Electromagnetic Energy Conversion - ELEC0431 : Introduction to exercise sessions

5 February 2021

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Contact and Organization

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Exercise sessions are organized on Friday mornings from 10.30 am to 12.30 am.

The exercise manual will be updated for each theme of the course.

Organization

Themes

- Useful reminders
- Transformers
- AC synchronous machines
- AC asynchronous machines
- DC machines
- Electronic control systems

Laboratory sessions

4 sessions

- Laboratory session 1: Transformers,
- Laboratory session 2: AC synchronous machines,
- Laboratory session 3: AC asynchronous machines.
- Laboratory session 4: DC machines,

The lab sessions will be provided through podcasts. At the end of the semester, face-to-face sessions might be organized if the sanitary conditions permit it.

The manual of laboratory will be updated concurrently with the podcasts.

Course Introduction

Wooclap code : HKEOZH

ELEC0431 : Exercise session 1

Reminders of phasors and power in the sinusoidal steady state

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Phasors and complex power

Wooclap code : OSKIYB

The phasor definition

$$f(t) = \underbrace{\sqrt{2}\,F_{\text{RMS}}}_{\text{amplitude}} \cos(\underbrace{\omega}_{\text{pulsation}} t + \underbrace{\theta}_{\text{phase angle}})$$

$$= \text{Re}\left(\sqrt{2}\,F_{\text{RMS}}\,e^{(j\omega t+\theta)}\right)$$

$$= \text{Re}\left(\sqrt{2}\,\underbrace{F_{\text{RMS}}\,e^{\theta}}_{\overline{F}}\,e^{j\omega t}\right)$$

$\overline{F}$, the phasor of $f(t)$ is a complex number, with its amplitude and its phase angle.

The phasor definition

The main assumptions of the phasor approximation are a constant frequency and linearity.

Linearity ensured as Kirchhoffs' laws and differential equations are linear.

Time domain		Frequency domain
$f(t)$	$\Longleftrightarrow$	$\overline{F}$
$a\,f(t) + b\,g(t)$	$\Longleftrightarrow$	$a\,\overline{F} + b\,\overline{G}$
$\dfrac{d\cdot}{dt}$	$\Longleftrightarrow$	$j\,\omega$
$\int \cdot\, dt$	$\Longleftrightarrow$	$\dfrac{1}{j\,\omega}$

RMS value and peak value

The Root Mean Square (RMS) value of a function, also called the effective value is defined by

$$F_{\text{RMS}} := \sqrt{\frac{1}{T}\int_0^T f(t)^2 \, dt} \tag{1}$$

In the case of a harmonic function of angular frequency ω, where the base period of the function is $T = \frac{2\pi}{\omega}$,

$$\boxed{F_{\text{peak}} = \sqrt{2}\, F_{\text{RMS}}} \tag{2}$$

which is the result of the following calculations :

$$\begin{aligned}\int_0^T f(t)^2 \, dt &= \int_0^T F_{\text{peak}}^2 \cos^2(\omega t + \theta) \, dt \\ &= \frac{F_{\text{peak}}^2}{2} T\end{aligned}$$

Product of 2 harmonic functions

In the continuity of the phasor definition, one can present the product of two harmonic functions as

$$\begin{aligned} f(t)\, g(t) &= F_{\text{peak}} \cos(\omega\, t + \theta) \quad G_{\text{peak}} \cos(\omega\, t + \psi) \\ &= 2\, F_{\text{RMS}}\, G_{\text{RMS}} \cos(\omega\, t + \theta) \cos(\omega\, t + \psi) \\ &= F_{\text{RMS}}\, G_{\text{RMS}} \left[\underbrace{\cos(\theta - \psi)}_{\text{DC, 0 Hz}} + \cos(2\,\omega\, t + \theta + \psi) \right] \end{aligned}$$

The simplification of the factor of 2 justifies the definition of the phasor as

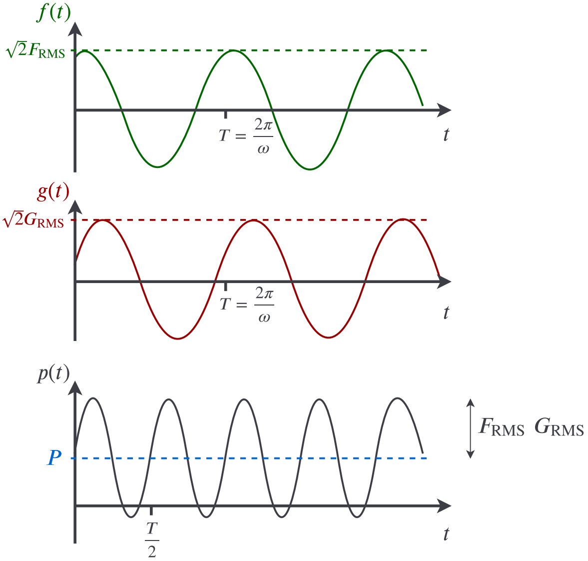
$$\boxed{\overline{F} = F_{\text{RMS}}\, e^{j\,\theta}} \tag{3}$$

