Electromagnetic Energy Conversion ELEC0431

Introduction to exercise sessions & Exercise session 1: Reminders

11 February 2022

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Exercise sessions on Friday mornings from 10.30 to 12.30 am.

Slides/exercise manual will be uploaded on the on the webpage of the course.

Laboratory sessions

4 sessions:

- Session 1: Transformers
- Session 2: AC synchronous machines
- Session 3: AC asynchronous machines
- Session 4: DC machines

The lab sessions are (so far) organized face-to-face. The lab manual will be available on the webpage of the course. More information later...

Reminders

Phasors and power in sinusoidal steady state

Introduction to the RMS value

- Dissipated power with DC current:

$$P = R\, I^2$$

- Dissipated power with AC current:

$$P(t) = Z\, i(t)^2 \quad \Rightarrow \quad P_{mean} = R\,\frac{1}{T}\int_0^T i(t)^2 dt = R\left(\underbrace{\sqrt{\frac{1}{T}\int_0^T i(t)^2 . dt}}_{I_{RMS}}\right)^2$$

$$= R\, {I_{RMS}}^2$$

The Root Mean Square value (or effective value) is equal to the value of the constant direct current (voltage) that would produce the same power **dissipation** in a resistive load.

RMS value and harmonic signal

$$F_{RMS} := \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt}$$

For a harmonic function with frequency f, pulsation $\omega = 2\pi f$ and period $T = \frac{1}{f}$:

$$f(t) = F_{max}\cos(\omega t)$$

$$F_{RMS} = \sqrt{\frac{1}{T}\int_0^T F_{max}^2 \cos^2(\omega t)\; dt} = \sqrt{\frac{F_{max}^2}{T}\int_0^T \cos^2(\omega t)\; dt}$$

$$= \sqrt{\frac{F_{max}^2}{T}\frac{T}{2}} = \frac{F_{max}}{\sqrt{2}}$$

Important note about notations

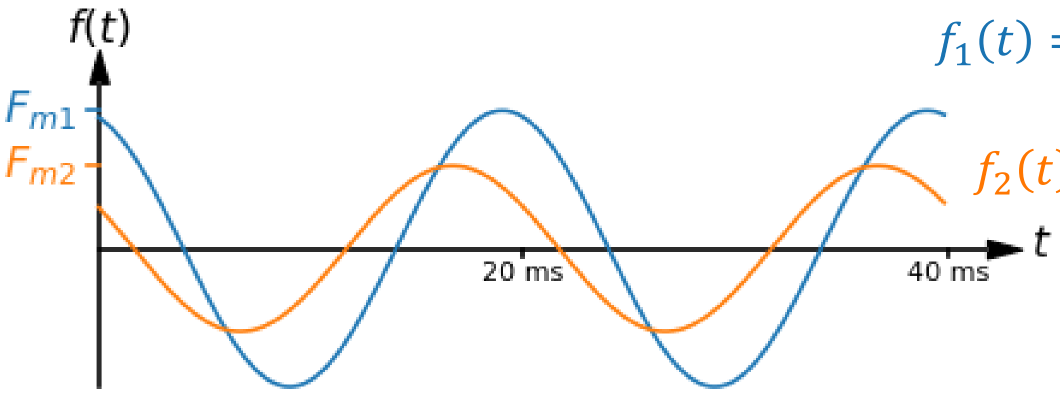
<u>Remark:</u>

The following conventions will be used in the rest of the course for RMS and max/peak values:

$$F_{RMS} \quad \Rightarrow \quad F$$

$$F_{max} \quad \Rightarrow \quad F_m$$

Introduction to phasors



$$f_1(t) = F_{m1}\cos(\omega t + \varphi_1)$$

$$f_2(t) = F_{m2}\cos(\omega t + \varphi_2)$$

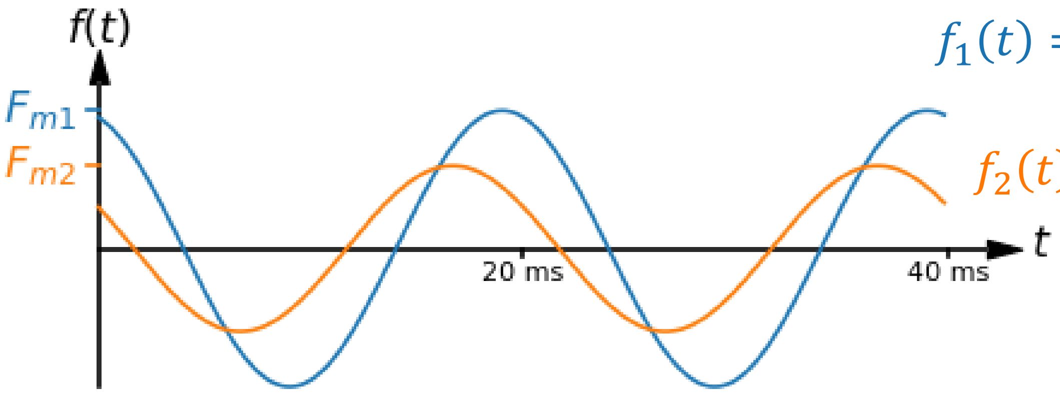
- F_{m2} → amplitude
- ω → pulsation $(= 2\pi f)$
- φ_2 → phase angle

