Electromagnetic Energy Conversion ELEC0431

Exercise session 2: Balanced three-phase systems

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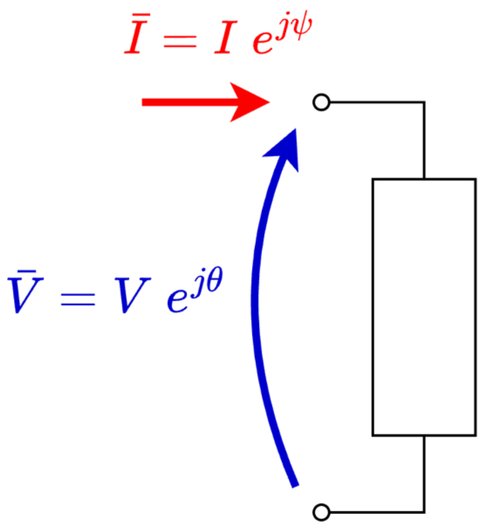
In this class…

- Producer or consumer?
- Exercise 3 (homework 11)
- Introduction to three-phase systems
- Phase and line current/voltages – Star and delta configurations
- Exercises 4 & 5

Producer or consumer ?

Using the **passive convention**, we consider a one-port crossed by a current $\bar{I} = I\, e^{j\psi}$ with a voltage $\bar{V} = V\, e^{j\theta}$.



The associated complex power is

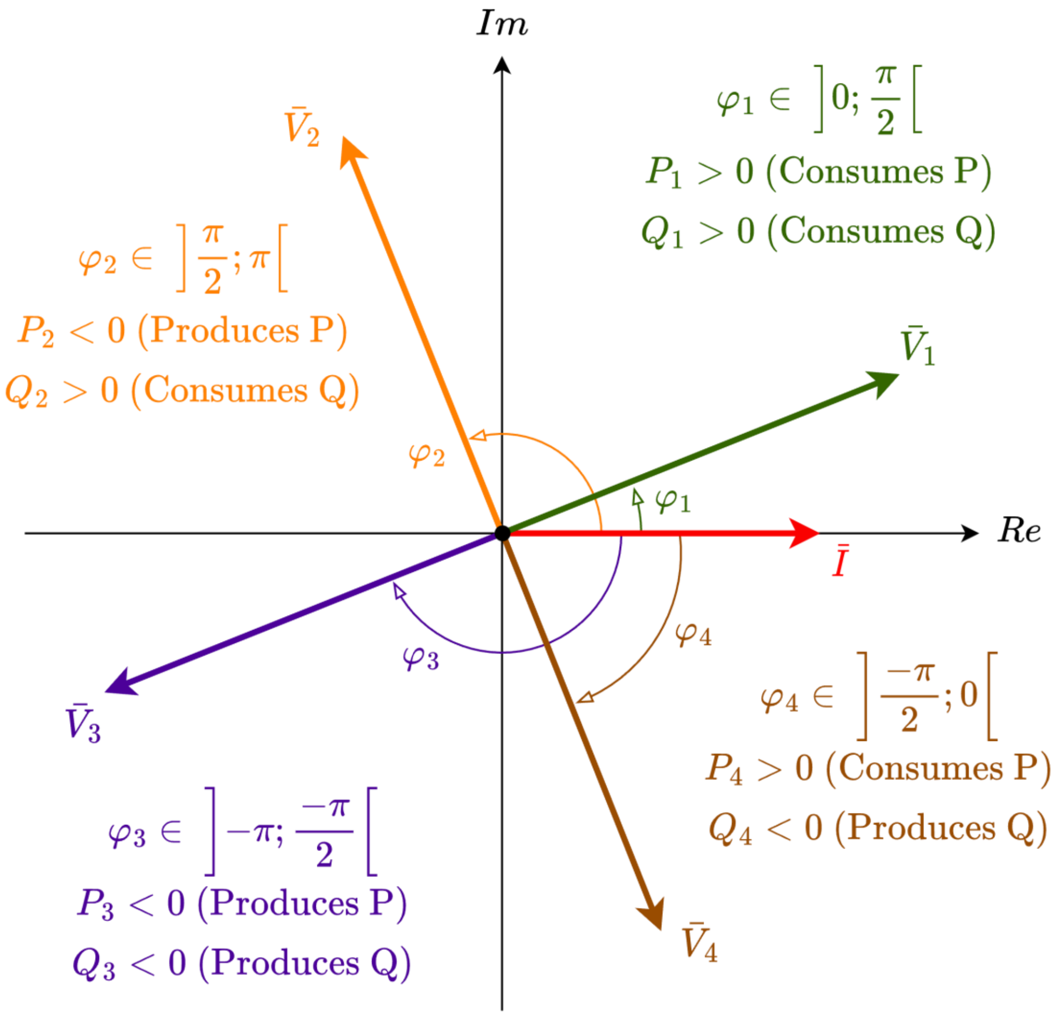
$$
\begin{aligned}
S &= \bar{V}\, \bar{I}^* = VI\, e^{j(\theta-\psi)} \\
&= VI\cos(\theta-\psi) + j\, VI\, sin(\theta-\psi) \\
&= P + j\, Q
\end{aligned}
$$

In case P is **positive**, we say that the one-port **consumes** active power.
In case P is **negative**, we say that the one-port **produces** active power.

In case Q is **positive**, we say that the one-port **consumes** reactive power.
In case Q is **negative**, we say that the one-port **produces** reactive power.

Producer or consumer ?