The instantaneous power $p(t)$

In the time domain, the definition of the instantaneous power $p(t)$ is expressed as the product

$$\begin{aligned} p(t) &= f(t)\, g(t) \\ &= \underbrace{F_{\text{RMS}}\, G_{\text{RMS}}\, \cos(\theta - \psi)}_{P} + \underbrace{F_{\text{RMS}}\, G_{\text{RMS}}\, \cos(2\,\omega\, t + \theta + \psi)}_{p_f(t)} \end{aligned}$$

The instantaneous power $p(t)$



The instantaneous power $p(t)$

The instantaneous power can be further developed

$$
\begin{aligned}
p(t) &= F_{\mathrm{RMS}}\, G_{\mathrm{RMS}}\, \left(\cos(\theta-\psi) + \cos(2\,\omega\, t + \theta + \psi)\right) \\
&= F_{\mathrm{RMS}}\, G_{\mathrm{RMS}}\, \Big(\cos(\theta-\psi) + \cos\,\left(2\,(\omega\, t + \theta) - (\theta - \psi)\right)\Big) \\
&= F_{\mathrm{RMS}}\, G_{\mathrm{RMS}}\, \Big(\cos(\theta-\psi) + \cos\,\left(2\,(\omega\, t + \theta)\right)\cos(\theta-\psi) \\
&\qquad\qquad + \sin\,\left(2\,(\omega\, t + \theta)\right)\sin(\theta-\psi)\Big) \\
&= \underbrace{F_{\mathrm{RMS}}\, G_{\mathrm{RMS}}\, \cos(\theta-\psi)}_{P,\ \text{the active power}}\Big(1 + \cos\,\left(2\,(\omega\, t + \theta)\right)\Big) \\
&\qquad + \underbrace{F_{\mathrm{RMS}}\, G_{\mathrm{RMS}}\, \sin(\theta-\psi)}_{Q,\ \text{the reactive power}}\ \sin\,\left(2\,(\omega\, t + \theta)\right)
\end{aligned}
$$

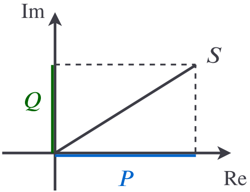
The power, here defined as the product of two harmonic functions, can be expressed in the frequency domain (*i.e.* with phasors). It is then called the complex power

$$S = \overline{F}\,\overline{G}^* \tag{4}$$

$$= F_{\mathrm{RMS}}\,G_{\mathrm{RMS}}\,\cos(\theta-\psi) + j\,F_{\mathrm{RMS}}\,G_{\mathrm{RMS}}\,\sin(\theta-\psi)$$

$$= \qquad P \qquad + j \qquad Q \tag{5}$$

where one can retrieve the active power P and the reactive power Q as previously defined.



Important note about the notations

Remark : The following conventions will be used in the rest of the course for RMS and peak value.

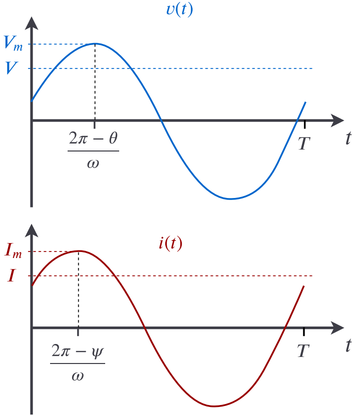
$$F_{\text{RMS}} \quad \Longrightarrow \quad F \quad (V \ \text{ and } \ I)$$

$$F_{\text{peak}} \quad \Longrightarrow \quad F_m \quad (V_m \ \text{ and } \ I_m)$$

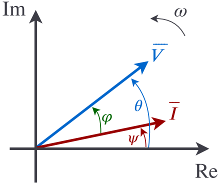
The concept of phasor can be applied to electrical quantities such as voltages and currents

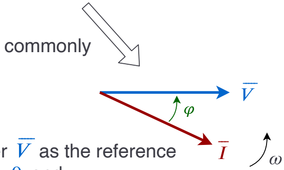
Time domain		Frequency domain
$v(t) = \sqrt{2}\, V \cos(\omega t + \theta)$	$\Longleftrightarrow$	$\overline{V} = V\, e^{j\,\theta}$
$i(t) = \sqrt{2}\, I \cos(\omega t + \psi)$	$\Longleftrightarrow$	$\overline{I} = I\, e^{j\,\psi}$

