Modeling Infinity...

ELEC 041-Modeling and design of electromagnetic systems

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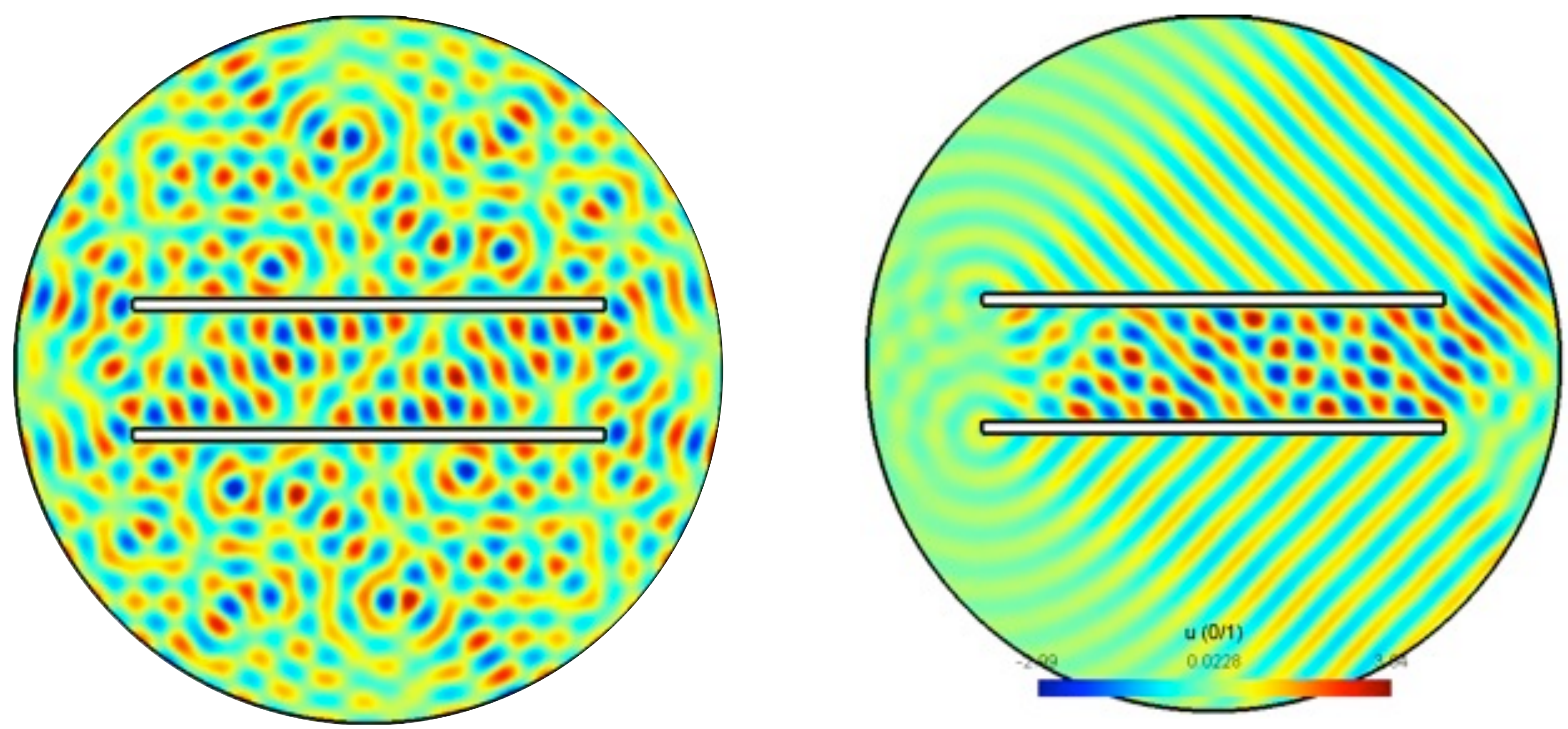
Infinite Domain and Truncation

- Electromagnetic problem defined in a unbounded domain
- A **fictitious boundary** Γ has to be introduced
- If arbitrary BC at finite distance, the radiated field is reflected towards the interior

➜ spurious fields



- A suitable boundary condition must be written on Γ
- Compromise between: accuracy, implementation and computational efficiency

