



# Electromagnetic Energy Conversion

## ELEC0431

### Exercise session 2: Balanced three-phase systems

13 February 2026

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# Reminder: Laboratories – Schedule and groups

To create the lab schedule, you are required to fill **by group** the doodle <https://beta.framadate.org/polls/465a07d3b8361e104f3d>:

- By group of **4 students**, select **6** available time slots (you may be given random sessions if less than 6 time slots were selected).
- In the space provided for names, write the **STUDENT ID NUMBERS** of all **4 MEMBERS** of the group (for example: “s161514, s171856, s164442, s179088”).
- A time slot can be selected by maximum six groups, **do not delay in completing this Doodle.**
- In addition to your selected time slots, your schedule could include laboratory sessions on the 17/04, 24/04, 8/05 and 15/05 mornings (Friday mornings in place of the traditional classes).

**IF AND ONLY IF** it is impossible for you to create a group of four students meeting the requirements, send me an email ([florent.purnode@uliege.be](mailto:florent.purnode@uliege.be)) without delay.

Make sure to complete the doodle by **23:59 on Friday, February 20**,  
(Students who would not have given their availabilities by this time will be given random time slots).

Quick reminder: Laboratories are **mandatory**  
In case of unexcused absence, an absence grade will be given for the entire course.

# Reminder: Test on phasors

- The two first exercise sessions are dedicated to the introduction of phasors.  
Phasors will be used throughout the class. Being able to work with them effortlessly is a must!
- A test, focusing on the material seen during the two first exercise sessions is scheduled on Friday 6<sup>th</sup> of March, at 9:00 am.  
It takes place in the usual classroom (B37 auditorium 02).  
It is a quick test of 30 minutes (we insist on the fact you should be able to manipulate phasors effortlessly).  
To take the test, you will need your calculator, ruler and protractor (“rapporteur” in French).
- At each exercise sessions, homeworks are available to prepare yourself for the test and the exam.  
A past test and past exams are also available on the webpage.
- The test accounts for 5 % of the final grade.
- The test will be followed by its correction and a normal exercise session.

# In this class...

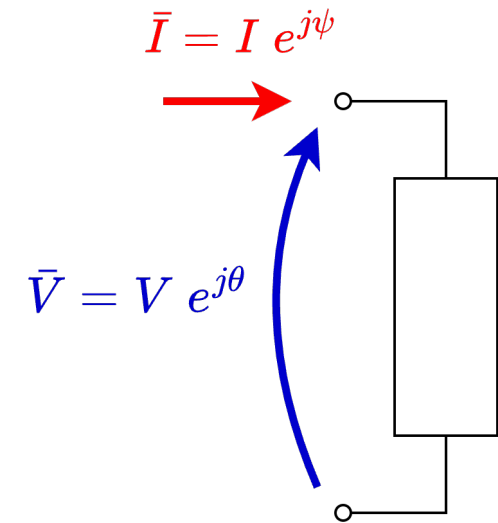
- Producer or consumer?
- Exercise 3 (homework 11)
- Introduction to three-phase systems
- Phase and line current/voltages – Star and delta configurations
- Exercises 4 & 5

# Producer or consumer?

Using the passive convention, we consider a one-port crossed by a current  $\bar{I} = I e^{j\psi}$  with a voltage  $\bar{V} = V e^{j\theta}$ .

The associated complex power is

$$\begin{aligned} S &= \bar{V} \bar{I}^* = VI e^{j(\theta - \psi)} \\ &= VI \cos(\theta - \psi) + j VI \sin(\theta - \psi) \\ &= P + j Q \end{aligned}$$



In case  $P$  is **positive**, we say that the one-port **consumes** active power.

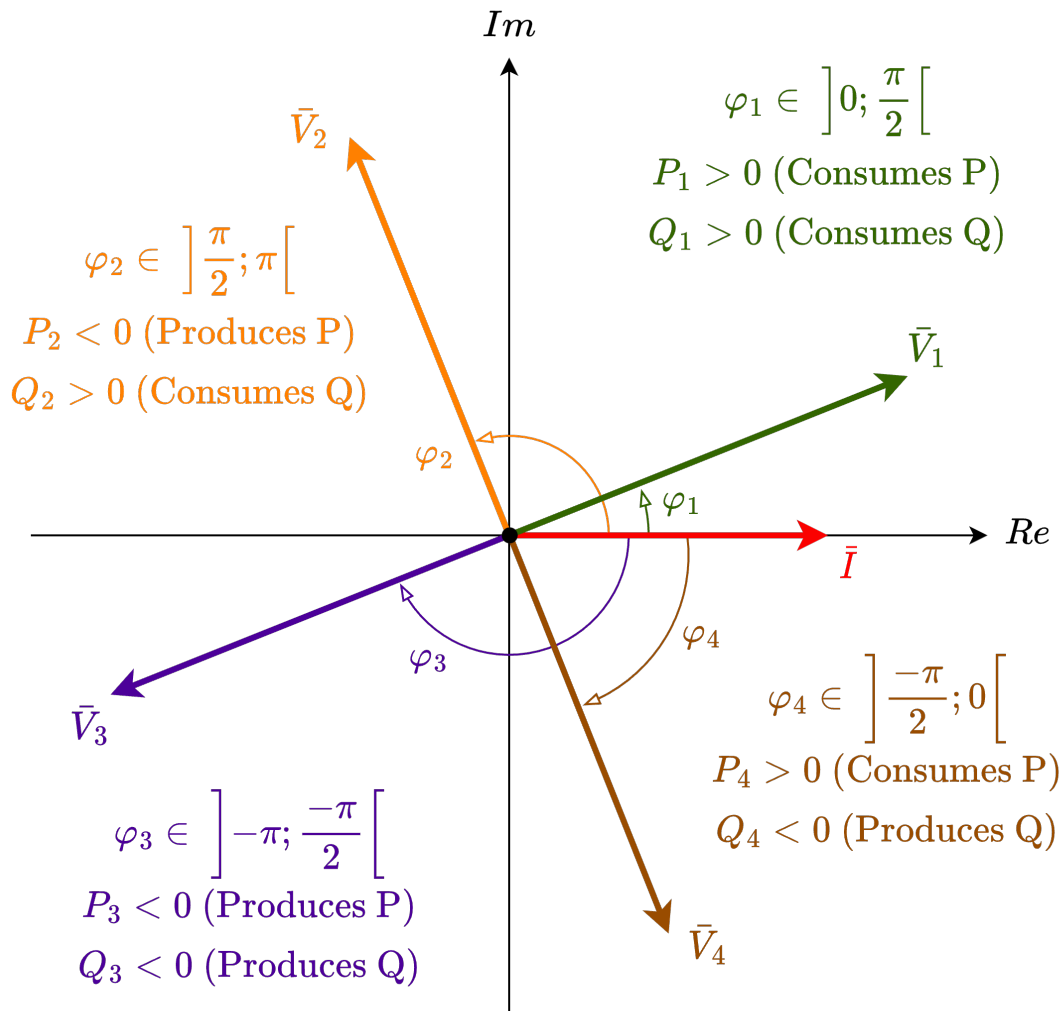
In case  $P$  is **negative**, we say that the one-port **produces** active power.