Alternating voltages and currents - Phasor diagrams



$\Longleftrightarrow$





Consider $\overline{V}$ as the reference
with $\theta = 0$ and $\varphi = -\psi$

$$\boxed{\varphi = \theta - \psi} \tag{6}$$

The complex power (for electrical quantities) and the apparent power

The definition of the complex power remains exactly the same for electrical quantities

$$\begin{aligned} S &= \overline{V}\ \overline{I}^* \\ &= V\, e^{j\,\theta}\ I\, e^{j\,(-\psi)} \\ &= V\ I\ e^{j\,(\theta-\psi)} \\ &= V\ I\ e^{j\,\varphi} \\ &= V\ I\ \cos(\varphi) + j\ V\ I\ \sin(\varphi) \\ &= \qquad P \qquad + j \quad Q \end{aligned} \tag{7}$$

Based on that definition, one also defines the apparent power as the norm of the complex power :

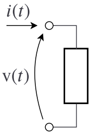
$$|S| = \sqrt{P^2 + Q^2} = V\ I \tag{8}$$

Power reference of the one-port ("le dipôle")

The signal $v(t)$ is the voltage accross the one-port.

The signal $i(t)$ is the current through the one-port.

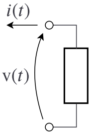
The "motor" convention



$i(t) > 0$ when it enters by the top of the one-port.

$p(t) = v(t)\, i(t)$ is the instantaneous power absorbed by the one-port.

The "generator" convention

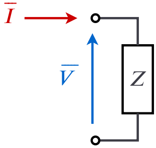


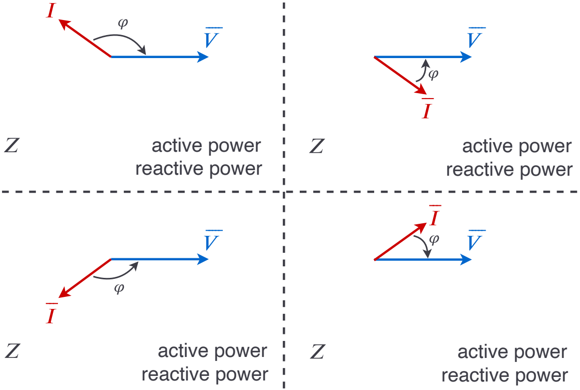
$i(t) > 0$ when it leaves by the top of the one-port.

$p(t) = v(t)\, i(t)$ is the instantaneous power produced by the one-port.

Consumer or producer ?

"Motor" convention :

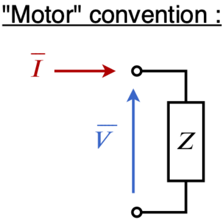




When $|\varphi| < \dfrac{\pi}{2}$: The load .... active power ($cos(\varphi) > 0$)

When $\underbrace{\varphi > 0}_{(\overline{I} \text{ is lagging } \overline{V})}$: The load .... reactive power ($sin(\varphi) > 0$)

Consumer or producer ?

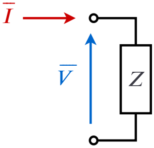


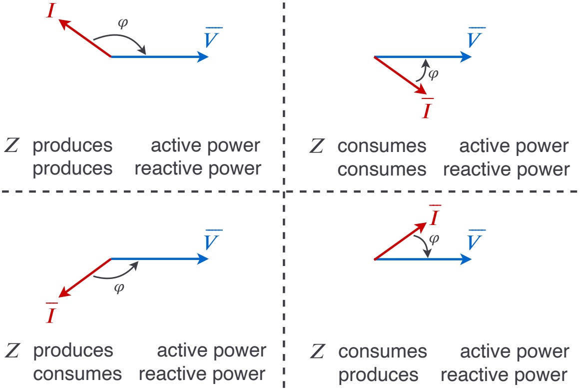
Consumer or Producer

Wooclap code : OXYARO

Consumer or producer - solution

"Motor" convention :





When $|\varphi| < \frac{\pi}{2}$: The load consumes active power $(\cos(\varphi) > 0)$

When $\underbrace{\varphi > 0}_{(\overline{I} \text{ is lagging } \overline{V})}$: The load consumes reactive power $(\sin(\varphi) > 0)$

