



Electromagnetic Energy Conversion

ELEC0431

Exercise session 1: Phasors and power in the sinusoidal steady state

6 February 2026

Florent Purnode (florent.purnode@uliege.be)

Montefiore Institute, Department of Electrical Engineering and Computer Science,
University of Liège, Belgium

Practical Organization

- Theoretical lectures with Professor Geuzaine
 - Practical lectures with myself (Florent Purnode)
 - Test on phasors in the sinusoidal steady state and three-phase systems
 - **4 lab sessions** to help you grasp the theoretical & practical concepts:
 - Transformers (mono-phase and three-phase)
 - Synchronous machines
 - Asynchronous machines
 - DC machines
 - Final exam
- ➔ The schedule, slides, manuals, etc. are available on the class webpage (<https://people.montefiore.uliege.be/geuzaine/ELEC0431>)

Test on phasors

- The two first exercise sessions are dedicated to the introduction of phasors.
Phasors will be used throughout the class. Being able to work with them effortlessly is a must!
- A test, focusing on the material seen during the two first exercise sessions is scheduled on Friday 6th of March, at 9:00 am.
It takes place in the usual classroom (B37 auditorium 02).
It is a quick test of 30 minutes (we insist on the fact you should be able to manipulate phasors effortlessly).
To take the test, you will need your calculator, ruler and protractor (“rapporteur” in French).
- At each exercise sessions, homeworks are available to prepare yourself for the test and the exam.
A past test and past exams are also available on the webpage.
- The test accounts for 5 % of the final grade.
- The test will be followed by its correction and a normal exercise session.

Laboratories – General information

- Each laboratory lasts at most 4 hours. They take place in the “pyramid” ([building in front of the cafeteria of Montéfiore](#)).
- Every laboratory comes with its own manual, available on the webpage.

Before each laboratory, **read the corresponding manual**.

It is advised to prepare the theoretical points in advance → Time saved and good preparation for the exam.

- Student monitors will be available during the laboratory sessions to help you.
- No report is asked. However, you will have to answer individually a quick evaluation at the end of each laboratory.

The evaluation focuses exclusively on the concepts seen during the laboratory. You can be asked to:

- Explain how a particular measurement was performed during the laboratory,
- Draw an equivalent circuit and use it for calculations,
- Draw phasor diagrams,
- Solve a problem encountered during the laboratory, etc.

Each test represents 3.75 % of your final grade. That is **15 % of the grade** for the four laboratories.

(5 % for the test, 15 % for the laboratories → 80 % for the exam)

- Laboratories are **mandatory** (In case of unexcused absence, an absence grade will be given for the entire course).

Laboratories – Schedule and groups

To create the lab schedule, you are required to fill **by group** the doodle <https://beta.framadate.org/polls/465a07d3b8361e104f3d>:

- By group of **4 students**, select **6** available time slots (you may be given random sessions if less than 6 time slots were selected).
- In the space provided for names, write the **STUDENT ID NUMBERS** of all **4 MEMBERS** of the group (for example: “s161514, s171856, s164442, s179088”).
- A time slot can be selected by maximum six groups, **do not delay in completing this Doodle.**
- In addition to your selected time slots, your schedule could include laboratory sessions on the 17/04, 24/04, 8/05 and 15/05 mornings (Friday mornings in place of the traditional classes).

IF AND ONLY IF it is impossible for you to create a group of four students meeting the requirements, send me an email (florent.purnode@uliege.be) without delay.

Make sure to complete the doodle by **23:59 on Friday, February 20,**
(Students who would not have given their availabilities by this time will be given random time slots).

Quick reminder: Laboratories are **mandatory**
In case of unexcused absence, an absence grade will be given for the entire course.

In this class...

Today's aim: Introduce a very important concept: **The phasors**

(pay particular attention to this class!)

- Reminders (one-ports and Kirchhoff's laws)
- Harmonic functions
- Root Mean Square value
- **The phasors**
- Exercise 1
- Product of two harmonic functions
- **Active, reactive and apparent power** (Very important too!)
- Exercises 2

Reminders

One-ports

Kirchhoff laws

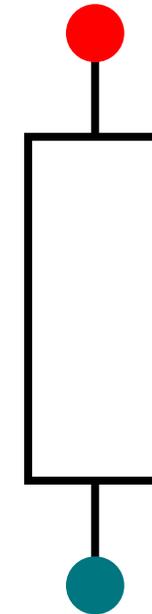
One-ports (or “dipoles” in French)

A one-port is a two-terminal electric component.

It can be:

- A resistor (of resistance R)
- An inductor (of inductance L)
- A capacitor (of capacitance C)
- A power generator
- A diode
- A combination of components
- Etc.

Terminal 1



Terminal 2

One-ports (or “dipoles” in French)

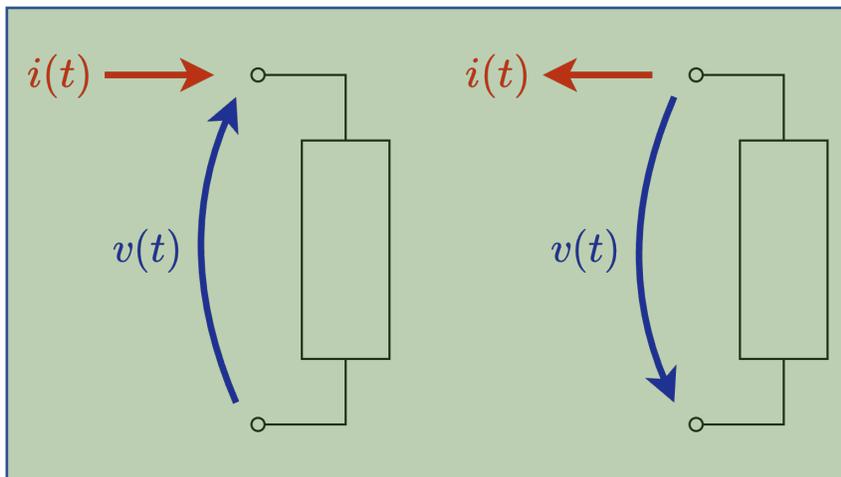
Each one-port is associated with a current $i(t)$ and a voltage $v(t)$.

- $i(t)$ is positive if the positive charges flow in the direction indicated by the arrows.
- $v(t)$ is positive if the tension at the head of the arrow is larger than the one at the tip.

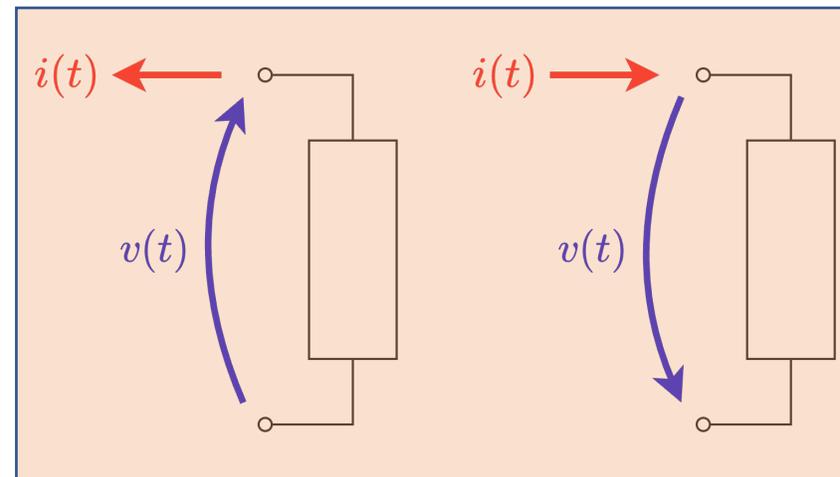
Four different configurations can thus be obtained.

➔ Two of them consume power and two of them deliver power.

Consume
Power
(like a **passive**
component)



Deliver
Power
(like an **active**
component)



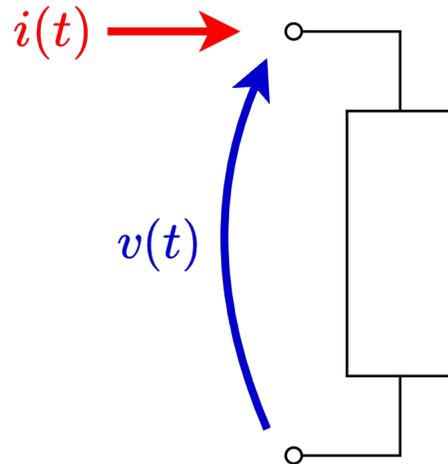
➔ This leads to two different conventions: the **passive** convention and the **active** convention.

Passive VS Active conventions

The passive convention

(also called receiver, motor or load convention)

The passive one-port **receives** the power $p(t) = v(t) i(t)$

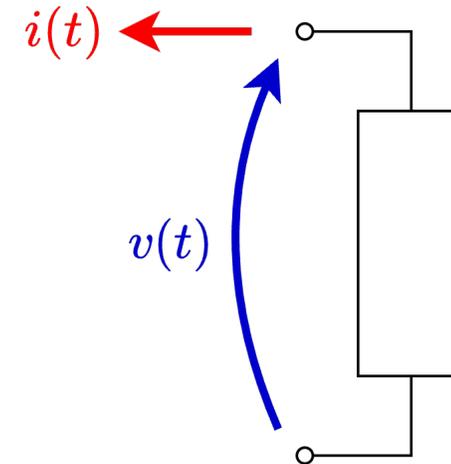


This convention is used for R , L , C , motors and primary side of transformers.

The active convention

(also called generator or source convention)

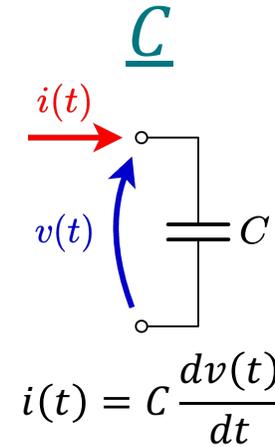
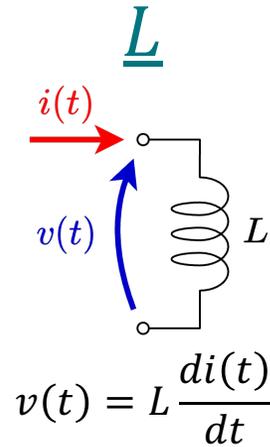
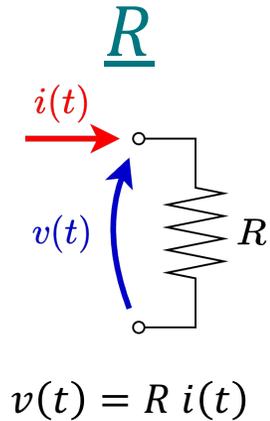
The active one-port **delivers** the power $p(t) = v(t) i(t)$



This convention is used for voltage sources, current sources, generator and secondary side of transformers.

