Electromagnetic Energy Conversion ELEC0431

Exercise session 2: Balanced three-phase systems

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In this class…

- Producer or consumer ?
- Exercise 3
- Introduction to three-phase systems
- Star and Delta configurations
- Exercise 6
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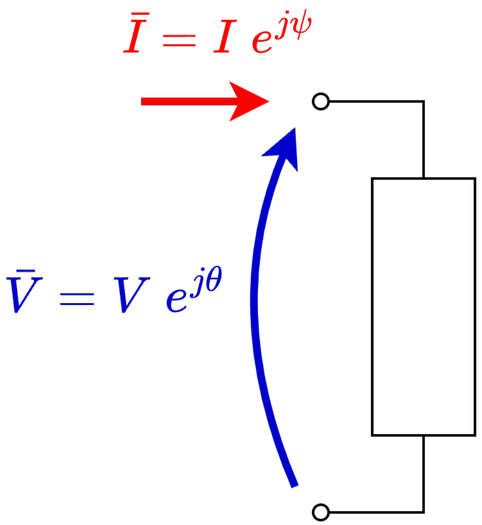
Producer or consumer ?

Using the **passive convention**, we consider a one-port crossed by a current $\bar{I} = I\, e^{j\psi}$ with a voltage $\bar{V} = V\, e^{j\theta}$.



The associated complex power is

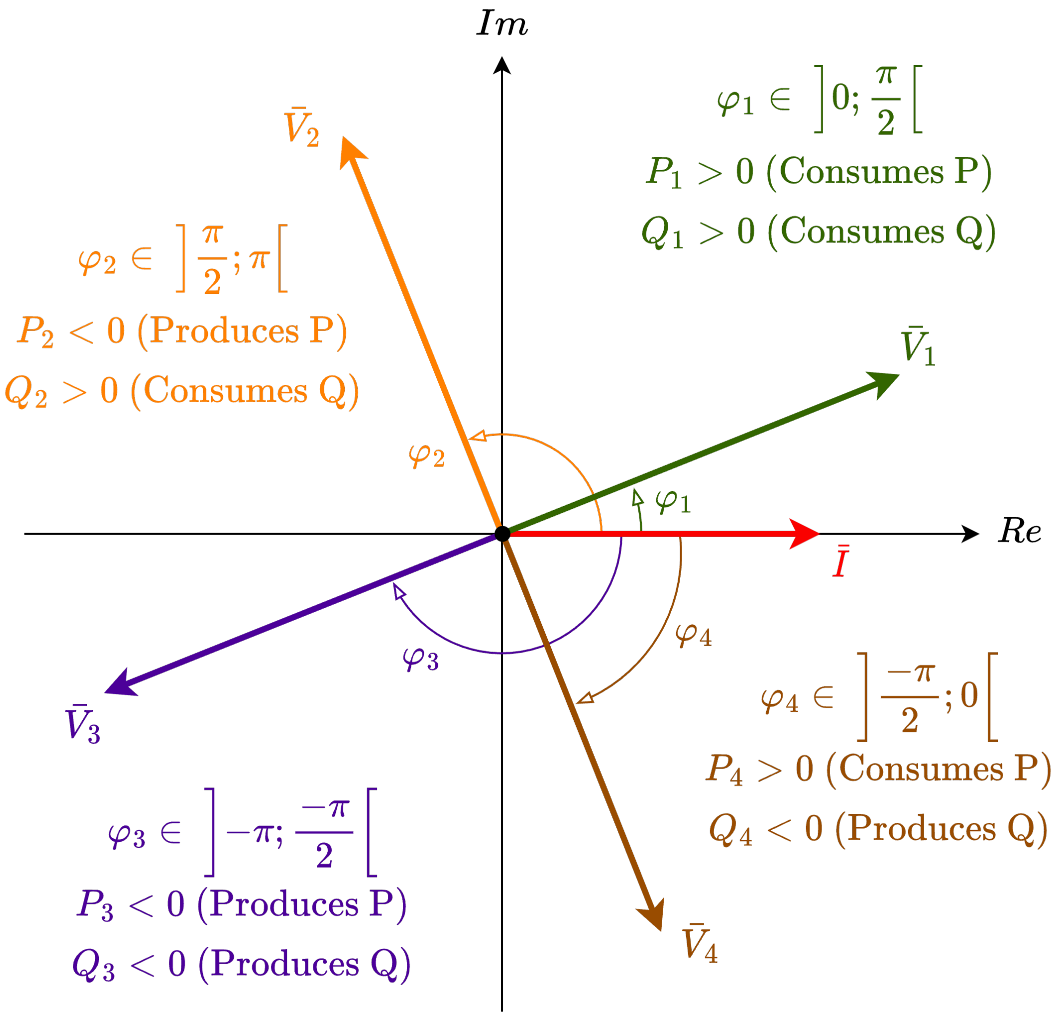
$$\begin{aligned} S &= \bar{V}\ \bar{I}^* = VI\ e^{j(\theta-\psi)} \\ &= VI\cos(\theta-\psi) + j\ VI\,sin(\theta-\psi) \\ &= P + j\ Q \end{aligned}$$

In case P is **positive**, we say that the one-port **consumes** active power.
In case P is **negative**, we say that the one-port **produces** active power.

In case Q is **positive**, we say that the one-port **consumes** reactive power.
In case Q is **negative**, we say that the one-port **produces** reactive power.

Producer or consumer ?

