Electromagnetic Energy Conversion ELEC0431

Exercise session 2: Reminders of balanced three-phase systems

18 February 2022

Florent Purnode (florent.purnode@uliege.be) – Nicolas Davister (ndavister@uliege.be)

Montefiore Institute, Department of Electrical Engineering and Computer Science,
University of Liège, Belgium

Looking back

Clarifications on exercise session 1

Exercise 1: manually computing $\bar{I}$

$\bar{I} = \frac{\bar{V}}{R + jX}$???

Option 1: use your calculator (win time)

Option 2: have fun with maths! (personal challenge ➔ happiness)

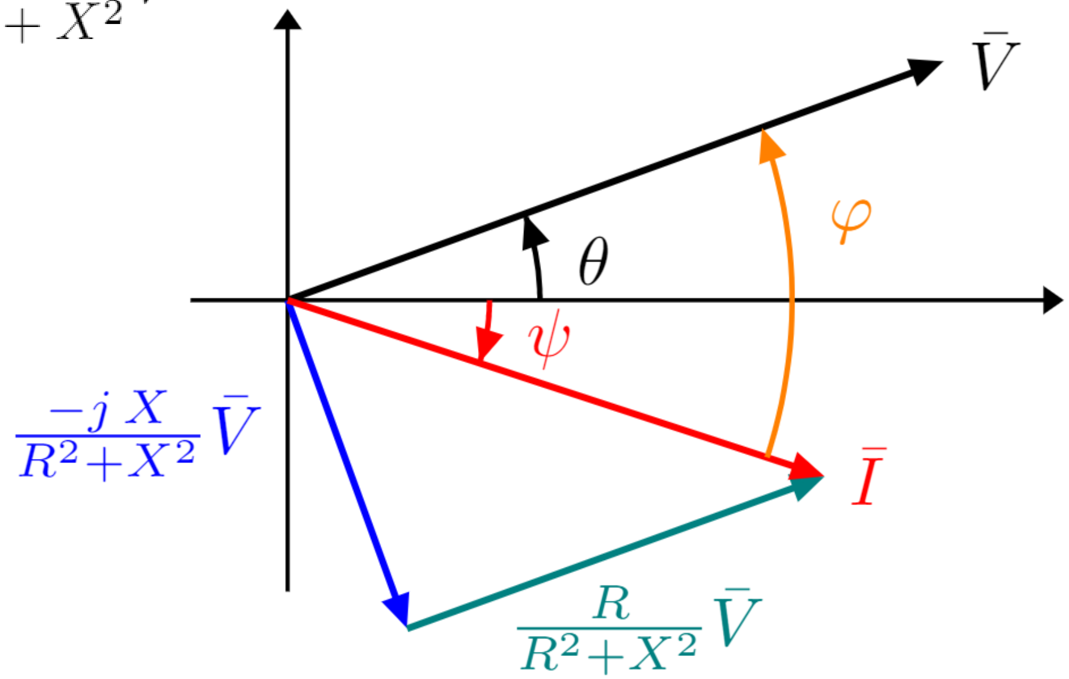
$$\bar{I} = \frac{\bar{V}}{R+jX} = \frac{\bar{V}}{R+jX}\frac{R-jX}{R-jX} = \frac{R}{R^2+X^2}\bar{V} - j\frac{X}{R^2+X^2}\bar{V}$$

$$|\bar{I}| = \sqrt{(\frac{R}{R^2+X^2})^2 + (\frac{X}{R^2+X^2})^2}\ V = 9.044\ A$$

$$\varphi = \arctan\frac{\frac{X}{R^2+X^2}}{\frac{R}{R^2+X^2}} = \arctan\frac{X}{R} = 38.146^\circ$$

$$\varphi = \theta - \psi \Leftrightarrow \psi = \theta - \varphi = 20 - 38.146 = -18.146^\circ$$

$$\bar{I} = 9.044\angle - 18.146^\circ$$



Exercise 2: Matching the reactive powers

$$Q_C = Q_{RL}$$

Reactive power produced by C — Reactive power consumed by L

➔ Ok if we consider both the motor and generator convention

In case only the motor convention is used (usual convention)

$$Q_C = -Q_{RL}$$

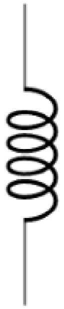
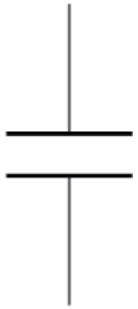
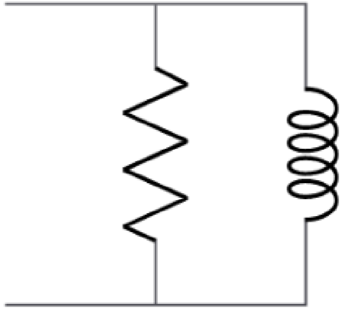
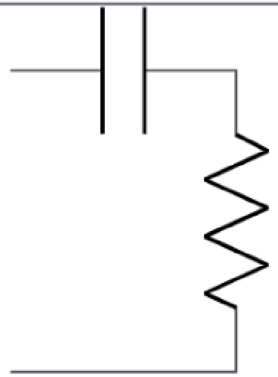
Correction ex. 3

One port small quiz

Exercise 3: one-port small quiz

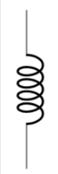
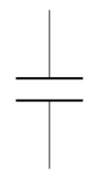
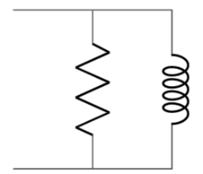
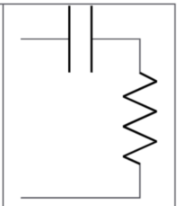
Fill the cells of the table below with the most appropriate answer among :

$=0$ <0 >0 $=1$ <1 $+\infty$ $-\infty$

one-port:					
active power consumed					
reactive power produced					
cos ϕ					
tan ϕ					

Exercise 3: R

$$\bar{V} = Z\,\bar{I}$$
$$Z = R$$
$$\bar{V} = R\,\bar{I}$$

one-port:					
active power consumed					
reactive power produced					
$\cos\phi$					
$\tan\phi$					

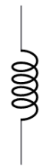
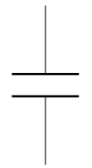
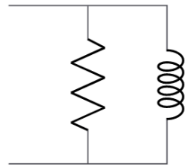
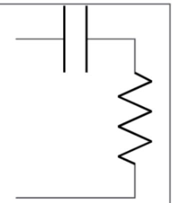
$$\cos(\varphi) = 1$$
$$\tan(\varphi) = 0$$

$$S = \bar{V}\,\bar{I}^* = RI^2$$

$$P > 0$$
$$Q = 0$$

Exercise 3: L

$$\bar{V} = Z\,\bar{I}$$
$$Z = j\omega L$$
$$\bar{V} = j\omega L\,\bar{I}$$

one-port:					
active power consumed	> 0				
reactive power produced	0				
$\cos\phi$	1				
$\tan\phi$	0				

$$\cos(\varphi) = 0$$
$$\tan(\varphi) = +\infty$$

$$S = \bar{V}\,\bar{I}^{*} = j\omega L I^{2}$$

$$P = 0$$
$Q > 0$ ➔ reactive power produced < 0

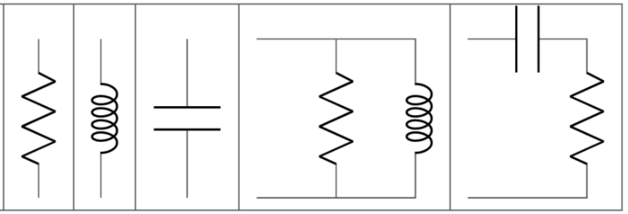
Exercise 3: C

$$\bar{V} = Z\,\bar{I}$$

$$Z = \frac{-j}{\omega C}$$

$$\bar{V} = \frac{-j}{\omega C}\,\bar{I}$$

$$\cos(\varphi) = 0$$
$$\tan(\varphi) = -\infty$$



one-port:					
active power consumed	> 0	0			
reactive power produced	0	< 0			
$\cos\phi$	1	0			
$\tan\phi$	0	$+\infty$			