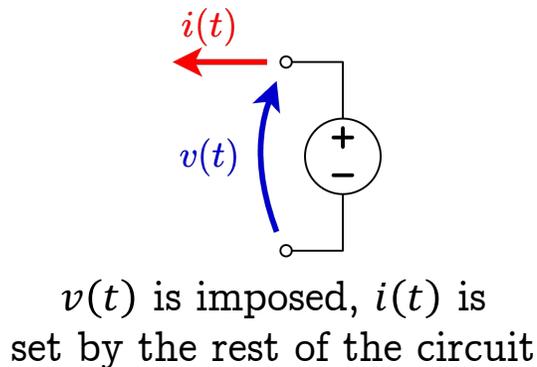
$v(t) - i(t)$ relationship of basic components

Passive convention

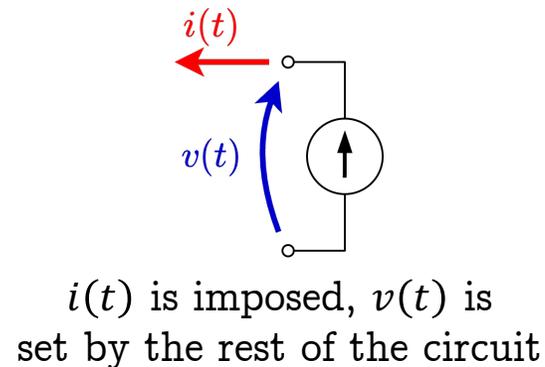


Voltage source

Active convention



Current source



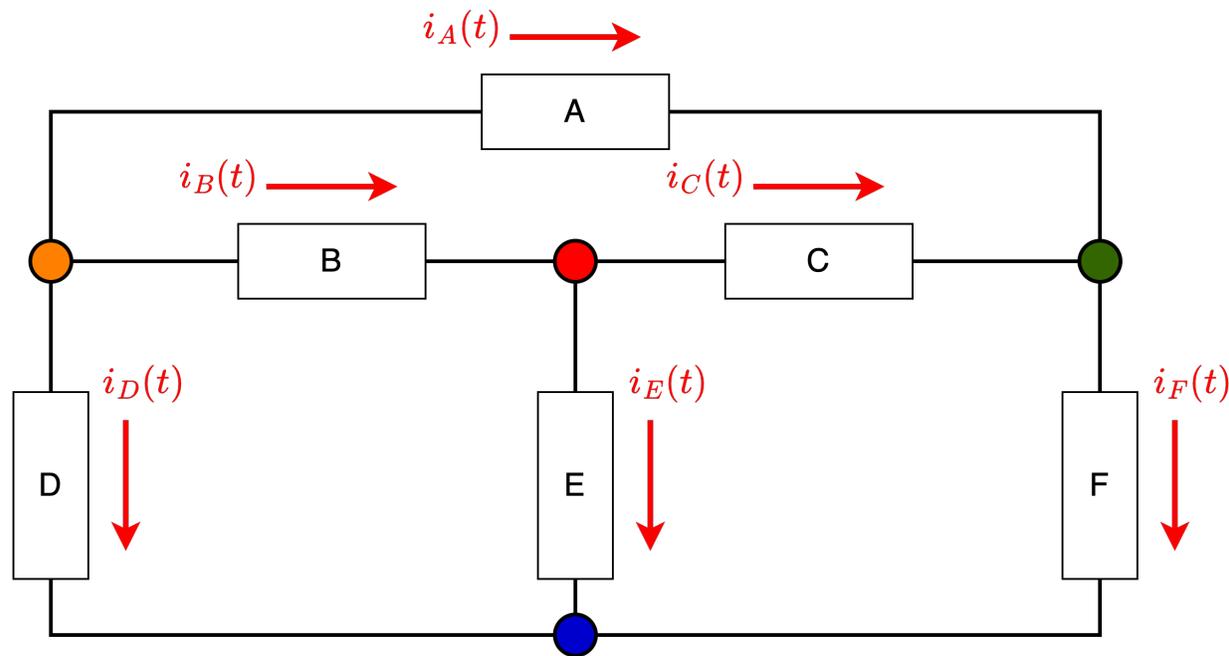
When connected together, use Kirchhoff laws

Kirchhoff's laws

Kirchhoff's first law

(also called Kirchhoff's junction rule or Kirchhoff's current law)

At any junction in the electrical circuit, the sum of currents flowing into the junction is equal to the sum of currents flowing out of the junction.



● $0 = i_A(t) + i_B(t) + i_D(t)$

● $i_B(t) = i_C(t) + i_E(t)$

● $i_A(t) + i_C(t) = i_F(t)$

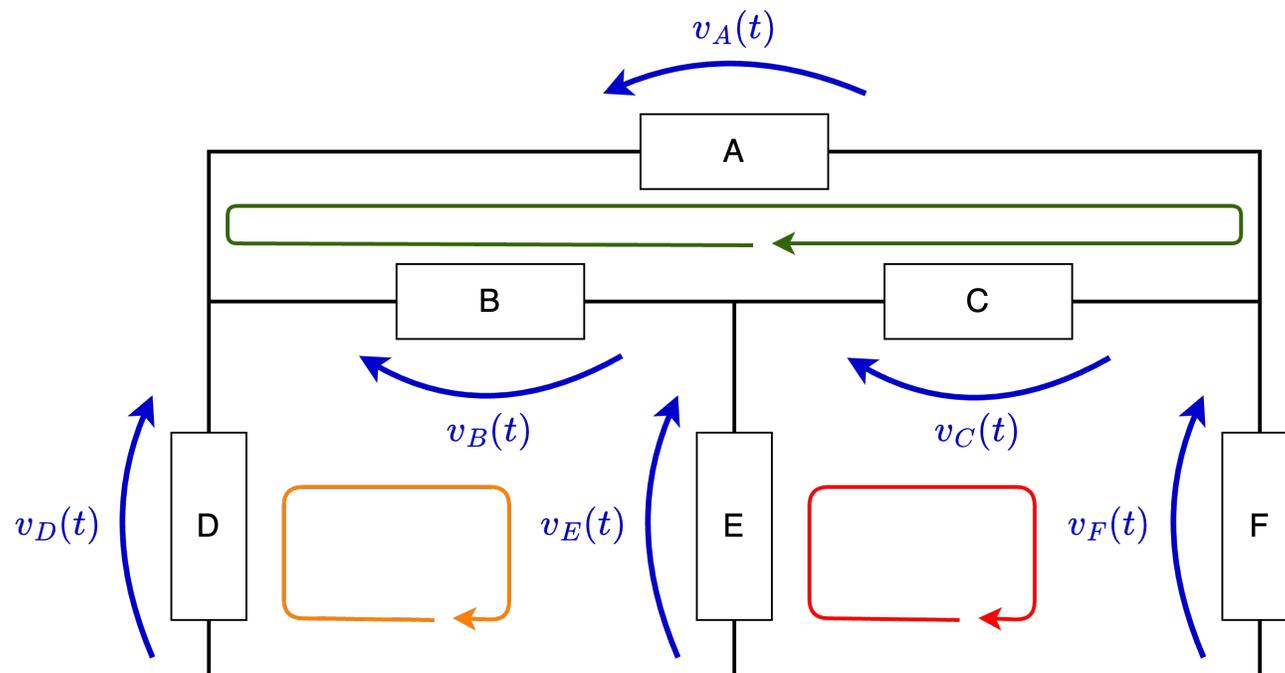
● $i_D(t) + i_E(t) + i_F(t) = 0$

Kirchhoff's laws

Kirchhoff's second law

(also called Kirchhoff's loop rule or Kirchhoff's voltage law)

Around any closed loop in a circuit, the directed sum of potential differences across components is zero.



$$\longrightarrow v_D(t) - v_B(t) - v_E(t) = 0$$

$$\longrightarrow v_E(t) - v_C(t) - v_F(t) = 0$$

$$\longrightarrow v_B(t) - v_A(t) + v_C(t) = 0$$

Introduction to phasors

Harmonic functions

Root Mean square value

The phasors

Harmonic function \Leftrightarrow Phasor equivalence

Phasors applied to circuits

Application: Exercise 1

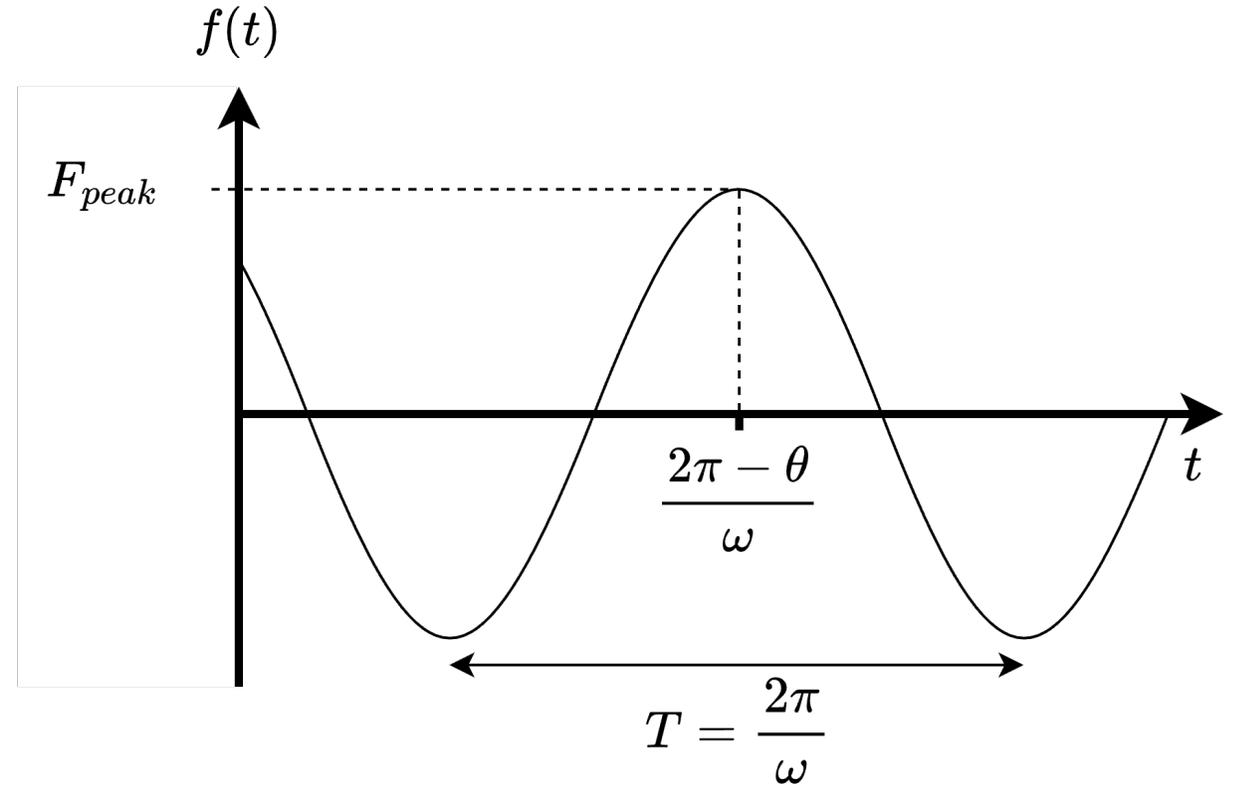
Harmonic functions

In this class, we deal with harmonic functions:

$$f(t) = F_{peak} \cos(\omega t + \theta)$$

where:

- F_{peak} is the amplitude,
- ω is the angular frequency,
- θ is the phase angle.