In case  $Q$  is **positive**, we say that the one-port **consumes** reactive power.

In case  $Q$  is **negative**, we say that the one-port **produces** reactive power.

# Producer or consumer ?

Taking  $\bar{I}$  as reference and  $\varphi \in [-\pi; \pi]$ :



$$P = VI \cos(\theta - \psi) = VI \cos(\varphi)$$

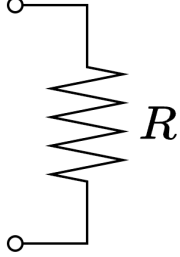
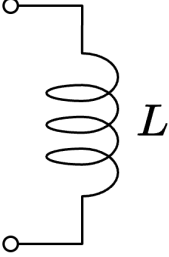
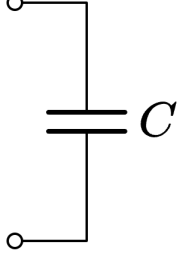
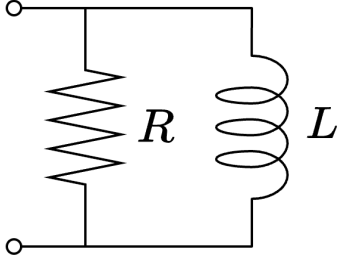
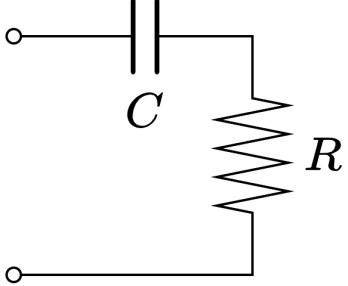
$$Q = VI \sin(\theta - \psi) = VI \sin(\varphi)$$

- If  $\varphi > 0$       The **voltage** leads the **current**  
The one-port consumes **reactive power**
- If  $\varphi < 0$       The **voltage** lags the **current**  
The one-port produces **reactive power**
- If  $|\varphi| < \frac{\pi}{2}$       The one-port consumes **active power**
- If  $|\varphi| > \frac{\pi}{2}$       The one-port produces **active power**

# Exercise 3 (Homework 11)

Fill the cells of the table below with the most appropriate answer among

$= 0$      $< 0$      $> 0$      $= 1$      $< 1$      $+\infty$      $-\infty$

					
Active power consumed	$> 0$	$= 0$	$= 0$	$> 0$	$> 0$
Reactive power produced	$= 0$	$< 0$	$> 0$	$< 0$	$> 0$
$\cos(\varphi)$	$= 1$	$= 0$	$= 0$	$> 0$	$> 0$
$\tan(\varphi)$	$= 0$	$+\infty$	$-\infty$	$> 0$	$< 0$