$$S = \bar{V}\,\bar{I}^* = \frac{-j}{\omega C} I^2$$

$$P = 0$$

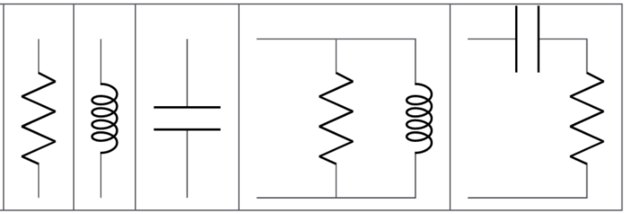
$Q < 0$ ➔ reactive power produced > 0

Exercise 3: R // L

$$\bar{V} = Z\,\bar{I}$$

$$Z = \left(\frac{1}{R} + \frac{1}{jX}\right)^{-1} = \frac{RX^2}{R^2+X^2} + j\,\frac{R^2X}{X^2+R^2}$$

$$\bar{V} = \frac{RX^2}{R^2+X^2}\bar{I} + j\,\frac{R^2X}{X^2+R^2}\,\bar{I}$$



one-port:					
active power consumed	> 0	0	0		
reactive power produced	0	< 0	> 0		
$\cos\phi$	1	0	0		
$\tan\phi$	0	$+\infty$	$-\infty$		

$\cos(\varphi) \in \,]0,1[$
$\tan(\varphi) \in \,]0,+\infty[$

$$S = \bar{V}\,\bar{I}^* = \frac{RX^2}{R^2+X^2}I^2 + j\,\frac{R^2X}{X^2+R^2}I^2$$

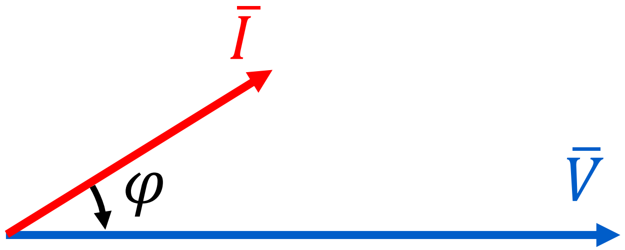
$P > 0$
$Q > 0$ ➔ reactive power produced < 0

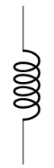
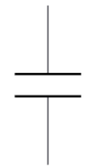
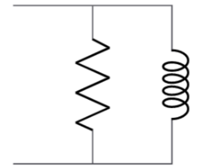
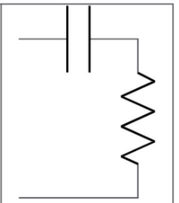
Exercise 3: RC

$$\bar{V} = Z\,\bar{I}$$

$$Z = R + \frac{-j}{\omega C}$$

$$\bar{V} = R\bar{I} - \frac{j}{\omega C}\,\bar{I}$$



one-port:					
active power consumed	> 0	0	0	> 0	
reactive power produced	0	< 0	> 0	< 0	
cos ϕ	1	0	0	$\in]0,1[$	
tan ϕ	0	$+\infty$	$-\infty$	$\in]0,+\infty[$	

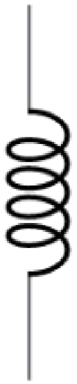
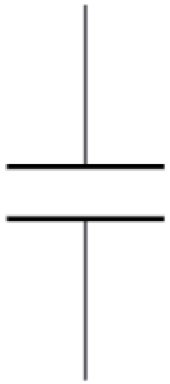
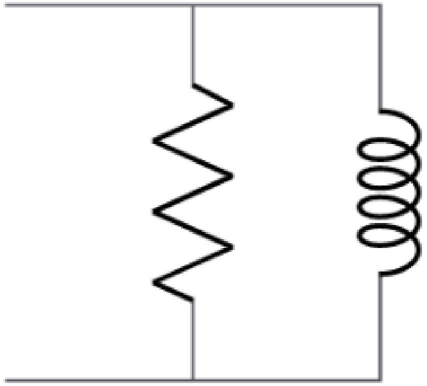
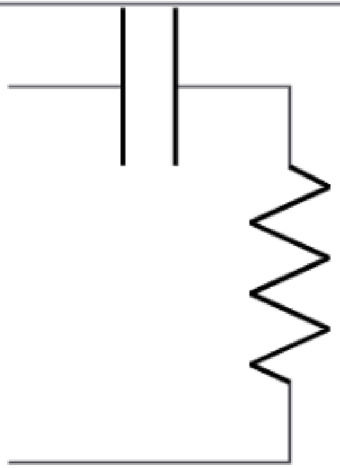
$$\cos(\varphi) \in]0,1[$$
$$\tan(\varphi) \in]-\infty, 0[$$

$$S = \bar{V}\,\bar{I}^* = RI^2 - \frac{j}{\omega C}\,I^2$$

$P > 0$

$Q < 0$ ➔ reactive power produced > 0

Exercise 3: summary

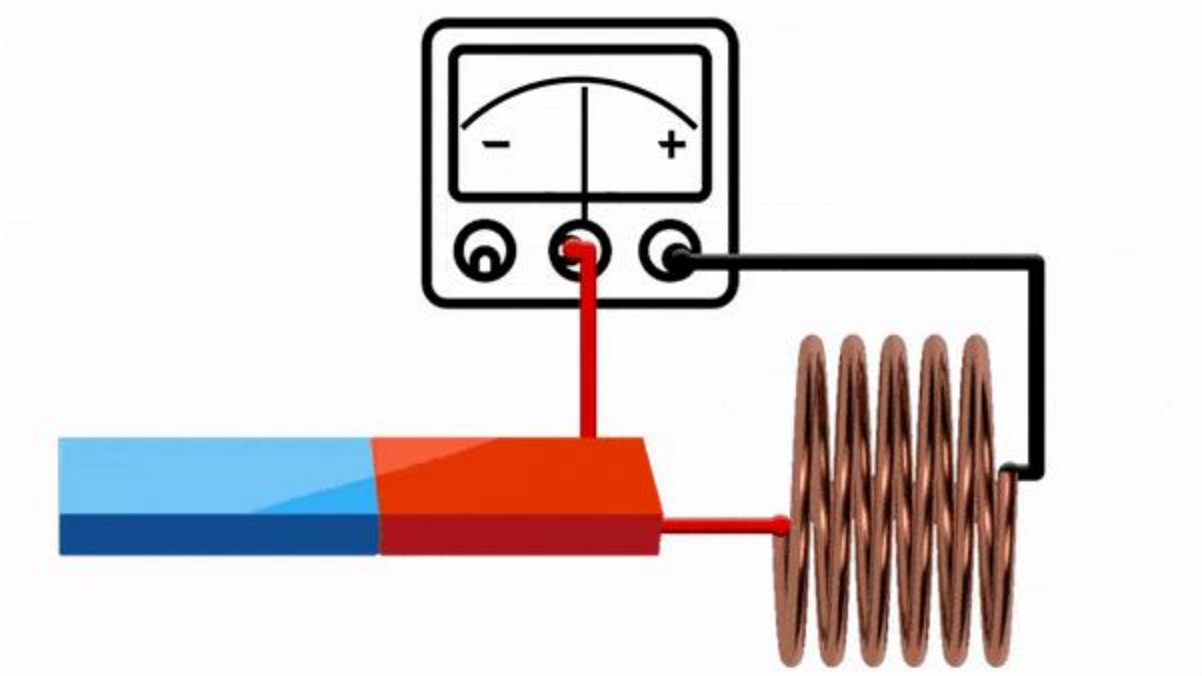
one-port:					
active power consumed	> 0	$= 0$	$= 0$	> 0	> 0
reactive power produced	$= 0$	< 0	> 0	< 0	> 0
$\cos\,\phi$	$= 1$	0	0	$\in\,]0,1[$	$\in\,]0,1[$
$\tan\,\phi$	$= 0$	$+\infty$	$= -\infty$	$\in\,]0,+\infty[$	$\in\,]-\infty,0[$