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Contents

Boundary conditions at infinity

- Low frequency applications - Shell transformations
- Wave propagation
  - Global boundary conditions
  - Local boundary conditions
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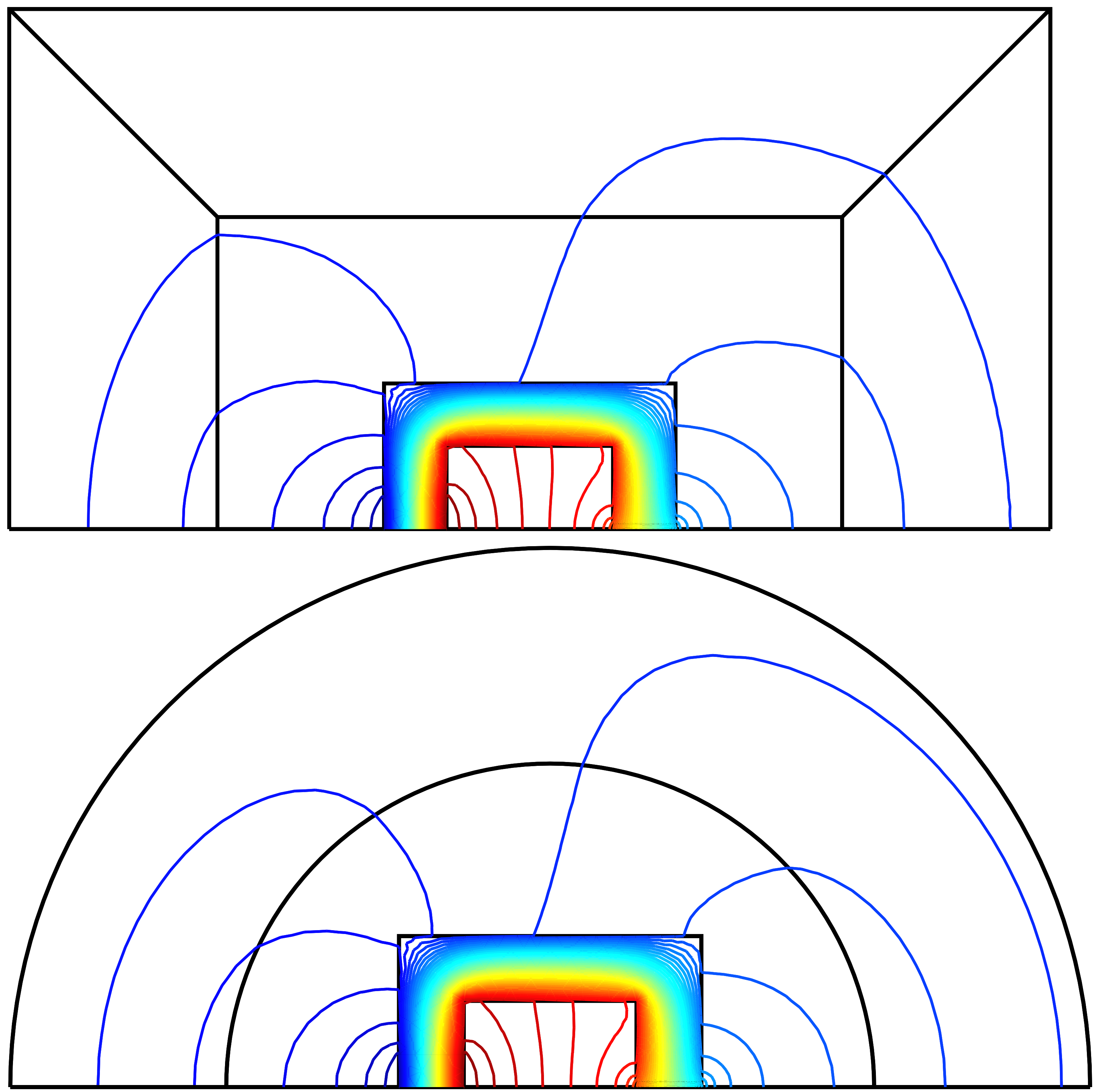
- Bonus
  - Pollution error
  - Iterative solution

Low Frequency: Shell Transformation



Low Frequency: Shell Transformation

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Boundary conditions must be imposed at infinity

➡ use of a shell transformation: $X^I - C^I = (y^i - C^j)\delta^I_j\, F(R_{int}, R_{ext}, r(y^j))$

$$F(R_{int}, R_{ext}, r) = \left(\frac{R_{int}(R_{ext} - R_{int})}{r(R_{ext} - r)}\right)^p \qquad \frac{dF}{dr} = -\theta\frac{F}{r}\,, \qquad \theta = \frac{R_{ext} - 2r}{p(R_{ext} - r)}$$

Jacobian matrix

$$n^i = \frac{y^i - C^i}{r}$$

$$\Lambda^I_j = \begin{pmatrix} 1 - \theta n^x \partial_x r & -\theta n^x \partial_y r & -\theta n^x \partial_z r \\ -\theta n^y \partial_x r & 1 - \theta n^y \partial_y r & -\theta n^y \partial_z r \\ -\theta n^z \partial_x r & -\theta n^z \partial_y r & 1 - \theta n^z \partial_z r \end{pmatrix}$$

This transformation applies to shells that are:

cylindrical

$$r(y^i) = \sqrt{(x - C^x)^2 + (y - C^y)^2}$$

parallelepipedic

$$r(y^i) = (y^k - C^k)$$

spherical

$$r(y^i) = \sqrt{(x - C^x)^2 + (y - C^y)^2 + (z - C^z)^2}$$

Generalities in Wave Propagation

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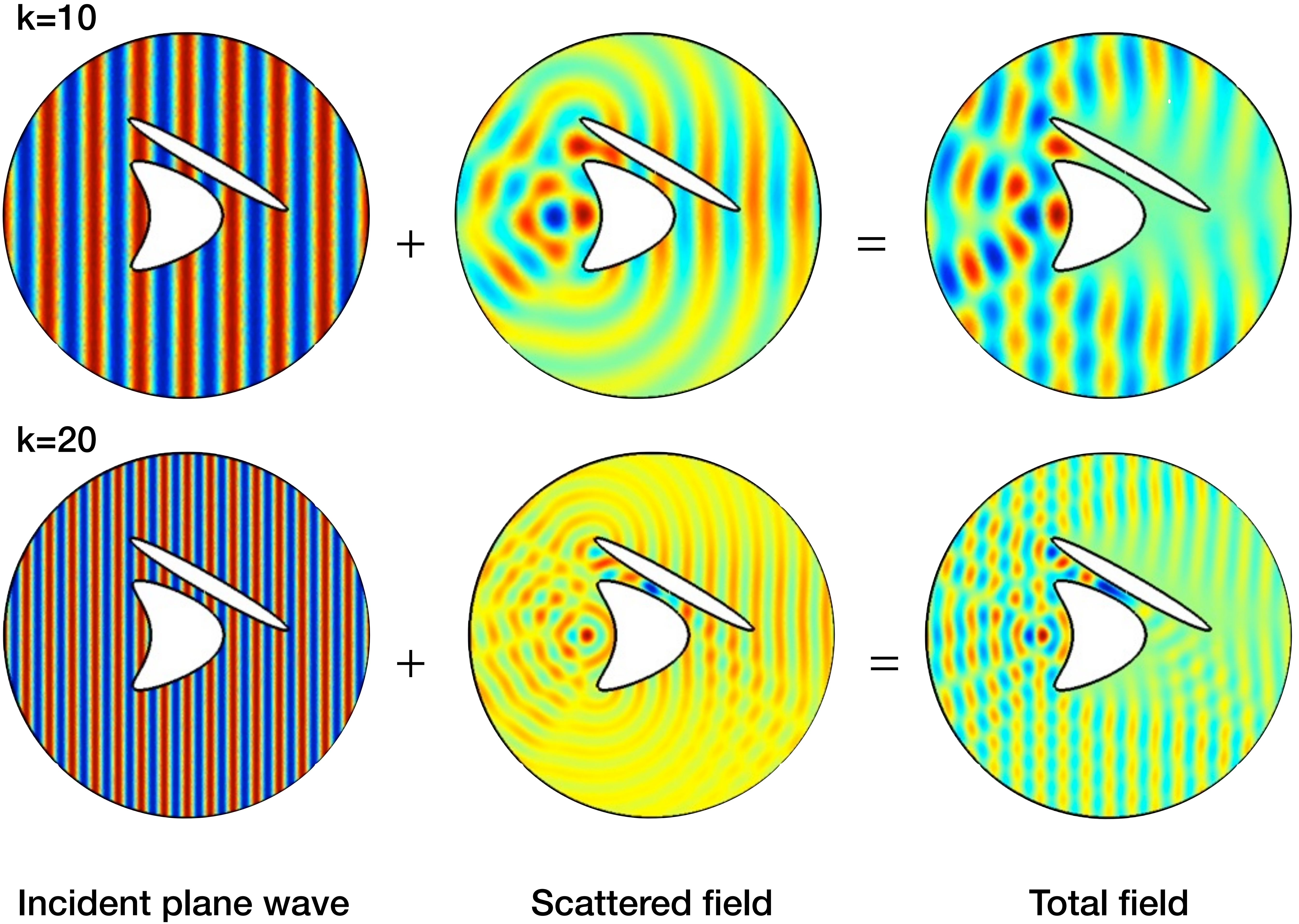
Basic steps for solving a wave propagation problem:

- formulations and numerical approximations (finite elements, finite differences, spectral methods, ...)
- truncation of the infinite of computation
  - global conditions
  - local conditions
  - absorbing layers
- iterative solver + preconditioning


k=10
+
=
k=20
+
=
Incident plane wave
Scattered field
Total field

Global Boundary Conditions

- Exact and non local
- It can be expressed as an integral operator set on the boundary Γ, e.g. through and integral representation formula
- Extremely expensive: while we are trying to solve a local PDE equation, the nonlocal form of the integral BC destroys the sparse matrix structure of the system
- Not applicable in practical cases
- Dirichlet-to-Neumann condition

Local Boundary Conditions

- Mostly approximations = Absorbing boundary conditions (ABC)
- They preserve the sparsity of the finite element matrix
- Examples:
  - Sommerfeld (Helmholtz) and Silver-Muller (Maxwell)
  - Including information about the shape of the boundary
    - Bayliss-Gunzburger-Turkel (BGT) (spherical/circular)
    - On-Surface Radiation Condition (convex boundaries)
  - High-order: Engquist-Majda

