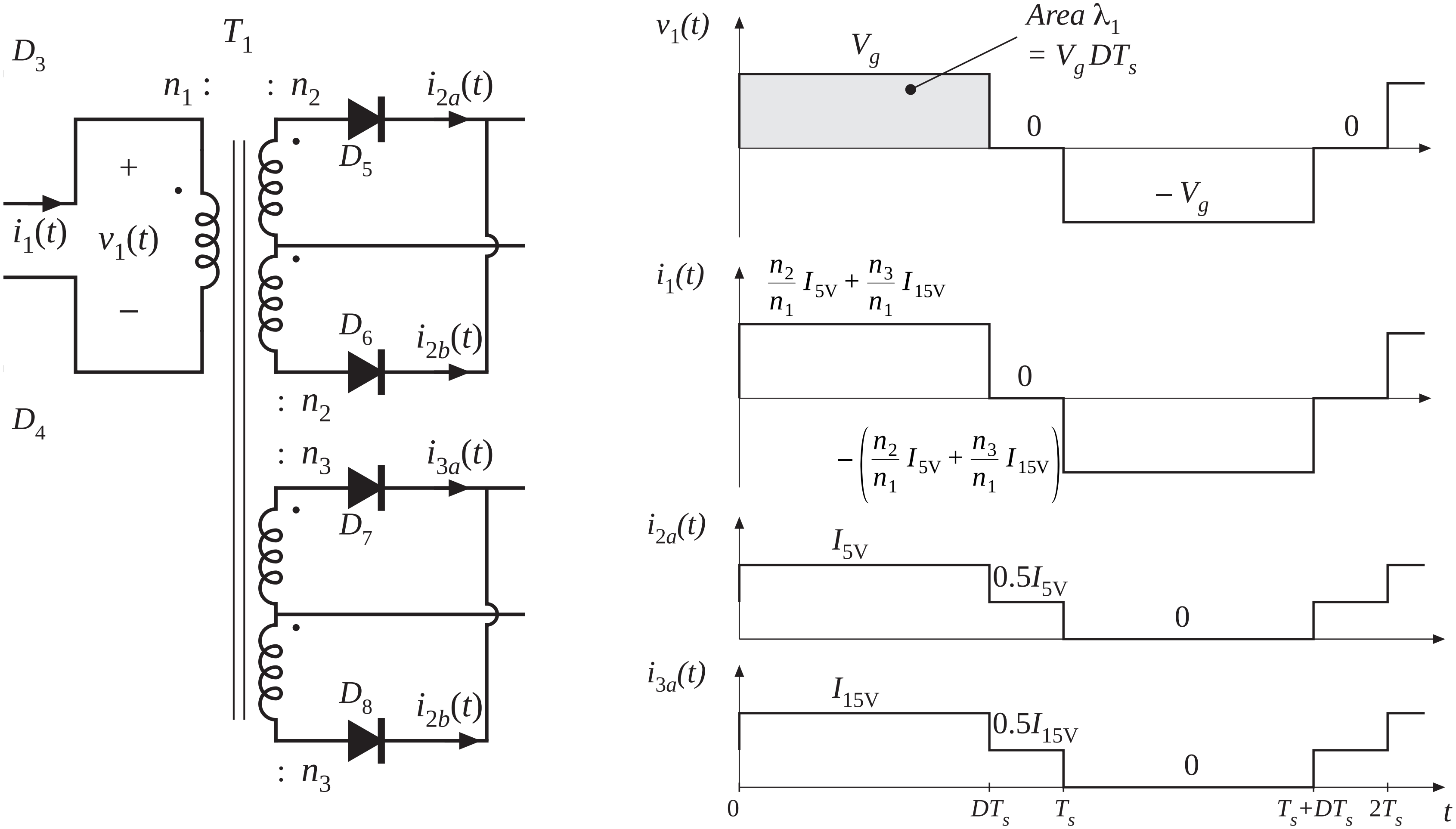
Allow transformer total power loss of

$P_{tot} = 4\ \mathrm{W}$ (approximately 0.5% of total output power)

