ELEC 0041: Homework assignments
Electromagnetic and thermal analysis of a high voltage cable

February 2, 2023

The objective of the homework is to study a three-phase power cable system using electric, magnetic and magneto-thermal models.

You start by choosing a power cable (high, medium voltage) from e.g. scientific papers, datasheets (single-core cables, three-core cables, magnetically shielded cables...):
NKT cables: 33 kV single core cable XLPE-AL
NKT cables: Medium Voltage Cables with XLPE Insulation
ABB cable systems

Note: You may choose a single or three core cable but in any case, you are asked to model a three-phase system.

You will perform an electromagnetic and thermal study of the cable you choose, see e.g. an example cable in Fig. 1. From the article/datasheets, you will need to get the dimensions, and the electromagnetic and the thermal properties, similar to data in the Tables 1 and 2. You will also need to place the cable in an environment: air, ground,... (with the corresponding properties) and bound the problem.

As starting point, you get a simple cable example (`cable.pro`, `cable.geo`) for linking the cable geometry (you have to make) to the FE formulations:

- electrodynamic formulation in `electrodynamic_formulation.pro`;
- magnetodynamic formulation in `magnetodynamic_formulation.pro`;
- coupled magneto-thermal formulation in `magneto-thermal_formulation.pro`.

The file `Jacobian_Integration.pro` contains the Jacobian transformation and the numerical integration details (number of Gauss points), common definitions for all the formulations.

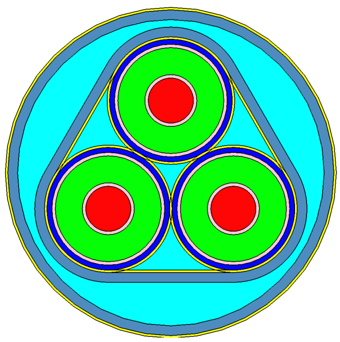

Figure 1: Left: Picture of a high voltage three-phase underground cable from NKTcables. Right: Geometrical model, the HV cable comprises: three copper stranded conductors (red), XLPE (green), semiconductors (pink and yellow), APL-sheath (dark blue), steel armour and pipe (grey), air (light blue).

The cable in Fig. 1 is a three-core cable with stranded copper conductors and cross linked polyethylene (XLPE) as insulating material. A semi-conductor layer surrounds the copper and the XLPE insulation. Each single core is surrounded by aluminum polyethylene laminated (APL) sheath. The group of the three cores is sheathed by polyethylene. Both wire armor and external pipe are made of steel. Finally, the external covering of the pipe is a polyethylene layer. Electromagnetic and thermal properties of the cable are given in Table 2.

Parameter	**dimensions [mm]**
Conductor diameter	18.4
Inner semiconductor thickness	1.4
XLPE-insulation thickness	11.0
Outer semiconductor thickness	1.4
APL-sheath thickness	2.0
Polyethylene-sheat thickness	1.0
Steel-armour thickness	2.0
Steel-pipe thickness	4.0
Polyethylene-covering thickness	1.0
Outer diameter	135.0

Table 1: Geometrical data.

The cable is placed at a depth of 1.2 m. The three-phase voltages and currents are balanced at frequency $f = 50\,\mathrm{Hz}$.

Material	ϵ_r	μ_r	σ [S/m]	κ [W/(m·K)]
copper	1	1	5.99e7	400
steel	1	4	4.7e6	50.2
aluminum	1	1	3.77e7	237
polyethylene	2.25	1	1.0e-18	0.46
semiconductor	2.25	1	2	10
XLPE	2.5	1	1.0e-18	0.46
soil (dry)	1	1	28	0.4

Table 2: Electromagnetic and thermal material properties.

1 Electrodynamic analysis – Due on March 24, 2023

The goal of this analysis is to determine the electric stress within the cable. Too high electric field in the cable can cause dielectric damage or early ageing. The problem can be solved by using an electrodynamic model accounting for the resistive and capacitive effects. Assume a time-harmonic steady-state problem (frequency f, angular frequency $\omega = 2\pi f$).

Answer the following questions:

- Define your computational domain, where do you truncate the domain? Link to the boundary conditions.
- Add a defect to the insulation of your cable (geometry) and perform the simulation with and without defect and check that the electric field respects the above limits (datasheet).
- Compute the per-unit-legth capacitance (μF/km) and compare with an analytical value (indicate assumptions if any).
- How do the results vary with the mesh refinement?
- Can you simplify the geometry of your cable without degrading the precision? (E.g. disregard details in the geometrical description)
- Can you improve the cable design?

2 Magnetodynamic analysis – Due on April 28, 2023

Let us add now the magnetic part. We may solve the quasi-static steady-state electromagnetic problem (frequency f, angular frequency $\omega = 2\pi f$) via a magnetic-vector-potential electric-scalar-potential formulation (file provided, see also templates).