For a given frequency, the values of F_m and φ completely define the functions. The time evolution is not of interest ➔ higher level representation: the **phasor**.

$$\begin{aligned} f(t) &= \sqrt{2}\,F\cos(\omega t + \varphi) \\ &= \mathbb{R}\left(\sqrt{2}\,F\,e^{j(\omega t+\varphi)}\right) \\ &= \mathbb{R}(\sqrt{2}\,\underbrace{F\,e^{j\varphi}}_{\bar{\mathrm{F}}}\,e^{j\omega t}) \end{aligned}$$

$\bar{\mathrm{F}}$ is the phasor of $f(t)$.
It is a complex number, with its amplitude and its phase angle.

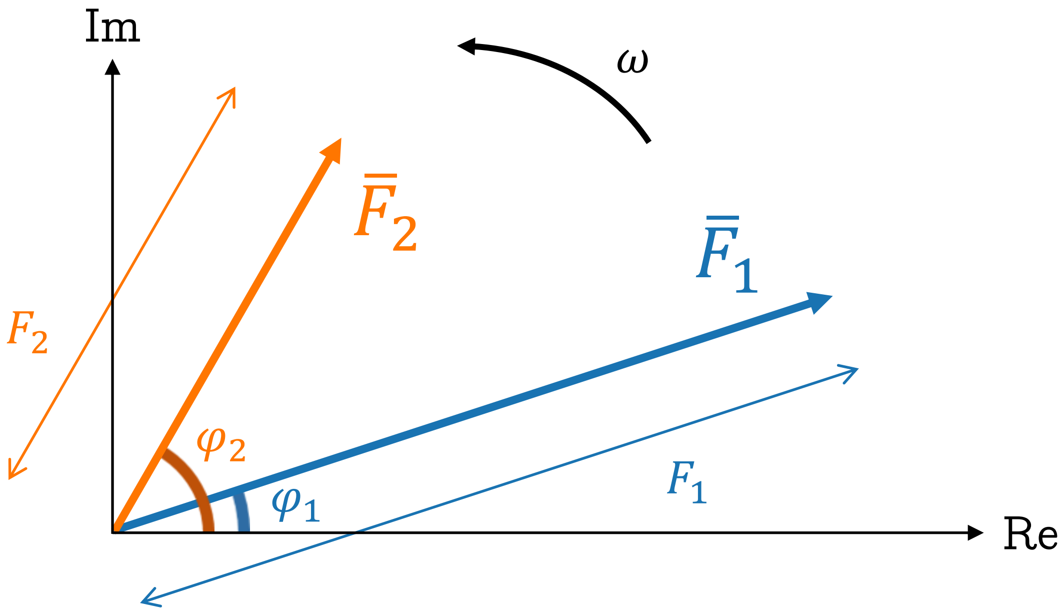
Phasor diagrams



$$f_1(t) = F_{m1}\cos(\omega t + \varphi_1)$$

$$f_2(t) = F_{m2}\cos(\omega t + \varphi_2)$$

Phasor diagram



Phasor properties

The main assumptions of the phasor approximation are a constant frequency and linearity.

Linearity ensured as Kirchhoff's laws and differential equations are linear.