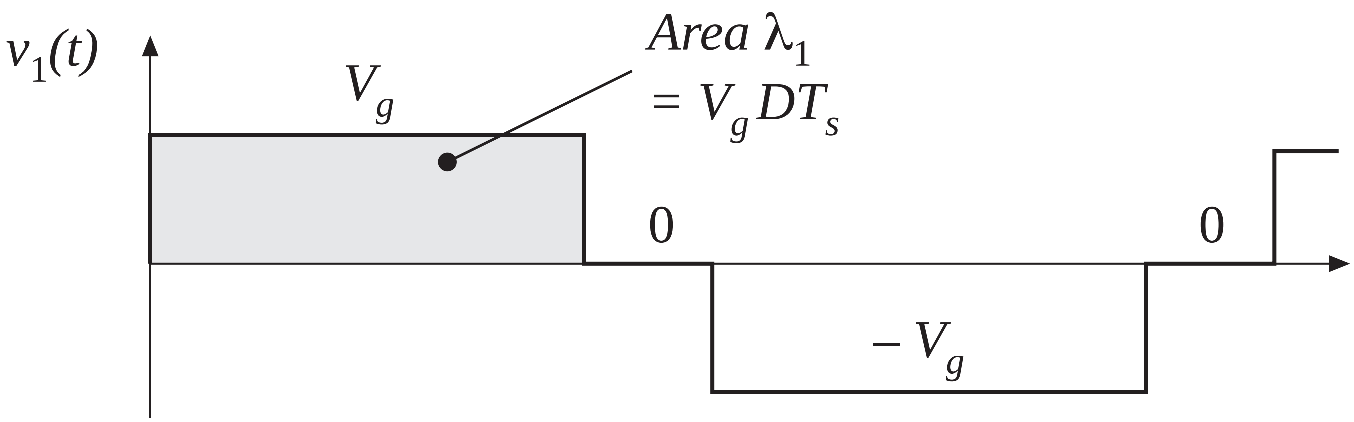
Use copper wire, with

$\rho = 1.724 \cdot 10^{-6}\ \Omega\text{-cm}$

Applied transformer waveforms

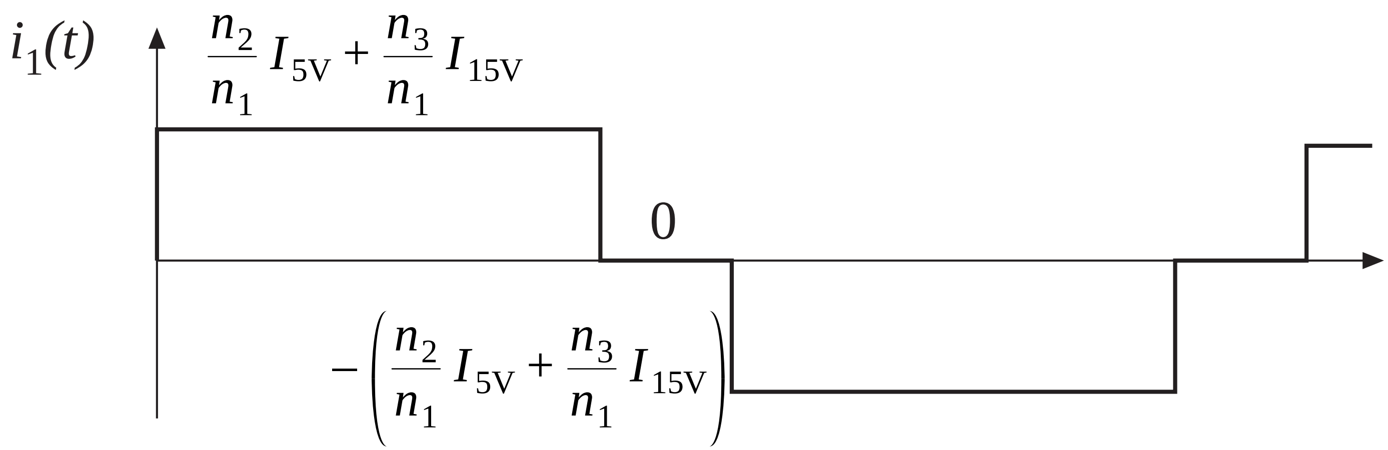


Applied primary volt-seconds



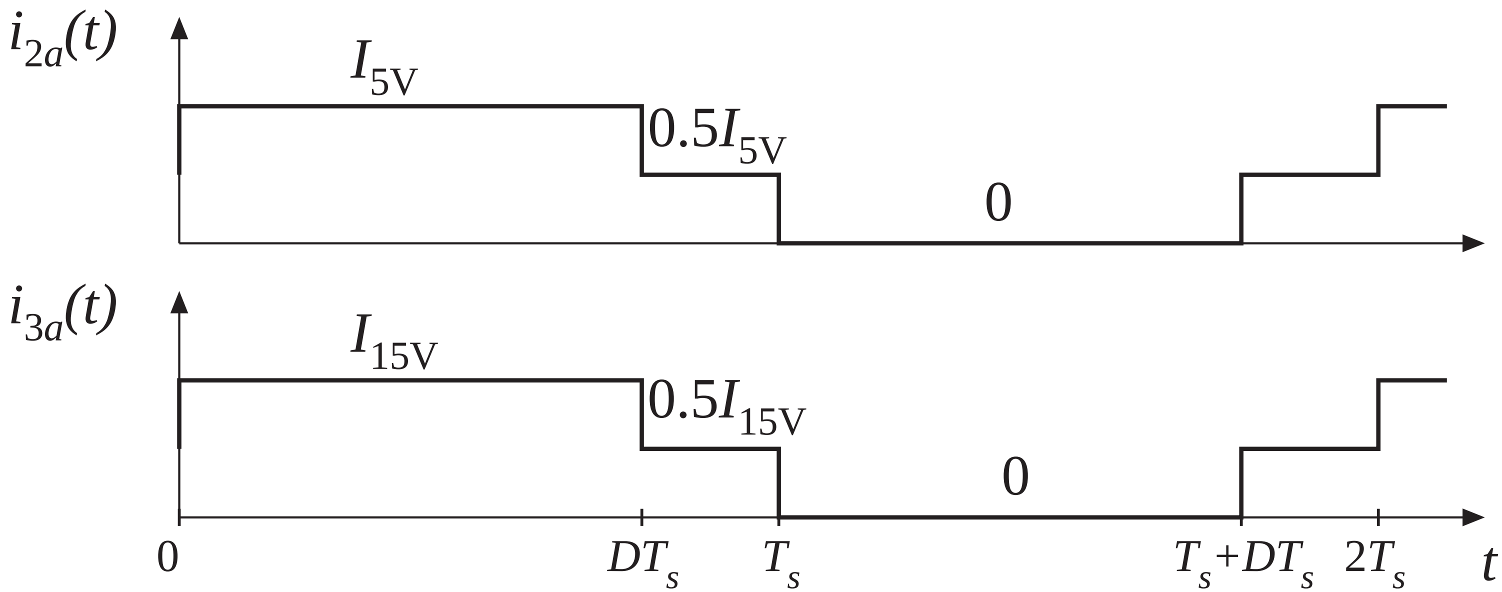
$$\lambda_1 = DT_s V_g = (0.75)\,(6.67\ \mu\text{sec})\,(160\ \text{V}) = 800\ \text{V–}\mu\text{sec}$$

Applied primary rms current



$$I_1 = \left(\frac{n_2}{n_1} I_{5V} + \frac{n_3}{n_1} I_{15V}\right)\sqrt{D} = 5.7 \text{ A}$$

Applied rms current, secondary windings



$$I_2 = \frac{1}{2} I_{5V} \sqrt{1 + D} = 66.1 \text{ A}$$

$$I_3 = \frac{1}{2} I_{15V} \sqrt{1 + D} = 9.9 \text{ A}$$

I_{tot}

RMS currents, summed over all windings and referred to primary

$$I_{tot} = \sum_{\substack{all\ 5\\ windings}} \frac{n_j}{n_1} I_j = I_1 + 2\,\frac{n_2}{n_1} I_2 + 2\,\frac{n_3}{n_1} I_3$$

$$= \left(5.7\text{ A}\right) + \frac{5}{110}\left(66.1\text{ A}\right) + \frac{15}{110}\left(9.9\text{ A}\right)$$