Taking $\bar{I}$ as reference:



$$P = VI\cos(\theta - \psi) = VI\cos(\varphi)$$
$$Q = VI\sin(\theta - \psi) = VI\sin(\varphi)$$

- If $\varphi > 0$

  The voltage **leads** the current

  The one-port consumes reactive power

- If $\varphi < 0$

  The voltage **lags** the current

  The one-port produces reactive power

- If $|\varphi| < \frac{\pi}{2}$

  The one-port consumes active power

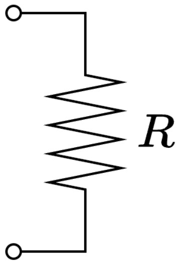
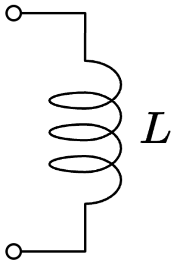
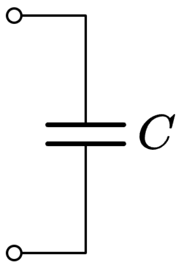
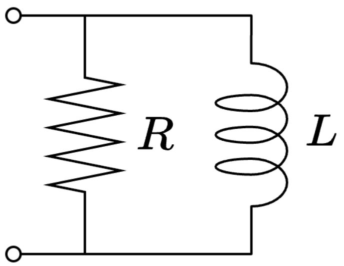
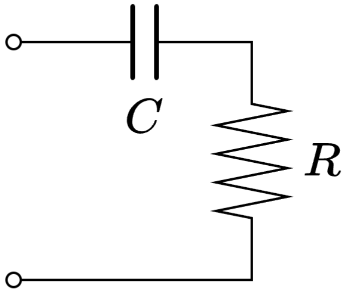
- If $|\varphi| > \frac{\pi}{2}$

  The one-port produces active power

Exercise 3

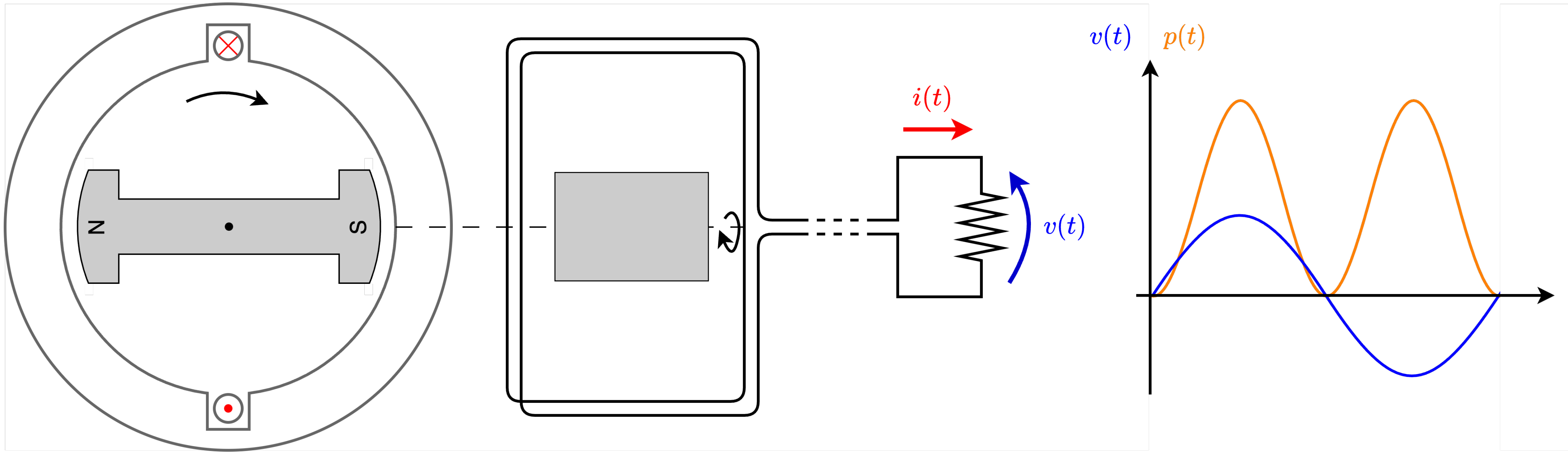
Fill the cells of the table below with the most appropriate answer among

$$= 0 \qquad < 0 \qquad > 0 \qquad = 1 \qquad < 1 \qquad +\infty \qquad -\infty$$

					
Active power **consumed**	> 0	$= 0$	$= 0$	> 0	> 0
Reactive power **produced**	$= 0$	< 0	> 0	< 0	> 0
$\cos(\varphi)$	$= 1$	$= 0$	$= 0$	> 0	> 0
$\tan(\varphi)$	$= 0$	$+\infty$	$-\infty$	> 0	< 0