Taking $\bar{I}$ as reference and $\varphi \in [-\pi; \pi]$:



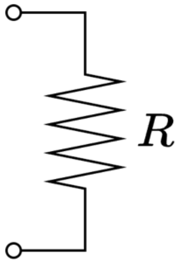
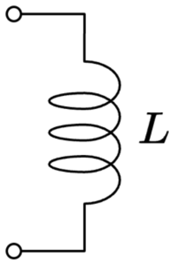
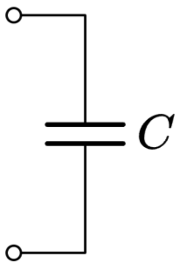
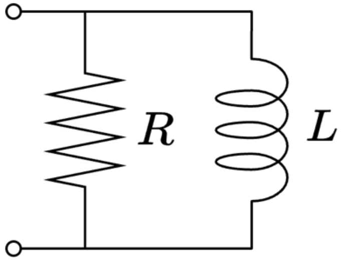
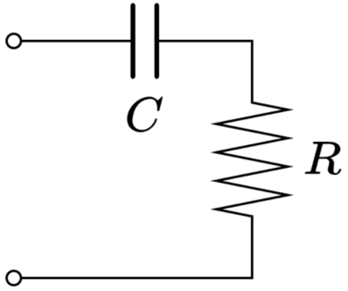
$$P = VI\cos(\theta - \psi) = VI\cos(\varphi)$$
$$Q = VI\sin(\theta - \psi) = VI\sin(\varphi)$$

- If $\varphi > 0$: The voltage **leads** the current. The one-port consumes reactive power
- If $\varphi < 0$: The voltage **lags** the current. The one-port produces reactive power
- If $|\varphi| < \frac{\pi}{2}$: The one-port consumes active power
- If $|\varphi| > \frac{\pi}{2}$: The one-port produces active power

Exercise 3 (Homework 11)

Fill the cells of the table below with the most appropriate answer among

$= 0 \qquad < 0 \qquad > 0 \qquad = 1 \qquad < 1 \qquad +\infty \qquad -\infty$

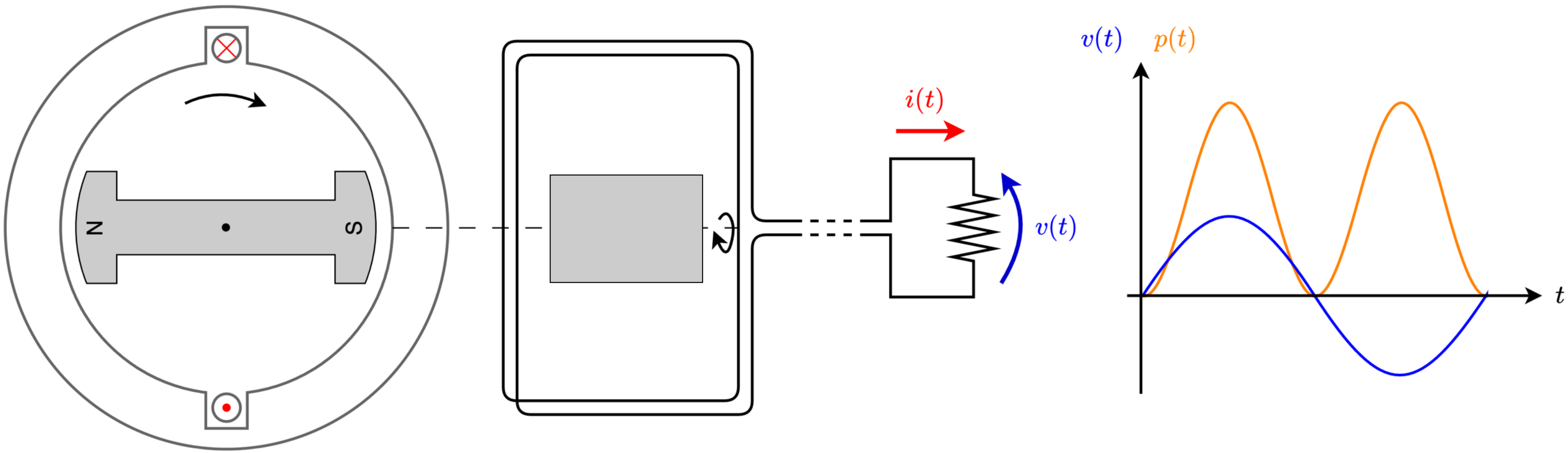
					
Active power **consumed**	> 0	$= 0$	$= 0$	> 0	> 0
Reactive power **produced**	$= 0$	< 0	> 0	< 0	> 0
$\cos(\varphi)$	$= 1$	$= 0$	$= 0$	> 0	> 0
$\tan(\varphi)$	$= 0$	$+\infty$	$-\infty$	> 0	< 0

Introduction to three-phase systems

One-phase generator
Two-phase generator
Three-phase generator
Phase and line current/voltages – Star and delta configurations
Exercises 4 & 5