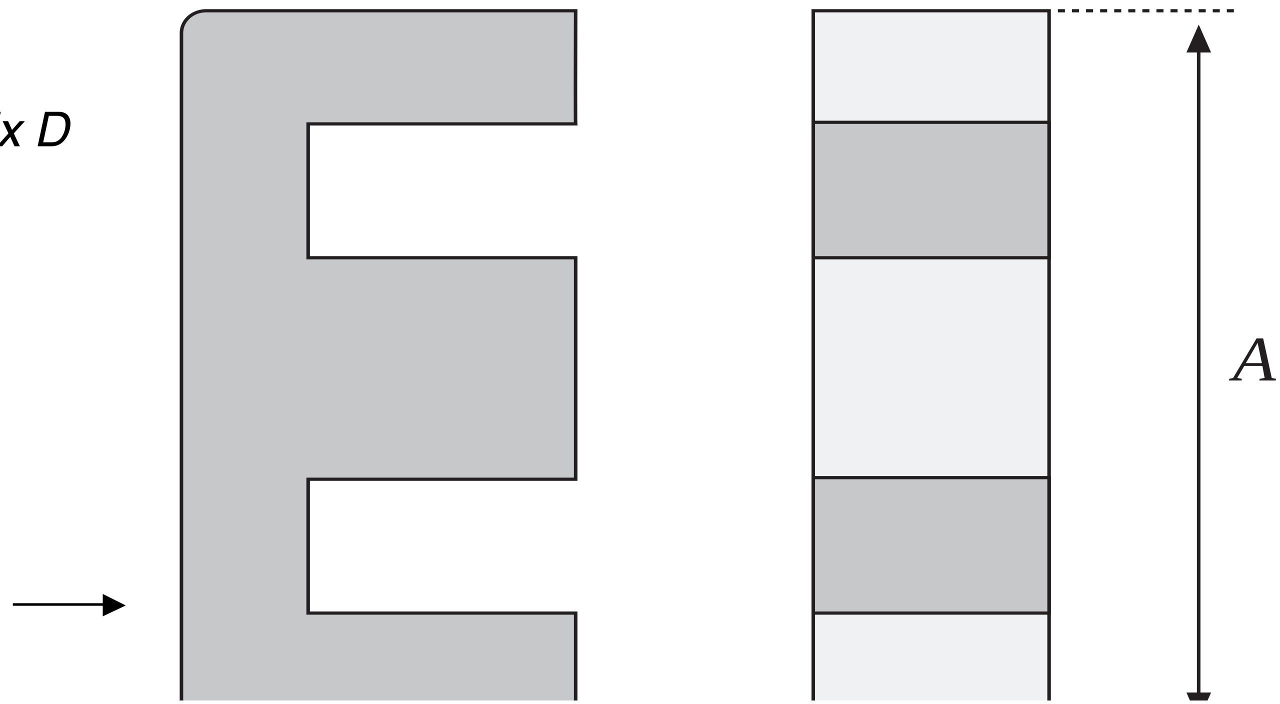
$$= 14.4\text{ A}$$

Select core size

$$K_{gfe} \geq \frac{(1.724\cdot10^{-6})(800\cdot10^{-6})^2(14.4)^2(7.6)^{(2/2.6)}}{4\,(0.25)\,(4)^{(4.6/2.6)}}\;10^8$$

$$= 0.00937$$

From Appendix D



Evaluate ac flux density ΔB

Eq. (15.20):

$$B_{max} = \left[10^8 \; \frac{\rho \lambda_1^2 I_{tot}^2}{2K_u} \; \frac{(MLT)}{W_A A_c^3 l_m} \; \frac{1}{\beta K_{fe}} \right]^{\left(\frac{1}{\beta + 2}\right)}$$

Plug in values:

$$\Delta B = \left[10^8 \, \frac{(1.724 \cdot 10^{-6})(800 \cdot 10^{-6})^2 (14.4)^2}{2(0.25)} \, \frac{(8.5)}{(1.1)(1.27)^3 (7.7)} \, \frac{1}{(2.6)(7.6)} \right]^{(1/4.6)}$$

$$= 0.23 \text{ Tesla}$$

This is less than the saturation flux density of approximately 0.35 T

Evaluate turns

Choose n_1 according to Eq. (15.21):

$$n_1 = \frac{\lambda_1}{2\Delta B A_c} \ 10^4$$

$$n_1 = 10^4 \frac{(800 \cdot 10^{-6})}{2(0.23)(1.27)}$$

$$= 13.7 \text{ turns}$$

Choose secondary turns according to desired turns ratios:

$$n_2 = \frac{5}{110} n_1 = 0.62 \text{ turns}$$

$$n_3 = \frac{15}{110} n_1 = 1.87 \text{ turns}$$

Rounding the number of turns

To obtain desired turns ratio of

110:5:15

we might round the actual turns to

22:1:3

Increased n_1 would lead to

- Less core loss
- More copper loss
- Increased total loss

Loss calculation
with rounded turns

With $n_1 = 22$, the flux density will be reduced to

$$\Delta B = \frac{(800 \cdot 10^{-6})}{2(22)(1.27)} 10^4 = 0.143 \text{ Tesla}$$