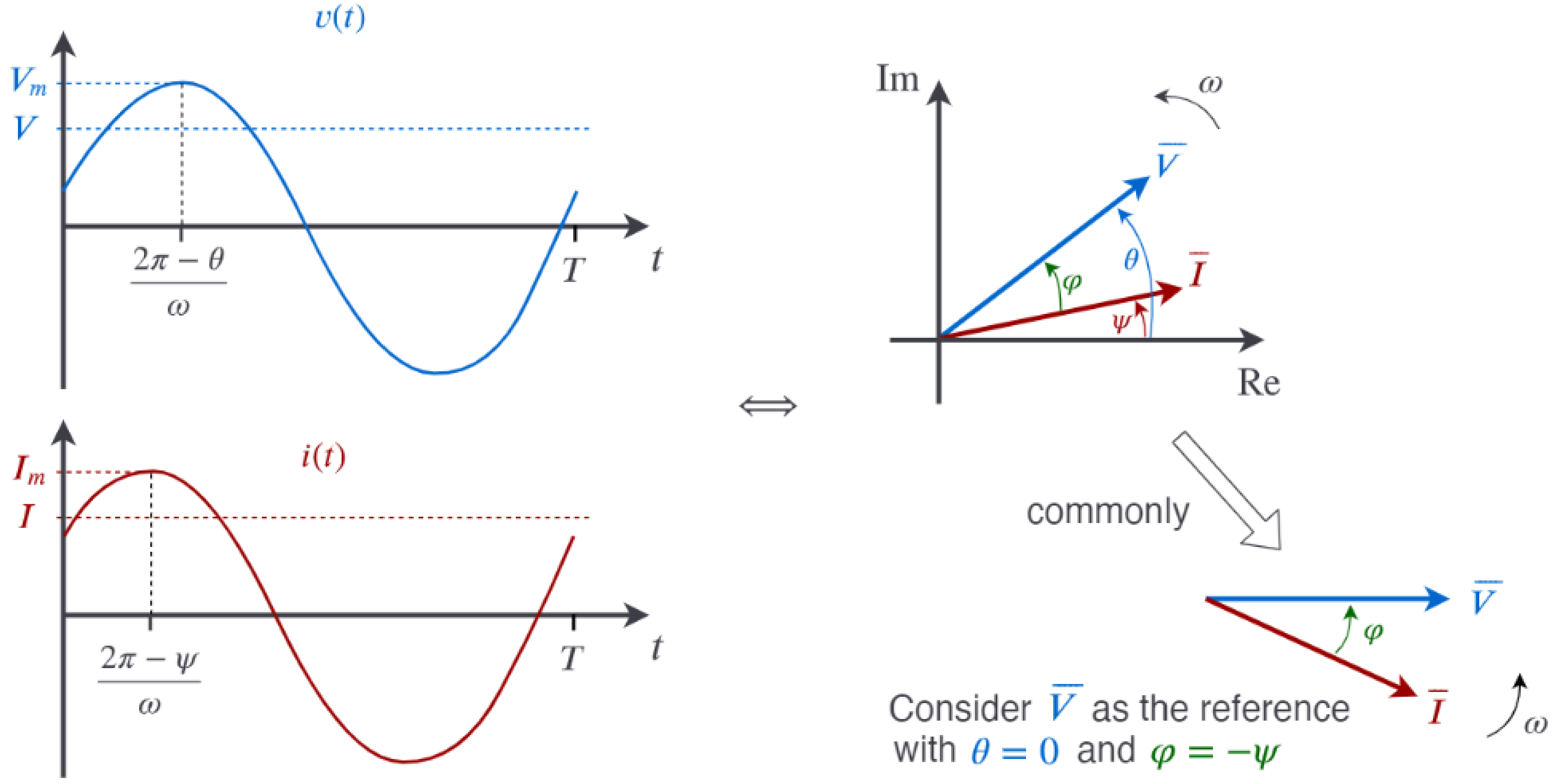
Time domain		Frequency domain
$f(t)$	$\Longleftrightarrow$	$\bar{F}$
$a\,f(t) + b\,g(t)$	$\Longleftrightarrow$	$a\,\bar{F} + b\,\bar{G}$
$\frac{d.}{dt}$	$\Longleftrightarrow$	$j\omega$
$\int .\,dt$	$\Longleftrightarrow$	$\frac{1}{j\omega}$

Application of the phasors

The concept of phasor can be applied to electrical quantities such as voltages and currents:

Time domain		Frequency domain
$v(t) = \sqrt{2}\, V \cos(\omega t + \theta)$	$\Longleftrightarrow$	$\bar{V} = V\, e^{j\theta}$
$i(t) = \sqrt{2}\, I \cos(\omega t + \psi)$	$\Longleftrightarrow$	$\bar{I} = I\, e^{j\psi}$