Reminders

synchronous machine and three-phase systems

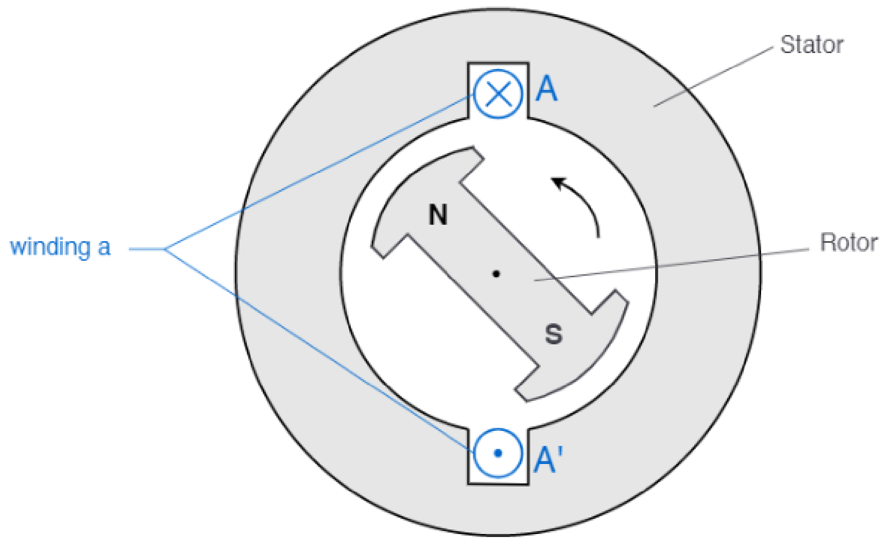
Faraday's law

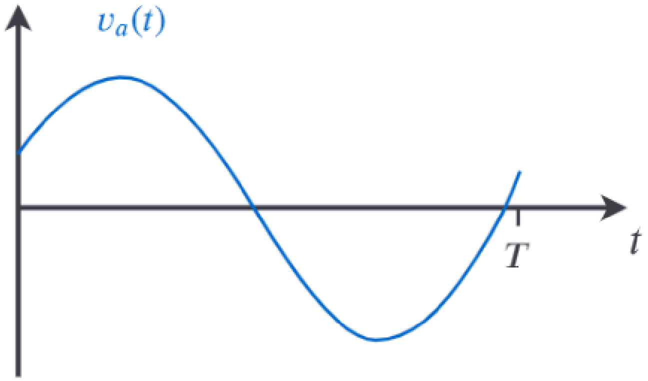
$$\varepsilon = -\frac{\partial \phi_B}{\partial t}$$



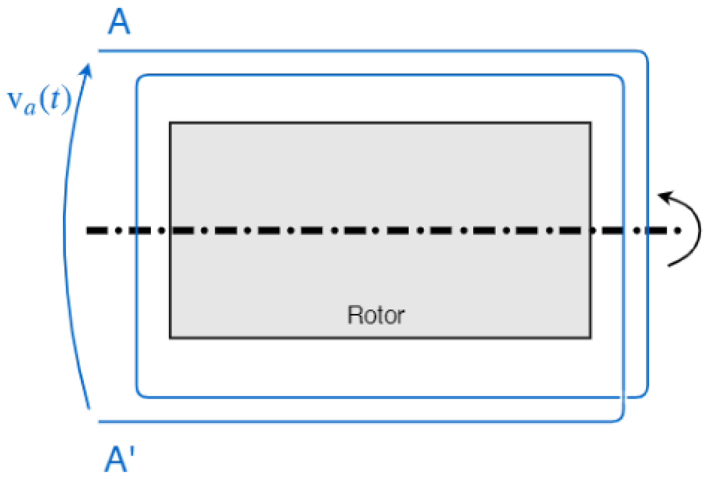
One phase generator

Front view





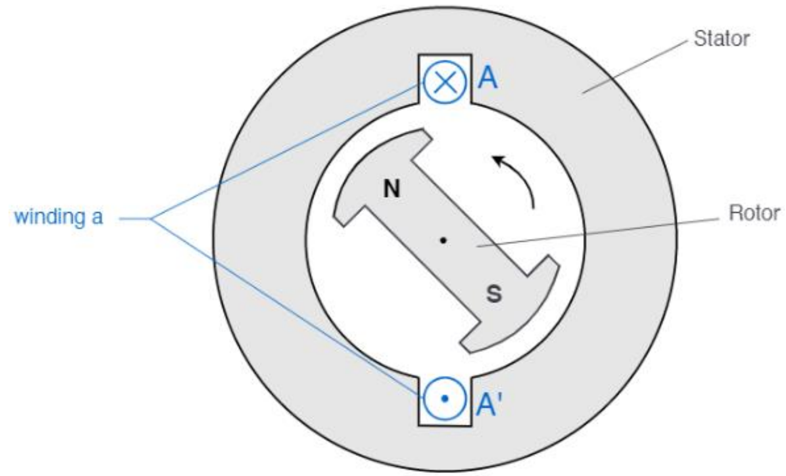
Side view

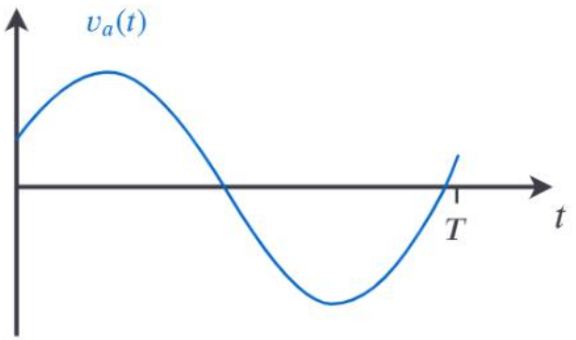


The voltage $v_a(t)$ is induced in the winding as the magnetic flux created by the rotor varies over time :

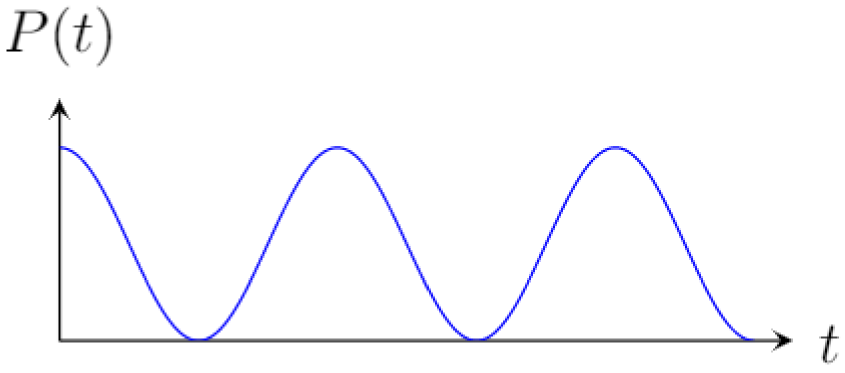
$$v_a(t) = -n_a \underbrace{\frac{d\,\phi(t)}{d\,t}}_{\text{varying flux}} \qquad (3)$$