The resulting losses will be

$$P_{fe} = (7.6)(0.143)^{2.6}(1.27)(7.7) = 0.47 \text{ W}$$

$$P_{cu} = \frac{(1.724 \cdot 10^{-6})(800 \cdot 10^{-6})^2 (14.4)^2}{4\,(0.25)} \frac{(8.5)}{(1.1)(1.27)^2} \frac{1}{(0.143)^2} 10^8$$

$$= 5.4 \text{ W}$$

$$P_{tot} = P_{fe} + P_{cu} = 5.9 \text{ W}$$

Which exceeds design goal of 4 W by 50%. So use next larger core size: EE50.

Calculations with EE50

Repeat previous calculations for EE50 core size. Results:

$$\Delta B = 0.14 \text{ T},\ n_1 = 12,\ P_{tot} = 2.3 \text{ W}$$

Again round n_1 to 22. Then

$$\Delta B = 0.08 \text{ T},\ P_{cu} = 3.89 \text{ W},\ P_{fe} = 0.23 \text{ W},\ P_{tot} = 4.12 \text{ W}$$

Which is close enough to 4 W.

Wire sizes for EE50 design

Window allocations

$$\alpha_1 = \frac{I_1}{I_{tot}} = \frac{5.7}{14.4} = 0.396$$

$$\alpha_2 = \frac{n_2 I_2}{n_1 I_{tot}} = \frac{5}{110}\,\frac{66.1}{14.4} = 0.209$$

$$\alpha_3 = \frac{n_3 I_3}{n_1 I_{tot}} = \frac{15}{110}\,\frac{9.9}{14.4} = 0.094$$

Wire gauges

$$A_{w1} = \frac{\alpha_1 K_u W_A}{n_1} = \frac{(0.396)(0.25)(1.78)}{(22)} = 8.0 \cdot 10^{-3}\ \text{cm}^2$$

$$\Rightarrow \text{AWG \#19}$$

$$A_{w2} = \frac{\alpha_2 K_u W_A}{n_2} = \frac{(0.209)(0.25)(1.78)}{(1)} = 93.0 \cdot 10^{-3}\ \text{cm}^2$$

$$\Rightarrow \text{AWG \#8}$$

$$A_{w3} = \frac{\alpha_3 K_u W_A}{n_3} = \frac{(0.094)(0.25)(1.78)}{(3)} = 13.9 \cdot 10^{-3}\ \text{cm}^2$$

$$\Rightarrow \text{AWG \#16}$$

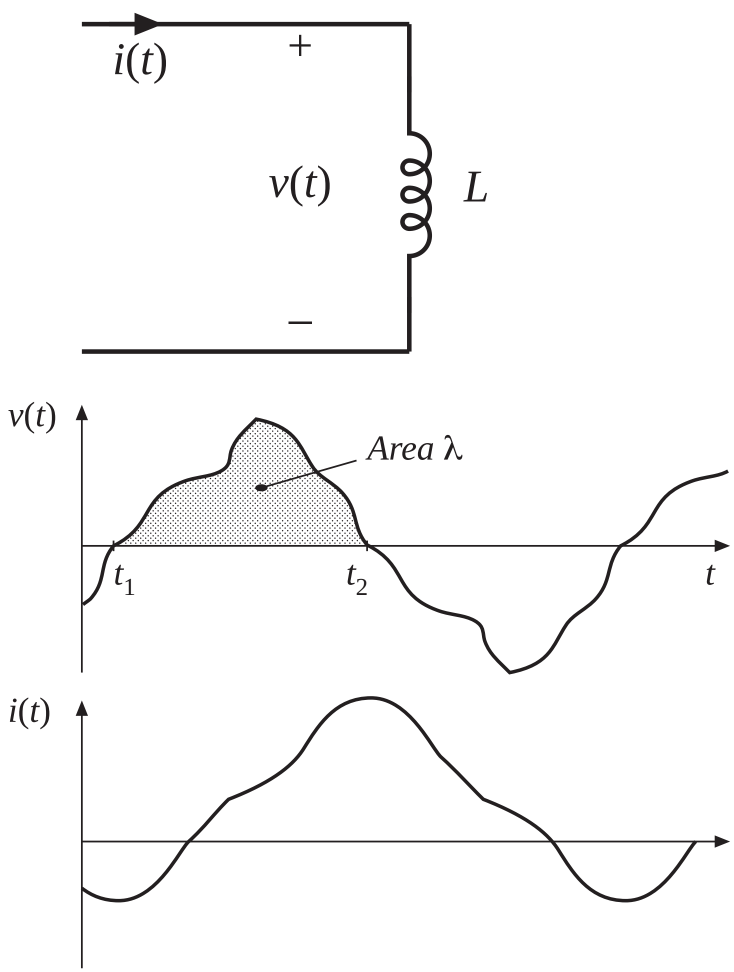
Might actually use foil or Litz wire for secondary windings