Alternating voltages and currents – Phasor diagrams



Consider $\overline{V}$ as the reference with $\theta = 0$ and $\varphi = -\psi$

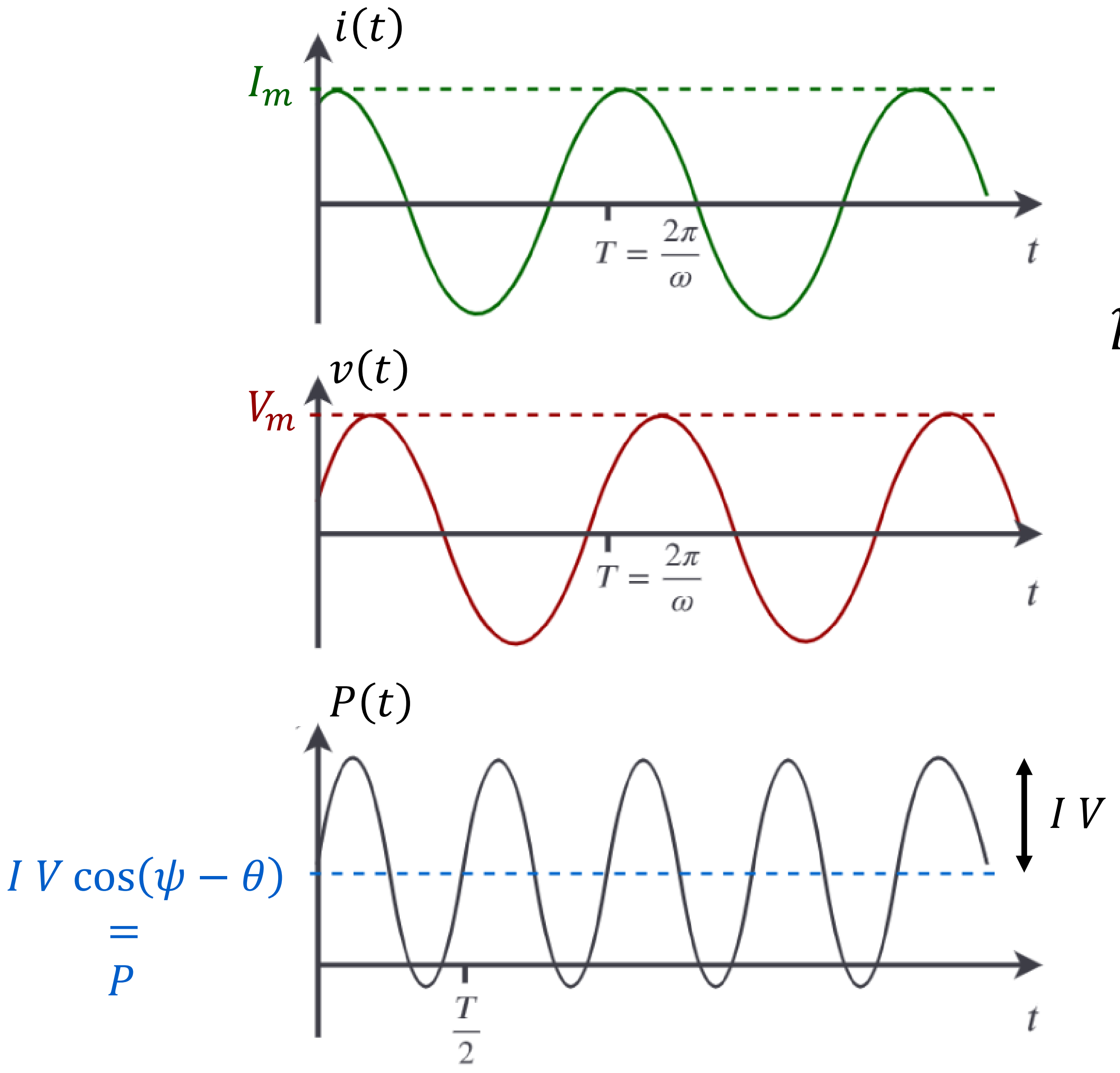
Product of 2 harmonic functions → instantaneous power

$$p(t) = i(t)v(t)$$

Development on the board

$$p(t) = I\,V\cos(\theta - \psi) + I\,V\cos(2\omega t + \psi + \theta)$$

Product of 2 harmonic functions → instantaneous power



$$p(t) = \underbrace{I\,V\cos(\theta - \psi)}_{\text{DC component}} + \underbrace{I\,V\cos(2\omega t + \psi + \theta)}_{\text{AC component}}$$

DC component
=
P
=
Active power

AC component
with double frequency

Power components

$$p(t) = I\,V\cos(\theta-\psi) + I\,V\cos(2\omega t+\psi+\theta)$$

Development on the board

$$p(t) = \underbrace{I\,V\cos(\theta-\psi)}_{\text{P, the active power}}\left[1+\cos\big(2(\omega t+\theta)\big)\right] + \underbrace{I\,V\sin(\theta-\psi)}_{\text{Q, the reactive power}}\sin\big(2(\omega t+\theta)\big)$$

P, the active power

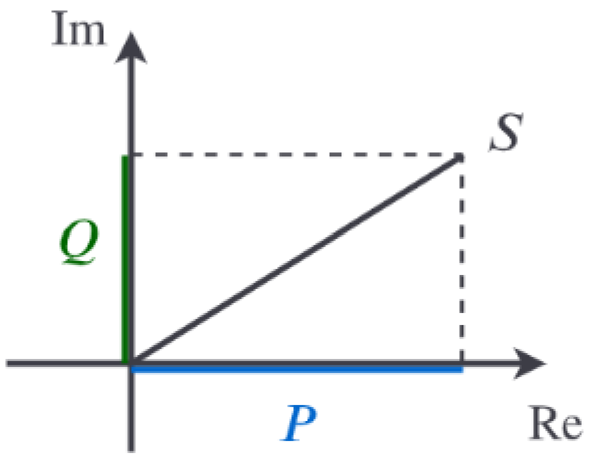
Q, the reactive power

Power in the frequency domain ➔ complex power S

$$S = \bar{V}\,\bar{I}^*$$

Development on the board

$$S = I\,V\cos(\theta-\psi) + j\,I\,V\sin(\theta-\psi)$$



Power components

$$S = \bar{V}\,\bar{I}^*$$

$$= P + j\,Q$$

Based on that definition, one also defines the apparent power as the norm of the complex power:

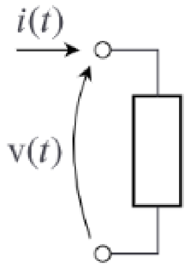
$$|S| = \sqrt{P^2 + Q^2} = V\,I$$

Power reference of the one-port ("le dipole")

The signal $v(t)$ is the voltage accross the one-port.

The signal $i(t)$ is the current through the one-port.

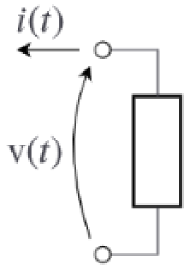
The "motor" convention



$i(t) > 0$ when it enters by the top of the one-port.