One-phase generator

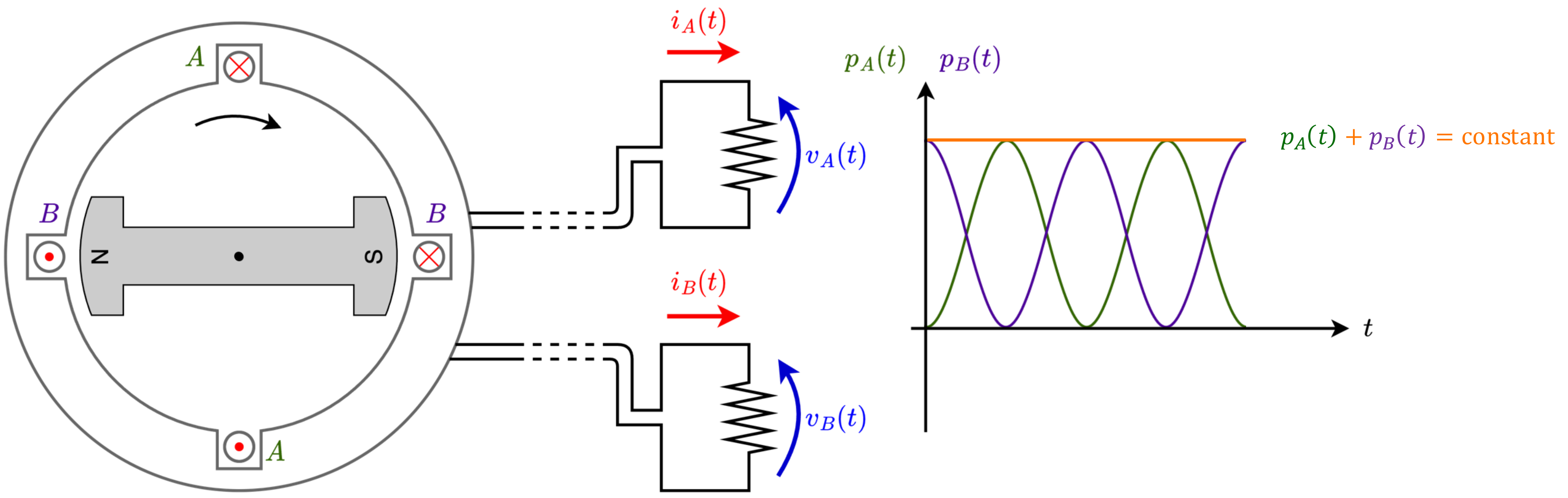
To build an AC generator, one could spin a magnet in a coil. A voltage $v(t)$ is induced in the winding as it perceives a magnetic flux varying over time (Lenz).



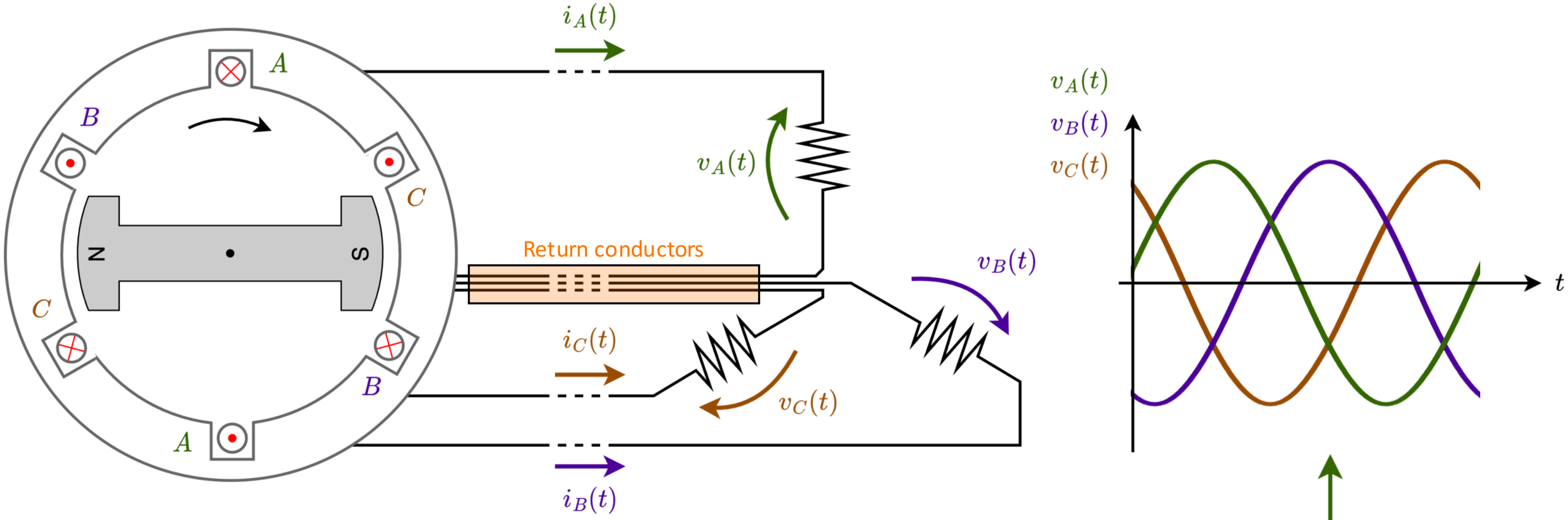
The output power $p(t)$ fluctuates so that the input torque fluctuates as well. However, we would like to have a constant input torque.

Two-phase generator

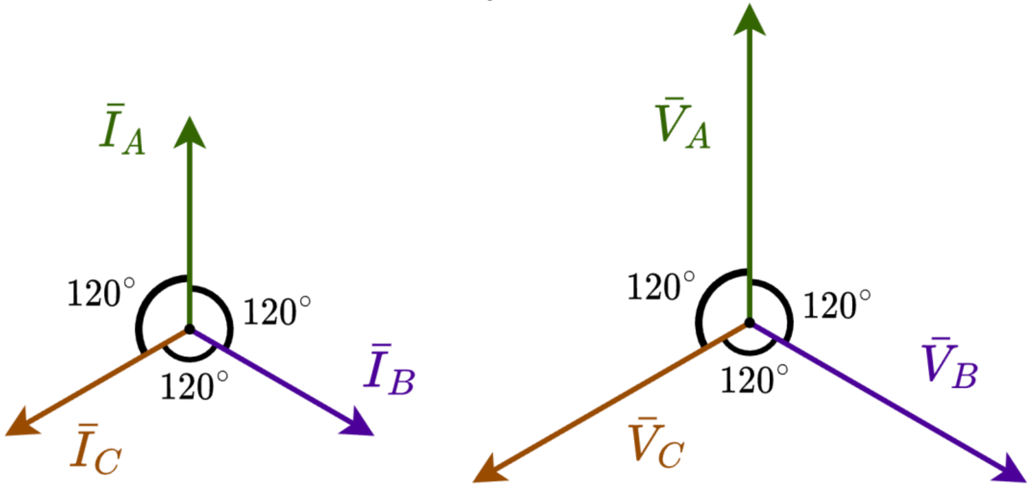
Using a second winding connected to **identical** loads, a constant input torque can be obtained.