# Introduction to three-phase systems

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One-phase generator

Two-phase generator

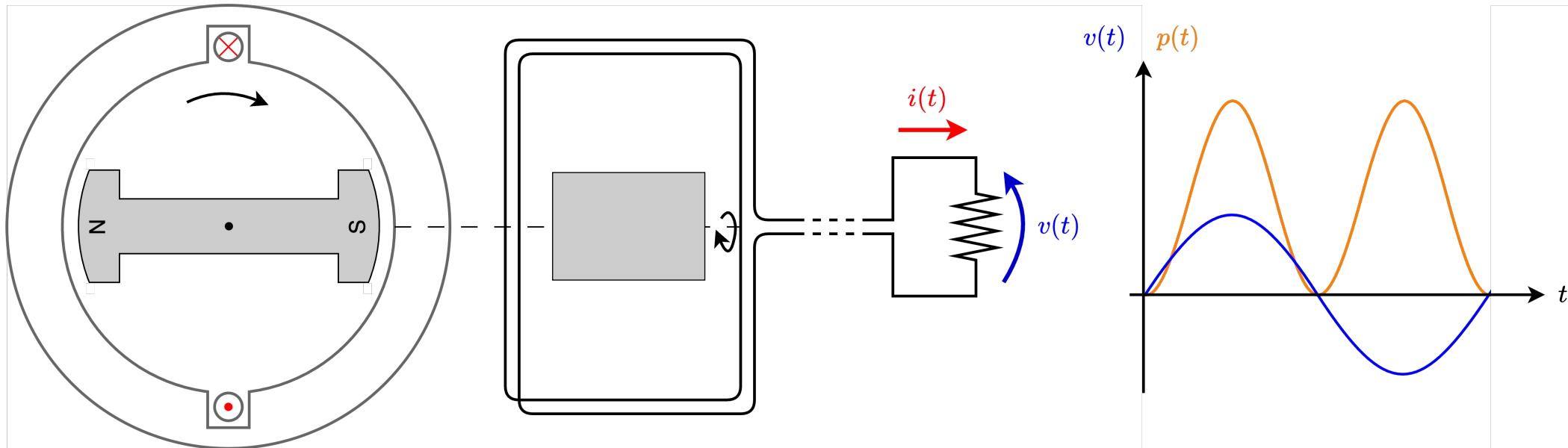
Three-phase generator

Phase and line current/voltages – Star and delta configurations

Exercises 4 & 5

# One-phase generator

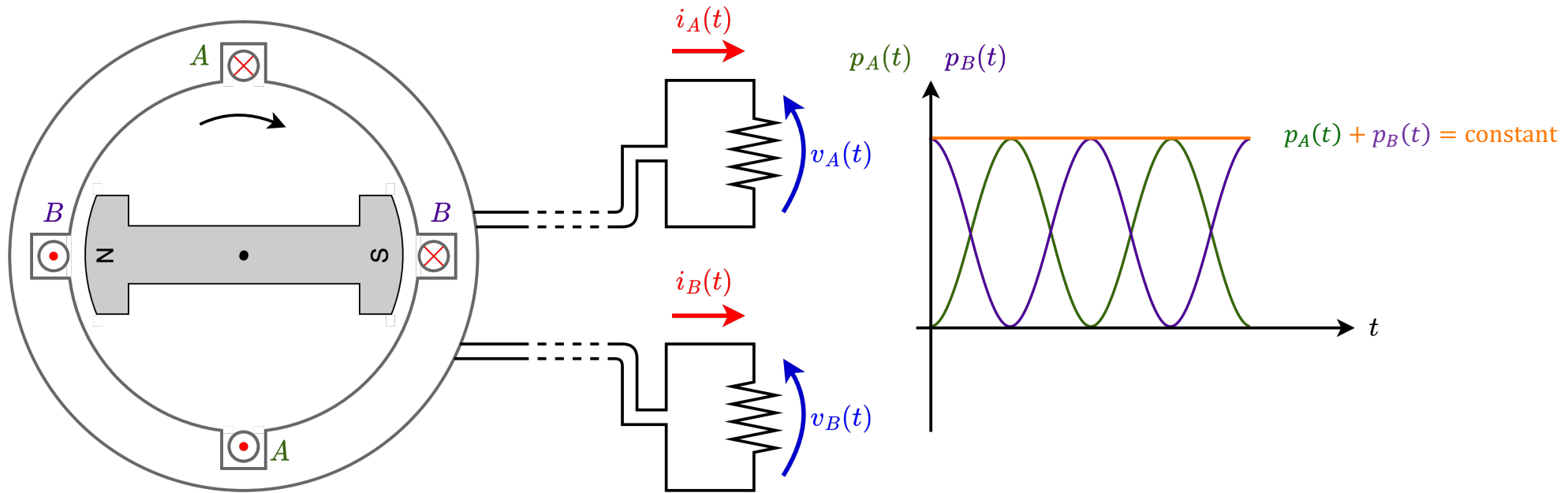
To build an AC generator, one could spin a magnet in a coil. A voltage  $v(t)$  is induced in the winding as it perceives a magnetic flux varying over time (Lenz).



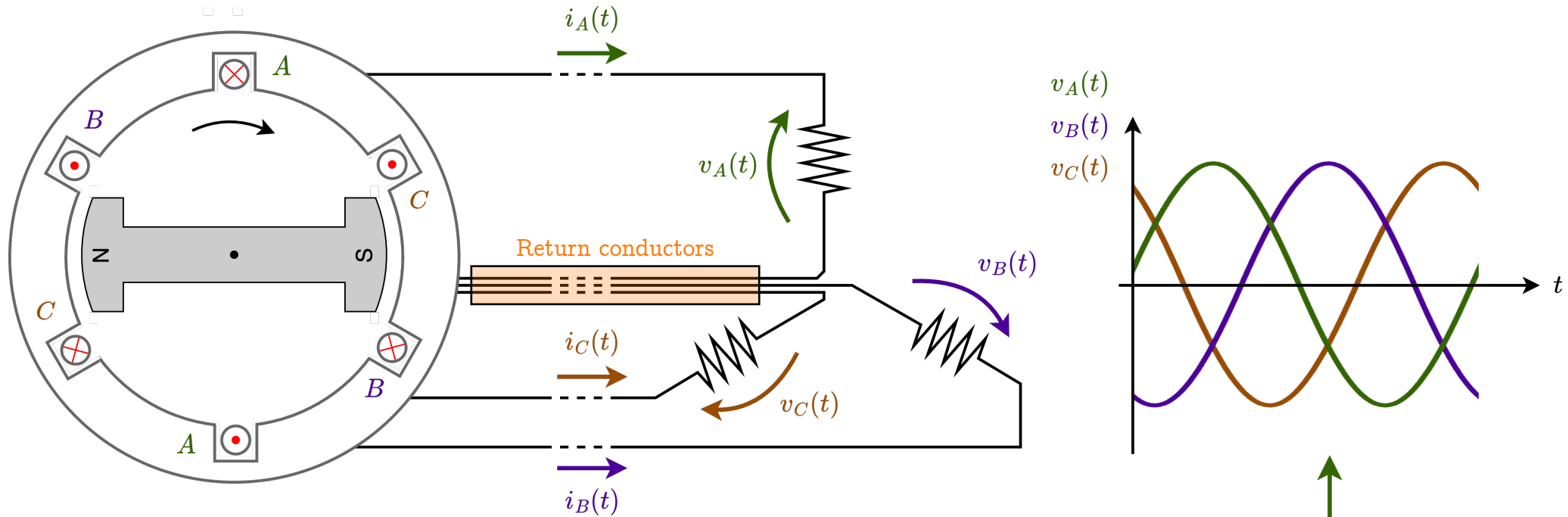
The output power  $p(t)$  fluctuates so that the input torque fluctuates as well. However, we would like to have a constant input torque.

# Two-phase generator

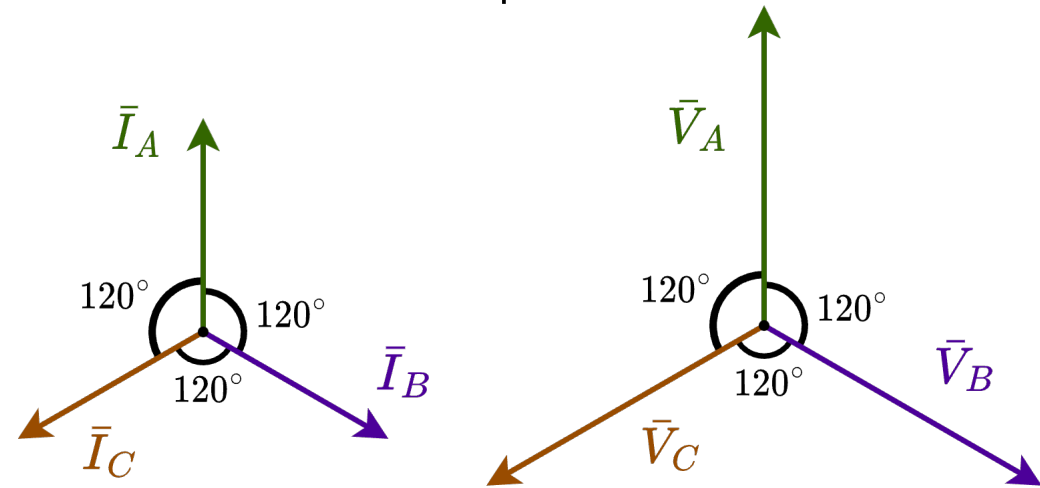
Using a two windings connected to identical loads, a constant input torque can be obtained.