$p(t) = v(t)\, i(t)$ is the instantaneous power absorbed by the one-port.

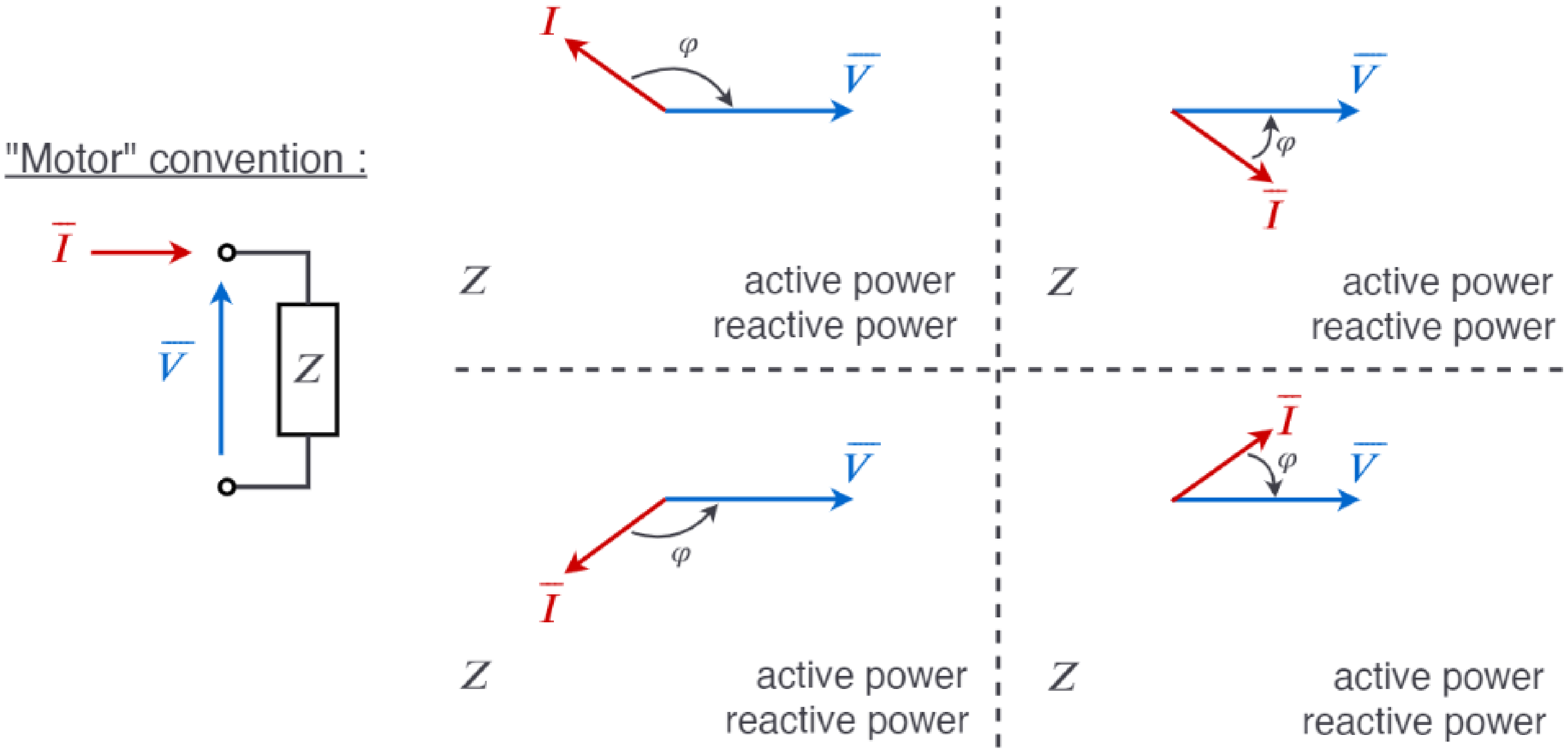
The "generator" convention



$i(t) > 0$ when it leaves by the top of the one-port.

$p(t) = v(t)\, i(t)$ is the instantaneous power produced by the one-port.