Three-phase generator



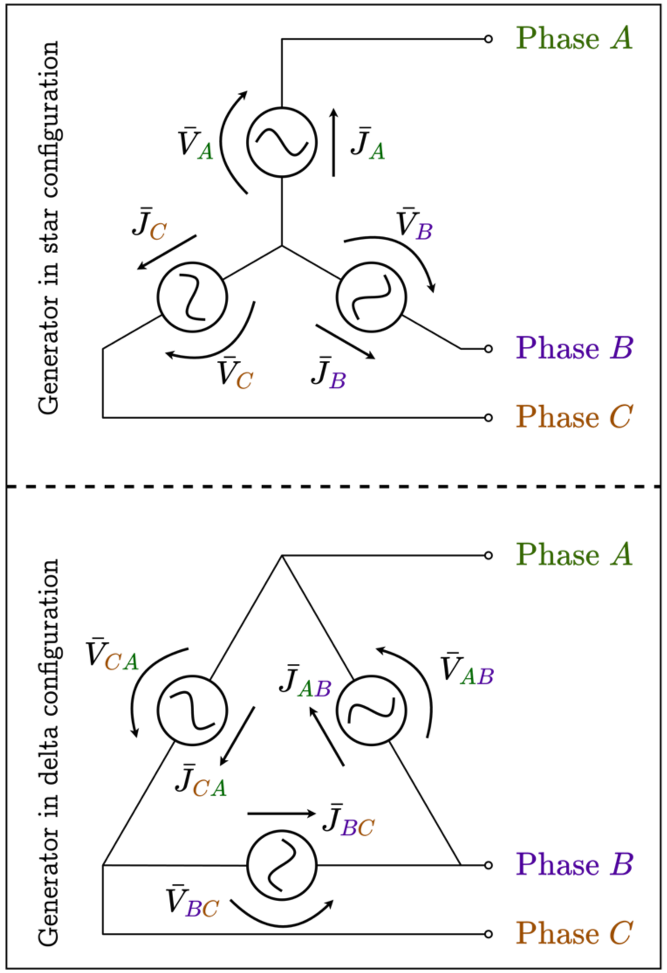
Using identical loads, the input torque also remains constant when using three phases. Moreover, plotting the current phase diagram, it appears that the sum of the currents is zero. The return conductors can thus be removed.

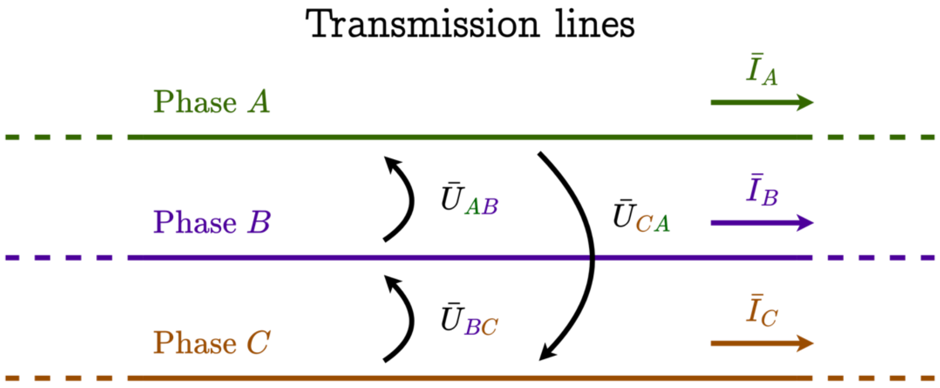


Three-phase circuits

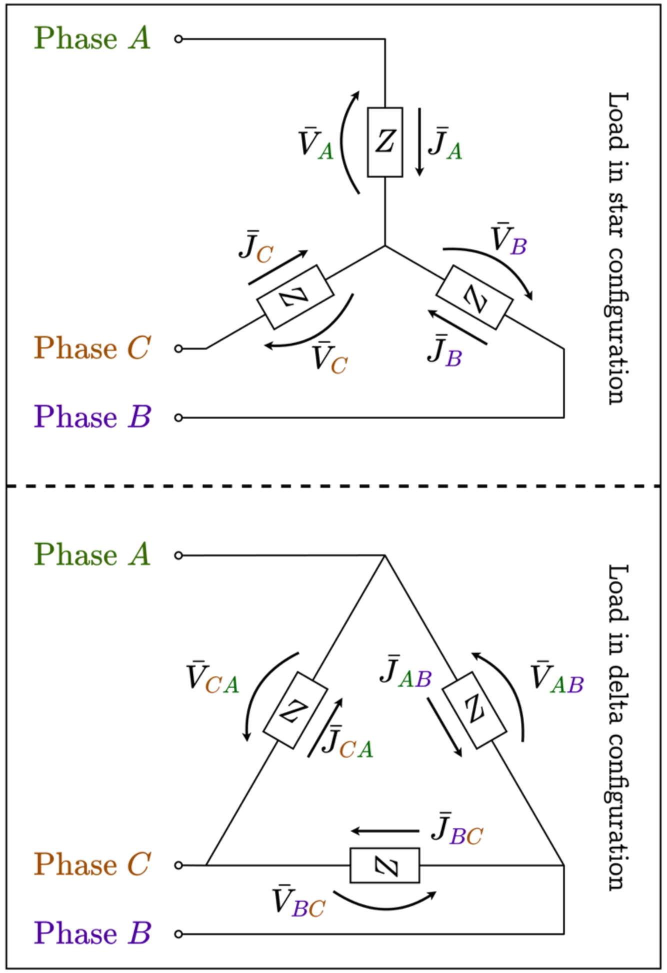
Generators and loads are connected in **star** ($\star$ or Y) or **delta** (Δ) configuration.