# Three-phase generator

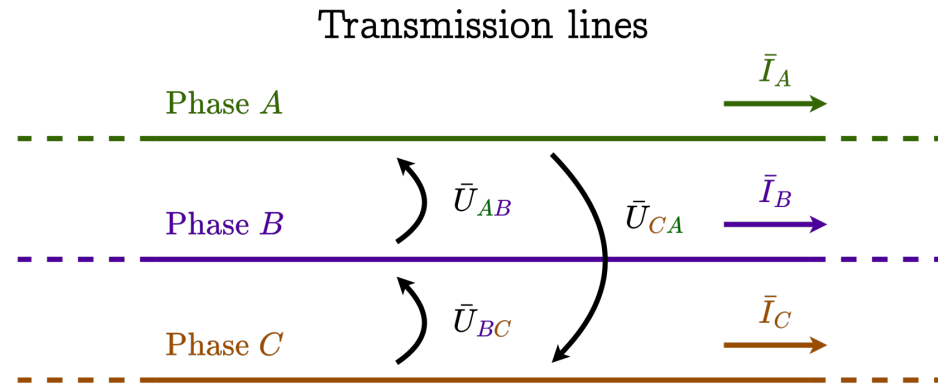
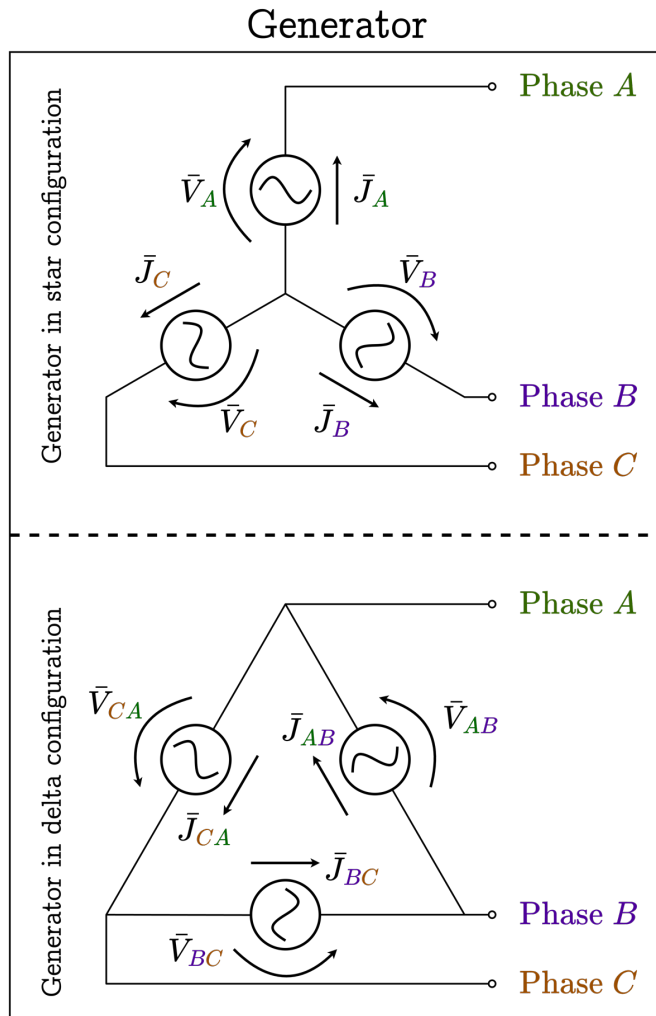


Using identical loads, the input torque also remains constant when using three phases. Moreover, plotting the current phase diagram, it appears that the sum of the currents is zero. The **return conductors** can thus be removed.

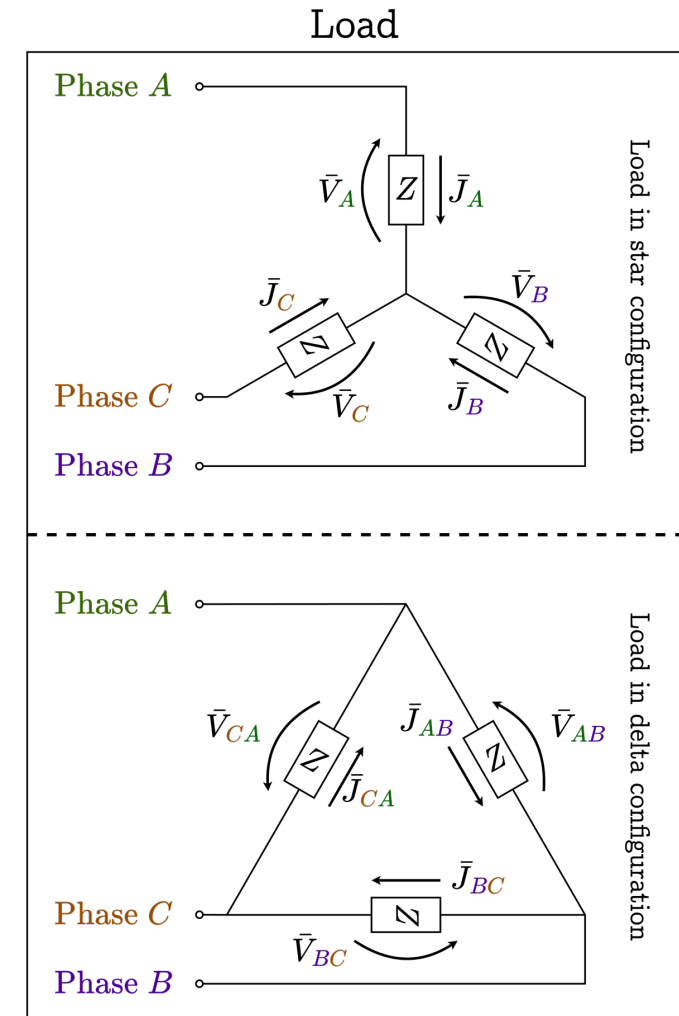


# Three-phase circuits

Generators and loads are connected in star ( $\star$  or  $Y$ ) or delta ( $\Delta$ ) configuration.