Consumer or producer

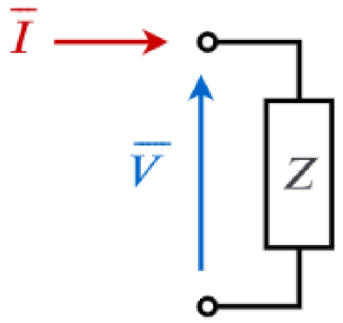


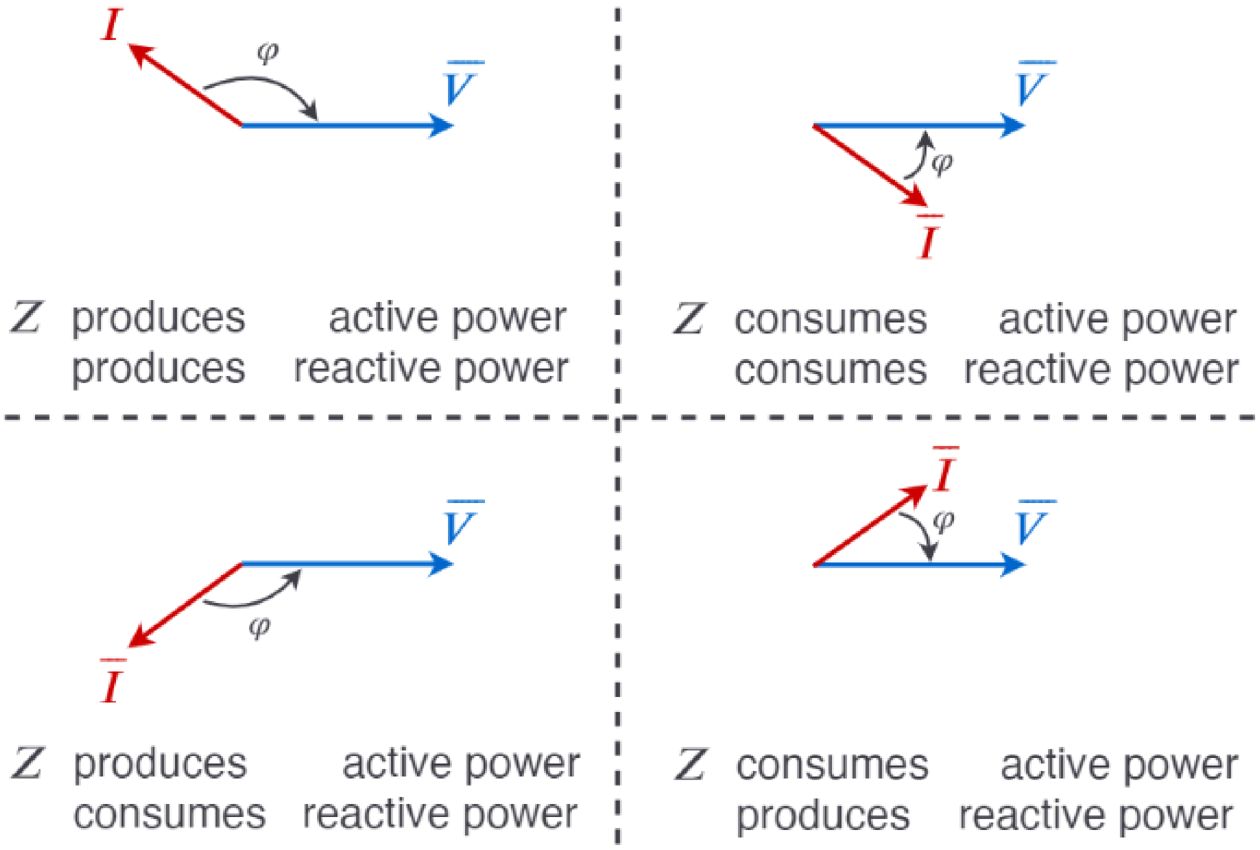
When $|\varphi| < \frac{\pi}{2}$: The load .... active power ($cos(\varphi) > 0$)

When $\underbrace{\varphi > 0}_{(\bar{I} \text{ is lagging } \bar{V})}$: The load .... reactive power ($sin(\varphi) > 0$)

Consumer or producer (solution)

"Motor" convention :





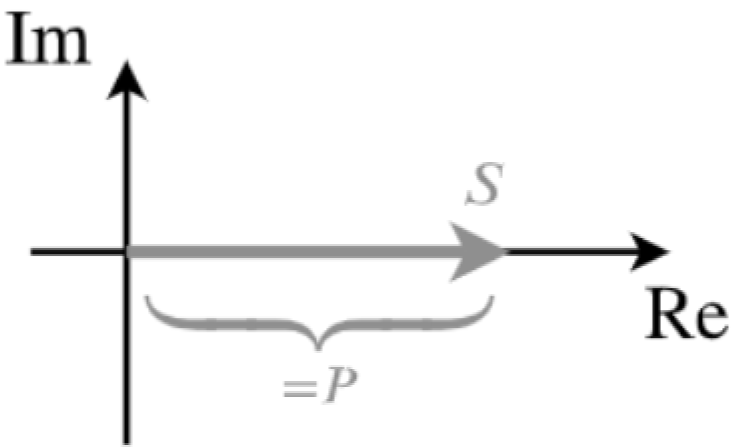
When $|\varphi| < \dfrac{\pi}{2}$: The load consumes active power ($cos(\varphi) > 0$)

When $\underbrace{\varphi > 0}_{(\overline{I} \text{ is lagging } \overline{V})}$: The load consumes reactive power ($sin(\varphi) > 0$)