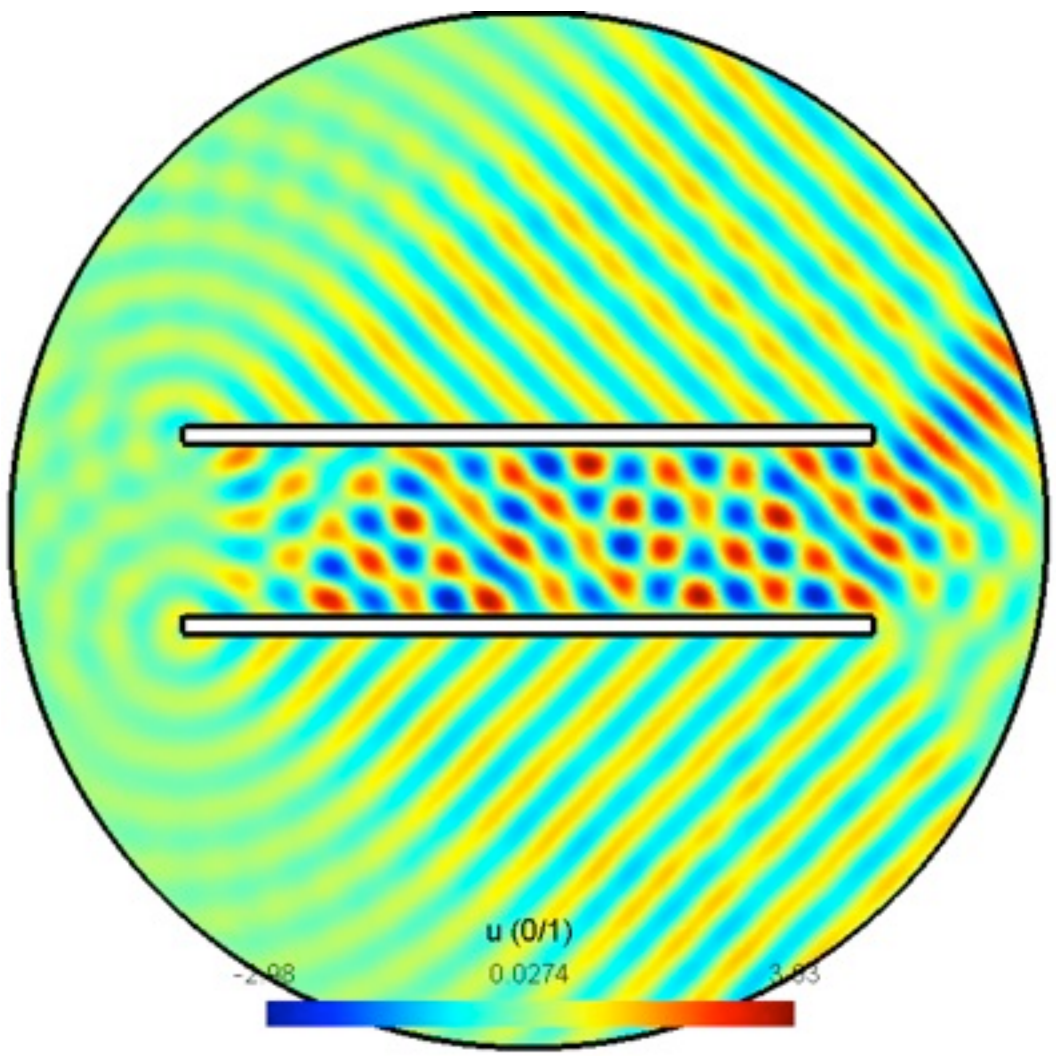
Local Boundary Conditions - Helmholtz

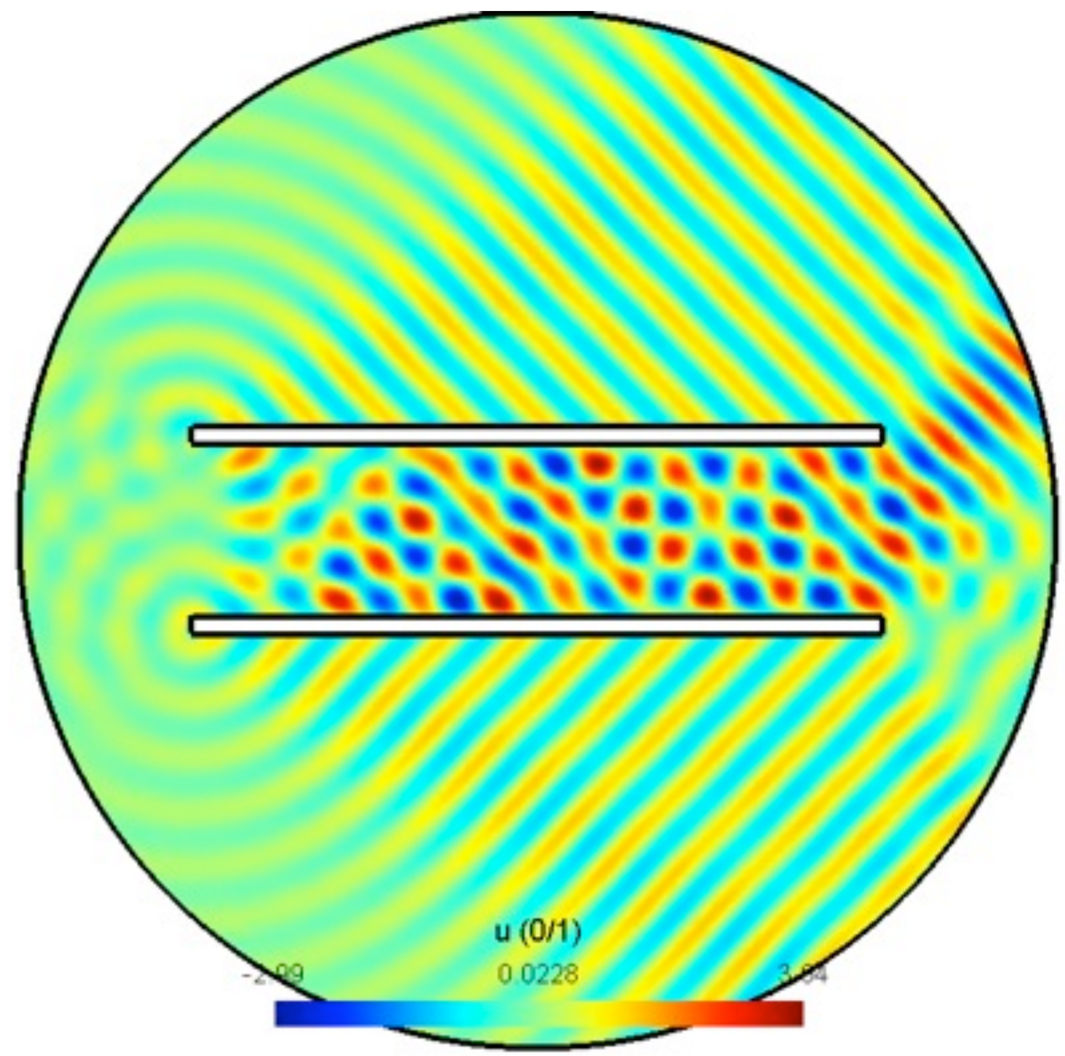
Sommerfeld ABC

$$\partial_{n_\Gamma} u = \imath k u$$

BGT ABC $\partial_{n_\Gamma} u = \imath k u - \alpha u - \beta u$

$$\alpha = \frac{1}{2R} - \frac{R^{-2}}{8(\imath k - R^{-1})} \qquad \beta = -\frac{1}{2(\imath k - R^{-1})}$$