We differentiate:  
 Phase voltage  $\bar{V}$  vs. Line voltage  $\bar{U}$ .  
 Phase current  $\bar{J}$  vs. Line current  $\bar{I}$ .



# Phase vs. line voltage/current

Some conventions for the practical lectures:

The line voltage  $U$  is the RMS voltage between any two transmission lines.

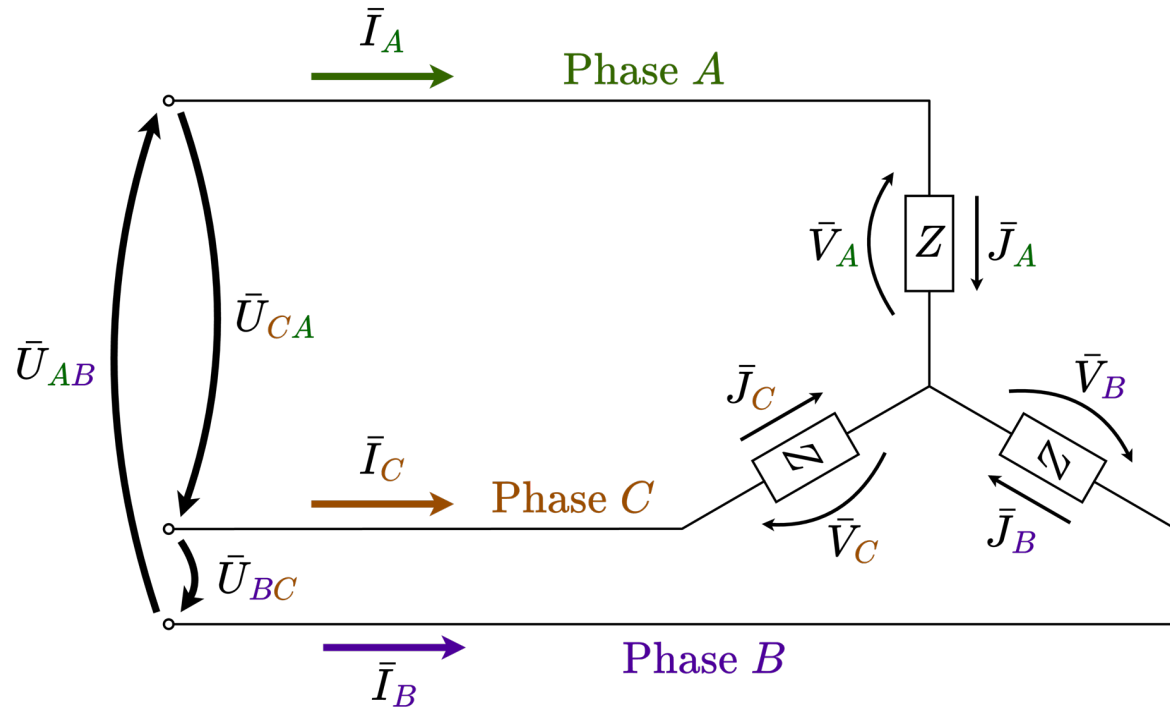
The line current  $I$  is the RMS current flowing in a transmission line.

VS.

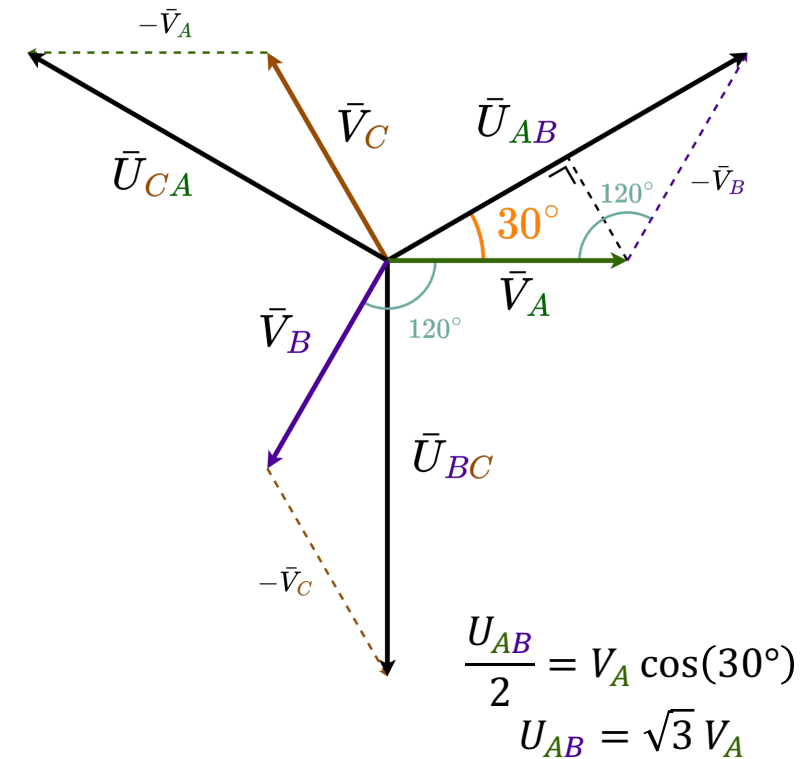
The phase voltage  $V$  is the RMS voltage across any of the generator/load leg.

The phase current  $J$  is the RMS current flowing in a generator/load leg.