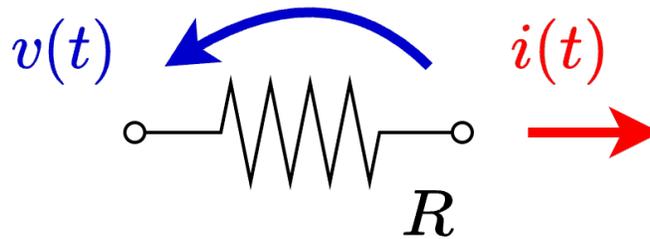


The angular frequency is linked to the frequency f and period T as

$$\omega = 2\pi f = \frac{2\pi}{T}.$$

Root Mean square (RMS) value

Let's consider a resistance R :



The diagram shows a resistor symbol with the letter R below it. A blue curved arrow above the resistor points from right to left, labeled $v(t)$. A red straight arrow below the resistor points from left to right, labeled $i(t)$.

The instantaneous power $p(t)$ dissipated in the resistance is

$$p(t) = v(t) i(t) = R i(t)^2 = \frac{v(t)^2}{R}$$

And the mean dissipated power is

$$P = \frac{1}{T} \int_0^T v(t) i(t) dt = R \underbrace{\frac{1}{T} \int_0^T i(t)^2 dt}_{I_{RMS}^2} = \frac{1}{R} \underbrace{\frac{1}{T} \int_0^T v(t)^2 dt}_{V_{RMS}^2}$$

Root Mean square (RMS) value

The Root Mean Square (RMS) value (or effective value) of $f(t)$ is defined as:

$$F_{RMS} = \sqrt{\frac{1}{T} \int_0^T f(t)^2 dt}.$$

With $f(t) = F_{peak} \cos(\omega t + \theta)$, we obtain:

$$\begin{aligned} F_{RMS} &= \sqrt{\frac{1}{T} \int_0^T F_{peak}^2 \cos^2(\omega t + \theta) dt} = F_{peak} \sqrt{\frac{1}{T} \int_0^T \cos^2(\omega t) dt} \\ &= F_{peak} \sqrt{\frac{1}{T} \int_0^T \frac{1 + \cos(2\omega t)}{2} dt} = F_{peak} \sqrt{\frac{1}{T} \int_0^T \frac{1}{2} dt} = \frac{F_{peak}}{\sqrt{2}} \end{aligned}$$

Root Mean square (RMS) value

Remark:

The following conventions will be used in the rest of the course for RMS and peak/max values:

$$F_{RMS} \Rightarrow F$$

$$F_{peak} \Rightarrow F_m$$

In this class, we deal only **with harmonic functions** $f(t) = F_m \cos(\omega t + \theta)$ for which we have

$$F_m = \sqrt{2} F$$

The phasors

The definition of the phasors is obtained by manipulating the expression of $f(t)$:

$$\begin{aligned} f(t) &= F_m \cos(\omega t + \theta) \\ &= \sqrt{2} F \cos(\omega t + \theta) \\ &= \mathbf{R}(\sqrt{2} F e^{j(\omega t + \theta)}) \\ &= \mathbf{R}(\underbrace{\sqrt{2} F e^{j\theta}}_{\bar{F}} e^{j\omega t}) \end{aligned}$$

$$\bar{F} = F e^{j\theta}$$

is the phasor of $f(t)$. It is a **complex** number with information on the signal amplitude (F) and the signal phase (θ). For a system operating at a **fixed frequency**, F and θ holds all the relevant information to describe the system.

The phasors

For signals evolving at a same angular frequency ω , phasors are like a snapshot representation containing all the necessary information (phase angle and amplitude).

$$f_1(t) = \sqrt{2} F_1 \cos(\omega t + \theta_1)$$

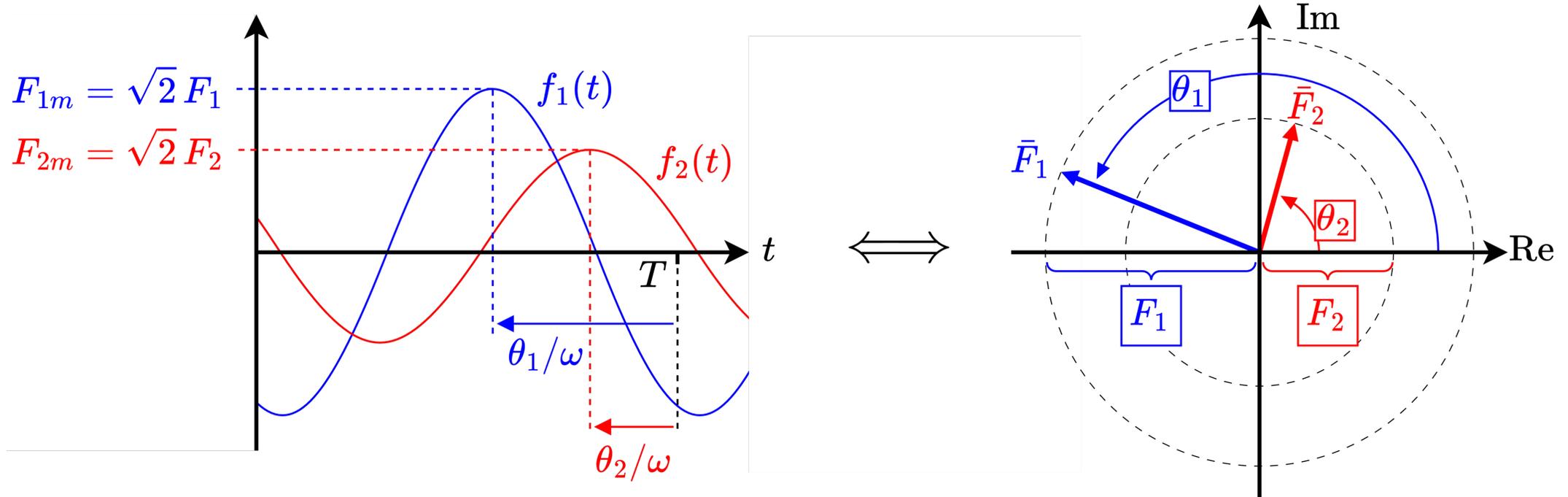
\Leftrightarrow

$$\bar{F}_1 = F_1 e^{j\theta_1} = F_1 \angle \theta_1$$

$$f_2(t) = \sqrt{2} F_2 \cos(\omega t + \theta_2)$$

\Leftrightarrow

$$\bar{F}_2 = F_2 e^{j\theta_2} = F_2 \angle \theta_2$$



Harmonic function \Leftrightarrow Phasor equivalence

Linearity:

$$\begin{aligned} a f_1(t) + b f_2(t) &= a F_{1m} \cos(\omega t + \theta) + b F_{2m} \cos(\omega t + \psi) \\ &= \sqrt{2} a F_1 \cos(\omega t + \theta) + \sqrt{2} b F_2 \cos(\omega t + \psi) \\ &= \mathbf{R}(\sqrt{2} a F_1 e^{j(\omega t + \theta)}) + \mathbf{R}(\sqrt{2} b F_2 e^{j(\omega t + \psi)}) \\ &= \mathbf{R}\left(\sqrt{2} e^{j\omega t} \underbrace{(a F_1 e^{j\theta} + b F_2 e^{j\psi})}_{a \bar{F}_1 + b \bar{F}_2}\right) \end{aligned}$$



$$a f_1(t) + b f_2(t) \quad \Leftrightarrow \quad a \bar{F}_1 + b \bar{F}_2$$

Harmonic function \Leftrightarrow Phasor equivalence

Differentiation:

$$\begin{aligned}\frac{df(t)}{dt} &= \frac{d}{dt} (F_m \cos(\omega t + \theta)) \\ &= \frac{d}{dt} \mathbf{R}(\sqrt{2} F e^{j(\omega t + \theta)}) \\ &= \mathbf{R}(\sqrt{2} F e^{j(\omega t + \theta)} j\omega) \\ &= \mathbf{R}(\underbrace{\sqrt{2} j\omega F e^{j\theta}}_{j\omega \bar{F}} e^{j\omega t})\end{aligned}$$



$$\boxed{\frac{df(t)}{dt} \Leftrightarrow j\omega \bar{F}}$$

Harmonic function \Leftrightarrow Phasor equivalence

Integration:

$$\begin{aligned}\int f(t) dt &= \int F_m \cos(\omega t + \theta) dt \\ &= \int \mathbf{R}(\sqrt{2} F e^{j(\omega t + \theta)}) dt \\ &= \mathbf{R}(\sqrt{2} F e^{j(\omega t + \theta)} 1/j\omega) \\ &= \mathbf{R}\left(\underbrace{\sqrt{2} 1/j\omega F e^{j\theta}}_{\bar{F}/j\omega} e^{j\omega t}\right)\end{aligned}$$



$$\int f(t) dt \Leftrightarrow \frac{\bar{F}}{j\omega}$$

Phasors applied to circuits

The phasor formalism can be exported to voltages and currents:

