ELEC0431 : Exercise session 2

Reminders of balanced three-phase systems

12 February 2021

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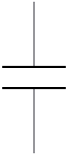
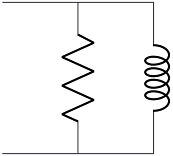
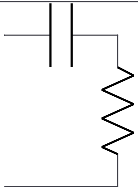
Exercise 3 : one-port small quiz

Fill the cells of the table below with the most appropriate answer among :

$=0$ <0 >0 $=1$ <1 $+\infty$ $-\infty$

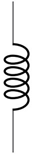
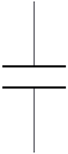
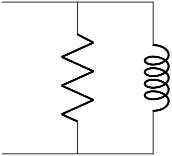
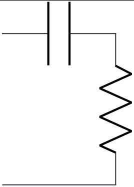
one-port:					
active power consumed					
reactive power produced					
cos ϕ					
tan ϕ					

One-port small quizz

One-port quizz

Wooclap code : JPMAJU

Exercise 3 : one-port small quiz - solution

one-port:					
active power consumed	> 0	$=0$	$=0$	> 0	> 0
reactive power produced	$=0$	< 0	> 0	< 0	> 0
$\cos\phi$	$=1$	$=0$	$=0$	< 1	< 1
$\tan\phi$	$=0$	$+\infty$	$-\infty$	> 0	< 0

Alternating voltages and currents

Alternating voltages and currents fluctuate over time as sinusoids (or cosinusoids) with a given frequency (AC voltages and currents).

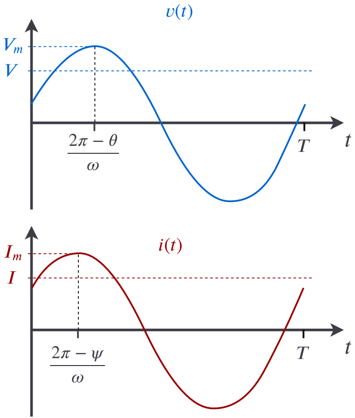
Time domain		Frequency domain
$v(t) = \sqrt{2}\,V\ \cos(\omega t + \theta)$	$\Longleftrightarrow$	$\overline{V} = V\,e^{j\,\theta}$
$i(t) = \sqrt{2}\,I\ \cos(\omega t + \psi)$	$\Longleftrightarrow$	$\overline{I} = I\,e^{j\,\psi}$

For sinusoidal waveforms, the ratio between maximum values and RMS values is

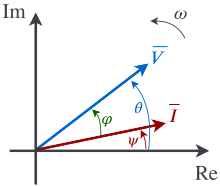
$$\boxed{\begin{aligned} V_{\mathrm{m}} &= \sqrt{2}\ V \\ I_{\mathrm{m}} &= \sqrt{2}\ I \end{aligned}}$$

As electrical quantities fluctuate at the same frequency, one can switch from the time domain to the frequency domain (with phasors).

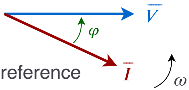
Alternating voltages and currents - Phasor diagrams



$\Longleftrightarrow$



commonly



Consider $\overline{V}$ as the reference with $\theta = 0$ and $\varphi = -\psi$

$$\boxed{\varphi = \theta - \psi} \quad (1)$$

The complex power and the apparent power

When a component is traveled by a current with a certain voltage accross its accesses, some electrical power is consumed (or produced) by this component. Defining the complex power

$$\begin{aligned} S &= \overline{V}\,\overline{I}^* \\ &= V\, I\, e^{j\varphi} \\ &= P + j\,Q \end{aligned}$$

Remark that only the phase angle difference φ is relevant for the power computation.

The norm of the complex power is called the apparent power :

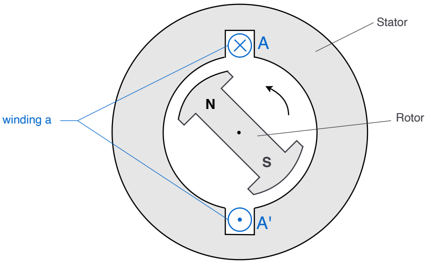
$$|S| = \sqrt{P^2 + Q^2} = V\ I \tag{2}$$