# Phase vs. line voltage/current – star configuration



$$\begin{aligned}\bar{I}_A &= \bar{J}_A \\ \bar{I}_B &= \bar{J}_B \\ \bar{I}_C &= \bar{J}_C \\ \bar{U}_{AB} &= \bar{V}_A - \bar{V}_B \\ \bar{U}_{BC} &= \bar{V}_B - \bar{V}_C \\ \bar{U}_{CA} &= \bar{V}_C - \bar{V}_A\end{aligned}$$

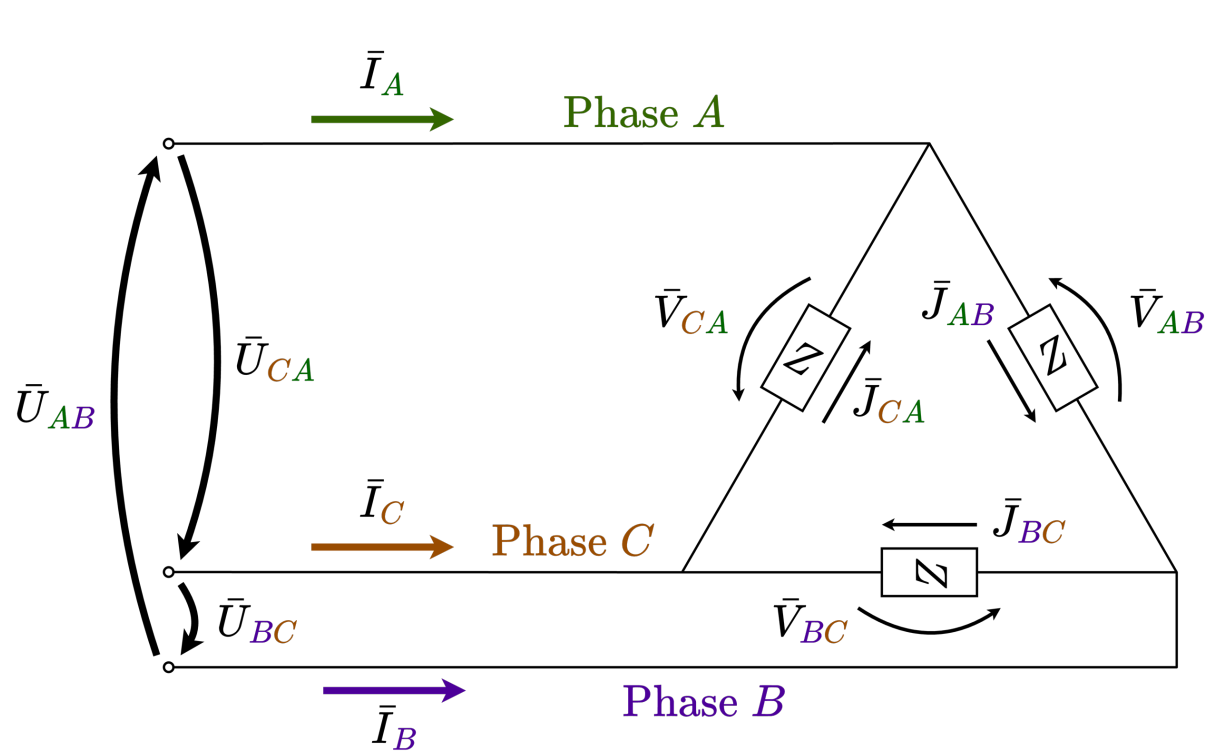


In an equilibrated three-phase system in star configuration:

The norms of the phase and line currents are equal  $\rightarrow I = J$

The norm of a line voltage is  $\sqrt{3}$  times the norm of a phase voltage  $\rightarrow U = \sqrt{3} V$

# Phase vs. line voltage/current – delta configuration



$$\bar{I}_A = \bar{J}_{AB} - \bar{J}_{CA}$$

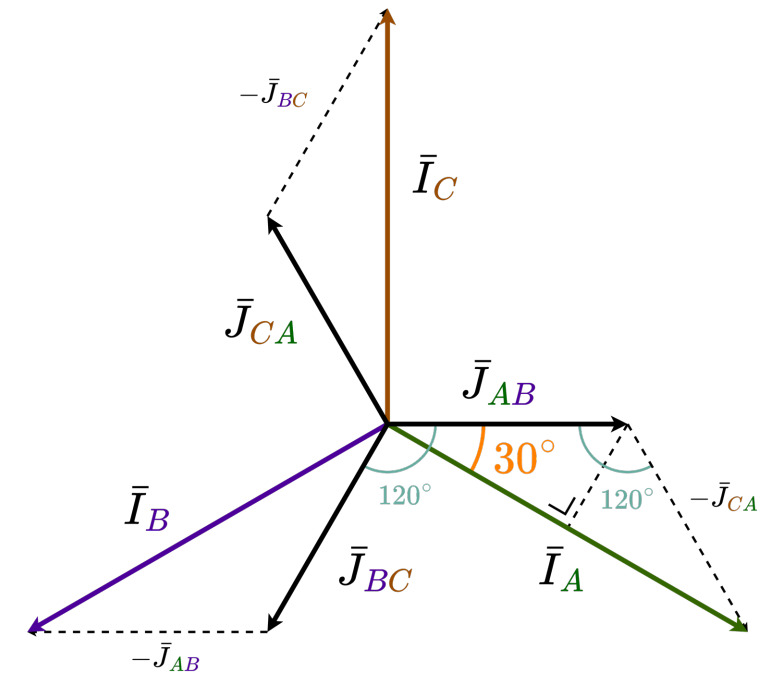
$$\bar{I}_B = \bar{J}_{BC} - \bar{J}_{AB}$$