$$v(t) = \sqrt{2} V \cos(\omega t + \theta)$$

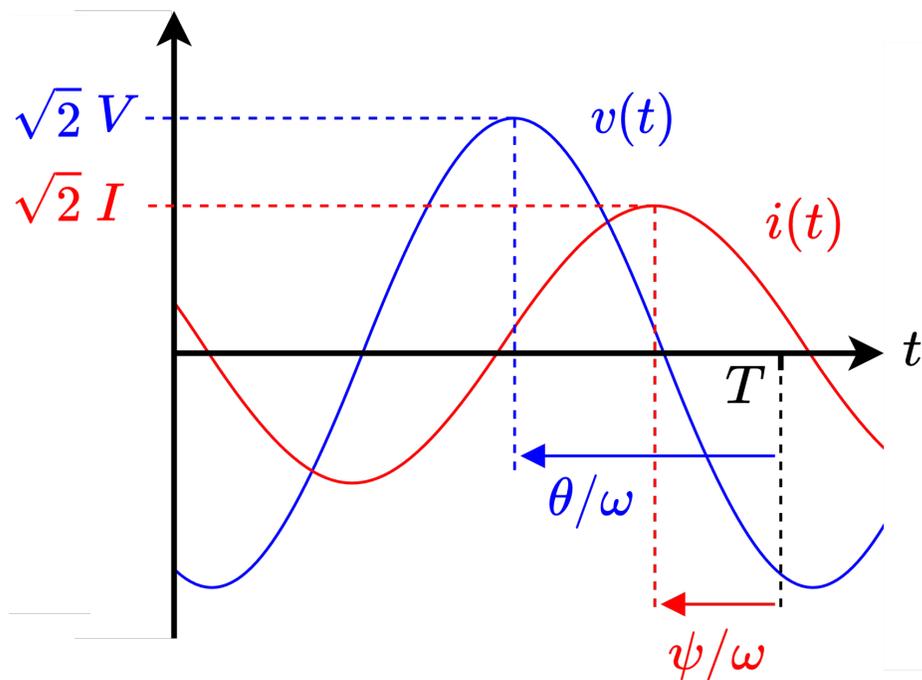
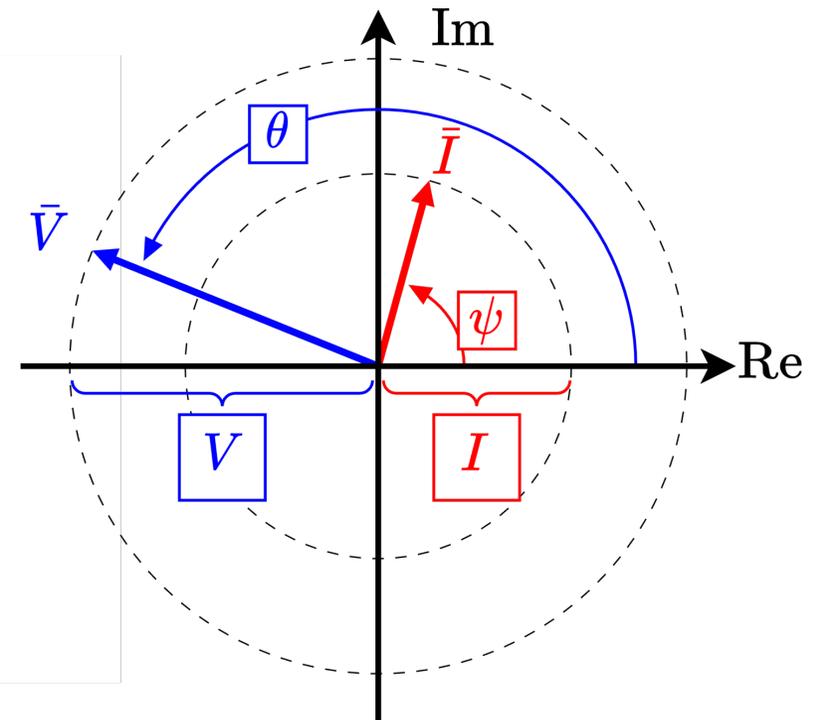
 \Leftrightarrow

$$\bar{V} = V e^{j\theta}$$

$$i(t) = \sqrt{2} I \cos(\omega t + \psi)$$

 \Leftrightarrow

$$\bar{I} = I e^{j\psi}$$

 \Leftrightarrow 

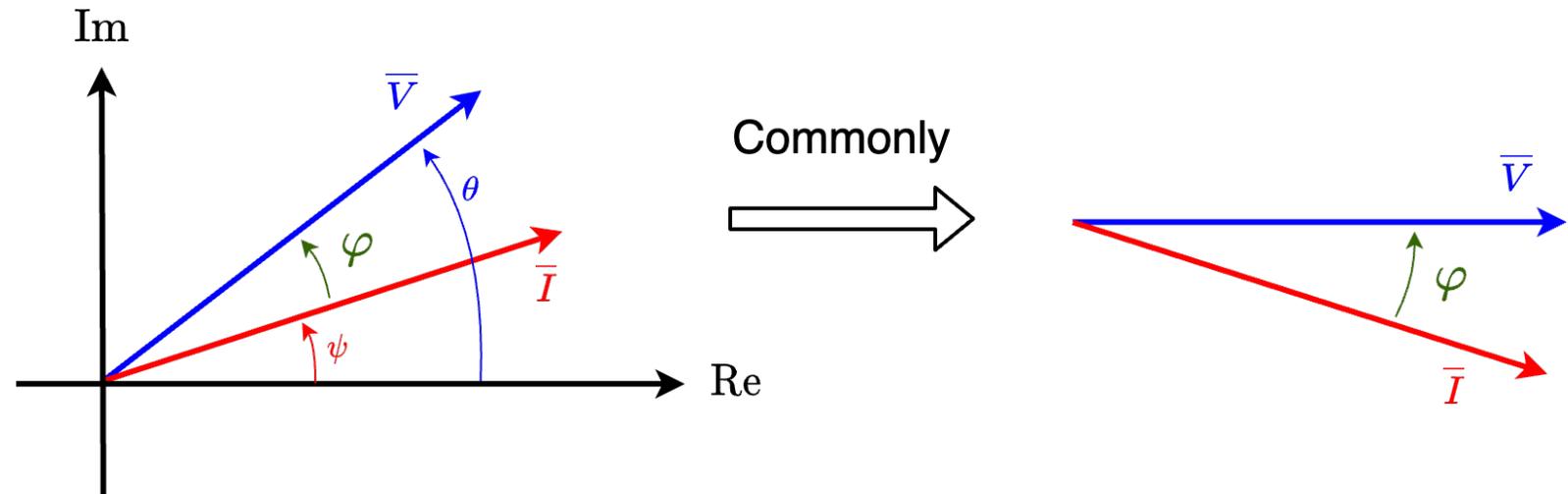
Phasors applied to circuits

The phase angle difference (or phase lag) is noted φ . It is defined as:

$$\varphi = \theta - \psi$$

(φ always goes from the current to the voltage !!!)

It is common to take \bar{V} as the reference with $\theta = 0$ and $\varphi = -\psi$.



Phasors applied to circuits

	<u>Time domain</u>	\Leftrightarrow	<u>Frequency domain</u>
	$f(t) = \sqrt{2} F \cos(\omega t + \theta)$	\Leftrightarrow	$\bar{F} = F e^{j\theta}$
	$a f_1(t) + b f_2(t)$	\Leftrightarrow	$a \bar{F}_1 + b \bar{F}_2$
	$df(t)/dt$	\Leftrightarrow	$j\omega \bar{F}$
	$\int f(t) dt$	\Leftrightarrow	$\bar{F}/j\omega$
Resistance:	$v(t) = R i(t)$	\Leftrightarrow	$\bar{V} = R \bar{I}$
Inductance:	$v(t) = L di(t)/dt$	\Leftrightarrow	$\bar{V} = j\omega L \bar{I}$
Capacitance:	$v(t) = (1/C) \int i(t) dt$	\Leftrightarrow	$\bar{V} = \bar{I}/j\omega C$

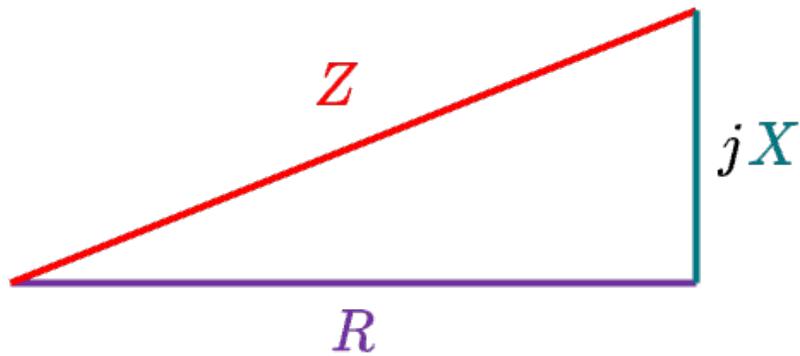
Thanks to the linearity equivalence, the Kirchhoff laws can be applied to phasors. In addition, **resistances**, **inductances** and **capacitances** can be replaced by complex impedances $Z = \frac{\bar{V}}{\bar{I}}$ of value respectively equal to R , $j\omega L$ and $\frac{1}{j\omega C}$.

Resistance, reactance and impedance

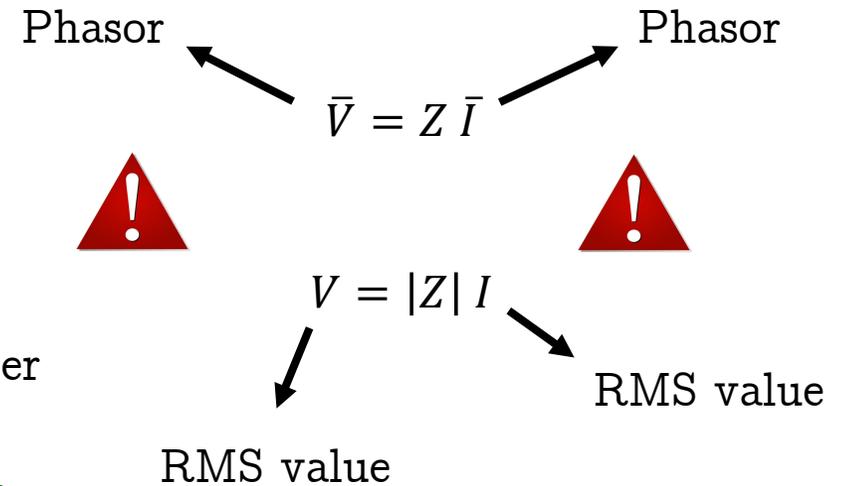
$$\bar{V} = R \bar{I} + j X \bar{I} = Z \bar{I}$$

Resistance R Reactance X Impedance Z

Resistance:	$\bar{V} = R \bar{I}$	R	0	R
Inductance:	$\bar{V} = j\omega L \bar{I}$	0	$X_L = \omega L$	$Z_L = j\omega L = jX_L$
Capacitance:	$\bar{V} = \bar{I}/j\omega C$	0	$X_C = -1/\omega C$ ★	$Z_C = -j/\omega C = jX_C$