One-phase generator

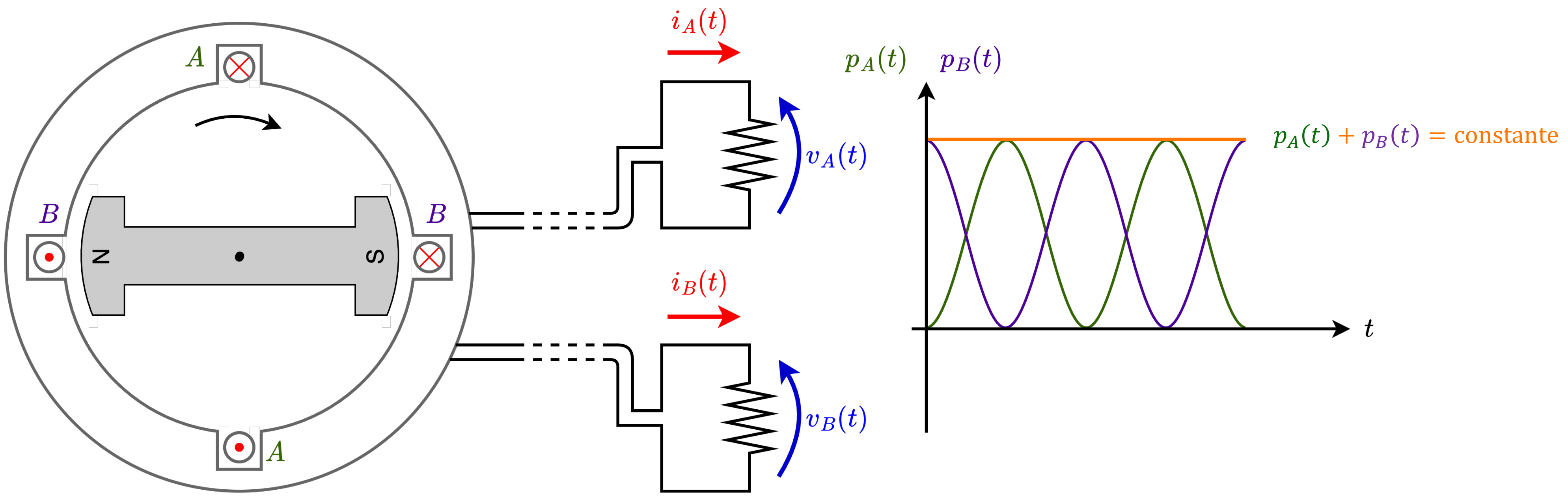
To build an AC generator, one could spin a magnet in a coil. A voltage $v(t)$ is induced in the winding as it perceives a magnetic flux varying over time (Lenz).



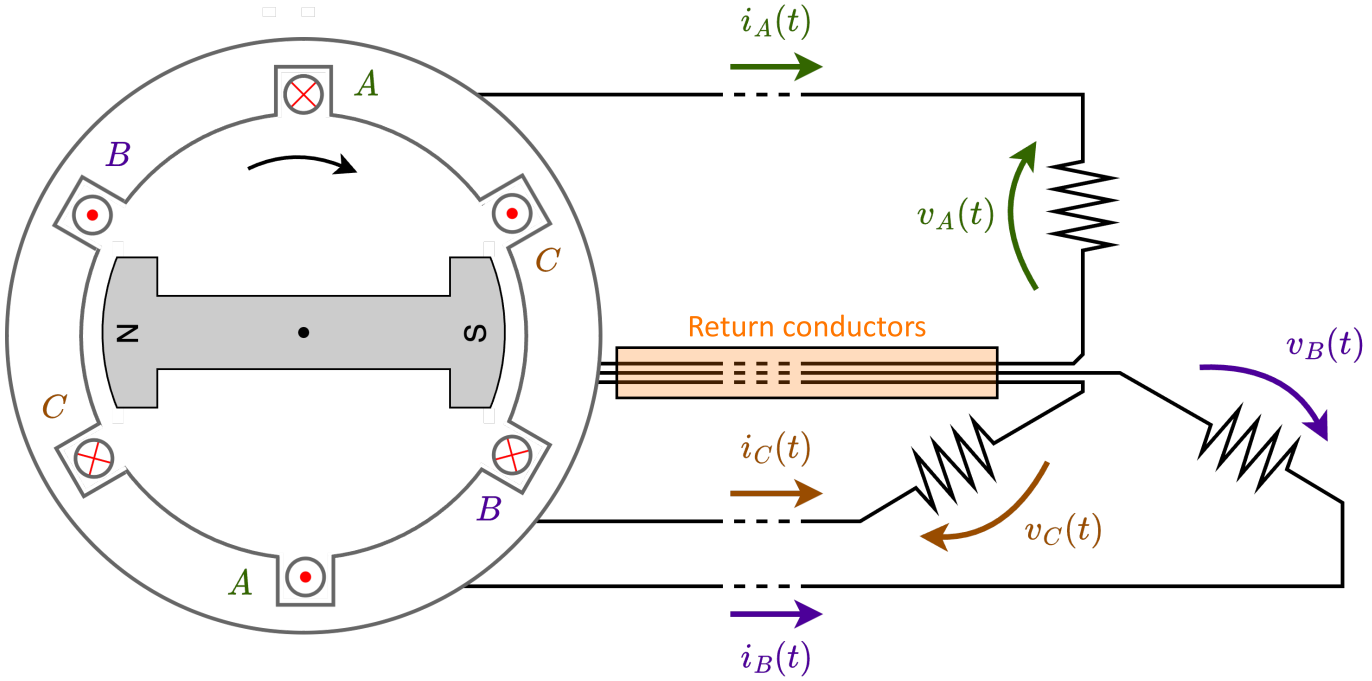
The output power $p(t)$ fluctuates so that the input torque fluctuates as well. However, we would like to have a constant input torque.