$$\bar{I}_C = \bar{J}_{CA} - \bar{J}_{BC}$$

$$\bar{U}_{AB} = \bar{V}_{AB}$$

$$\bar{U}_{BC} = \bar{V}_{BC}$$

$$\bar{U}_{CA} = \bar{V}_{CA}$$



$$\frac{I_A}{2} = J_{AB} \cos(30^\circ)$$

$$I_A = \sqrt{3} J_{AB}$$

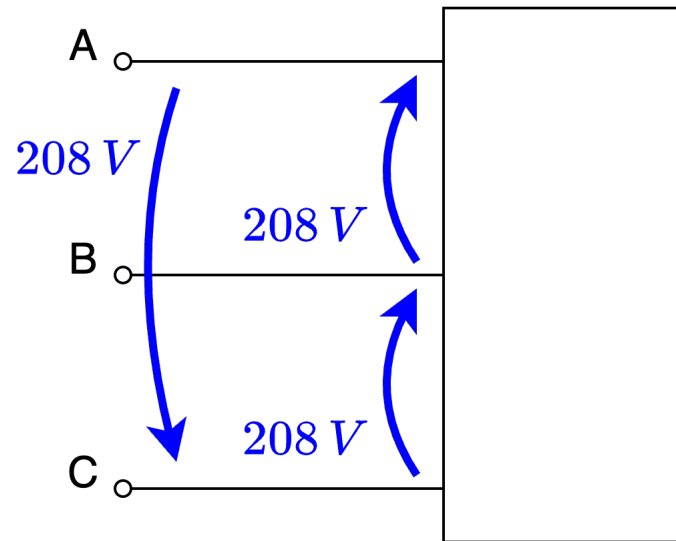
In an equilibrated three-phase system in delta configuration:

The norms of the phase and line voltages are equal  $\rightarrow U = V$

The norm of a line current is  $\sqrt{3}$  times the norm of a phase current  $\rightarrow I = \sqrt{3} J$

# Exercise 4

Consider an electrical heater that dissipates  $15 \text{ kW}$  of power when connected to a three-phase power system of  $208 \text{ V}$ . As a first approximation, the heater is modelled as a purely resistive three-phase load.

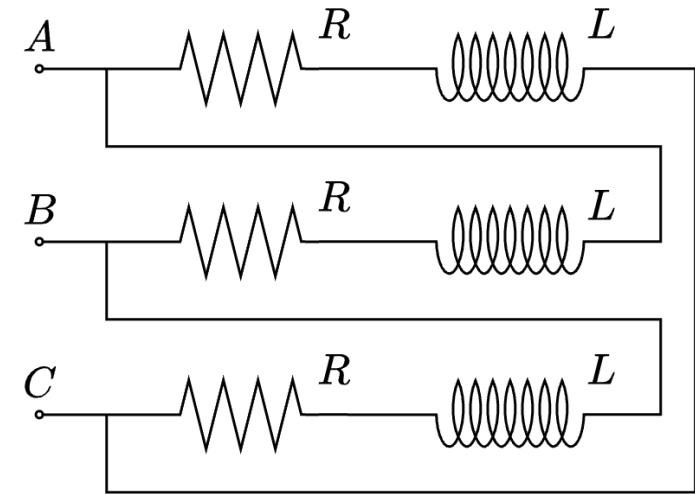


1. If no additional information is provided about the voltage, does the  $208 \text{ V}$  correspond to the peak or to the RMS value ?
2. Compute the line current if the resistive loads are connected in  $\star$ .
3. If the resistors are connected in  $\star$ , compute the resistance of each.
4. Compute the line current if the resistive loads are connected in  $\Delta$ .
5. If the resistors are connected in  $\Delta$ , compute the resistance of each.

# Exercise 5

Consider the equilibrated 3-phase load showed on the right.  
To determine the value of the resistances and the value of the inductances, two tests are performed:

- A DC voltage of  $150\text{ V}$  is applied between terminals  $A$  and  $B$  for a measured current of  $1.95\text{ A}$ , with terminal  $C$  left unconnected.
- An AC voltage of RMS value  $230\text{ V}$ , oscillating at  $50\text{ Hz}$ , is applied between terminals  $A$  and  $B$  for a measured RMS current of  $2.81\text{ A}$ , with terminal  $C$  left unconnected.



1. Determine the values of  $R$  and  $L$ .

The load is now connected in  $A$ ,  $B$ ,  $C$  to an equilibrated 3-phase power supply working at  $50\text{ Hz}$ .

2. What is the phase shift  $\varphi$  between phase currents and corresponding phase voltages?
3. Draw on a phasor diagram the phase voltages, the phase currents, the line voltages and the line currents.

# Homework 12

A load is connected to a 50 Hz single-phase generator of 20 V for an active power of 10 W with  $\cos \varphi = \frac{\sqrt{2}}{2}$ .

1. What is the reactive power  $Q$ ?
2. Assuming the load is composed of a resistor and of an inductor in parallel, what are their values  $R$  and  $L$ ?
3. The frequency is increased to 100 Hz. What is now the active and reactive powers consumed by the load?
4. A capacitor is placed in parallel of the load. Considering a frequency of 50 Hz, for which value  $C$  does the reactive power is 0?
5. In case the capacitor is placed in series of the load, derive the expression of the total equivalent impedance. In this configuration, derive the expression of the value of  $C$  for which the total reactive power is zero?

*Answer:*

1.  $Q = 10 \text{ var}$
2.  $R = 40 \Omega, L = 127.324 \text{ mH}$
3.  $P = 10 \text{ W}, Q = 5 \text{ var}$
4.  $C = 79.577 \mu\text{F}$
5.  $Z = \frac{1}{j\omega C} + \left(\frac{1}{R} + \frac{1}{j\omega L}\right)^{-1}, C = \frac{R^2 + \omega^2 L^2}{R^2 \omega^2 L} = 159.155 \mu\text{F}$

# Homework 13

A load is composed of an inductor  $L = 10 \text{ mH}$  in series with a resistor  $R = 5 \Omega$ .

1. Assuming a frequency of 50 Hz, what is the phase shift  $\varphi$ ?

Three of these loads (identical) are set in a star configuration and connected to an equilibrated three-phase power supply working at 50 Hz. The amplitude of the line voltage  $U$  is 400 V.

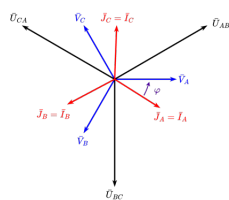
2. What is the amplitude of the phase voltage  $V$ , of the line current  $I$  and of the phase current  $J$ ?

3. Taking one phase voltage as reference, draw on a phasor diagram the phase voltages, the phase currents, the line voltages and the line currents.