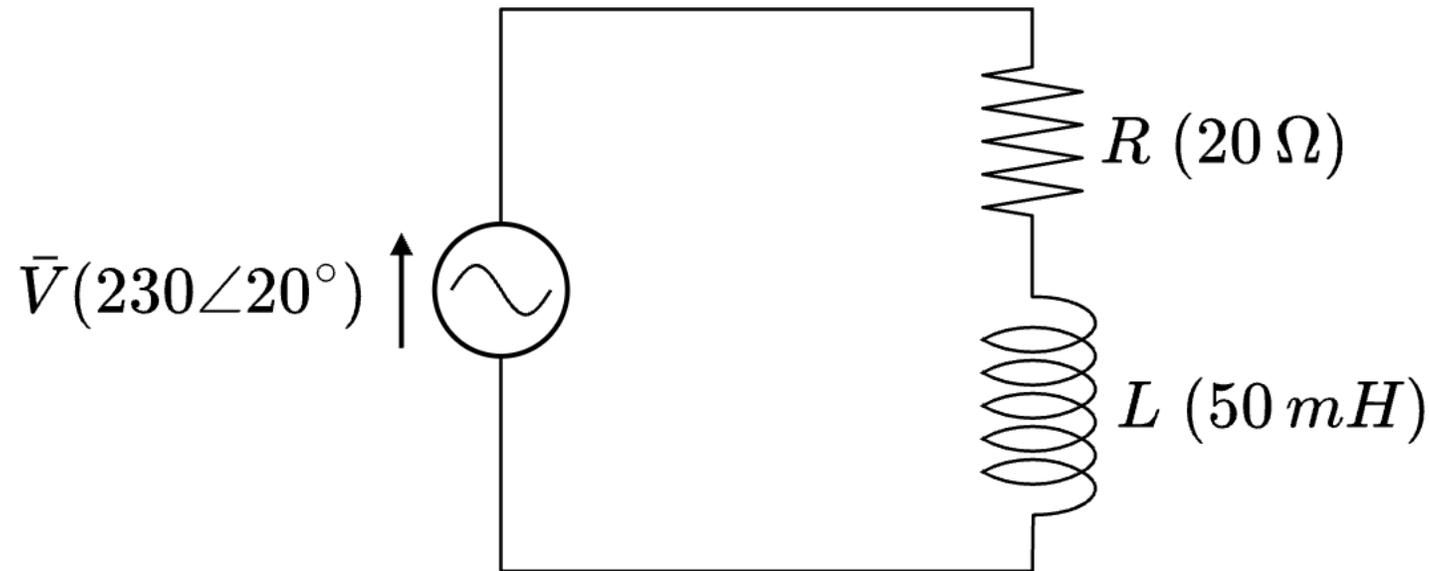
- R : Real number
- X : Real number
- Z : Imaginary number
- $|Z| = \sqrt{R^2 + X^2}$: Real number



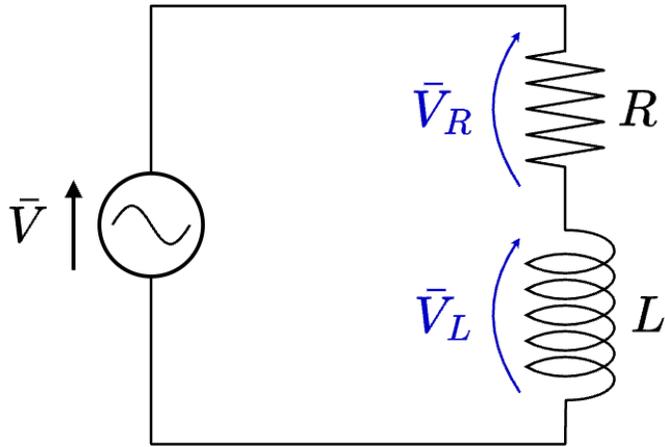
★ X_C is defined as a negative number, but it is often given as a positive magnitude. In that case, the impedance is written $Z_C = -jX_C$. For example, if $X_C = 2\Omega$ (magnitude), then $Z_C = -j2\Omega$.

Application: Exercise 1

For the hereunder circuit powered with a 50 Hz sinusoidal voltage generator, find the voltages and currents across all components (magnitude and phase angle). Represent all of them on a phasor diagram.



Application: Exercise 1: Watch out !



What the student writes on the exam copy:

$$\bar{V} = \bar{V}_R + \bar{V}_L$$

The teaching assistant face when correcting:



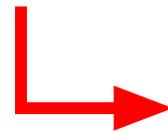
Student score at the question:

$$\geq 0$$

~~$$V = V_R + V_L$$~~



$$= 0$$

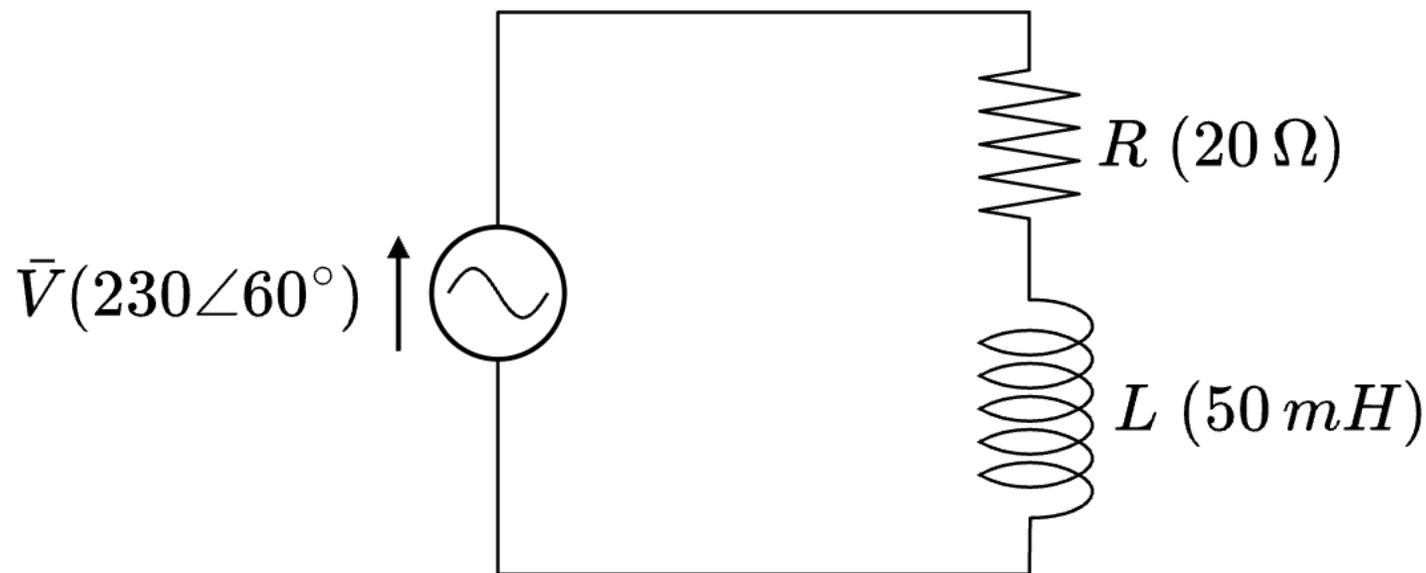


\bar{V} , \bar{V}_R and \bar{V}_L are complex numbers,
 V , V_R and V_L are their respective magnitudes/norms.

The sum of the norms of individual complex numbers usually differs from the norm of the sum of those complex numbers.

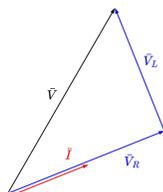
Homework 1

For the hereunder circuit powered with a 50 Hz sinusoidal voltage generator, find the voltages and currents across all components (magnitude and phase angle). Represent all of them on a phasor diagram.



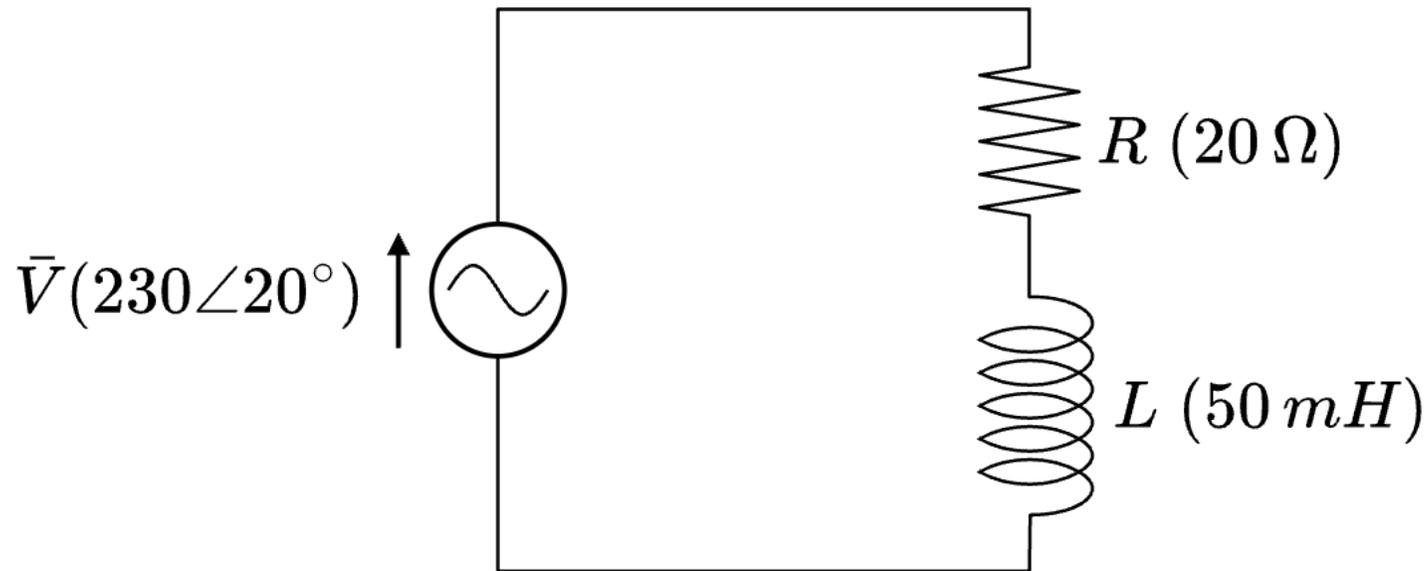
Answer:

$I = 9.044 \angle 21.854^\circ$
 $V_R = 180.88 \angle 21.854^\circ$
 $V_L = 142.06 \angle 111.854^\circ$
(rotation of 40° compared to Ex. 1.)



Homework 2

For the hereunder circuit powered with a 200 *Hz* sinusoidal voltage generator, find the voltages and currents across all components (magnitude and phase angle). Represent all of them on a phasor diagram.