Complex power and phasors for a resistor

$$\bar{V} = Z\,\bar{I} \quad \& \quad Z = R$$

$$\Rightarrow S = \bar{V}\,\bar{I}^* = \mathrm{R}\,\bar{I}\,\bar{I}^* = R\,I^2$$

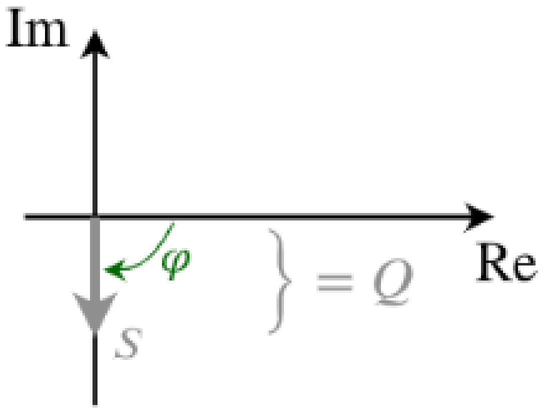


Only active power

Complex power and phasors for a capacitor

$$\bar{V} = Z\,\bar{I} \quad \& \quad Z = \frac{1}{j\omega C}$$

$$\Rightarrow S = \bar{V}\,\bar{I}^* = \frac{1}{j\omega C}\,\bar{I}\,\bar{I}^* = -j\frac{1}{\omega C}\,I^2$$

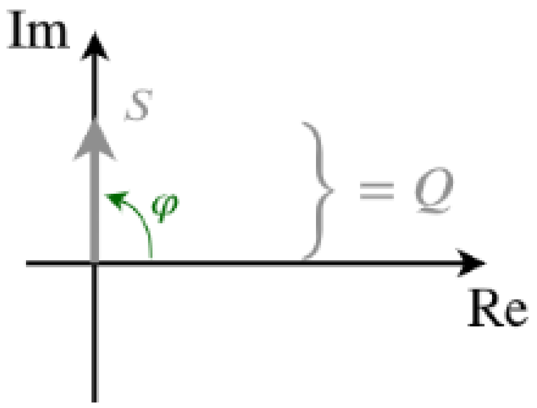


Only reactive power

Complex power and phasors for an inductance

$$\bar{V} = Z\,\bar{I} \quad \& \quad Z = j\omega L$$

$$\Rightarrow S = \bar{V}\,\bar{I}^* = j\,\omega L\,\bar{I}\,\bar{I}^* = j\,\omega L\,I^2$$



Only reactive power

Expressions relative to one-ports

$$\overline{V} = Z\,\overline{I} = (R + j\,X)\,\overline{I}$$

$$\overline{I} = Y\,\overline{V} = (G + j\,B)\,\overline{V}$$

Z	the impedance	[Ω]
R	the resistance	[Ω]
X	the reactance	[Ω]

Y	the admittance	[S]
G	the conductance	[S]
B	the susceptance	[S]

$$S = Z\,I^2 \quad [\text{VA}]$$
$$P = R\,I^2 \quad [\text{W}]$$
$$Q = X\,I^2 \quad [\text{var}]$$

$$S = Y^*\,V^2 \quad [\text{VA}]$$
$$P = G\,V^2 \quad [\text{W}]$$
$$Q = \text{-}B\,V^2 \quad [\text{var}]$$

Recap complex power basic components

	R	L	C
φ	0	$\frac{\pi}{2}$	$-\frac{\pi}{2}$
P	$\mathrm{R}I^2 = \frac{V^2}{\mathrm{R}}$	0	0
Q	0	$\mathrm{X}I^2 = \omega\, L\, I^2 = \frac{V^2}{\omega\, L}$	$-\mathrm{B}V^2 = -\omega\, C\, I^2 = -\frac{I^2}{\omega\, C}$

Exercise 1: voltage distribution

The circuit of Figure 1 presents a resistive-inductive load powered with an AC generator of sinusoidal voltage (230 V, 50 Hz). Find the voltages accross R and L (magnitude and phase angle) and represent all the voltages on a phasor diagram.

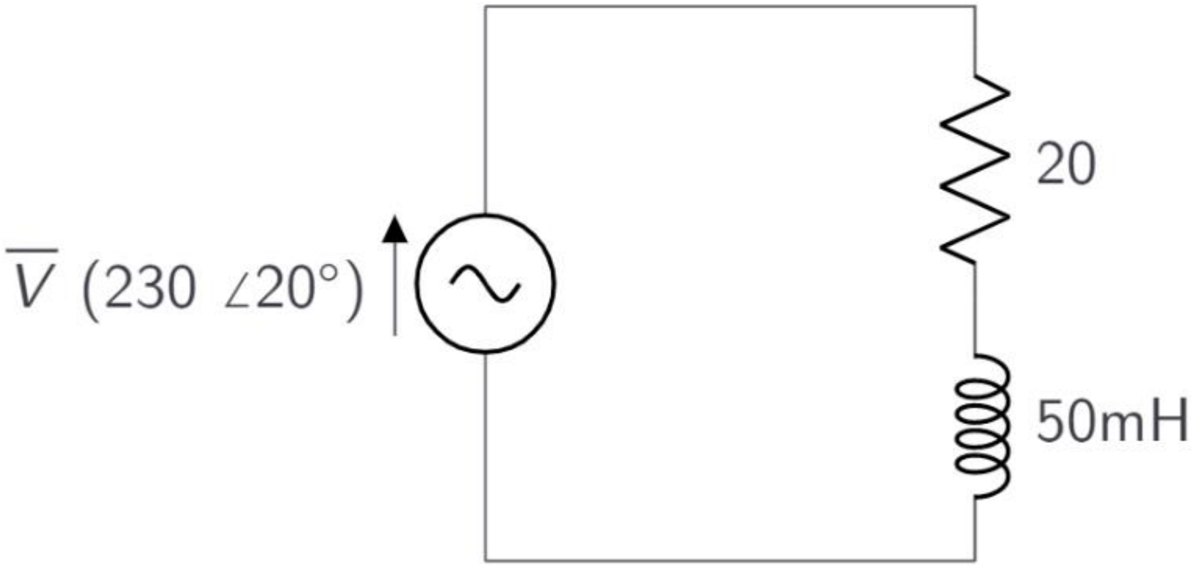


Figure: Resistive-inductive circuit.