Generator





Load



We differentiate:

Phase voltage $\bar{V}$ vs. Line voltage $\bar{U}$.

Phase current $\bar{J}$ vs. Line current $\bar{I}$.

Phase vs. line voltage/current

Some conventions for the practical lectures:

The line voltage U is the voltage between any two transmission lines.

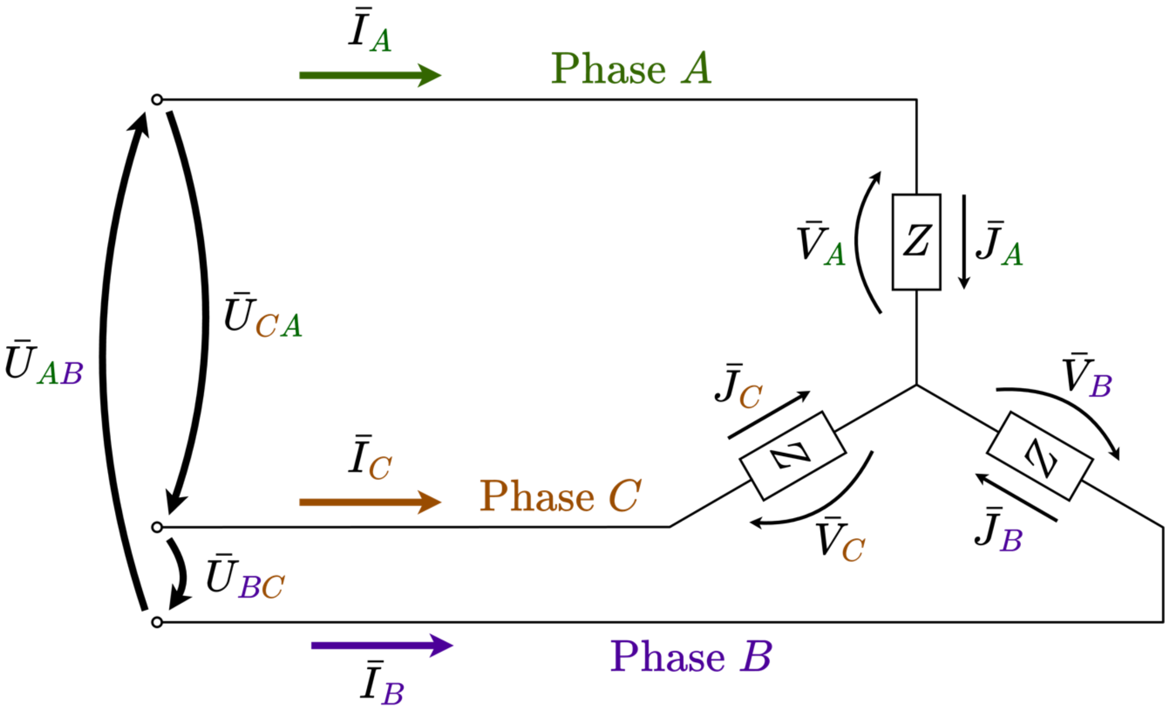
The line current I is the current flowing in a transmission line.

VS.

The phase voltage V is the voltage across any of the generator leg and/or load leg.

The phase current J is the current flowing in a generator leg and/or load leg.