One phase generator – power



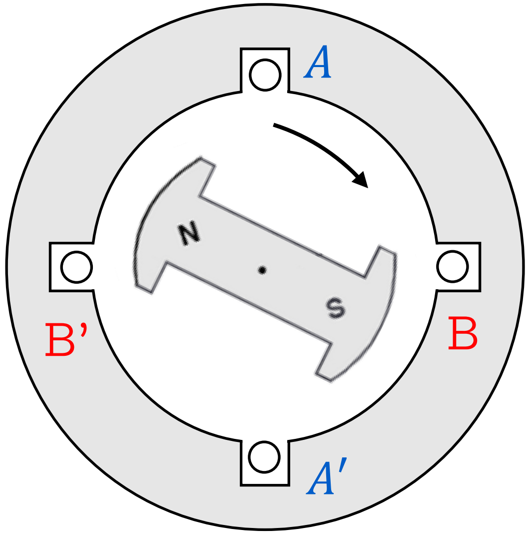


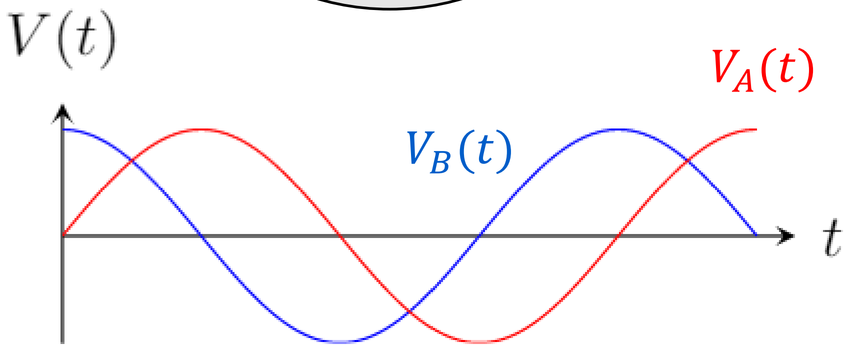
Output power



The total output power fluctuates ➔ the input torque fluctuates ☹

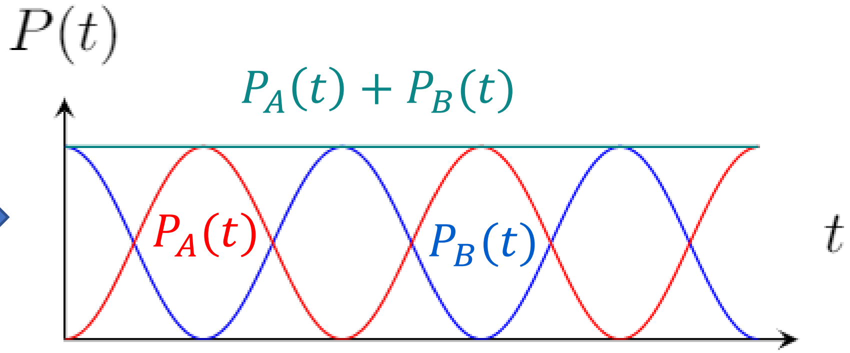
Two phase generator





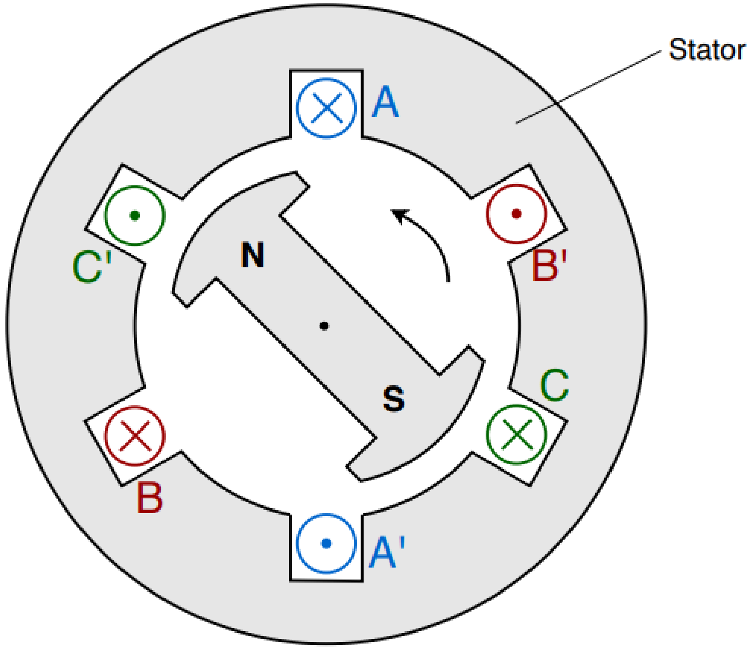
Output power

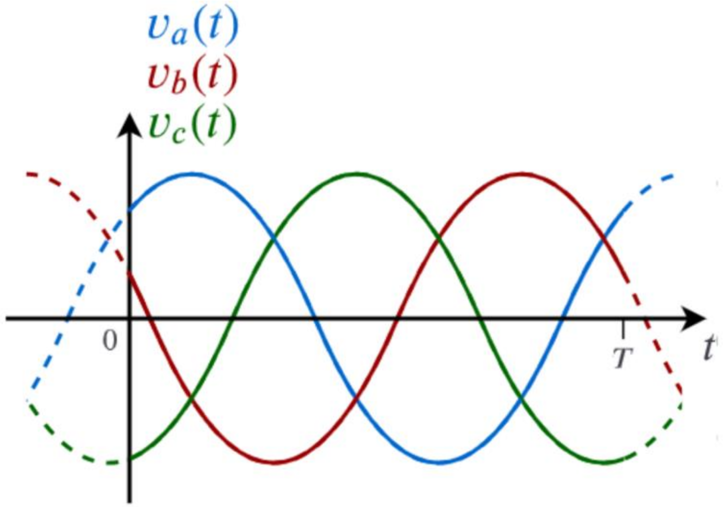