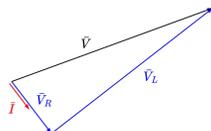
Answer:

$$I = 3.488 \angle -52.34^\circ$$

$$V_R = 69.762 \angle -52.34^\circ$$

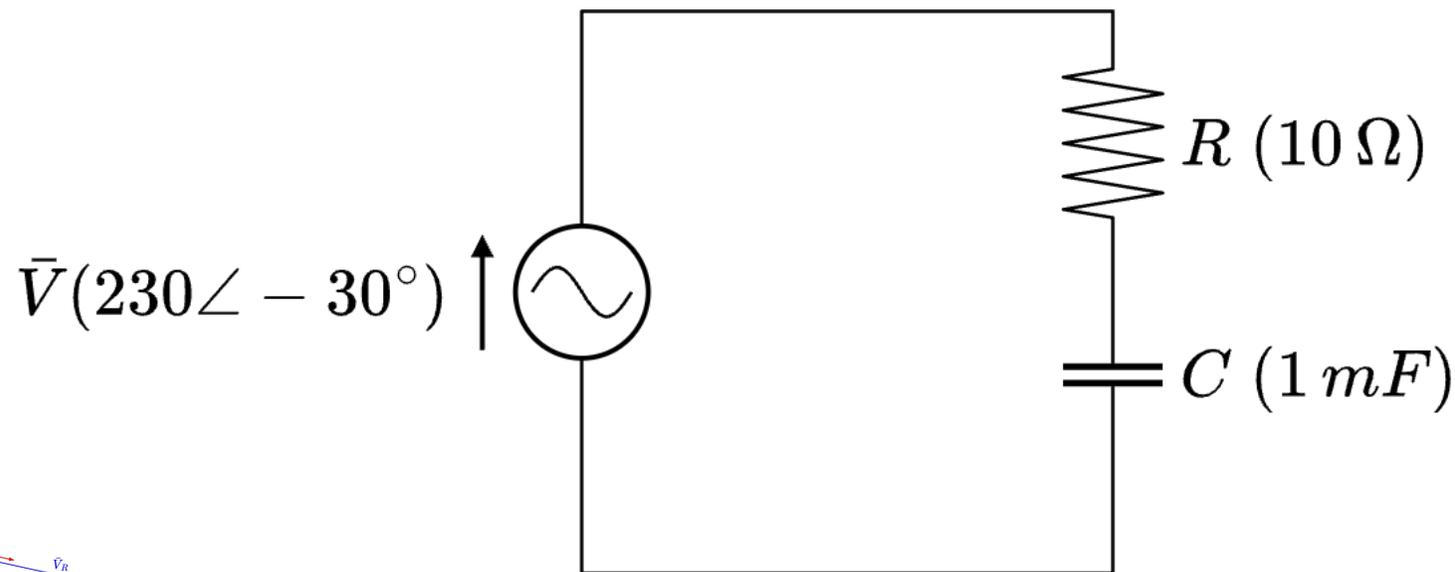
$$V_L = 219.165 \angle 37.66^\circ$$

(Try to understand the impact of the frequency by comparison to Ex. 1.)



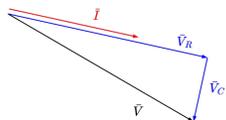
Homework 3

For the hereunder circuit powered with a 50 Hz sinusoidal voltage generator, find the voltages and currents across all components (magnitude and phase angle). Represent all of them on a phasor diagram.



Answer:

$$\begin{aligned} I &= 21.916 \angle -12.34^\circ \\ V_R &= 219.16 \angle -12.34^\circ \\ V_C &= 69.762 \angle -102.34^\circ \end{aligned}$$

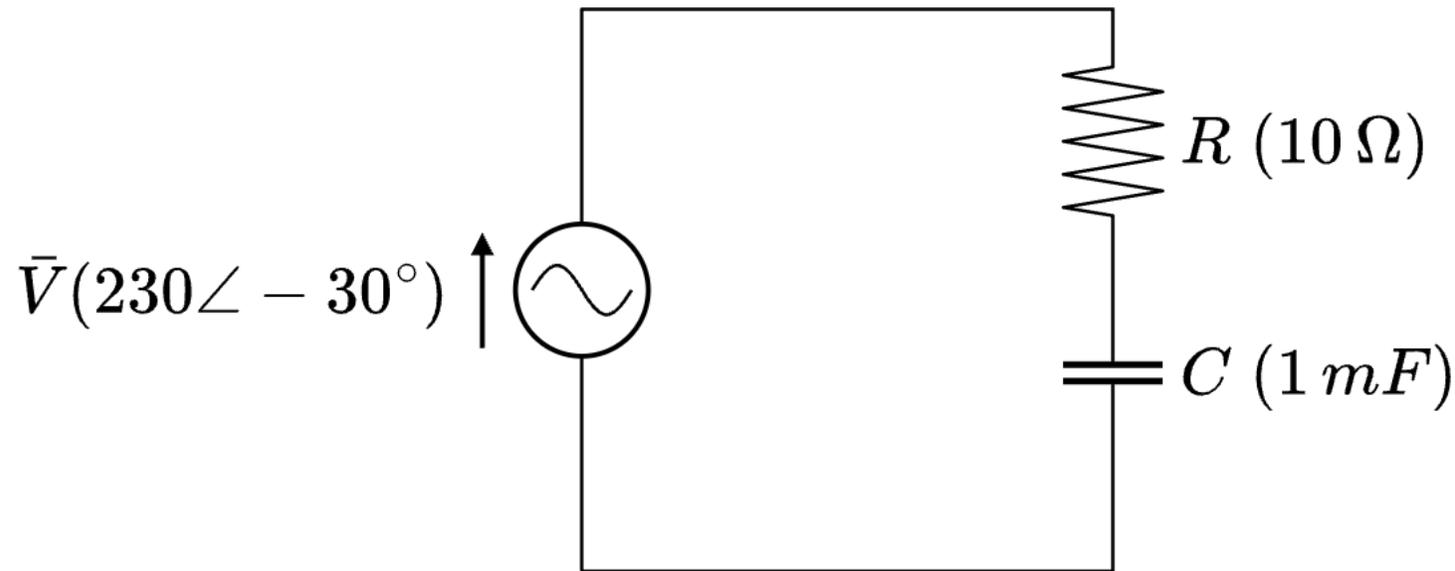


(Compare the phasor diagram with the one of Ex. 1, Hw. 1 and Hw. 2:

- for a capacitor, the voltage is always delayed by 90° compared to the current through it;
- for an inductor, the voltage is always in advance by 90° compared to the current through it.)

Homework 4

For the hereunder circuit powered with a 200 *Hz* sinusoidal voltage generator, find the voltages and currents across all components (magnitude and phase angle). Represent all of them on a phasor diagram.



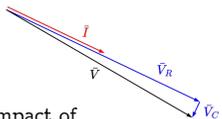
Answer:

$$I = 22.928 \angle -25.45^\circ$$

$$V_R = 229.28 \angle -25.45^\circ$$

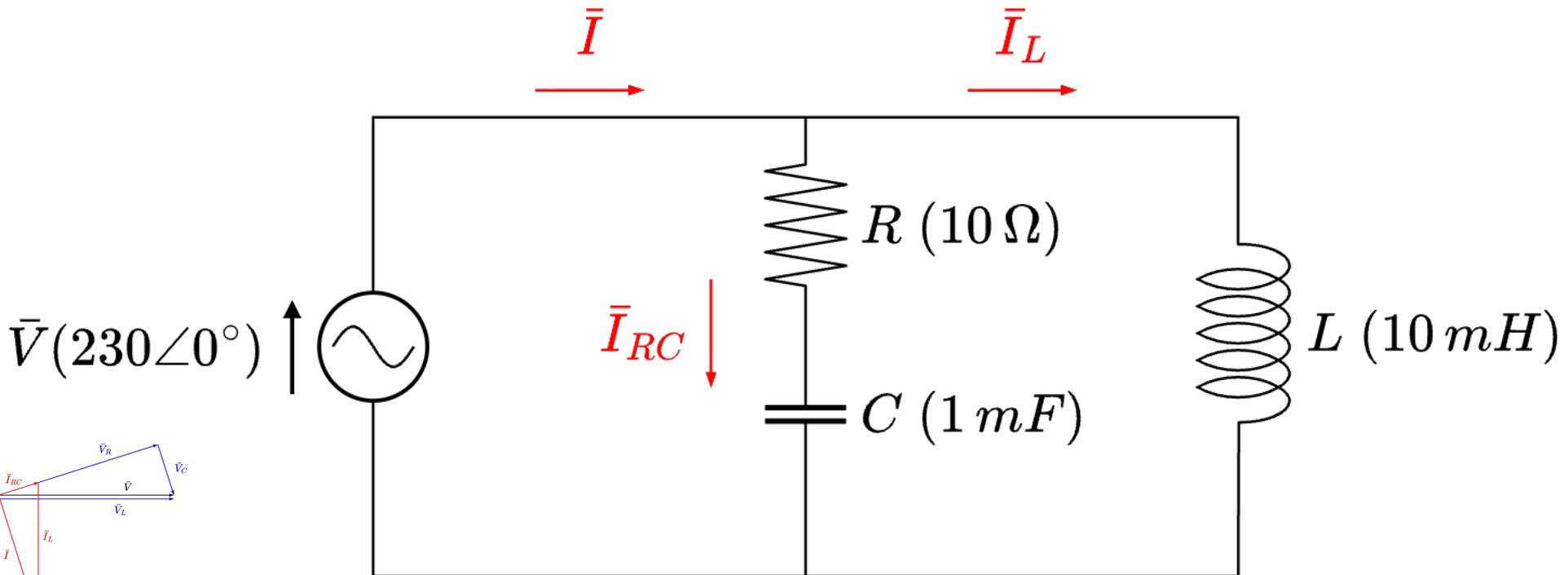
$$V_C = 18.245 \angle -115.45^\circ$$

(Try to understand the impact of the frequency by comparison to Hw. 3.)



Homework 5

For the hereunder circuit powered with a 50 Hz sinusoidal voltage generator, find the voltages and currents across all components (magnitude and phase angle). Represent all of them on a phasor diagram.



Answer:

$$\begin{aligned} I_{RC} &= 21.915 \angle 17.657^\circ \\ I_L &= 73.211 \angle -90^\circ \\ I &= 69.763 \angle -72.581^\circ \\ V_R &= 219.15 \angle 17.657^\circ \\ V_C &= 69.762 \angle -72.343^\circ \\ V_L &= 230 \angle 0^\circ \end{aligned}$$

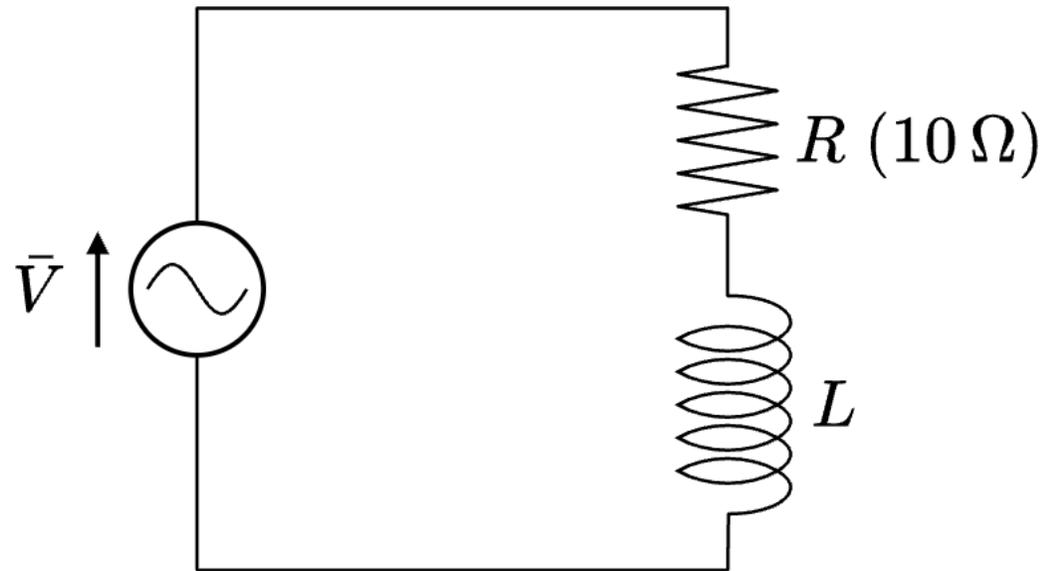
(Note that $I = I_L + I_{RC}$ but $I \neq I_L + I_{RC} \rightarrow$ see slide 24)

Homework 6

A resistor $R = 10 \Omega$ and an inductor L are connected in series to a 50 Hz sinusoidal voltage generator of RMS voltage $V = 230 \text{ V}$. An RMS voltage of 100 V is measured on the terminals of the resistor. What is the value of L ?