Phase vs. line voltage/current – star configuration



$$\bar{I}_A = \bar{J}_A$$

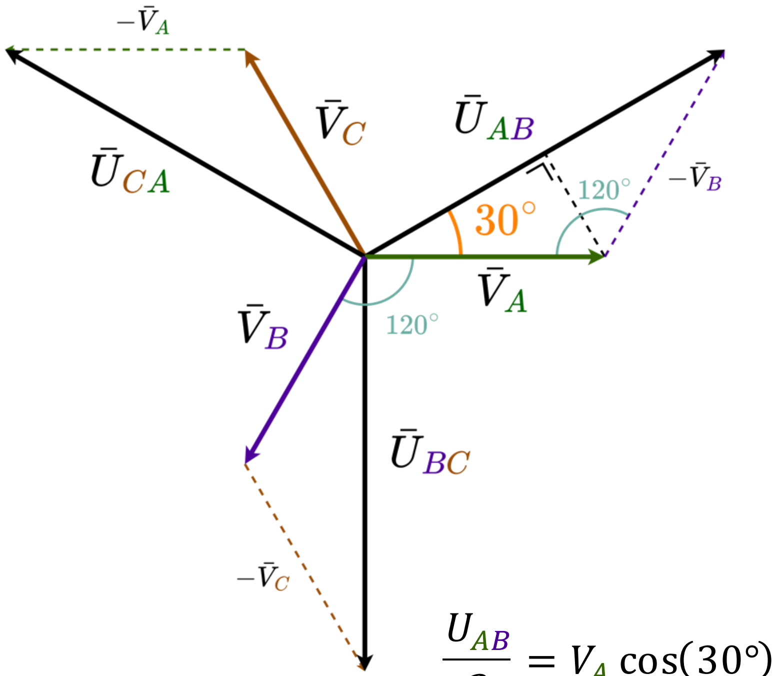
$$\bar{I}_B = \bar{J}_B$$

$$\bar{I}_C = \bar{J}_C$$

$$\bar{U}_{AB} = \bar{V}_A - \bar{V}_B$$

$$\bar{U}_{BC} = \bar{V}_B - \bar{V}_C$$

$$\bar{U}_{CA} = \bar{V}_C - \bar{V}_A$$



$$\frac{U_{AB}}{2} = V_A \cos(30°)$$

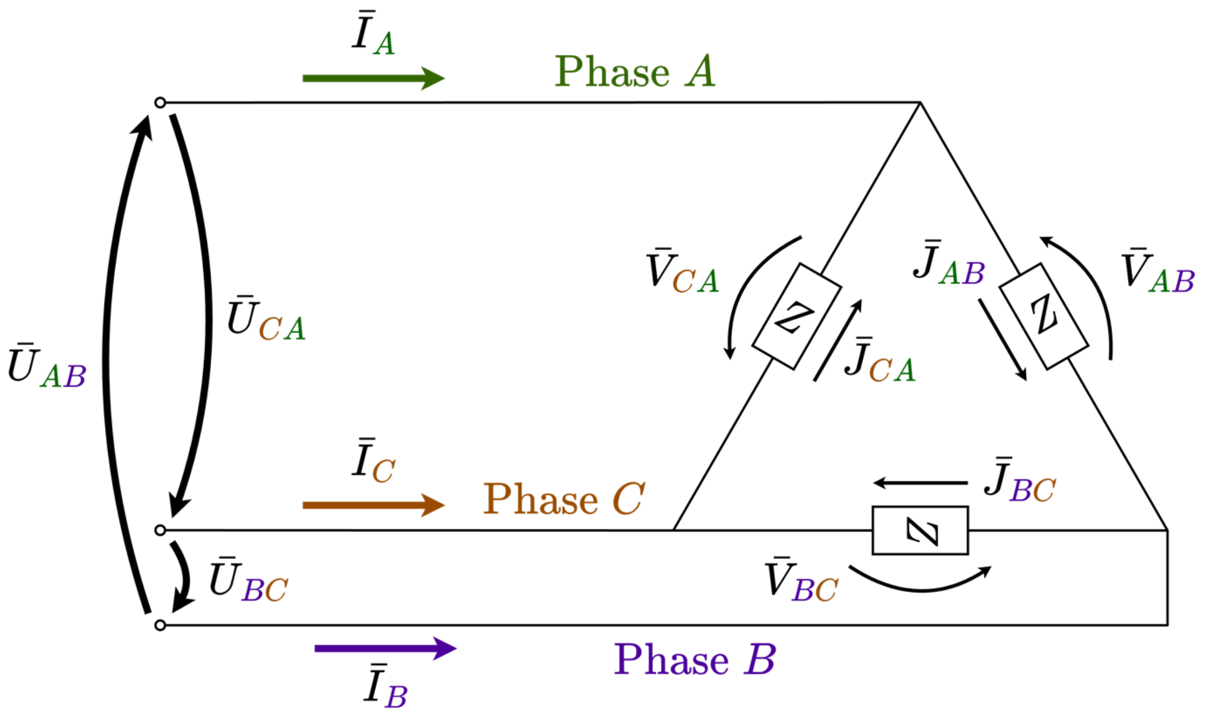
$$U_{AB} = \sqrt{3}\, V_A$$

In an equilibrated three-phase system in **star** configuration:

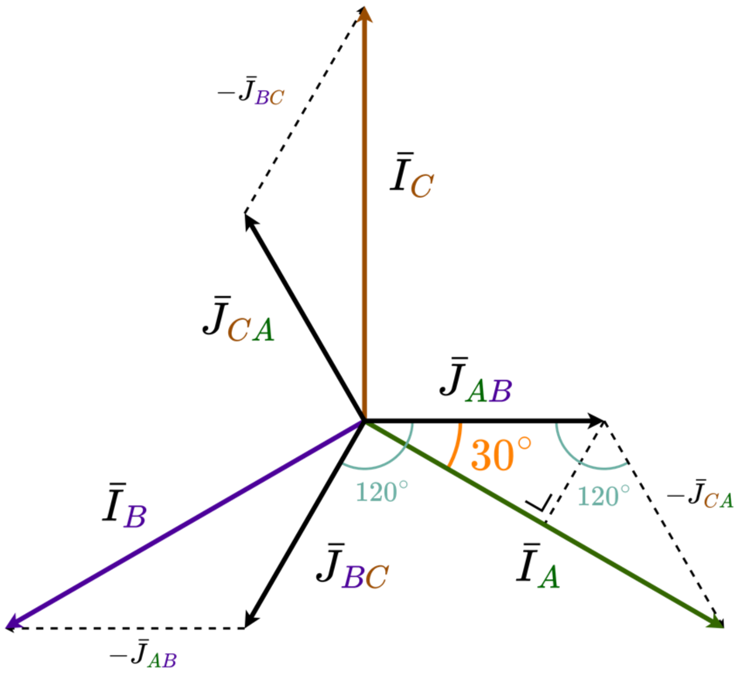
The norms of the phase and line currents are equal ➔ $I = J$

The norm of a line voltage is $\sqrt{3}$ times the norm of a phase voltage ➔ $U = \sqrt{3}\, V$

Phase vs. line voltage/current – delta configuration



$$\bar{I}_A = \bar{J}_{AB} - \bar{J}_{CA}$$
$$\bar{I}_B = \bar{J}_{BC} - \bar{J}_{AB}$$
$$\bar{I}_C = \bar{J}_{CA} - \bar{J}_{BC}$$
$$\bar{U}_{AB} = \bar{V}_{AB}$$
$$\bar{U}_{BC} = \bar{V}_{BC}$$
$$\bar{U}_{CA} = \bar{V}_{CA}$$