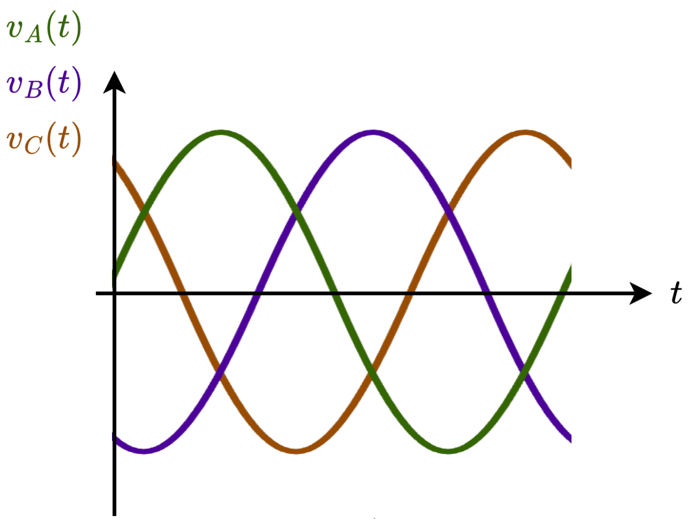
Two-phase generator

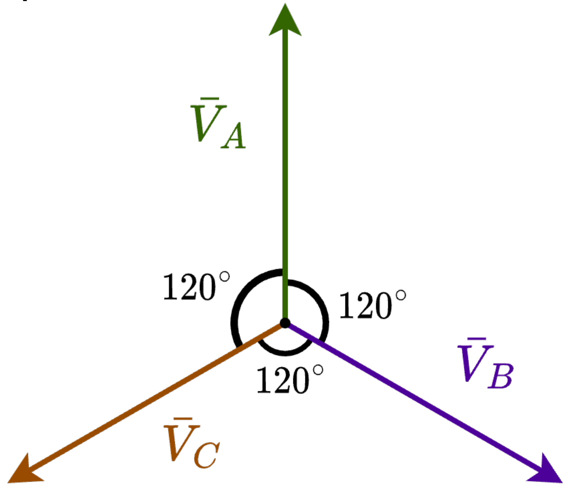
Using a second winding connected to an **identical** load, a constant input torque can be obtained.



Three-phase generator







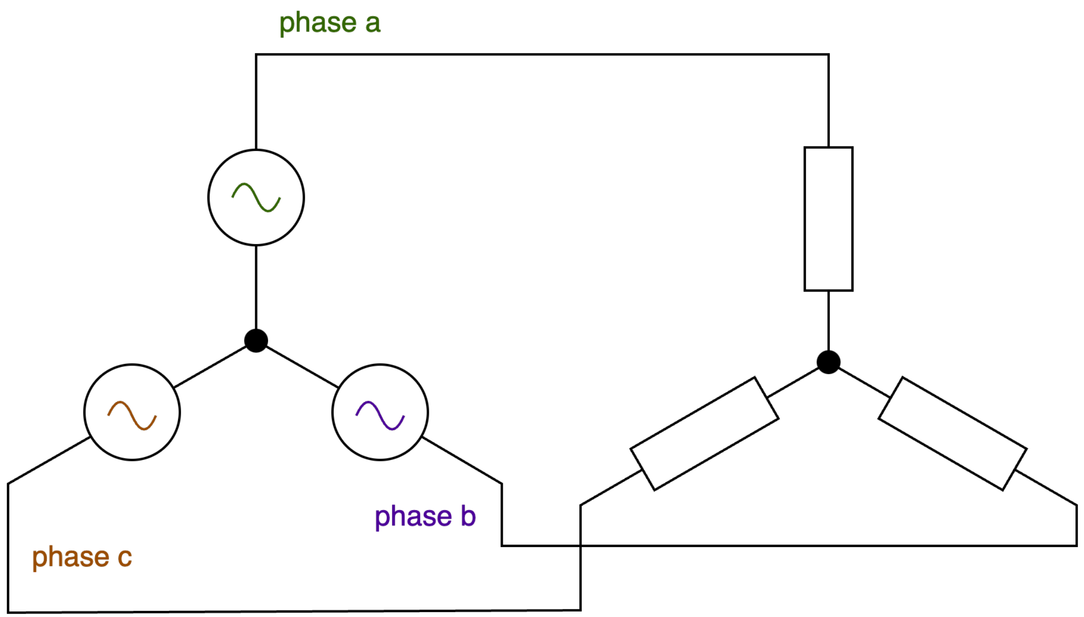
Using identical loads, the input torque also remains constant when using three phases. Moreover, plotting the current phase diagram, it appears that the sum of the currents is zero. The return wires can thus be removed.

Balanced three phase circuit

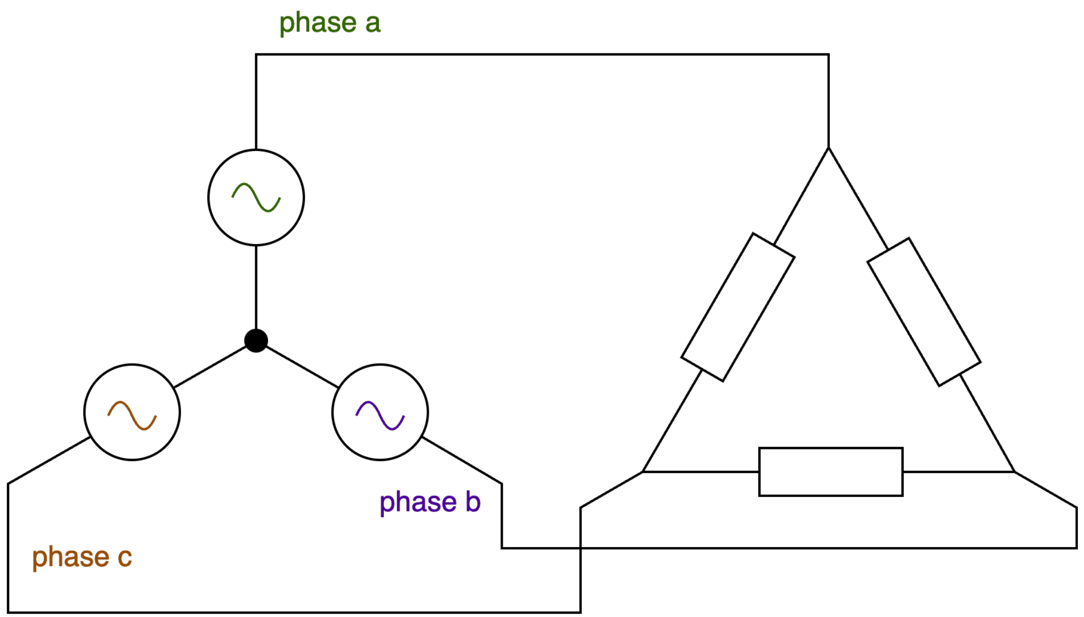
A balanced three-phase circuit is the assembly of three identical circuits. From one circuit to the other, currents and voltages are out of phase by $120°$.