Hint:

Use the norm of $R + j\omega L$.

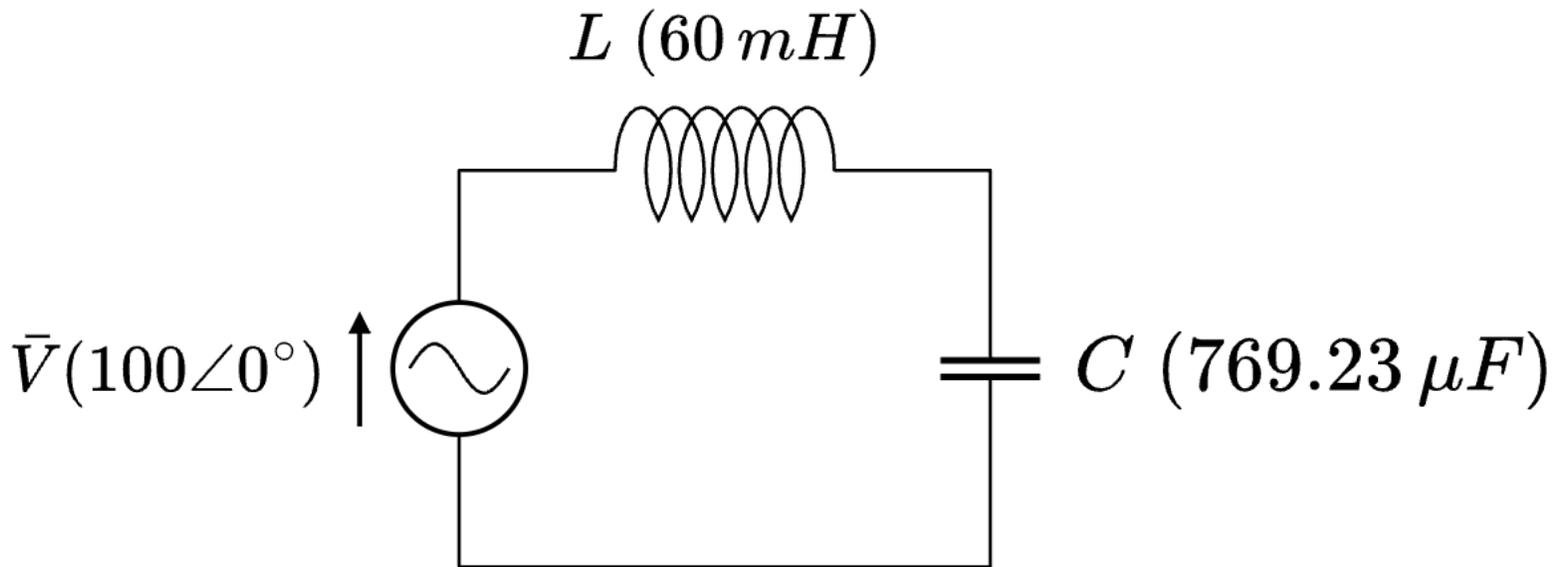


Answer:

$L = 65.929 \text{ mH}$

Homework 7

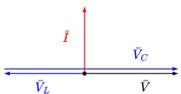
Consider the circuit hereunder operating with a pulsation $\omega = 2\pi f = 100 \text{ rad/s}$. Find the voltages and currents across all components (magnitude and phase angle). Represent all of them on a phasor diagram. Do the same in case the inductance is increased to 200 mH .



Answer:

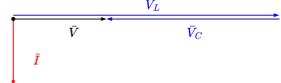
$L = 60 \text{ mH}$:

$$\begin{aligned} I &= 14.286 \angle 90^\circ \\ V_C &= 185.714 \angle 0^\circ \\ V_L &= 85.714 \angle 180^\circ \end{aligned}$$



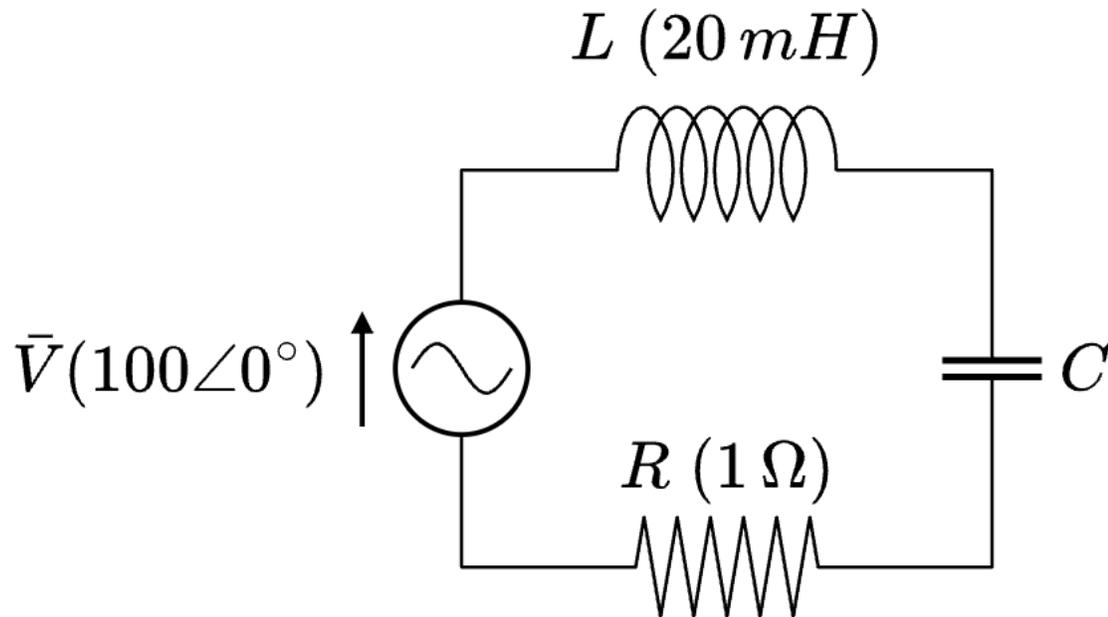
$L = 200 \text{ mH}$:

$$\begin{aligned} I &= 14.286 \angle -90^\circ \\ V_C &= 185.714 \angle 180^\circ \\ V_L &= 285.715 \angle 0^\circ \end{aligned}$$



Homework 8

Consider the circuit hereunder operating at 50 Hz . What are the two possible values of C if the RMS current is measured to be 20 A ? From these two capacitance values, select the larger one and calculate the voltage and current (magnitude and phase angle) across all components. Represent all of them on a phasor diagram.



Answer:

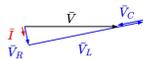
$$C = 2.3\text{ mF (or } 284.7\ \mu\text{F)}$$

$$I = 20\ \angle -78.464^\circ$$

$$V_R = 20\ \angle -78.464^\circ$$

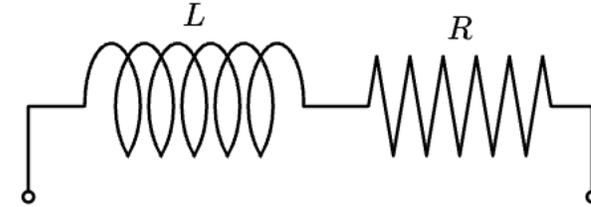
$$V_C = 27.678\ \angle -168.464^\circ$$

$$V_L = 125.658\ \angle 11.536^\circ$$



Homework 9

Consider the load showed on the right.



To determine the value of the resistance and the value of the inductance, two tests are performed:

- A DC voltage of 150 V is applied between the load terminals for a measured current of 1.95 A .
- An AC voltage of RMS value 230 V , oscillating at 50 Hz , is applied between the load terminals for a measured RMS current of 2.81 A .

What are the values of R and L ?

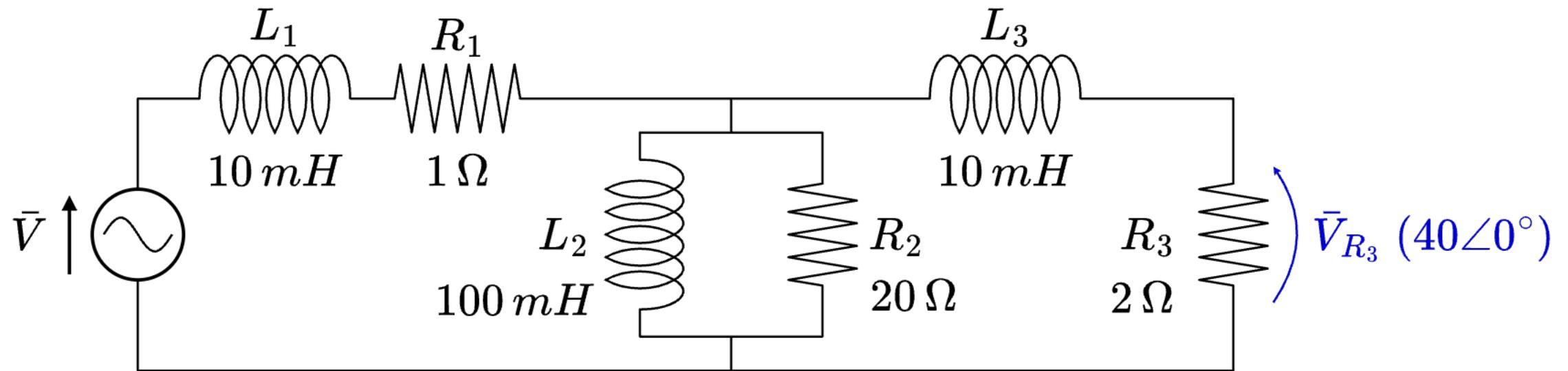
Answer:

$$R = 76.923\ \Omega$$

$$L = 89\text{ mH}$$

Homework 10

Considering the circuit depicted hereunder, find the input voltage \bar{V} (magnitude and phase angle) considering a frequency of 50 Hz.