$$\frac{I_A}{2} = J_{AB}\cos(30°)$$
$$I_A = \sqrt{3}\,J_{AB}$$

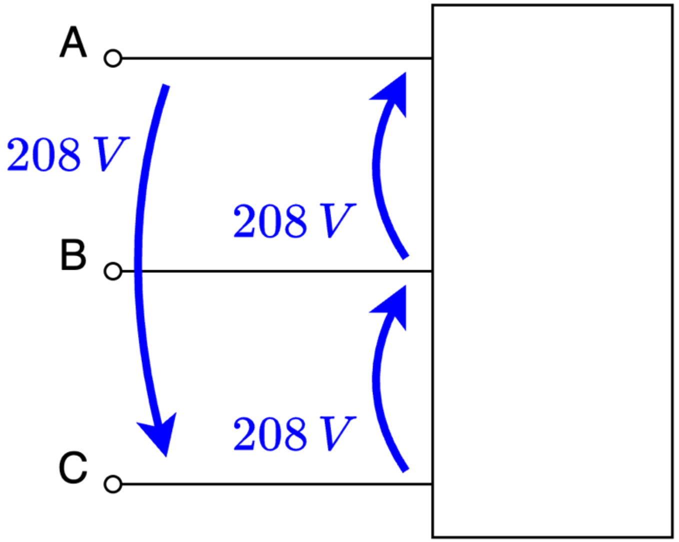
In an equilibrated three-phase system in **delta** configuration:

The norms of the phase and line voltages are equal ➔ $\mathrm{U} = V$

The norm of a line current is $\sqrt{3}$ times the norm of a phase current ➔ $\mathrm{I} = \sqrt{3}\,J$

Exercise 4

Consider an electrical heater that dissipates $15\ kW$ of power when connected to a three-phase power system of $208\ V$. As a first approximation, the heater is modelled as a purely resistive three-phase load.

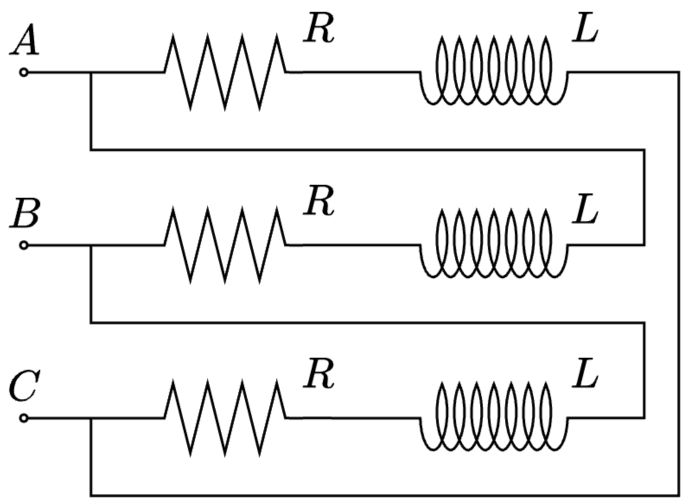


1. If no additional information is provided about the voltage, does the $208\ V$ correspond to the peak or to the RMS value ?
2. Compute the line current if the resistive loads are connected in $\star$.
3. If the resistors ae connected in $\star$, compute the resistance of each.
4. Compute the line current if the resistive loads are connected in Δ.
5. If the resistors ae connected in Δ, compute the resistance of each.

Exercise 5

Consider the equilibrated 3-phase load showed on the right.
To determine the value of the resistances and the value of the inductances, two tests are performed:

- A DC voltage of $150\ V$ is applied between terminals A and B for a measured current of $1.95\ A$.
- An AC voltage of RMS value $230\ V$, oscillating at $50\ Hz$, is applied between terminals A and B for a measured RMS current of $2.81\ A$.



1. Determine the values of R and L.

The load is now connected in A, B, C to an equilibrated 3-phase power supply working at $50\ Hz$.

2. What is the phase shift φ between phase currents and corresponding phase voltages?

3. Draw on a phasor diagram the phase voltages, the phase currents, the line voltages and the line currents.

Homework 12

A load is connected to a 50 Hz single-phase generator of 20 V for an active power of 10 W with $\cos\varphi = \frac{\sqrt{2}}{2}$.

1. What is the reactive power Q?

2. Assuming the load is composed of a resistor and of an inductor in parallel, what are their values R and L?

3. The frequency is increased to 100 Hz. What is now the active and reactive powers consumed by the load?

4. A capacitor is placed in parallel of the load. Considering a frequency of 50 Hz, for which value C does the reactive power is 0?

5. In case the capacitor is placed in series of the load, derive the expression of the total equivalent impedance. In this configuration, derive the expression of the value of C for which the total reactive power is zero?

Answer:

1. $Q = 10\ var$
2. $R = 40\ \Omega, L = 127.324\ mH$
3. $P = 10\ W, Q = 5\ var$
4. $C = 79.577\ \mu F$
5. $Z = \frac{1}{j\omega C} + \left(\frac{1}{R} + \frac{1}{j\omega L}\right)^{-1}, C = \frac{R^2+\omega^2 L^2}{R^2\omega^2 L} = 159.155\ \mu F$

Homework 13

A load is composed of an inductor $L = 10$ mH in series with a resistor $R = 5\ \Omega$.

1. Assuming a frequency of 50 Hz, what is the phase shift φ?

Three of these loads (identical) are set in a star configuration and connected to an equilibrated three-phase power supply working at 50 Hz. The amplitude of the line voltage U is 400 V.

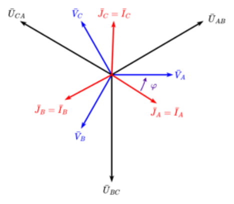
2. What is the amplitude of the phase voltage V, of the line current I and of the phase current J?

3. Taking one phase voltage as reference, draw on a phasor diagram the phase voltages, the phase currents, the line voltages and the line currents.

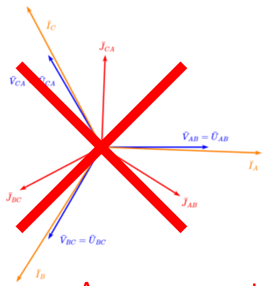
4. Repeat point 2. and 3. in case the load is now connected in delta configuration.