Exercise 2: Reactive power compensation

Your colleague suggests to add a 50 μF capacitor in parallel of the RL load. It is supposed to compensate the reactive power consumed by the inductive load. Is it a good idea ? If so, what would be the exact value of C needed ?

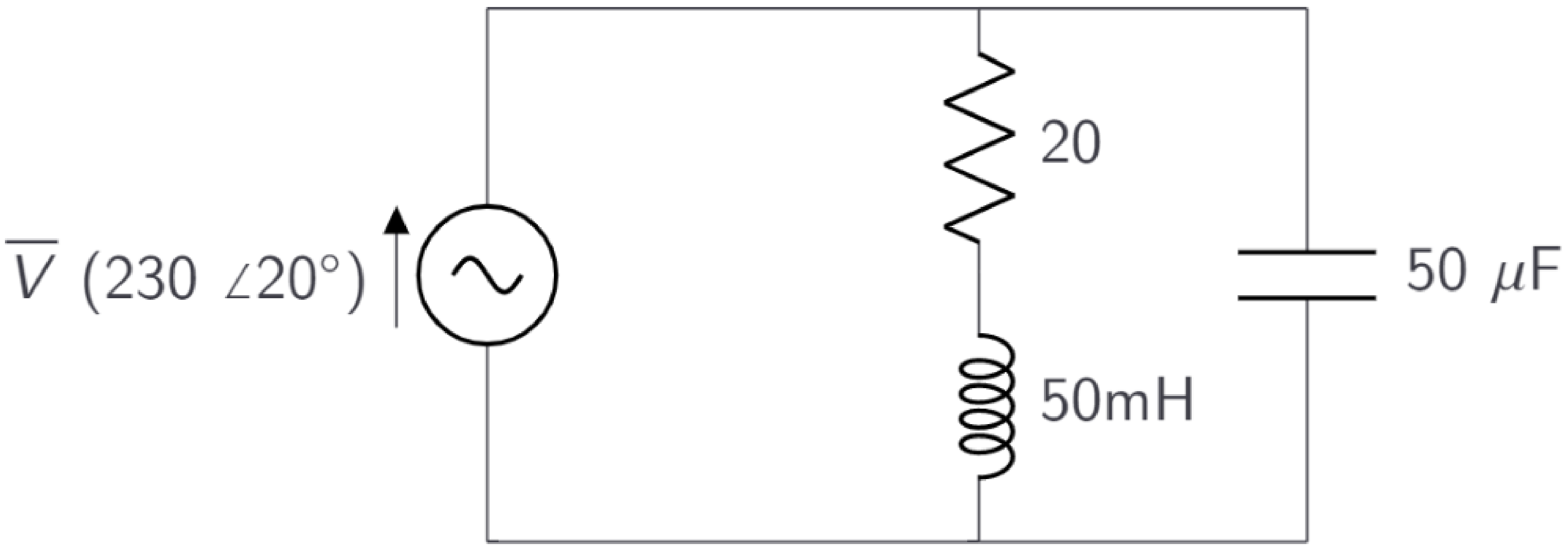
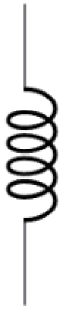
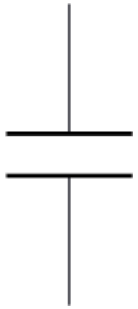
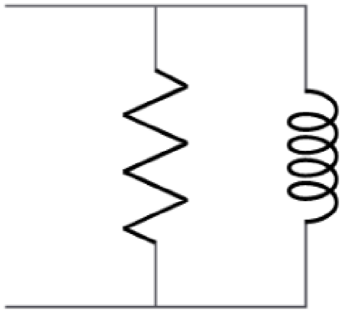
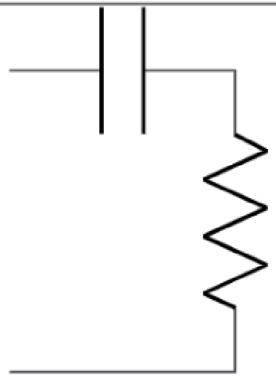


Figure: Inductive circuit with compensation capacitor.

Exercise 3: one-port small quiz

Fill the cells of the table below with the most appropriate answer among :

=0   <0   >0   =1   <1 $+\infty$ $-\infty$

one-port:					
active power consumed					
reactive power produced					
cos ϕ					
tan ϕ					