Expressions relative to one-ports

$$\overline{V} = Z\,\overline{I} = (R + j\,X)\,\overline{I}$$

$$\overline{I} = Y\,\overline{V} = (G + j\,B)\,\overline{V}$$

Z	the impedance	[Ω]
R	the resistance	[Ω]
X	the reactance	[Ω]

Y	the admittance	[S]
G	the conductance	[S]
B	the susceptance	[S]

$S = \mathrm{Z}\,I^2$	[VA]
$P = \mathrm{R}\,I^2$	[W]
$Q = \mathrm{X}\,I^2$	[var]

$S = \mathrm{Y}^*\,V^2$	[VA]
$P = \mathrm{G}\;V^2$	[W]
$Q = -\mathrm{B}\,V^2$	[var]

Power consumption of basic one-ports

	R	L	C
φ	0	$\frac{\pi}{2}$	$-\frac{\pi}{2}$
P	$\mathrm{R}I^2 = \frac{V^2}{\mathrm{R}}$	0	0
Q	0	$\mathrm{X}I^2 = \omega\, L\, I^2 = \frac{V^2}{\omega\, L}$	$-\mathrm{B}V^2 = -\omega\, C\, I^2 = -\frac{I^2}{\omega\, C}$

Exercises

Exercises of session 1

Wooclap code : QBSBER

Exercise 1 : voltage distribution

The circuit of Figure 1 presents a resistive-inductive load powered with an AC generator of sinusoidal voltage (230 V, 50 Hz). Find the voltages accross R and L (magnitude and phase angle) and represent all the voltages on a phasor diagram.

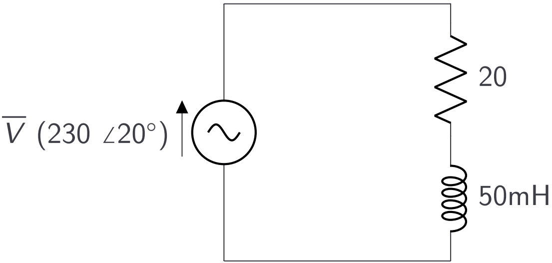
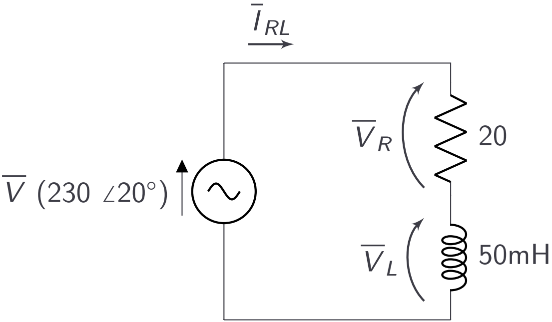


Figure: Resistive-inductive circuit.

Exercise 1 : voltage distribution - solution

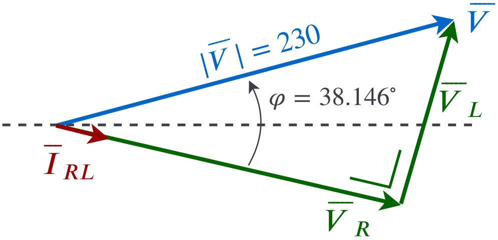


$$\overline{V} = \underbrace{R\,\overline{I}_{RL}}_{\overline{V}_R} + \underbrace{j\,X_L\,\overline{I}_{RL}}_{\overline{V}_L} = (R + j\,X_L)\,\overline{I}_{RL} \tag{9}$$

$$\overline{I}_{RL} = \frac{\overline{V}}{(R + j\,X_L)} = \frac{230\angle 20^\circ}{20 + j\,15.708} = 9.044\angle - 18.146^\circ \tag{10}$$

$$\overline{V}_R = \mathrm{R}\ \overline{I}_{\mathrm{RL}} = 180.881 \angle -18.146^\circ \quad [\mathrm{V}] \tag{11}$$

$$\overline{V}_L = j\ \mathrm{X}_L\ \overline{I}_{\mathrm{RL}} = 142.064 \angle 71.854^\circ \quad [\mathrm{V}] \tag{12}$$



The reactive power consumed by the load is

$$Q = V\ I_{\mathrm{RL}}\ \sin(\varphi) = 230\times 9.044\times\sin(38.146^\circ) = 1284.82\ [\mathrm{var}] \tag{13}$$

Exercise 2 : reactive power compensation

Your colleague suggests to add a 50 μF capacitor in parallel of the RL load. It is supposed to compensate the reactive power consumed by the inductive load. Is it a good idea ? If so, what would be the exact value of C needed ?