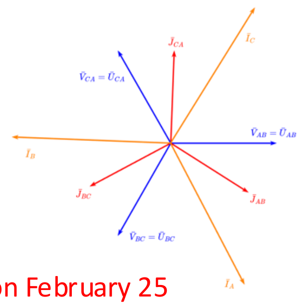
Answer:

1. $\varphi = 32.142°$
2. $V = 230.94\,V, J = I = 39.109\,A$
3.



4. $V = U = 400\,V, J = 67.739\,A, I = 117.327\,A$





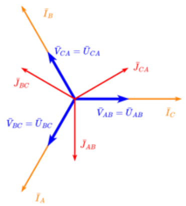
Answer corrected on February 25

Homework 14

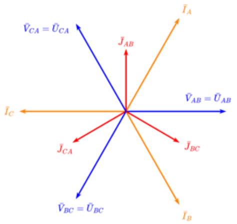
A three-phase load in delta configuration is connected to an equilibrated three-phase power supply working at 50 Hz. Each branch of the load is composed of an inductor ($L = 10$ mH) in series with a capacitor.

1. What is the phase shift φ in case $\mathrm{C} = 1.6$ mF?
2. The amplitude of the line current is 10 A. Compute the amplitudes of the phase current J, of the line voltage U and of the phase voltage V.
3. Taking one phase voltage as reference, draw on a phasor diagram the phase voltages, the phase currents, the line voltages and the line currents.
4. Repeat point 1., 2. and 3. in case $\mathrm{C} = 600\ \mu F$.

Answer:

1. $\varphi = 90°$
2. $J = 5.774\,A, V = U = 6.652\,V$
3. 

4. $\varphi = -90°, J = 5.774\,A, V = U = 12.491\,V$



Homework 15

A three-phase load is connected to an equilibrated three-phase power supply working at 50 Hz. Each branch of the load is composed of a resistor in parallel with a capacitor.

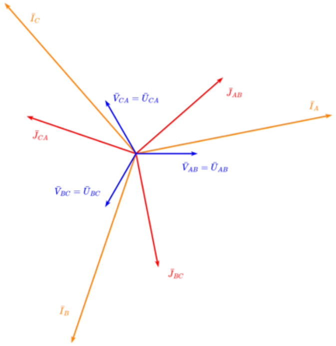
1. In case the load is connected in a delta configuration, a total three-phase active power $P_{3-\varphi}$ of 4.5 kW is measured for an RMS line voltage of 230 V and an RMS line current of 15 A. Find the values of C, R and φ.

2. Taking one phase voltage as reference, draw on a phasor diagram the phase voltages, the phase currents, the line voltages and the line currents.

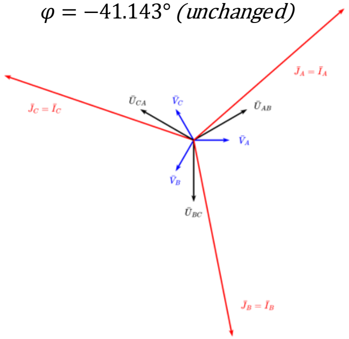
3. Repeat point 1. and 2. in case the load is connected in a star configuration. How does this modification impact the values of C, R and φ?

Answer:

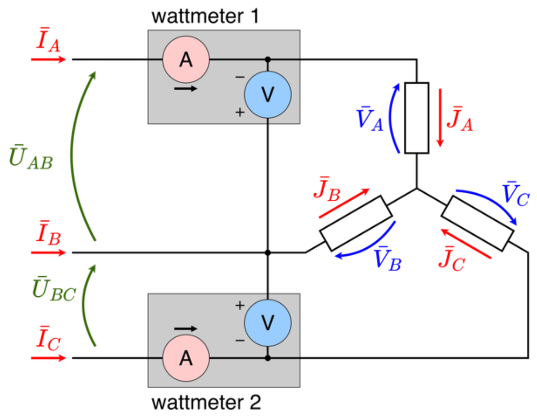
1. $C = 78.857\,\mu F, R = 35.267\,\Omega, \varphi = -41.143°$
2.



3. $C = 236.556\,\mu F$ *(multiplied by 3),*
$R = 11.756\,\Omega$ *(divided by 3),*
$\varphi = -41.143°$ *(unchanged)*



Homework 16

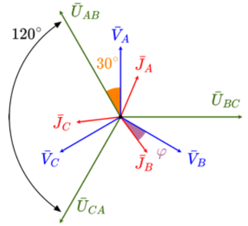


Two wattmeters can be used to compute the active and reactive powers of an equilibrated three-phase system. The first wattmeter measures the current flowing in the first line $\bar{I}_A$ and the voltage from the first to the second line $\bar{U}_{BA} = -\bar{U}_{AB}$. The second wattmeter measures the current flowing in the third line $\bar{I}_C$ and the voltage from the third to the second line $\bar{U}_{BC}$.

1. Considering a resistive-inductive load in star configuration and taking $\bar{U}_{BC}$ as reference, draw a phasor diagram showing the phase currents, the phase voltages and the line voltages.
2. Express the complex powers measured by the two wattmeters.
3. Show that one can compute the total three-phase active power as the opposite of the sum of the active powers measured by the two wattmeters ($P_{3\varphi} = -P_1 - P_2$).
4. Show that one can compute the total three-phase reactive power as $\sqrt{3}$ times the difference of the active powers measured by the two wattmeters ($Q_{3\varphi} = \sqrt{3}(P_1 - P_2)$).

Answer:

1.



2. $S_1 = -UI e^{j(\varphi+30°)}$
$S_2 = -UI e^{j(\varphi-30°)}$

3. $P_{3\varphi} = 3VJ\cos\varphi = \sqrt{3}\,UI\cos\varphi$

4. $Q_{3\varphi} = 3VJ\sin\varphi = \sqrt{3}\,UI\sin\varphi$

NB: The complete solution of this exercise will be available in the manual of lab 3.