Voltages and frequencies

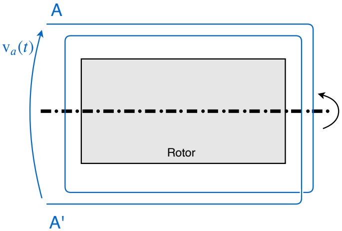
Wooclap code : NZKDFJ

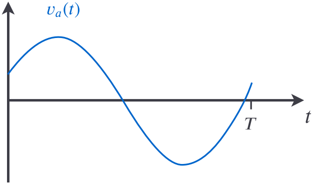
1 Phase generator

Front view



Side view

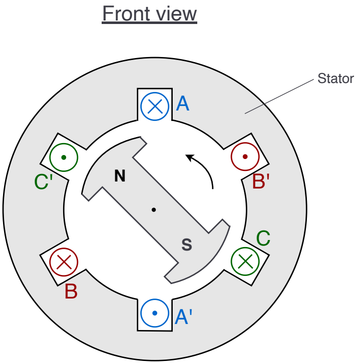


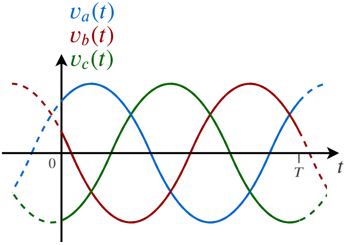


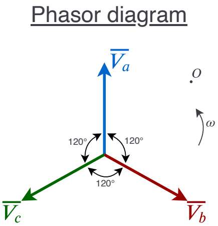
The voltage $v_a(t)$ is induced in the winding as the magnetic flux created by the rotor varies over time :

$$v_a(t) = -n_a \underbrace{\frac{d\,\phi(t)}{d\,t}}_{\text{varying flux}} \qquad (3)$$

3 Phase generator

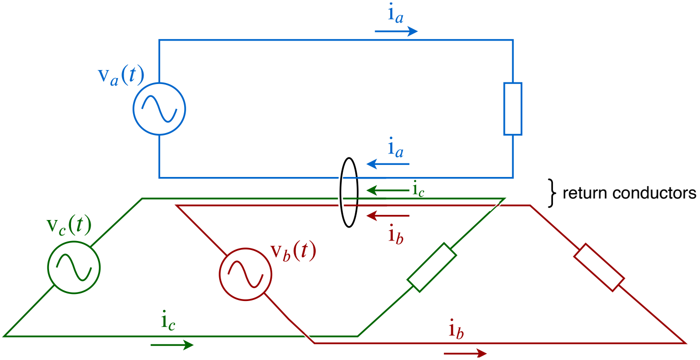






The three-phase generator now possesses 3 windings on its stator.

3 Phase circuit



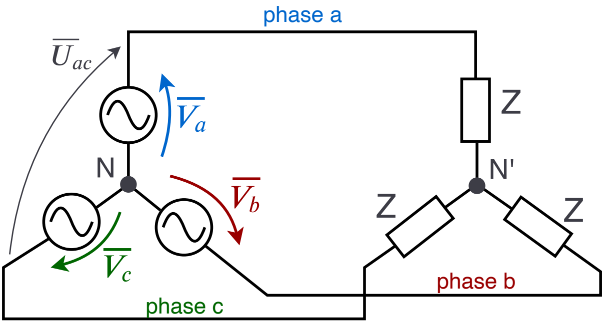
The 3 return conductors can be merged into one return conductor. This conductor carries a current

$$i_a + i_b + i_c = 0 \tag{4}$$

and can be removed.

3 Phase circuit

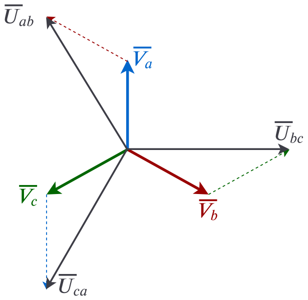
The balanced three-phase circuit is the assembly of three identical circuits called phases. The 3 signals are dephased by 120°.



Line voltages

Line voltages

$$\overline{U}_{ab} = \overline{V}_a - \overline{V}_b$$
$$\overline{U}_{bc} = \overline{V}_b - \overline{V}_c$$
$$\overline{U}_{ca} = \overline{V}_c - \overline{V}_a$$

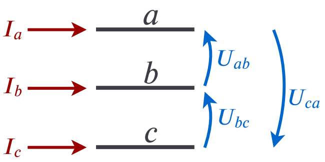


$\overline{V}_a$, $\overline{V}_b$ and $\overline{V}_c$ are the phase voltages, measured between a phase and the neutral N.

$\overline{U}_{ab}$, $\overline{U}_{bc}$ and $\overline{U}_{ca}$ are the line voltages (or phase-to-phase voltages), measured between two phases.

Introduction to balanced three-phase systems

On a three-phase network without the neutral conductor, one can only measure the line voltages and the line currents.