In case loads are the same

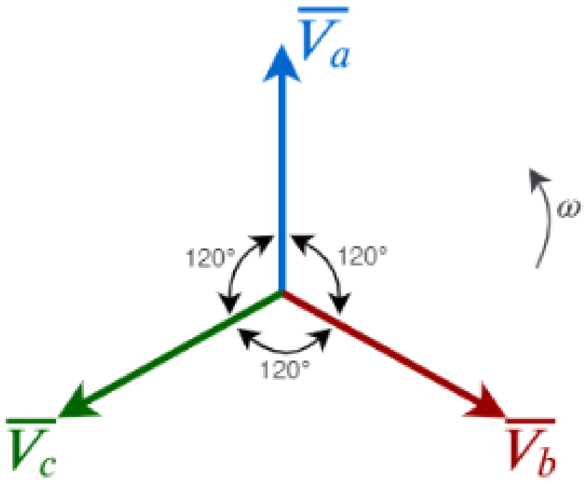


The total output power is constant ➔ the input torque is constant ☺

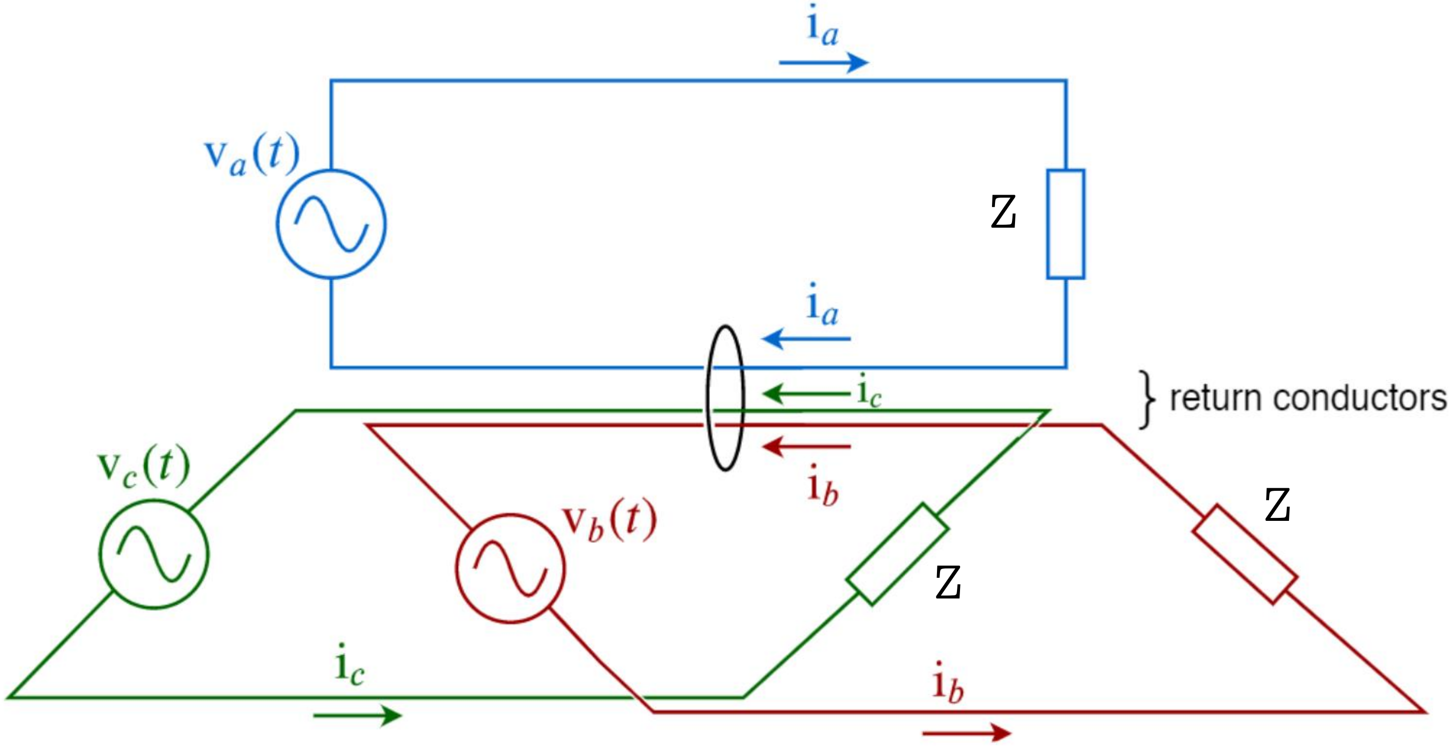
Three phase generator

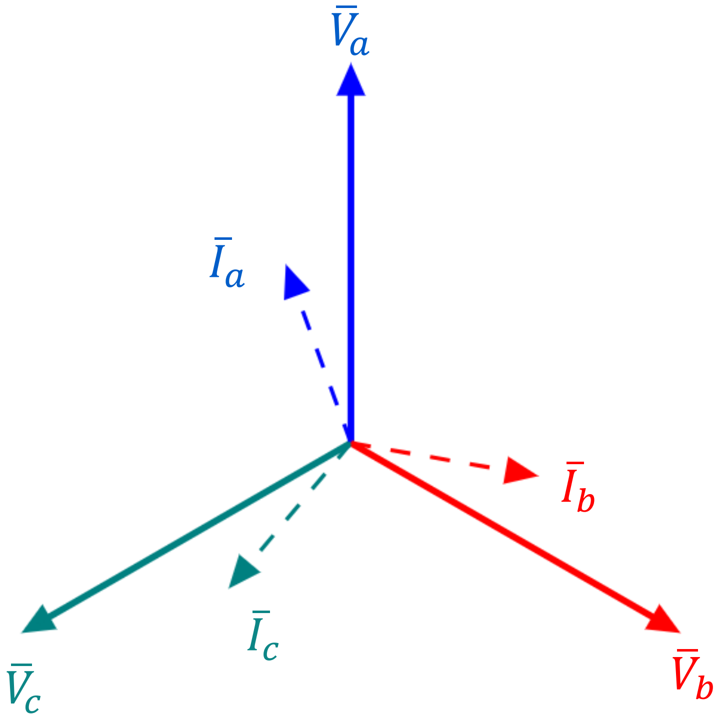






Three phase circuit

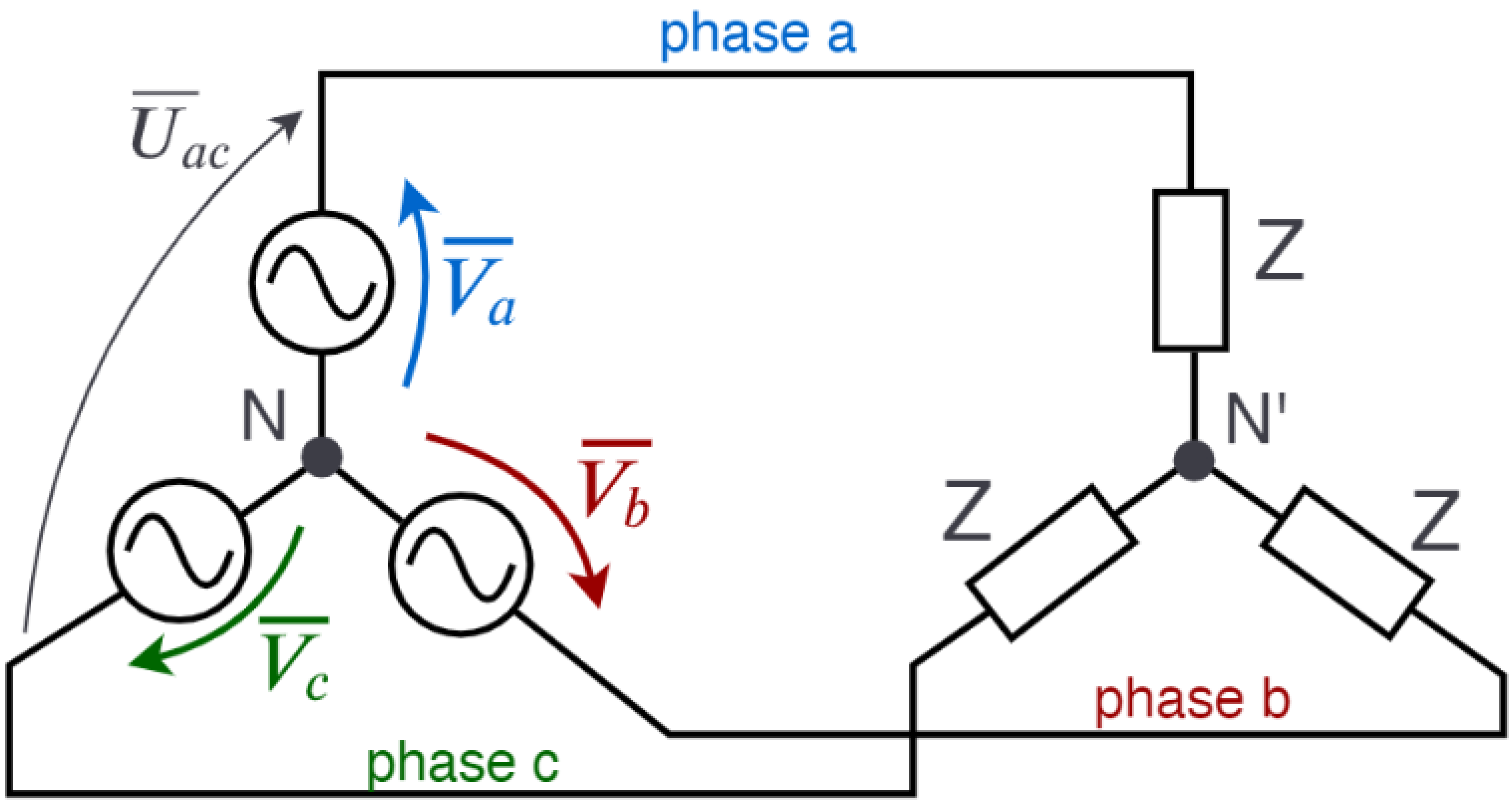




$\bar{I}_a + \bar{I}_c + \bar{I}_c = 0$ (for identical load Z)