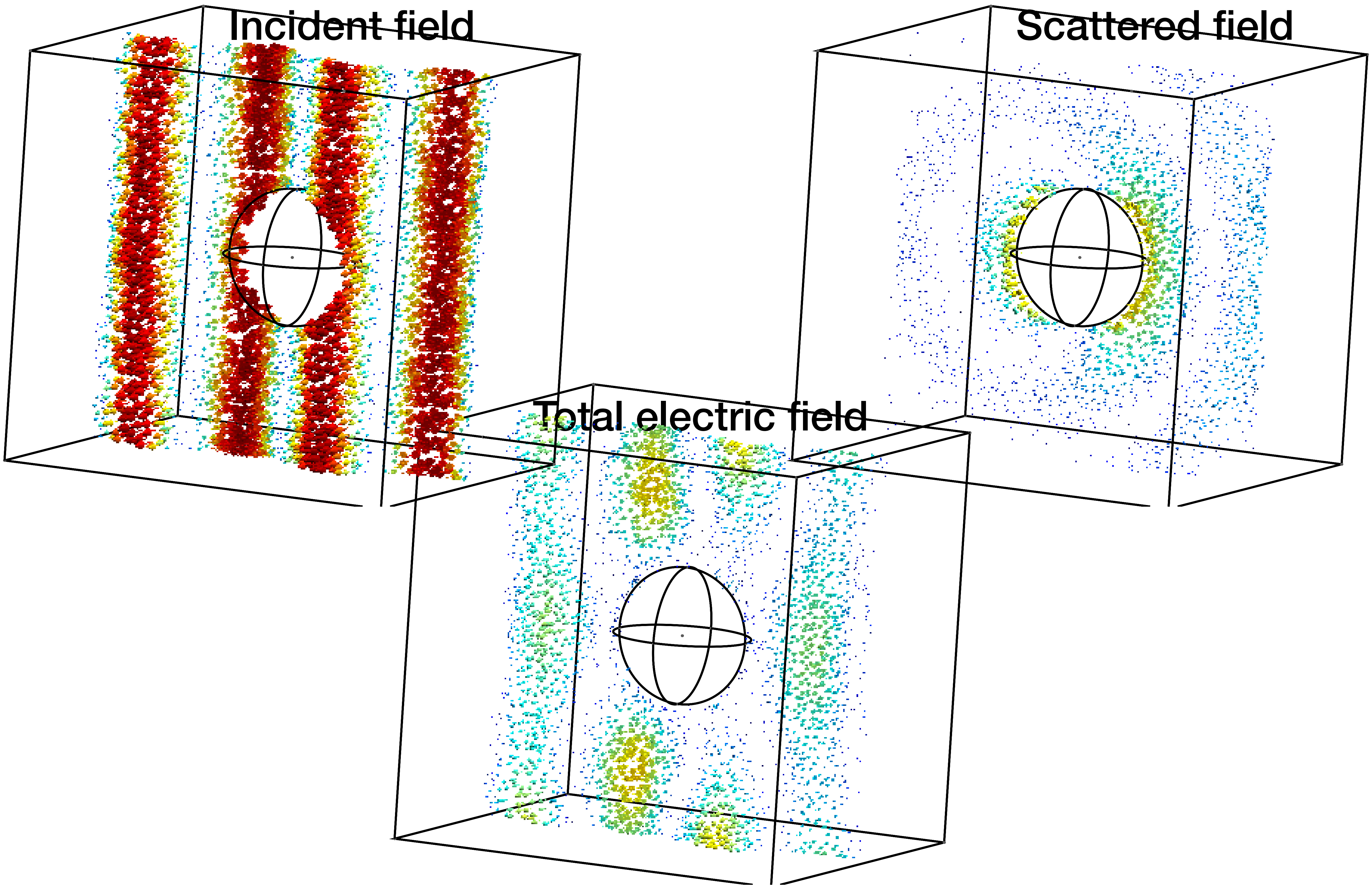




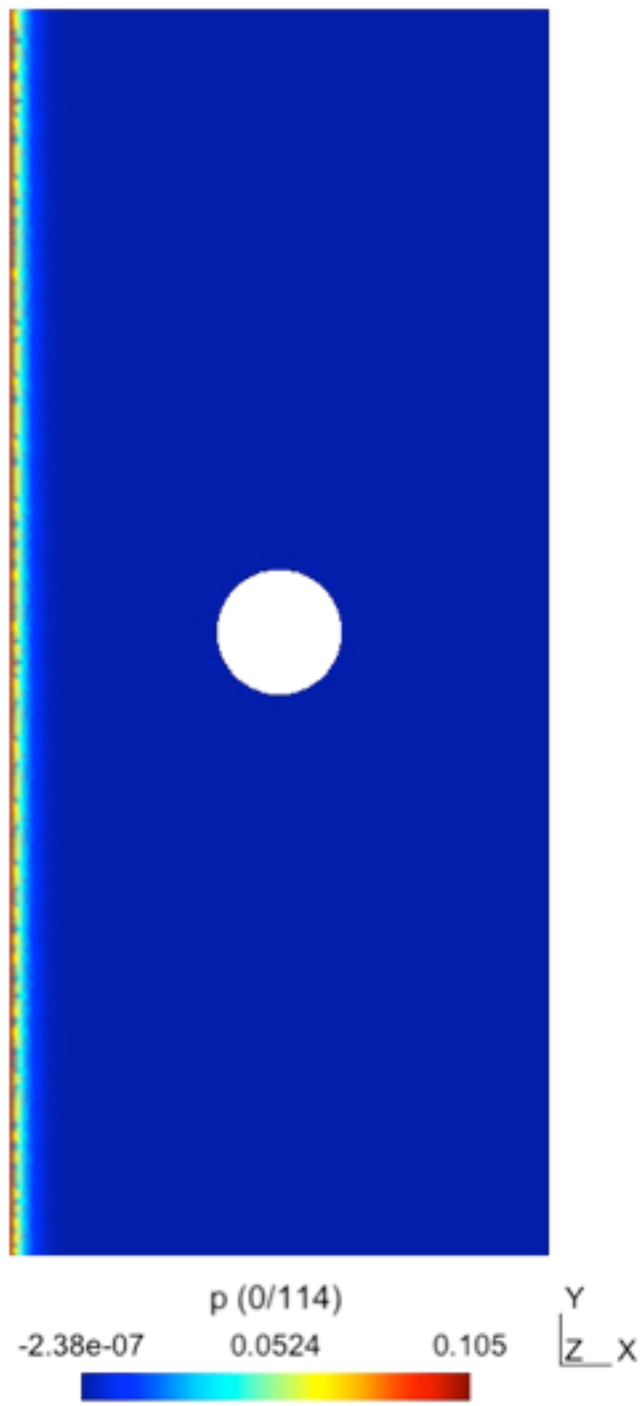
http://www.onelab.info/wiki/Multiple_scattering_with_Sommerfeld_absorbing_condition