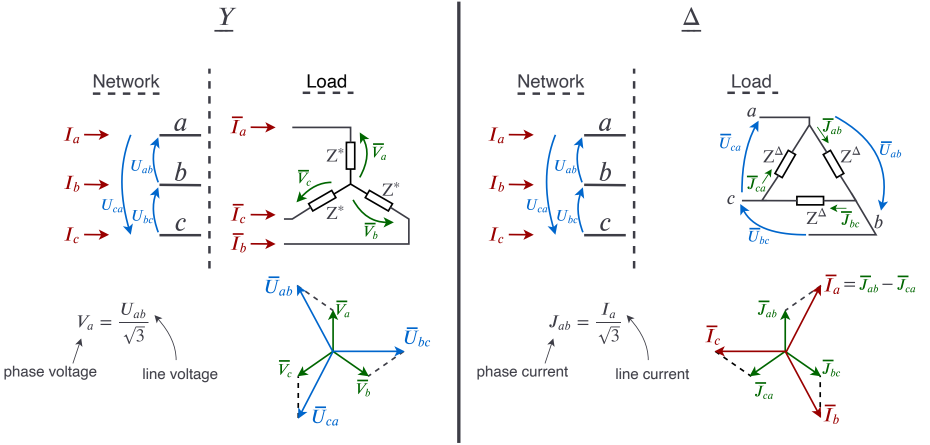
and the three-phase apparent power will be

$$S = 3\ V\ I \tag{5}$$

$$= \sqrt{3}\ U\ I \tag{6}$$

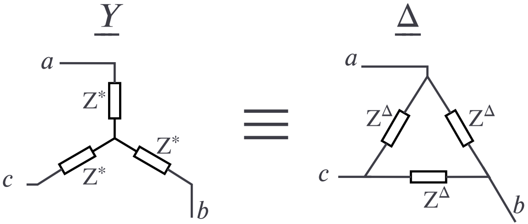
Three-phase loads under Y and Δ connections (étoile-triangle)

Three-phase loads can be arranged either in star (Y) or in triangle (Δ). The voltages and currents are splitted as following :



Y and Δ equivalents

From the network perspective, the Y or Δ arrangements are equivalent as long as they provide the same line voltages and currents.



$$\boxed{Z^{\Delta} = 3\ Z^{*}} \tag{7}$$

Under condition (7), the two loads are equivalents and consume (produce) the same power because $Z^{*} = \frac{V}{I}$ and $Z^{\Delta} = \frac{U}{J}$.

Exercises

Exercises of session 2

Wooclap code : ZYTYKI

Exercise 4 : 2-ports characterization

Characterize the 2-ports of Figure 1. In that context, two tests have been performed: a short circuit test and an open circuit test.

- 559 mV and 1.118 A are measured at the access 1 while the access 2 is shorted (short circuit test at 50 Hz).
- 5 V and 4.472 A are measured at the access 1 while the access 2 is left open (open circuit test at 50 Hz).

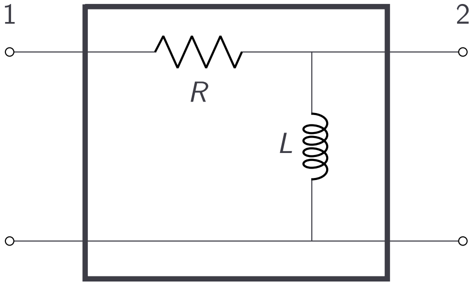


Figure: 2-Ports circuit

Consider that the tests are performed at 50 Hz. The 2-ports could be fully caracterized by only one of the two tests if the active power was measured during the tests. The active power can be measured with a wattmeter.

1. Determine the value of R and L with the information above.
2. Which test would be necessary ?
3. During that test, an active power of 9.99392 W has been measured. Prove that it gives the correct value of R and L.

Exercise 4 : 2-ports characterization - solution

Short circuit test :

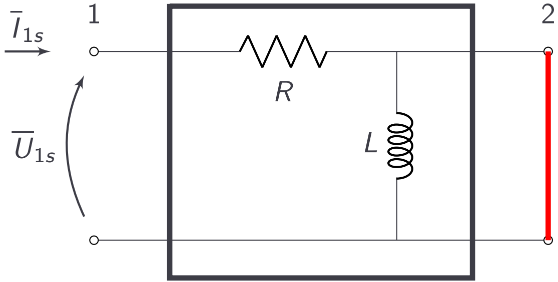


Figure: 2-Ports under short circuit test

$$R = \frac{U_{1s}}{I_{1s}} = \frac{0.559}{1.118} = 0.5 \quad [\Omega] \tag{8}$$

Exercise 4 : 2-ports characterization - solution

Open circuit test :

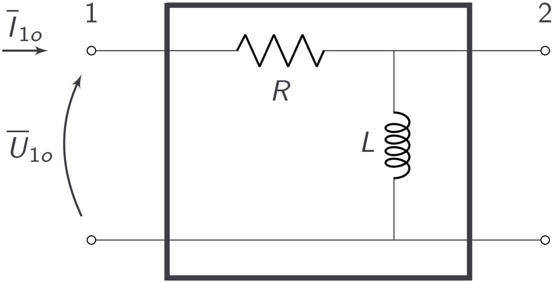


Figure: 2-Ports under open circuit test

$$|Z| = \frac{U_{1o}}{I_{1o}} = \frac{5}{4.472} = 1.118 \quad [\Omega] \tag{9}$$

$$X = \sqrt{|Z|^2 - R^2} = \sqrt{1.118^2 - 0.5^2} = 1 \quad [\Omega] \tag{10}$$

Exercise 4 : 2-ports characterization - solution

If a wattmeter had been used, only one manipulation would have been required.