➔ The 3 return conductors can be removed ☺☺☺

Balanced three phase circuit

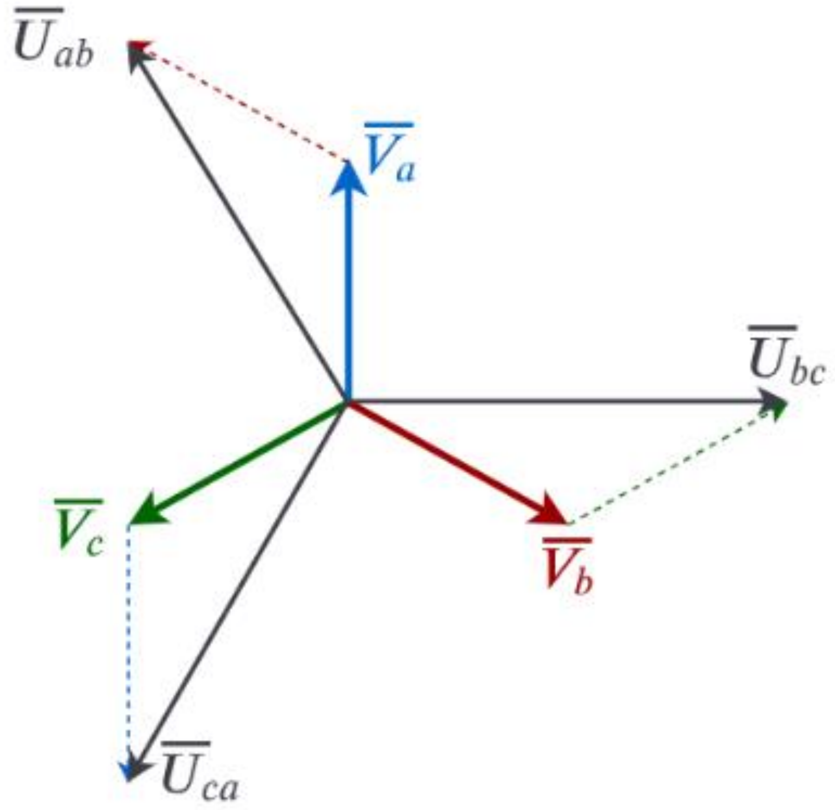


The balanced three-phase circuit is the assembly of three identical **circuits called phases. The 3 signals are dephased by 120°.**

Line and phase voltages

Line voltages

$$\overline{U}_{ab} = \overline{V}_a - \overline{V}_b$$
$$\overline{U}_{bc} = \overline{V}_b - \overline{V}_c$$
$$\overline{U}_{ca} = \overline{V}_c - \overline{V}_a$$

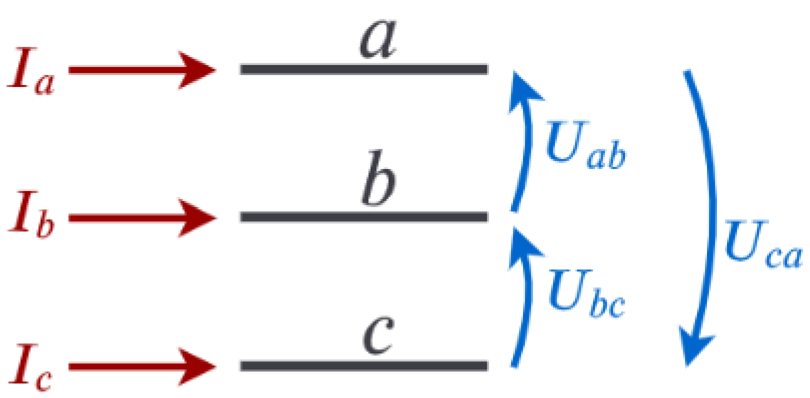


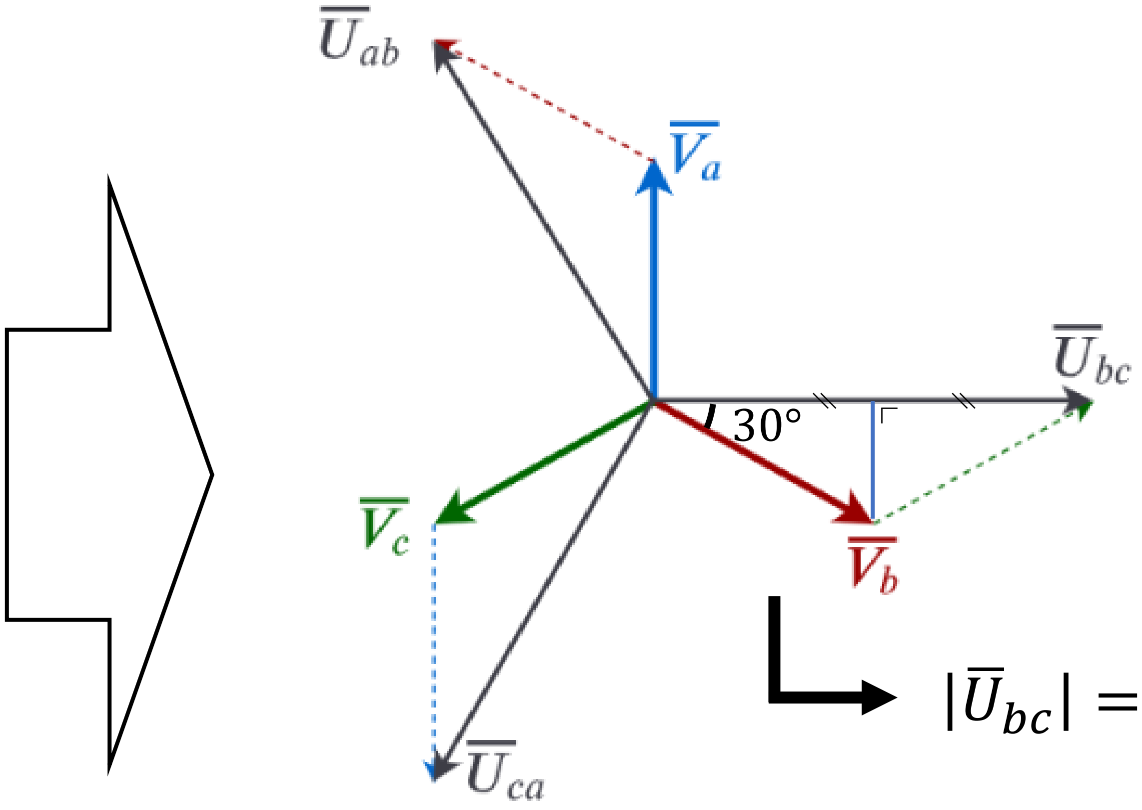
$\overline{V}_a$, $\overline{V}_b$ and $\overline{V}_c$ are the phase voltages, measured between a phase and the neutral N.