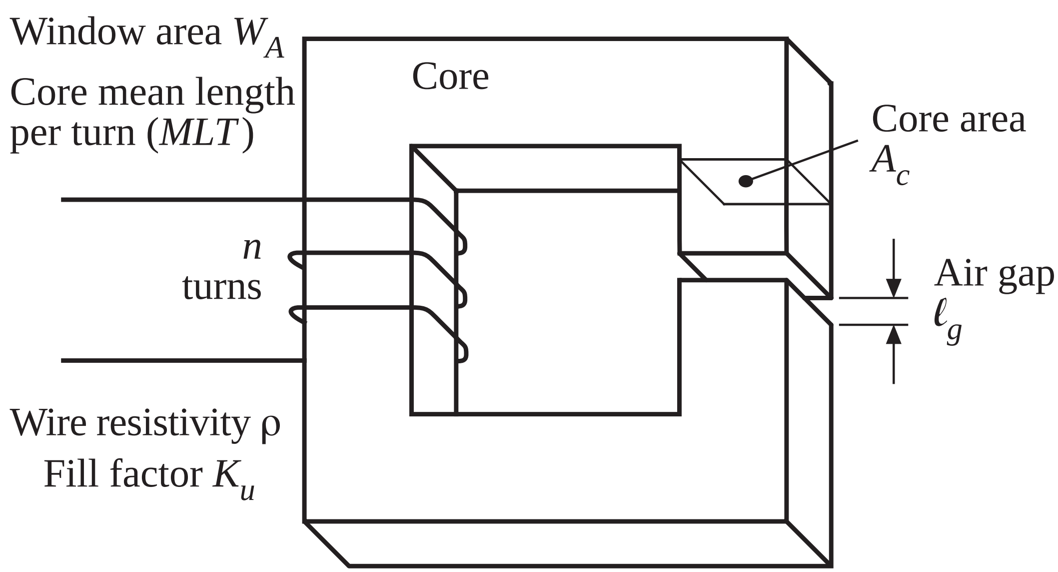
Discussion: Transformer design

- Process is iterative because of round-off of physical number of turns and, to a lesser extent, other quantities
- Effect of proximity loss
  - Not included in design process yet
  - Requires additional iterations
- Can modify procedure as follows:
  - After a design has been calculated, determine number of layers in each winding and then compute proximity loss
  - Alter effective resistivity of wire to compensate: define

    $\rho_{eff} = \rho \cdot P_{cu}/P_{dc}$ where P_{cu} is the total copper loss (including proximity effects) and P_{dc} is the copper loss predicted by the dc resistance.
  - Apply transformer design procedure using this effective wire resistivity, and compute proximity loss in the resulting design. Further iterations may be necessary if the specifications are not met.

15.4 AC Inductor Design





Design a single-winding inductor, having an air gap, accounting for core loss

(note that the previous design procedure of this chapter did not employ an air gap, and inductance was not a specification)

Outline of key equations

Obtain specified inductance:

$$L = \frac{\mu_0 A_c n^2}{\ell_g}$$

Relationship between applied volt-seconds and peak ac flux density:

$$\Delta B = \frac{\lambda}{2 n A_c}$$

Copper loss (using dc resistance):

$$P_{cu} = \frac{\rho n^2 (MLT)}{K_u W_A} I^2$$

Total loss is minimized when

$$\Delta B = \left[\frac{\rho \lambda^2 I^2}{2K_u} \frac{(MLT)}{W_A A_c^3 \ell_m} \frac{1}{\beta K_{fe}} \right]^{\left(\frac{1}{\beta + 2}\right)}$$

Must select core that satisfies

$$K_{gfe} \geq \frac{\rho \lambda^2 I^2 K_{fe}^{(2/\beta)}}{2K_u \left(P_{tot}\right)^{\left((\beta + 2)/\beta\right)}}$$

See Section 15.4.2 for step-by-step design equations