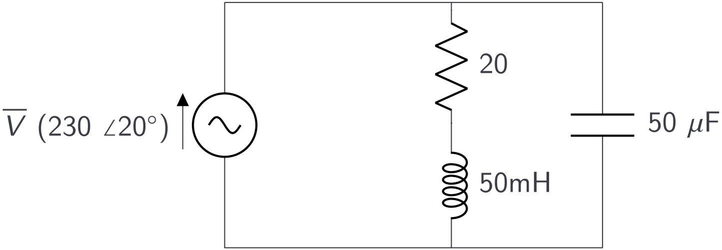
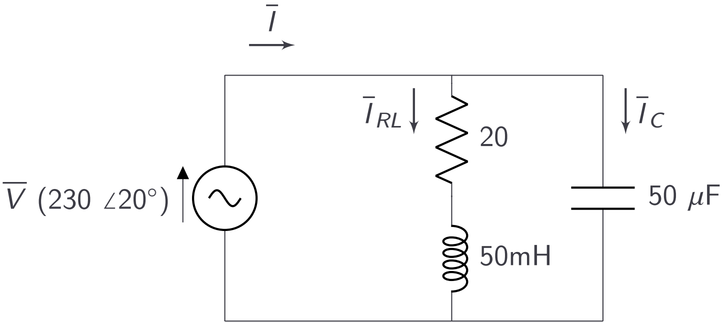


Figure: Inductive circuit with compensation capacitor.

Exercise 2 : reactive power compensation - solution



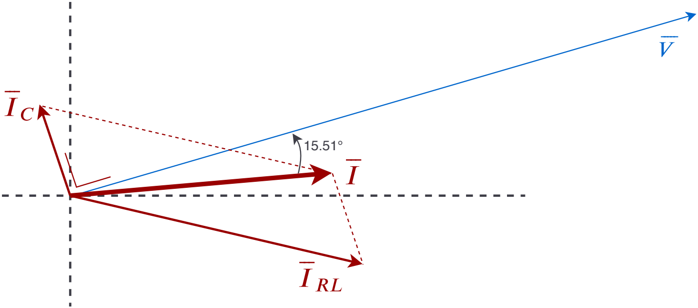
The voltage accross the RL load remains the same ($\overline{V} = 230\ \angle 20^\circ$).
Then, the current in RL is still $\overline{I}_{RL} = 9.044\angle - 18.146^\circ$ [A] and

$$\overline{I}_C = \frac{\overline{V}}{Z_C} = Y_C\ \overline{V} = j\ \omega C\ \overline{V} = 3.61\angle 110^\circ \tag{14}$$

Exercise 2 : reactive power compensation - solution

Apply the KCL (Kirchhoff Circuit Law) and obtain

$$\overline{I} = \overline{I}_{RL} + \overline{I}_C = 7.381\angle 4.49^\circ \quad (15)$$



The reactive power consumed by the load is

$$Q = V\ I\ \sin(\varphi) = 230 \times 7.381 \times \sin(15.51^\circ) = 453.957\ \text{[var]} \quad (16)$$

Exercise 2 : reactive power compensation - solution

The exact value of capacitance to be used must cancel the reactive power consumption of the inductance, such that the reactive power produced by C matches the reactive power consumed by L,

$$Q_C = -Q_{RL} \tag{17}$$

$$-B_C \; V^2 = B_{RL} \; V^2 \tag{18}$$

$$-\omega \; C \; V^2 = -\frac{X}{R^2 + X^2} \; V^2 \tag{19}$$

with $B_C = \omega \mathrm{C}$ and $B_{RL} = -\frac{\omega \, L}{R^2+(\omega \, L)^2}$ because

$$\mathrm{Y}_{RL} = \frac{1}{\mathrm{Z}_{RL}} = \frac{1}{\mathrm{R} + j \, \mathrm{X}} = \frac{\mathrm{R} - j \, \mathrm{X}}{\mathrm{R}^2 + \mathrm{X}^2} \tag{20}$$

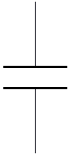
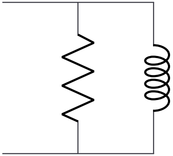
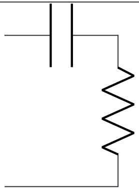
Thus,

$$\mathrm{C} = \frac{\mathrm{L}}{\mathrm{R}^2 + (\omega \mathrm{L})^2} = 77.31[\;\mu\mathrm{F}] \tag{21}$$

Exercise 3 : one-port small quiz

Fill the cells of the table below with the most appropriate answer among :

$=0$ <0 >0 $=1$ <1 $+\infty$ $-\infty$

one-port:					
active power consumed					
reactive power produced					
$\cos\phi$					
$\tan\phi$					

Exercise 3 : one-port small quiz - solution