Answer:

$$V_{R_2} = 74.484 \angle 57.518^\circ$$

$$I_{R_1} = 24.073 \angle 4.45^\circ$$

$$V = 151.68 \angle 67.465^\circ$$

Active, reactive, complex and apparent power

Product of two harmonic functions

Active, reactive, complex and apparent power

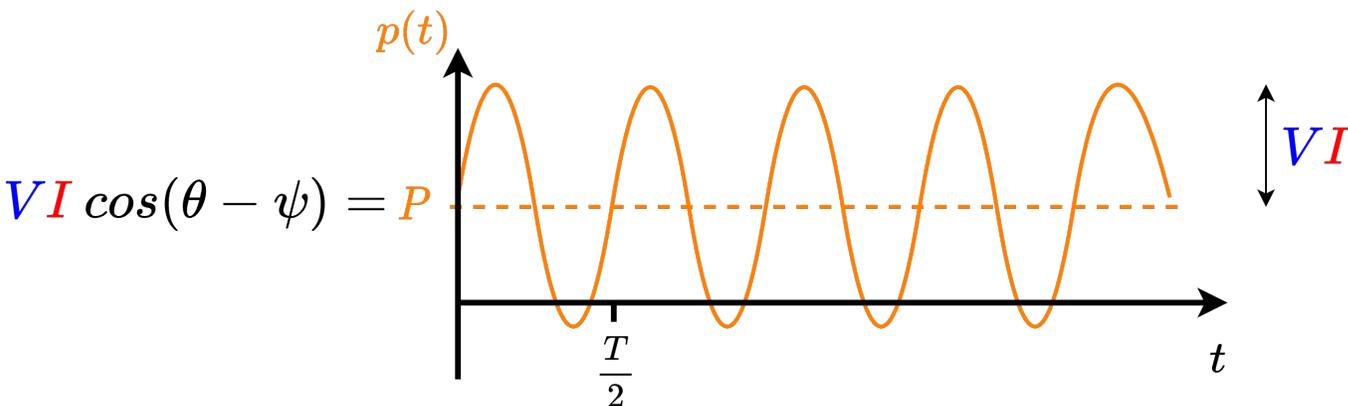
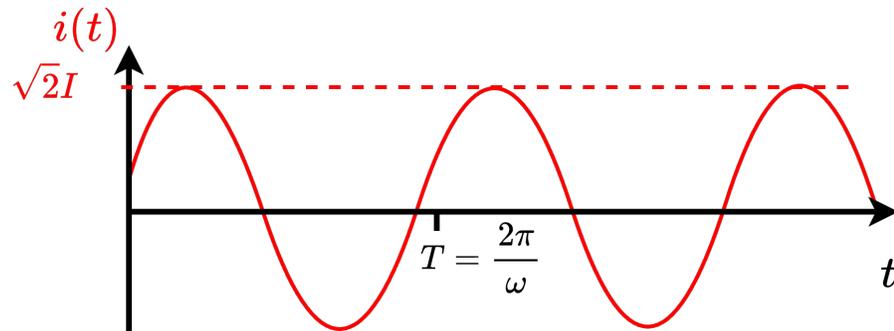
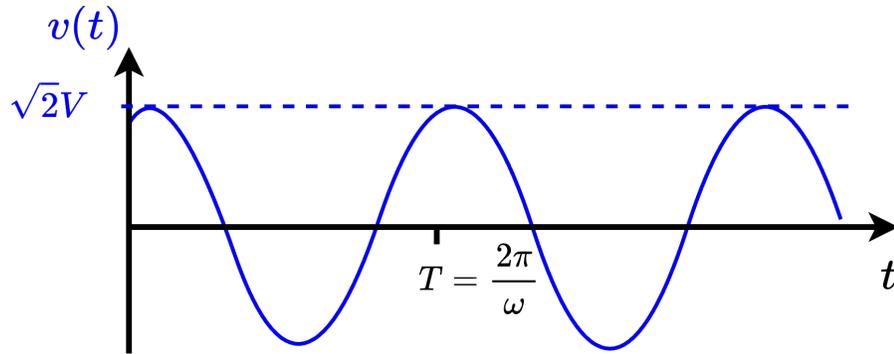
Application: Exercise 2

Product of two harmonic functions

The instantaneous power $p(t)$ consumed by passive components (or delivered by active components) is the product of the instantaneous voltage $v(t)$ with the instantaneous current $i(t)$:

$$\begin{aligned} p(t) &= v(t) i(t) \\ &= \sqrt{2} V \cos(\omega t + \theta) \sqrt{2} I \cos(\omega t + \psi) \\ &= 2 V I \cos(\omega t + \theta) \cos(\omega t + \psi) \\ &= V I \left[\underbrace{\cos(\theta - \psi)}_{\text{DC component}} + \underbrace{\cos(2\omega t + \theta + \psi)}_{\text{AC component (at twice the original frequency)}} \right] \end{aligned}$$

Product of two harmonic functions



$$p(t) = V I [\cos(\theta - \psi) + \cos(2\omega t + \theta + \psi)]$$

The instantaneous power can be split in two parts:

- $VI \cos(\theta - \psi)$: A DC component called the **Active power P**.
- $VI \cos(2\omega t + \theta + \psi)$: An AC component oscillating at **twice** the original frequency.

Active, reactive, complex and apparent power

$$\begin{aligned} p(t) &= v(t) i(t) \\ &= V I [\cos(\theta - \psi) + \cos(2\omega t + \theta + \psi)] \\ &= V I [\cos(\theta - \psi) + \cos(2(\omega t + \theta) - (\theta - \psi))] \\ &= V I [\cos(\theta - \psi) + \cos(2(\omega t + \theta)) \cos(\theta - \psi) + \sin(2(\omega t + \theta)) \sin(\theta - \psi)] \\ &= \underbrace{V I \cos(\theta - \psi)}_P \underbrace{[1 + \cos(2(\omega t + \theta))]}_{\geq 0} + \underbrace{V I \sin(\theta - \psi)}_Q \underbrace{\sin(2(\omega t + \theta))}_{\text{Oscillating } +-} \end{aligned}$$

$p(t)$ can be decomposed into:

- A flow of energy going always in the same direction with an average value $P = VI \cos(\theta - \psi)$. The value P is called the **active power**.
- A fluctuating flow of energy, exchanged back and forth twice a period, with an average value $Q = VI \sin(\theta - \psi)$. The value Q is called the **reactive power**.

Active, reactive, complex and apparent power

Although $p(t) = P [1 + \cos(2(\omega t + \theta))] + Q \sin(2(\omega t + \theta))$ is a real number, one defines the complex power S as:

$$S = P + jQ$$



S IS NOT A PHASOR, it is just a convenient complex number allowing to describe easily and completely the instantaneous power (real number).

One also shows that it is easy to determine S using phasors:

$$\begin{aligned} S &= P + jQ = VI \cos(\theta - \psi) + j VI \sin(\theta - \psi) = VI [\cos(\theta - \psi) + j \sin(\theta - \psi)] \\ &= VI e^{j(\theta - \psi)} = V e^{j\theta} I e^{-j\psi} = \bar{V} \bar{I}^* \end{aligned}$$



$$S = \bar{V} \bar{I}^*$$

Active, reactive, complex and apparent power

One also defines the apparent power:

$$\begin{aligned} |S| &= \sqrt{P^2 + Q^2} \\ &= \sqrt{V^2 I^2 \cos^2(\theta - \psi) + V^2 I^2 \sin^2(\theta - \psi)} \\ &= VI \end{aligned}$$

And the power factor:

$$PF = \cos(\theta - \psi) = \cos(\varphi) = \frac{P}{|S|}$$

Let's now suppose that we want to deliver an active power P to a load. What current I should we inject for a fixed voltage V ?

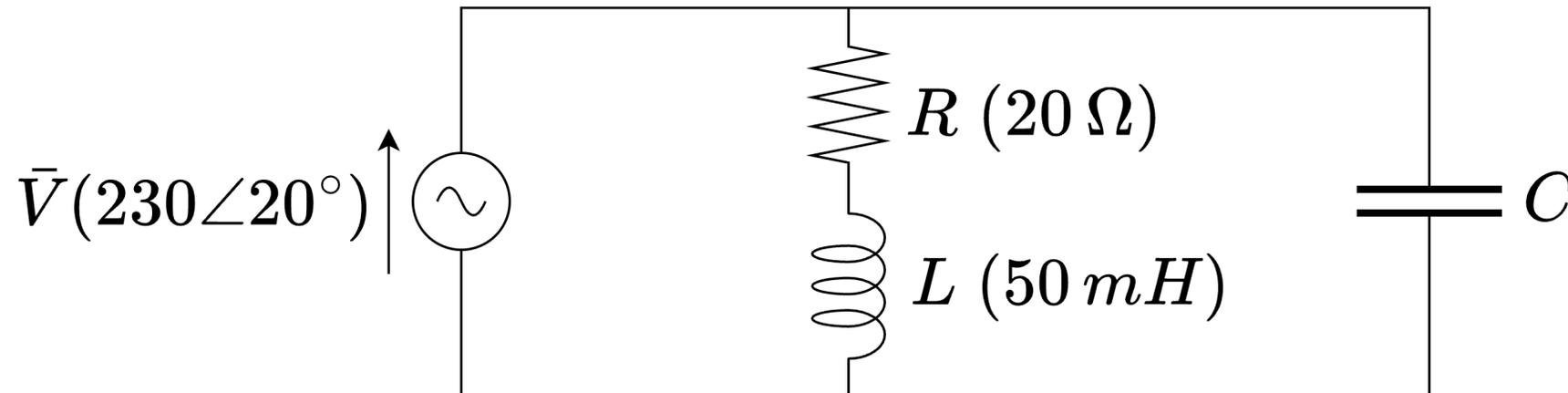
$$I = \frac{|S|}{V} = \frac{P}{V \cos(\varphi)}$$

I is larger when $\cos(\varphi)$ is small \rightarrow If $\cos(\varphi)$ is small, there are more transmission losses.

\rightarrow We want $\cos(\varphi)$ to be 1 \rightarrow To avoid transmission losses, we want the reactive power Q to be as small as possible.

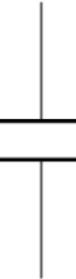
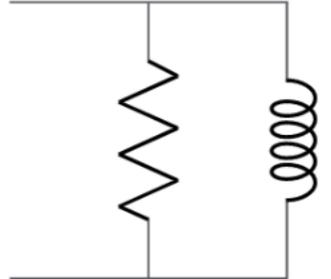
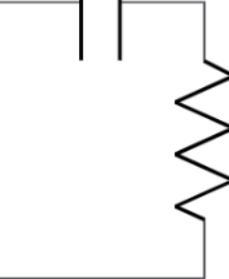
Application: Exercise 2

Your colleague suggests to add a $50 \mu F$ capacitor in parallel of the RL load. It is supposed to compensate the reactive power consumed by the inductive load. Is it a good idea ? If so, what would be the exact value of C needed ?



Homework 11

Fill the cells of the table below with the most appropriate answer among $= 0$, < 0 , > 0 , $= 1$, < 1 , $+\infty$ and $-\infty$.

one-port:					
active power consumed					
reactive power produced					
$\cos \phi$					
$\tan \phi$					

Answer:

					
Active power consumed	> 0	$= 0$	$= 0$	> 0	> 0
Reactive power produced	$= 0$	< 0	> 0	< 0	> 0
$\cos(\phi)$	$= 1$	$= 0$	$= 0$	> 0	> 0
$\tan(\phi)$	$= 0$	$+\infty$	$-\infty$	> 0	< 0