$\overline{U}_{ab}$, $\overline{U}_{bc}$ and $\overline{U}_{ca}$ are the line voltages (or phase-to-phase voltages), measured between two phases.

Line and phase voltages

On a three-phase network without the neutral conductor, one can only measure the line voltages and the line currents.



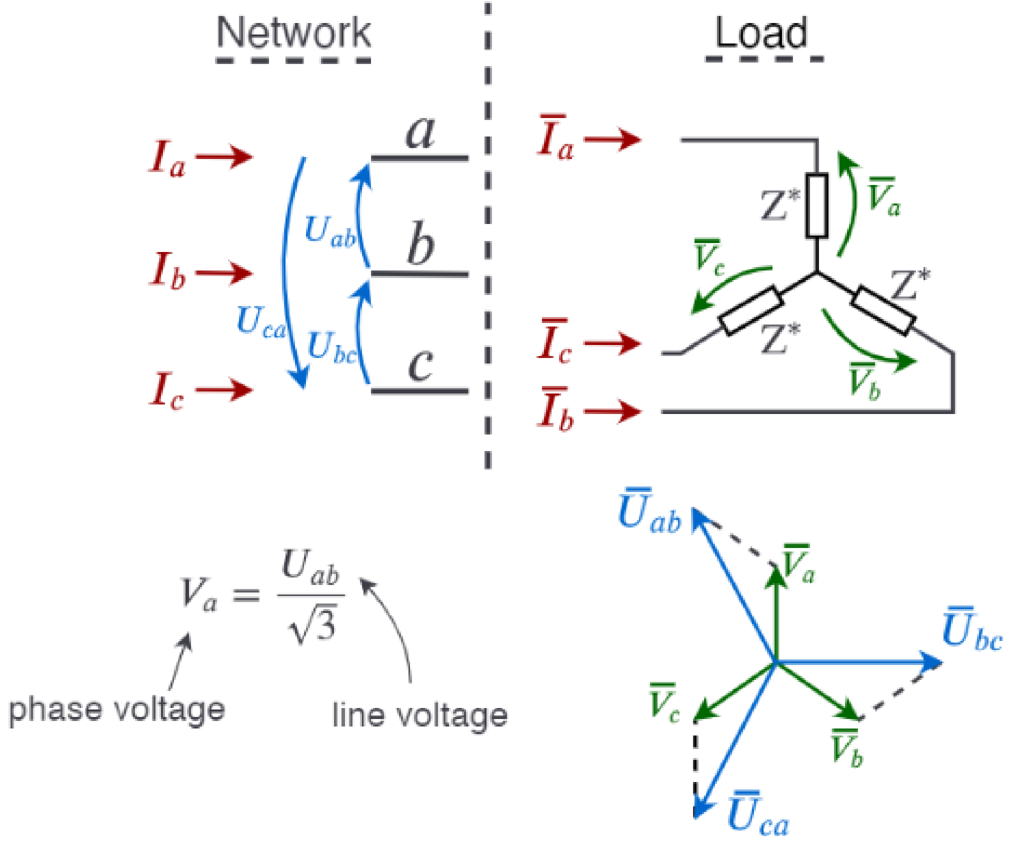


$$|\overline{U}_{bc}| = \sqrt{3}\,|\overline{V}_b|$$

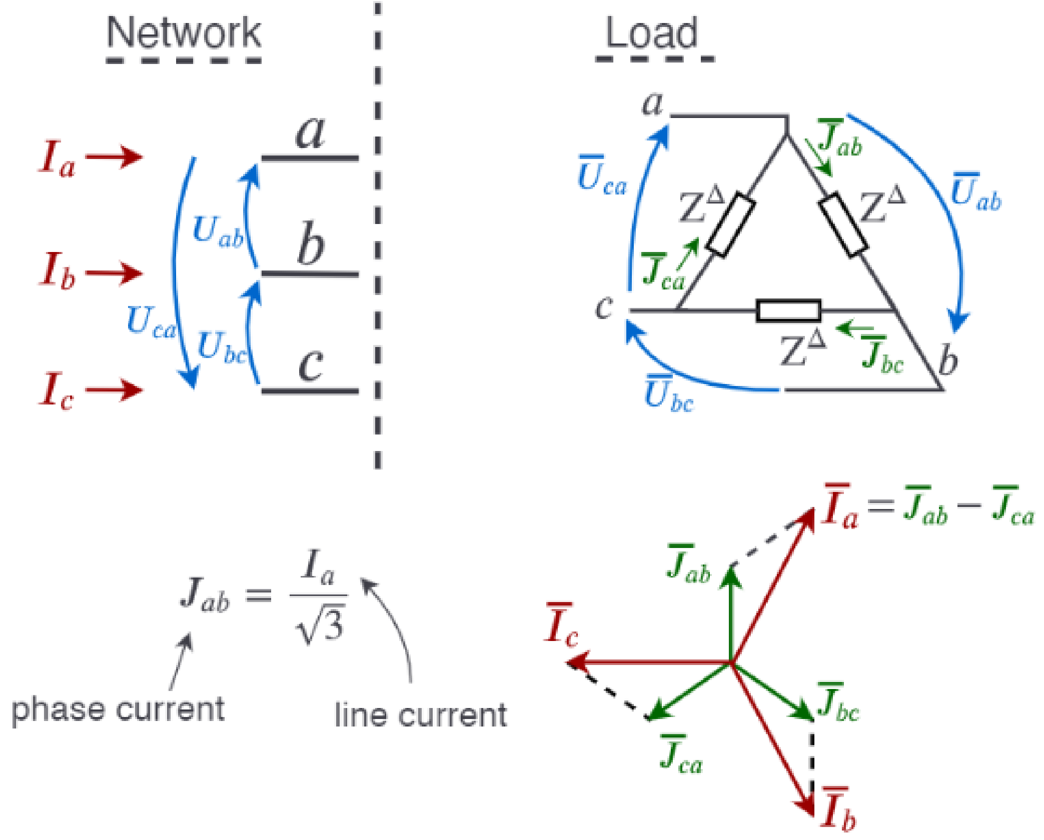
Star and triangle connections

Three-phase loads can be arranged either in star (Y) or in triangle (Δ)