Provide the following results:

1. Define your computational domain, where do you truncate the domain? Link to the boundary conditions.
2. Get the magnetic flux density (or induction) in your cable.
3. Get the current density in the conducting parts of your cable.
4. Get the joule losses (kW/km) in the conducting parts of your cable.
5. Get the per-unit AC-resistance (Ω/km).
6. Get the per-unit inductance (mH/km).
7. How do the results vary with the mesh refinement?
8. Can you simplify the geometry withouth degrading the precision? (E.g. disregard details in the geometrical description)
9. Can you improve the cable design to reduce the losses?

3 Coupled magneto-thermal analysis – Due on May 19, 2023

The losses computed via the magnetodynamic model are the source for a simple steady-state heat conduction problem. The steady-state heat conduction equation to consider is:

$$-\mathrm{div}\,(\kappa\,\mathrm{grad}\,T) = Q\,, \tag{1}$$

with T the temperature distribution (K), κ the thermal conductivity W/(m·K), and Q the heat source (W/m^3). The heat source Q is given by the local joule losses $Q = p$ (**??**) [1]. You need to define the ambient temperature (e.g. $T_{\text{amb}} = 15°\text{C}$) and pay attention to the operating temperature of the cable given in the datasheets.

Solve the coupled problem and determined the temperature distribution in the cable and surroundings. Concretely, answer the following questions:

1. Define your computational domain, where do you truncate the domain? Link to the boundary conditions.
2. Get the temperature distribution in the cable. Is it in the operating temperature limits (check datasheets)?
3. Get the temperature distribution around the cable. What is the maximum temperature at the interface ground/air?
4. How do the results vary with the mesh refinement?

Till this point, we have consider that the conductivity σ does not depend on the temperature T. Let us account for an electrical conductivity that varies with the temperature:

$$\sigma(T) = \frac{\sigma_0}{1 + \alpha(T - T_{\text{ref}})}\,, \tag{2}$$

with σ_0 the conductivity at $T_{\text{ref}} = 20°\text{C}$, given in Table 2, and α equal to 0.00386K^{-1} for the copper and 0.00390K^{-1} for the aluminum.

Solve the coupled nonlinear problem, iterating between the electromagnetic and the thermal problems. Obtain the following quantities, field distributions and compare with the linear case (σ invariant):

[1] The thermal source is computed from the complex MVP {a}, without Dof. Notation: <a>[my_function] must appear in front of any function applied to a in the thermal formulation to indicated that the result of the operation is a real quantity.

1. How many nonlinear iterations do you need for achieving convergence (stop criterion $1e-6$)? Is this convergence influenced by the mesh density?
2. Get the magnetic flux density (or induction) in the cable.
3. Get the current density in the aluminum shealth, the steel armour and the steel pipe.
4. Get the joule losses (kW/km) in the conducting parts of your cable.
5. Get the AC-resistance-per-unit length (Ω/km).
6. Get the inductance-per-unit length (mH/km).
7. Get the temperature distribution in the cable. Is it in the operating temperature limits?
8. Get the temperature distribution around the cable. What is the maximum temperature at the interface ground/air?

4 To include in the report

The `geo` and `pro` files you have created and/or modified.

Electrodynamic analysis:

- Post-processing map of the electric potential in the cable area.
- Post-processing map of the electric field in the cable area (with and without defect).
- Post-processing map of the displacement current in the cable area (with and without defect).
- Electric field stress of the cable in a phase (cut in the radial direction) when adding a defect in the insulation. Comparison with the case without.
- Per-unit-capacitance (μF/km), comparison with analytical capacitance.

Magnetodynamic analysis:

- Post-processing map of the magnetic flux density norm in the cable area.
- Post-processing map of the current density norm in the conducting parts of your cable.
- Joule losses (kW/km) in the conducting parts of your cable.
- Per-unit AC-resistance (Ω/km).
- Per-unit inductance (mH/km).

Coupled magneto-thermal analysis

- Post-processing map of the temperature in the cable.
- Post-processing map of the temperature outside the cable.

For the nonlinear coupled case with $\sigma(T)$, the new maps and values asked in previous section.

5 Additional investigation (optional)

However, depending on the type of cable system you choose, it maybe useful to adapt e.g.:

- An under-sea cable may be laying in a pipe, buried in soil or directly in water, what would be impact of the different environment?
- You may consider different cooling conditions, e.g. include convection at the air-soil interface, vary the depth at which the cable is buried, or the distance between the cable and soil if you are dealing with an aerian cable...
- Effect of the cable design on the temperature distribution.
- You could even consider multiple cables laying on a cable tray, what is then the best cable arrangement for a given specification?
- Any idea you may get when working on this cable project!

Once you have chosen your cable system and delivered your first assignment, the teaching team will indicate the precise additional investigation to perform. It could be one of possibilities above or something similar.