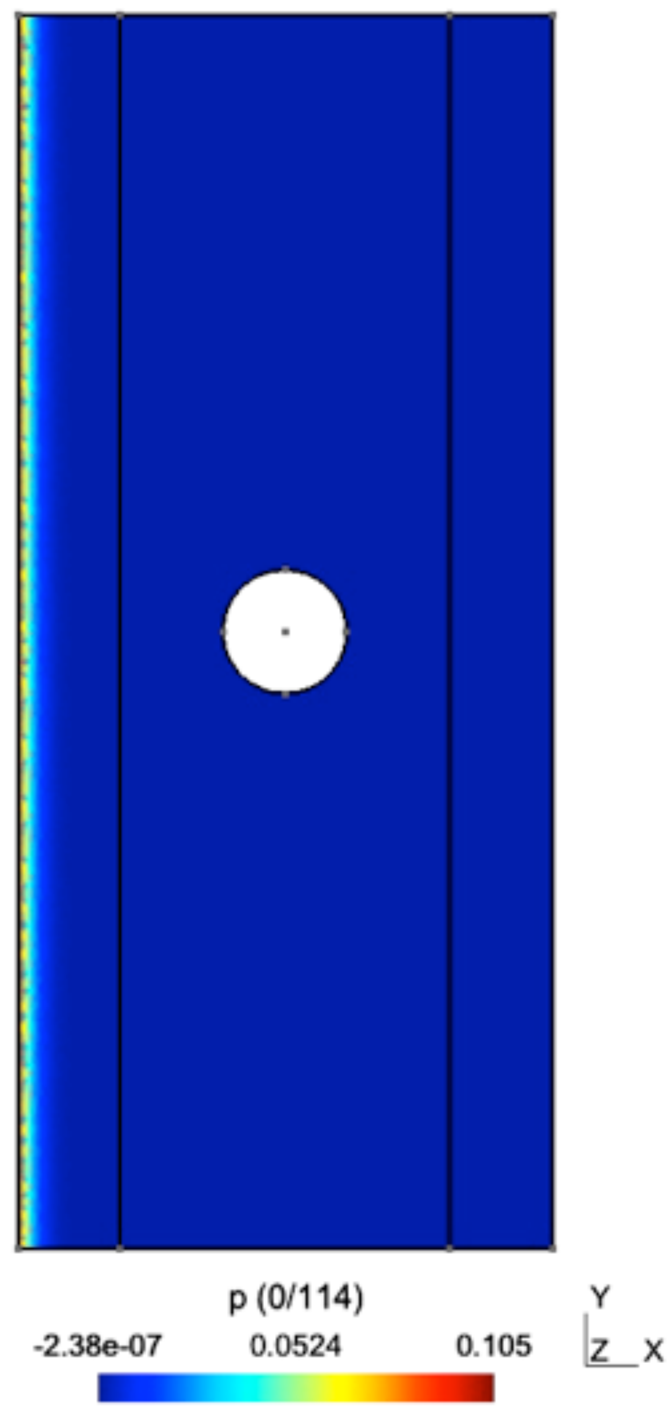
Local Boundary Conditions - Maxwell

Silver-Muller $\underline{n} \times (\underline{e}^{tot} - \underline{e}^{inc}) + \sqrt{\frac{\mu}{\varepsilon}}\, \underline{n} \times (\underline{n} \times (\underline{h}^{tot} - \underline{h}^{inc})) = 0$



PML vs ABC





Absorbing Layers

- Domain bounded by dissipative layer = “absorbing shell”
- Perfectly Matched Layers
  - Perfect wave transmission at interface, whatever the incidence
  - Media with modified EM characteristics: non physical
  - In the case of the 1D Helmholtz equation in the PML reads:

Helmholtz	modified Helmholtz in PML
$\partial^2_{xx} u + k^2 u = 0$	$\partial_x (\frac{1}{S_x} \partial_x u) + S_x k^2 u = 0$
$u = e^{\imath k x}$	$u = e^{\imath k x} e^{-\alpha x} e^{-\imath \beta x}$

http://www.onelab.info/wiki/Multiple_scattering_with_a_Perfectly_Matched_Layer_(PML)