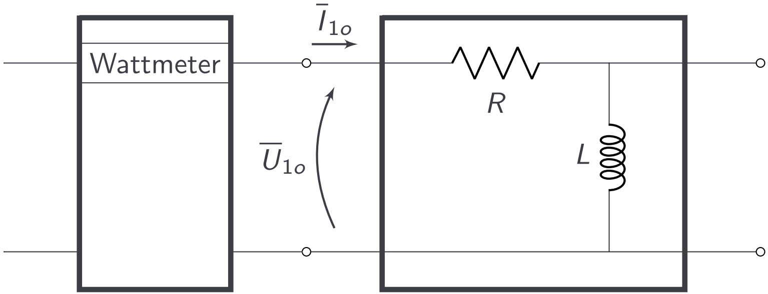


Figure: 2-Ports under open circuit test with wattmeter

The active power measured would be $P_{1o} = 9.99392$ [W] and the apparent power $S_{1o} = U_{1o}\ I_{1o} = 5 \times 4.472 = 22.36$ [VA].

From the knowledge of the apparent power and the active power (both measured in open circuit for this test), one can deduce the reactive power

$$Q_{1o} = \sqrt{S_{1o}^2 - P_{1o}^2} = 20.002 \text{ [var]} \tag{11}$$

$$P = R\,I^2 \quad \Longrightarrow \qquad R = \frac{P_{1o}}{I_{1o}^2} = \frac{9.99392}{4.472^2} = 0.5 \text{ } [\Omega]$$

$$Q = X\,I^2 \quad \Longrightarrow \qquad X = \frac{Q_{1o}}{I_{1o}^2} = \frac{20.002}{4.472^2} = 1 \text{ } [\Omega]$$

Exercise 5 : electric heater

Consider an electrical heater that dissipates 15 kW of power when connected to a three-phase power system of 208 V. As a first approximation, the heater is modelled as a purely resistive three-phase load.

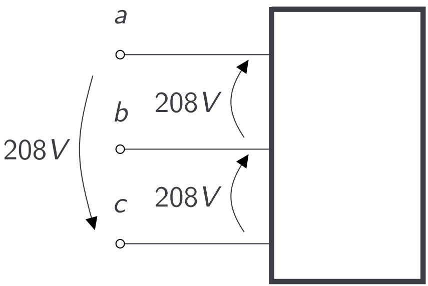


Figure: Three-phase Electrical Heater

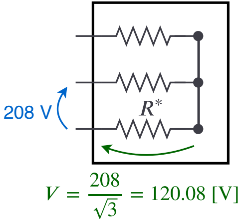
Exercise 5 : electric heater

1. If no additionnal information is provided about the voltage, does the 208 V correspond to the peak or the RMS value ?
2. Compute the line current if the resistive loads are connected in **Y**.
3. If the resistors are connected in **Y**, compute the resistance of each.
4. Compute the line current if the resistive loads are connected in Δ.
5. If the resistors are connected in Δ, compute the resistance of each.

Exercise 5 : electric heater - solution

1. The value of 208 V corresponds to an RMS value of the line voltages (phase-to-phase). The peak value is $\sqrt{2} \times 208 = 294.1 V$
2. The power of the three-phase system is $P_{3\varphi} = 15$ kW, then, the power of one phase is $P_{1\varphi} = \frac{P_{3\varphi}}{3} = 5$ kW. The line current is

$$I = \frac{P_{1\varphi}}{V} = \frac{5000}{120.08} = 41.635 \text{ [A]} \quad (12)$$



3. If the resistors are connected in star configuration, their value must be

$$R^* = \frac{V_{a,b,c}}{I_{a,b,c}} = \frac{120.08}{41.635} = 2.884 \quad [\Omega] \tag{13}$$

4. Even if the resistors are connnected in Δ, in this case, the line currents remain the same because the power consumption and voltages are kept constant : $I = 41.635$ [A]

   Remark that the current through the loads is different now : $J = \frac{I}{\sqrt{3}} = 24.038$ [A]

Exercise 5 : electric heater - solution

5. If the resistors are known to be connected in Δ, the resistance of each is

$$R^{\Delta} = \frac{U}{J} = \frac{208}{24.038} = 8.6528 \quad [\Omega] \tag{14}$$

Also remark that $Z^{\Delta} = 3\, Z^{*}$ which is the expected results if the two loads consume the same amount of power.