Loads are typically connected in either a **star** ($\star$ or Y) configuration or in a **delta** (Δ) configuration:

Star configuration:



Delta configuration:

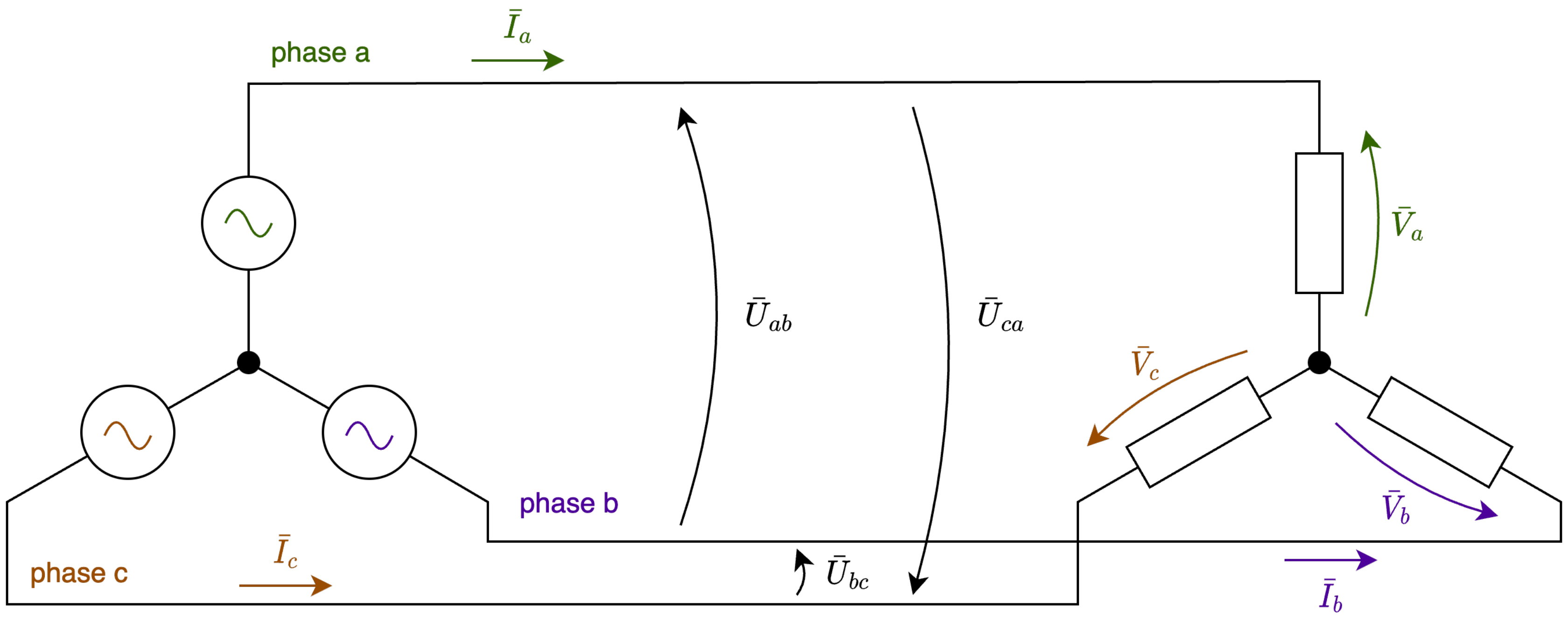


Star configuration

In a star configuration, the current flowing in the transmission line z is the current flowing through the load z.

However, we differentiate:

- The **line voltage** $\bar{U}_{xy}$ which is the voltage between the phase x and the phase y.
- The **phase voltage** $\bar{V}_z$ which is the voltage across the load z.



$$\bar{U}_{ab} = \bar{V}_a - \bar{V}_b$$
$$\bar{U}_{bc} = \bar{V}_b - \bar{V}_c$$
$$\bar{U}_{ca} = \bar{V}_c - \bar{V}_a$$

Star configuration

$$\bar{U}_{ab} = \bar{V}_a - \bar{V}_b$$
$$\bar{U}_{bc} = \bar{V}_b - \bar{V}_c$$
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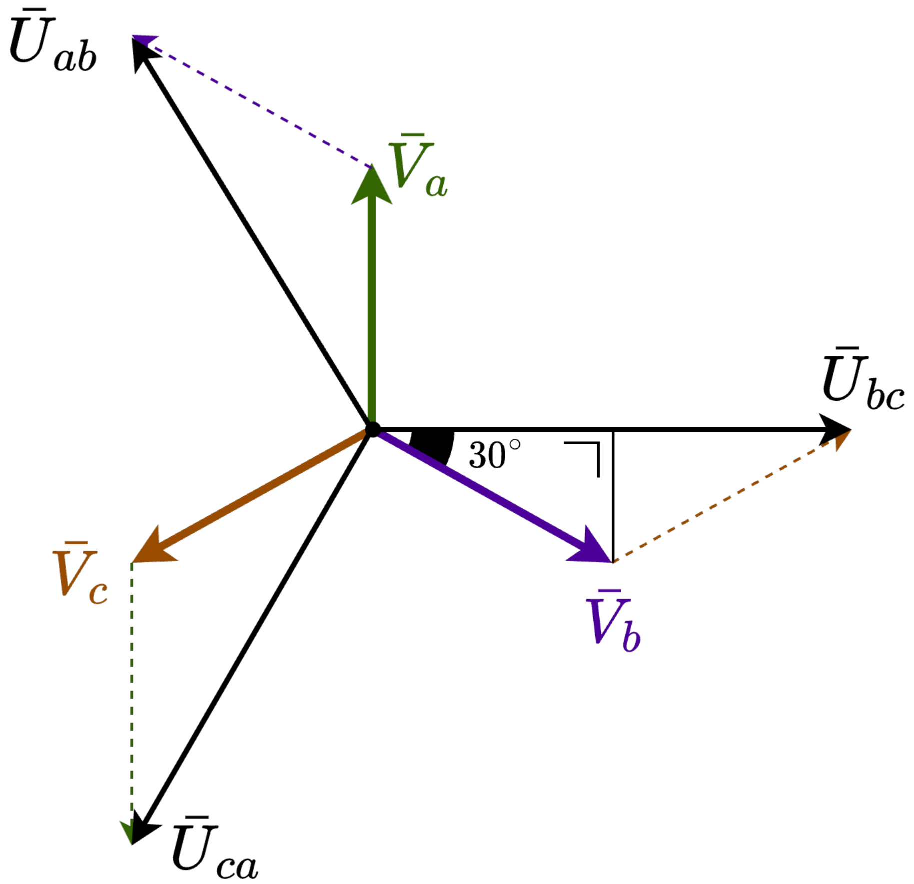