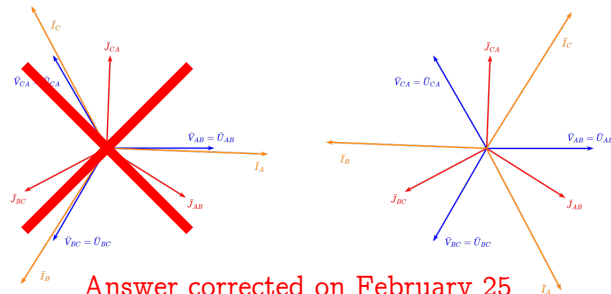
4. Repeat point 2. and 3. in case the load is now connected in delta configuration.

Answer:

1.  $\varphi = 32.142^\circ$
2.  $V = 230.94 \text{ V}, J = I = 39.109 \text{ A}$
- 3.



4.  $V = U = 400 \text{ V}, J = 67.739 \text{ A}, I = 117.327 \text{ A}$



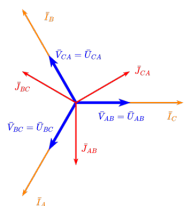
# Homework 14

A three-phase load in delta configuration is connected to an equilibrated three-phase power supply working at 50 Hz. Each branch of the load is composed of an inductor ( $L = 10$  mH) in series with a capacitor.

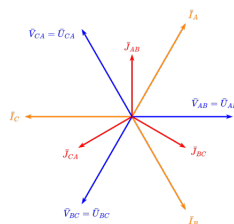
1. What is the phase shift  $\varphi$  in case  $C = 1.6$  mF?
2. The amplitude of the line current is 10 A. Compute the amplitudes of the phase current  $J$ , of the line voltage  $U$  and of the phase voltage  $V$ .
3. Taking one phase voltage as reference, draw on a phasor diagram the phase voltages, the phase currents, the line voltages and the line currents.
4. Repeat point 1., 2. and 3. in case  $C = 600$   $\mu$ F.

Answer:

1.  $\varphi = 90^\circ$
2.  $J = 5.774$  A,  $V = U = 6.652$  V
- 3.



4.  $\varphi = -90^\circ$ ,  $J = 5.774$  A,  $V = U = 12.491$  V



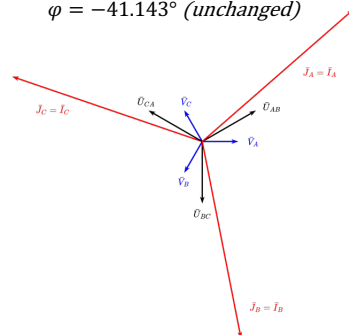
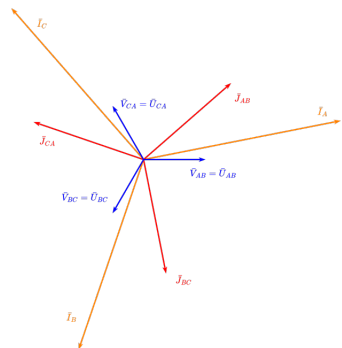
# Homework 15

A three-phase load is connected to an equilibrated three-phase power supply working at 50 Hz. Each branch of the load is composed of a resistor in parallel with a capacitor.

1. In case the load is connected in a delta configuration, a total three-phase active power  $P_{3-\varphi}$  of 4.5 kW is measured for an RMS line voltage of 230 V and an RMS line current of 15 A. Find the values of C, R and  $\varphi$ .
2. Taking one phase voltage as reference, draw on a phasor diagram the phase voltages, the phase currents, the line voltages and the line currents.
3. Repeat point 1. and 2. in case the load is connected in a star configuration. How does this modification impact the values of C, R and  $\varphi$ ?

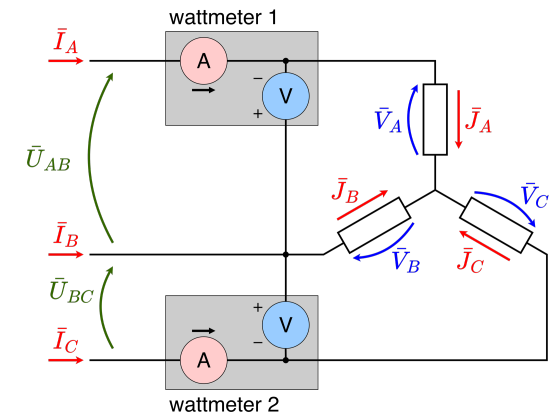
Answer:

1.  $C = 78.857 \mu\text{F}$ ,  $R = 35.267 \Omega$ ,  $\varphi = -41.143^\circ$
- 2.
3.  $C = 236.556 \mu\text{F}$  (multiplied by 3),  
 $R = 11.756 \Omega$  (divided by 3),  
 $\varphi = -41.143^\circ$  (unchanged)



# Homework 16

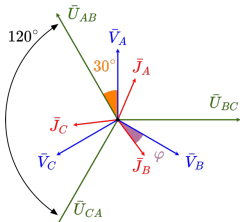
Two wattmeters can be used to compute the active and reactive powers of an equilibrated three-phase system. The first wattmeter measures the current flowing in the first line  $\bar{I}_A$  and the voltage from the first to the second line  $\bar{U}_{BA} = -\bar{U}_{AB}$ . The second wattmeter measures the current flowing in the third line  $\bar{I}_C$  and the voltage from the third to the second line  $\bar{U}_{BC}$ .



1. Considering a resistive-inductive load in star configuration and taking  $\bar{U}_{BC}$  as reference, draw a phasor diagram showing the phase currents, the phase voltages and the line voltages.
2. Express the complex powers measured by the two wattmeters.
3. Show that one can compute the total three-phase active power as the opposite of the sum of the active powers measured by the two wattmeters ( $P_{3\varphi} = -P_1 - P_2$ ).
4. Show that one can compute the total three-phase reactive power as  $\sqrt{3}$  times the difference of the active powers measured by the two wattmeters ( $Q_{3\varphi} = \sqrt{3}(P_1 - P_2)$ ).

Answer:

1.



2.  $S_1 = -UIe^{j(\varphi+30^\circ)}$     3.  $P_{3\varphi} = 3VJ \cos \varphi = \sqrt{3} UI \cos \varphi$   
 $S_2 = -UIe^{j(\varphi-30^\circ)}$     4.  $Q_{3\varphi} = 3VJ \sin \varphi = \sqrt{3} UI \sin \varphi$

NB: The complete solution of this exercise will be available in the manual of lab 3.