Absorbing Layers

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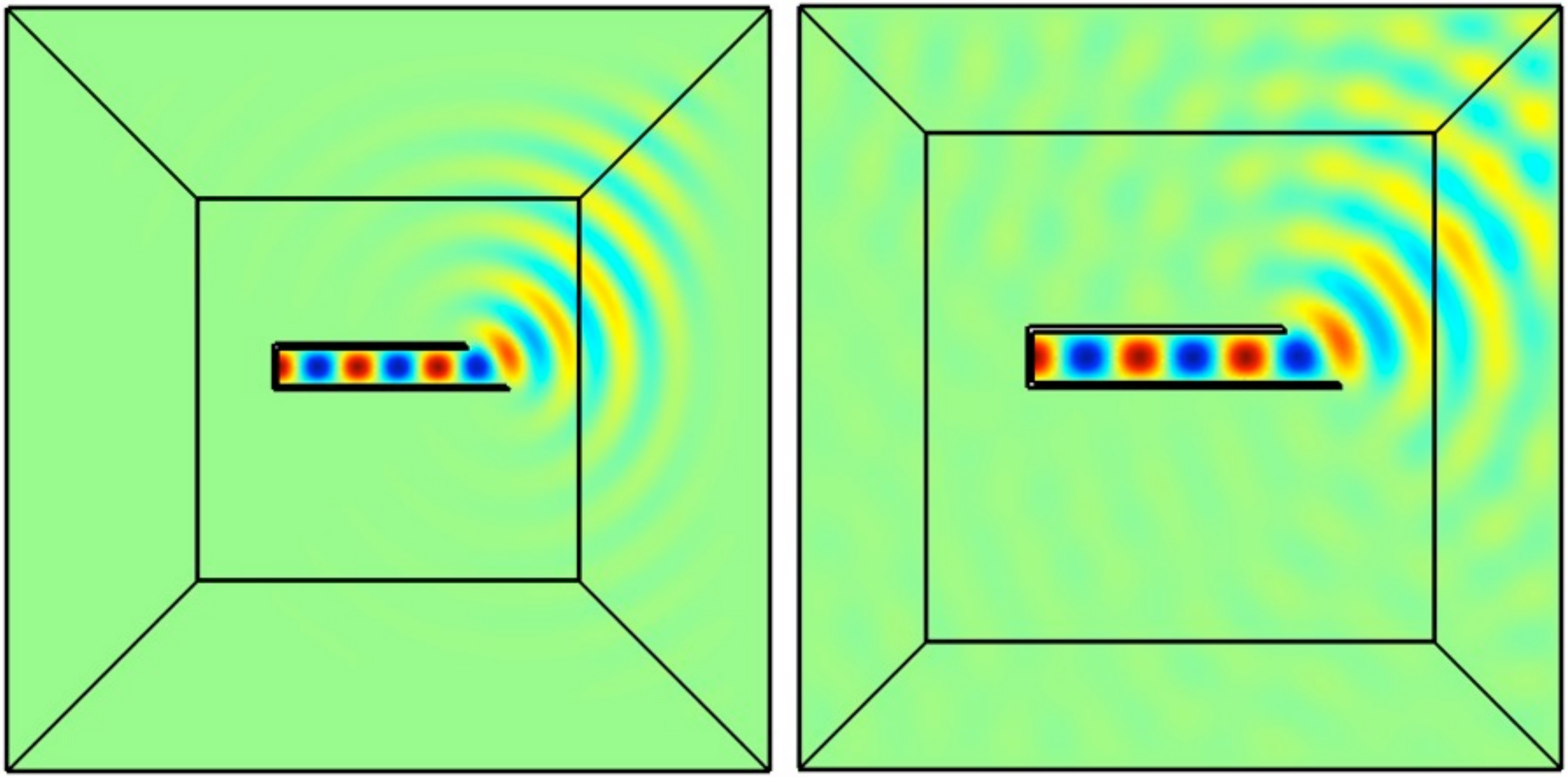
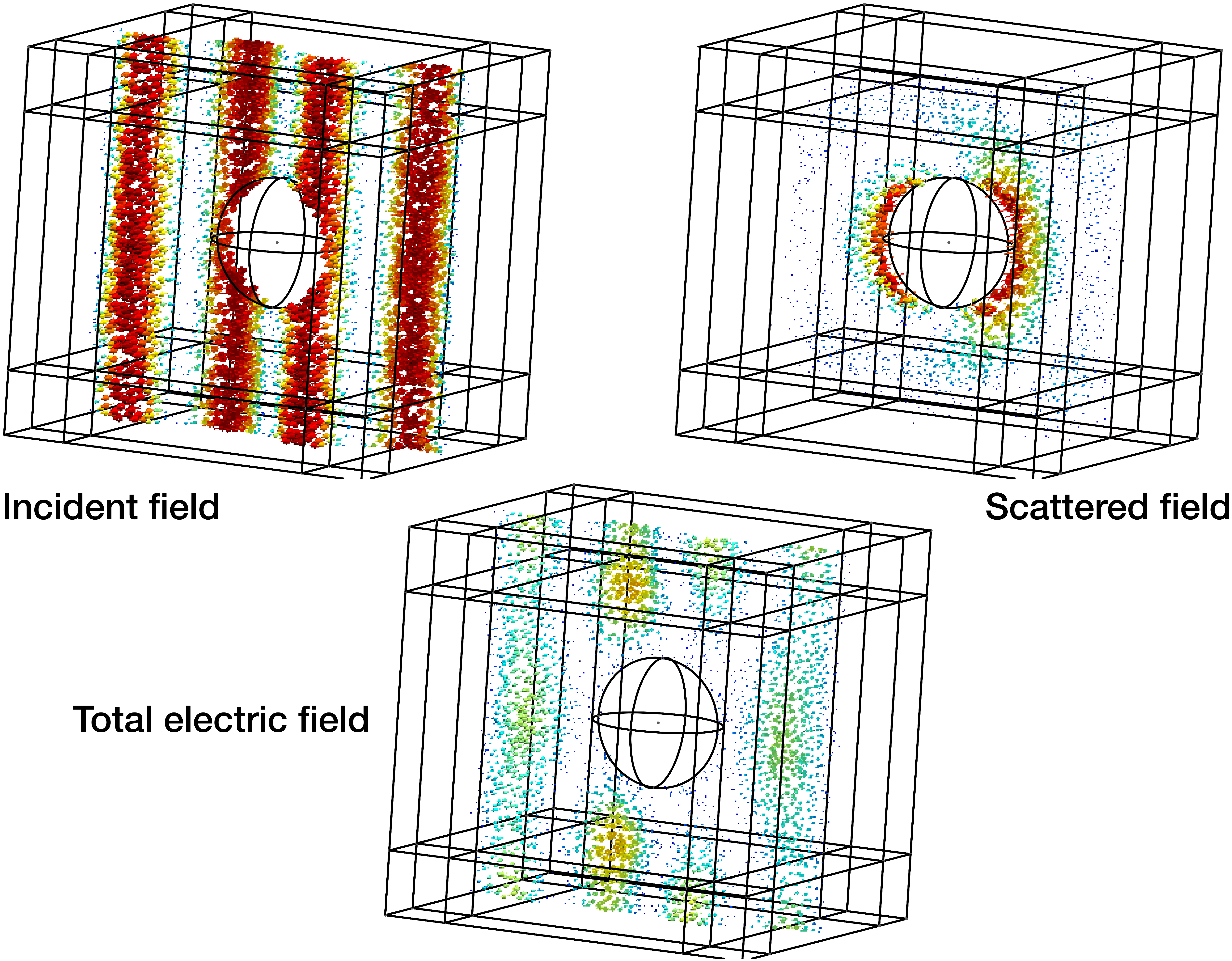
The choice of the PML parameters is crucial for a good performance



Good

Bad



Incident field
Scattered field
Total electric field