$$\frac{U_{bc}}{2} = \cos(30°)\ V_b = \frac{\sqrt{3}}{2} V_b$$

$$\Rightarrow U_{bc} = \sqrt{3}\ V_b$$

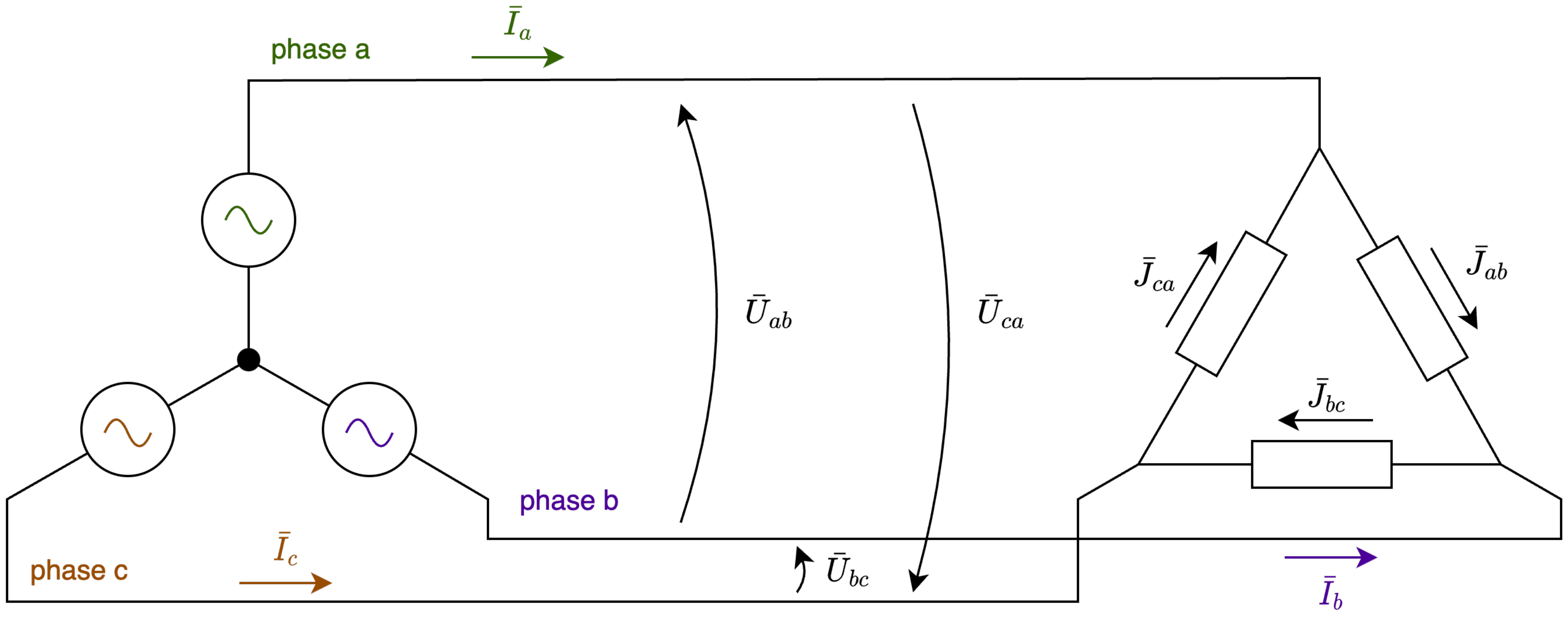
In a star configuration, the line voltage is $\sqrt{3}$ the phase voltage.

Delta configuration

In a star configuration, the voltage between the phase x and the phase y is the voltage across the load xy.

However, we differentiate:

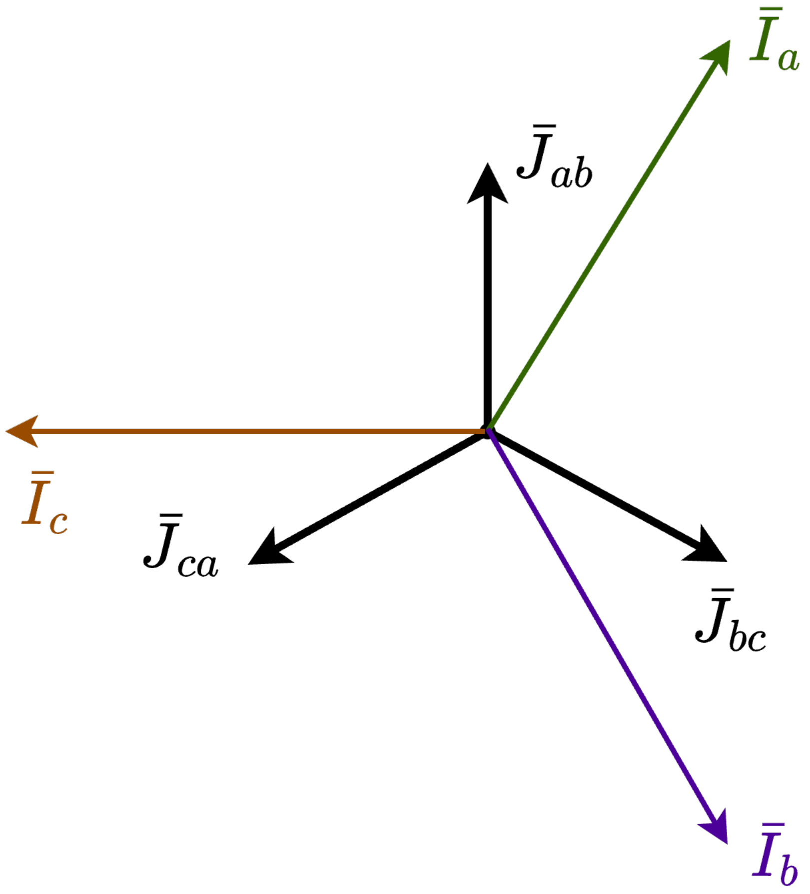
- The **line current** $\bar{I}_z$ which is the current flowing in the transmission line z.
- The **phase current** $\bar{J}_{xy}$ which is the current flowing in the load xy.



$$\bar{I}_a = \bar{J}_{ab} - \bar{J}_{ca}$$
$$\bar{I}_b = \bar{J}_{bc} - \bar{J}_{ab}$$
$$\bar{I}_c = \bar{J}_{ca} - \bar{J}_{bc}$$

Star configuration

$$\bar{I}_a = \bar{J}_{ab} - \bar{J}_{ca}$$
$$\bar{I}_b = \bar{J}_{bc} - \bar{J}_{ab}$$
$$\bar{I}_c = \bar{J}_{ca} - \bar{J}_{bc}$$



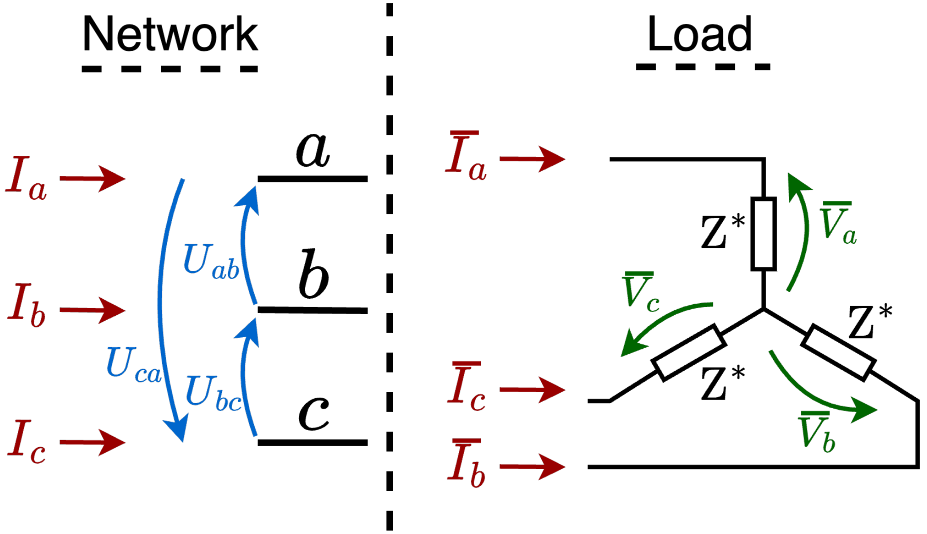
$$\frac{I_b}{2} = \cos(30°)\ J_{bc} = \frac{\sqrt{3}}{2}\ J_{bc}$$

$$\Rightarrow I_b = \sqrt{3}\ J_{bc}$$

In a delta configuration, the line current is $\sqrt{3}$ the phase current.