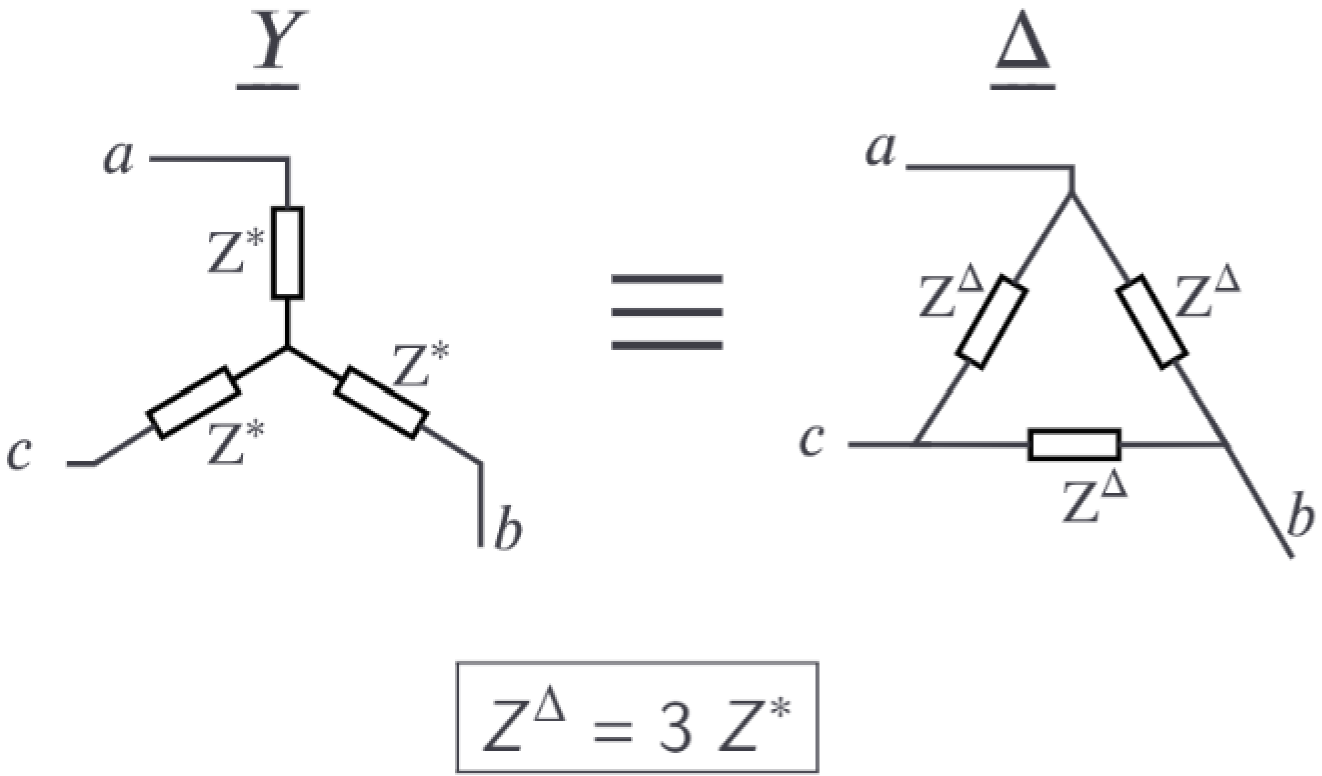
Y



Δ



Line and phase voltages



From the network perspective, the Y and Δ arrangements ae equivalent as long as they provide the same line voltages and current. This is achieved in case $Z^{\Delta} = 3\,Z^{*}$.

Exercises

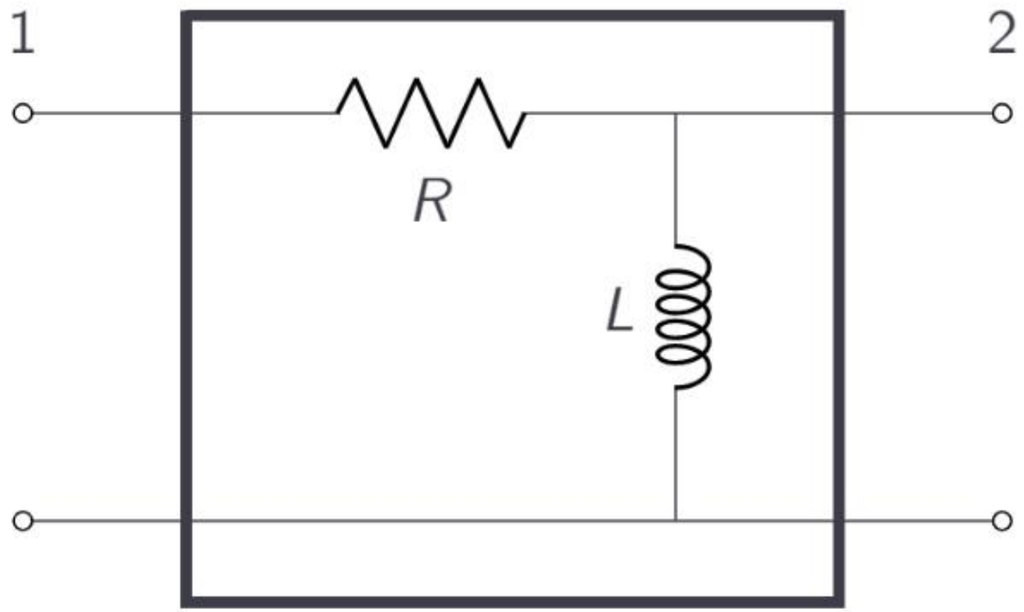
Exercise 4: Two-ports characterization

Exercise 5: Electric heater

Exercise 4: Two-ports characterization

Characterize the 2-ports of Figure 1. In that context, two tests have been performed: a short circuit test and an open circuit test.

- 559 mV and 1.118 A are measured at the access 1 while the access 2 is shorted (short circuit test at 50 Hz).
- 5 V and 4.472 A are measured at the access 1 while the access 2 is left open (open circuit test at 50 Hz).



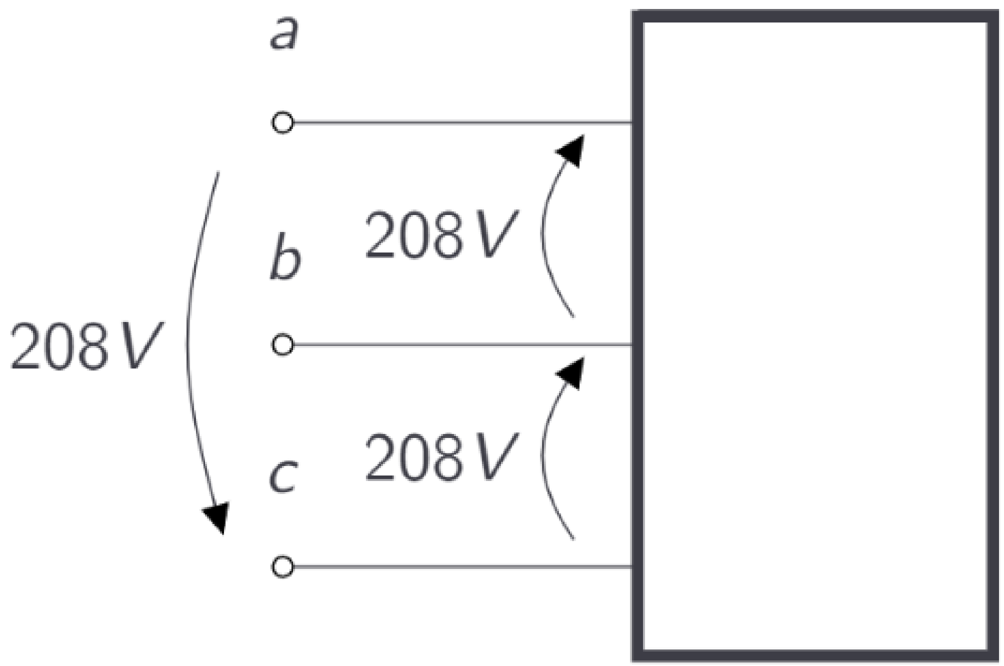
Exercise 4: Two-ports characterization

Consider that the tests are performed at 50 Hz. The 2-ports could be fully caracterized by only one of the two tests if the active power was measured during the tests. The active power can be measured with a wattmeter.

1. Determine the value of R and L with the information above.
2. Which test would be necessary ?
3. During that test, an active power of 9.99392 W has been measured. Prove that it gives the correct value of R and L.

Exercise 5: Electric heater

Consider an electrical heater that dissipates 15 kW of power when connected to a three-phase power system of 208 V. As a first approximation, the heater is modelled as a purely resistive three-phase load.



Exercise 5: Electric heater

1. If no additionnal information is provided about the voltage, does the 208 V correspond to the peak or the RMS value ?
2. Compute the line current if the resistive loads are connected in **Y**.
3. If the resistors are connected in **Y**, compute the resistance of each.
4. Compute the line current if the resistive loads are connected in Δ.
5. If the resistors are connected in Δ, compute the resistance of each.