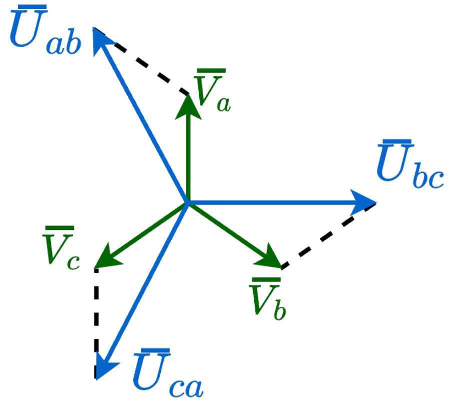
Star and Delta configurations: recap

$\underline{Y}$

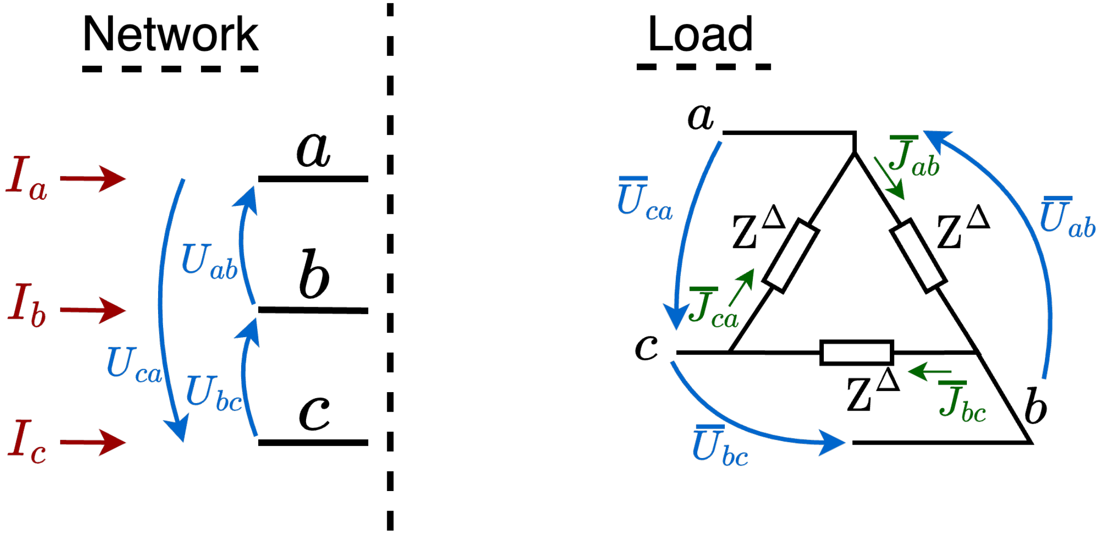


$$V_a = \frac{U_{ab}}{\sqrt{3}}$$

phase voltage — line voltage

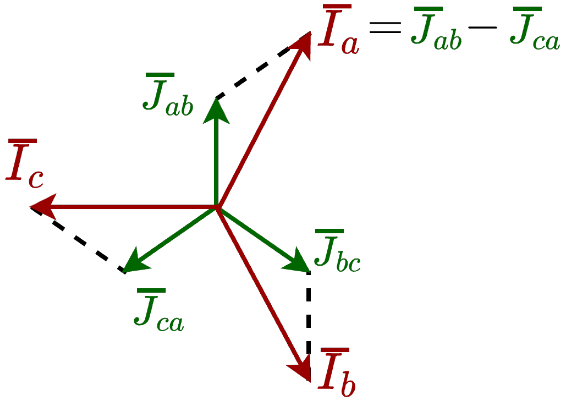


$\underline{\Delta}$



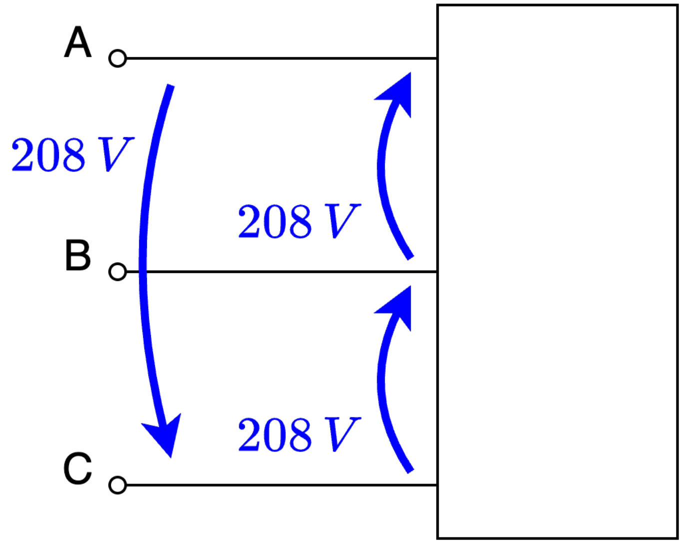
$$J_{ab} = \frac{I_a}{\sqrt{3}}$$

phase current — line current



Exercise 6

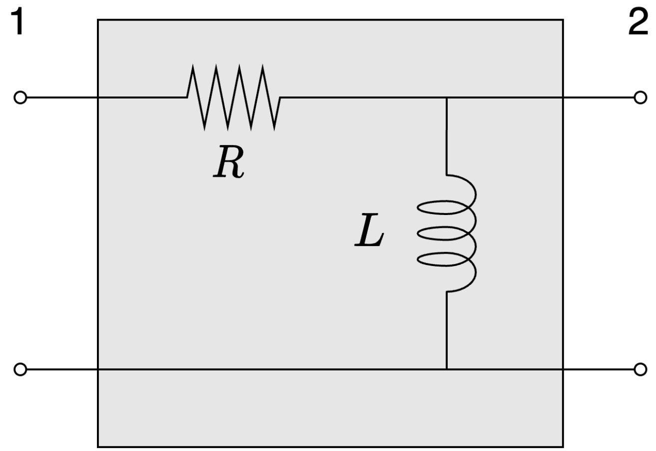
Consider an electrical heater that dissipates $15\ kW$ of power when connected to a three-phase power system of $208\ V$. As a first approximation, the heater is modelled as a purely resistive three-phase load.



1. If no additional information is provided about the voltage, does the $208\ V$ correspond to the peak or to the RMS value ?
2. Compute the line current if the resistive loads are connected in $\star$.
3. If the resistors ae connected in $\star$, compute the resistance of each.
4. Compute the line current if the resistive loads are connected in Δ.
5. If the resistors ae connected in Δ, compute the resistance of each.

Exercise 4

Characterize the 2-port hereunder. Two tests have been performed: a short circuit test and an open circuit test:



- $559\ mV$ and $1.118\ A$ are measured at the access 1 while the access 2 is shorted (short circuit test at $50\ Hz$).
- $5\ V$ and $4.472\ A$ are measured at the access 1 while the access 2 is left open (open circuit test at $50\ Hz$).

The 2-port could be fully characterized by only one of the two tests if the active power was measured during the tests. The active power can be measured with a wattmeter.

1. Determine the value of R and L with the information above.
2. Which test would be necessary ?
3. During that test, an active power of $9.99392\ W$ has been measured. Prove that it gives the correct value R and L.