ELEC0431 : Exercise session 3

Magnetic circuits and transformers

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Transformer illustrations

Transformer

Wooclap code : POSMDD

Reluctances and magnetic circuits

From Maxwell's equations, one can retrieve the Ampere law as

$$\oint_{C} \vec{h} \cdot d\vec{l} = \mathcal{F} \tag{1}$$

which can be simplified into

$$\boxed{\mathcal{F} = h\,l \quad = n\,i} \tag{2}$$

Similarly, one can define the magnetic flux ϕ as the quantity of magnetic induction $\vec{b}$ crossing a surface S

$$\phi = \int_{S} \vec{b} \cdot \vec{n}\, dS \tag{3}$$

which we will also simplify into

$$\boxed{\phi = b\,S} \tag{4}$$

Reluctances and magnetic circuits

Equations (2) and (4) can be linked with the help of the magnetic constitutive law

$$\boxed{\vec{b} = \mu\, \vec{h}} \tag{5}$$

where μ is

$$\mu = \underbrace{\mu_0}_{=4\pi\times10^{-7}} \mu_r \tag{6}$$

and μ_r is the relative permeability of the material (for iron 1000-2000).

Finally one can define the reluctance of a magnetic circuit as

$$\boxed{\mathcal{R} = \frac{1}{\mu}\frac{l}{S}} \tag{7}$$

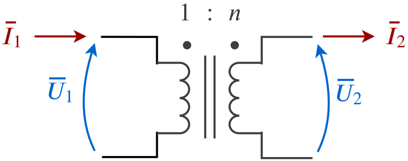
Reluctances and magnetic circuits

The reluctance $\mathcal{R}$ makes the link between the magnetomotive force to the magnetic flux by

$$\boxed{\mathcal{F} = \mathcal{R}\,\phi} \tag{8}$$

One parallel can be made with Ohm's law as $V = R\,I$ is similar to Equation (8) with the resistance expressed from Pouillet's law $R = \frac{1}{\sigma}\,\frac{l}{S}$.

Ideal transformer



$$\boxed{\begin{aligned} \overline{U}_2 &= n\,\overline{U}_1 \\ \overline{I}_2 &= \frac{\overline{I}_1}{n} \end{aligned}}$$

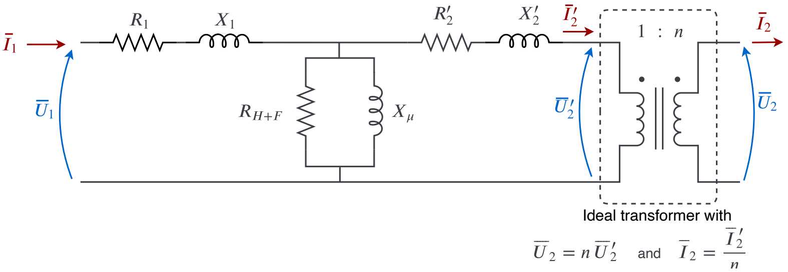
Remark that the transformer ratio n can also be defined from the secondary to the primary, there is no convention here.

The apparent power is conserved in the ideal transformer :

$$S_1 = S_2$$
$$\overline{U}_1 \, \overline{I}_1^* = \overline{U}_2 \, \overline{I}_2^*$$
$$P_1 + j \, Q_1 = P_2 + j \, Q_2$$

The real transformer model

A more complete model of practical transformers, including losses, can be represented as



The different components represent physical phenomenon such as

R_1 : the joule losses in the primary winding

X_1 : the leakage flux of the primary

R_2' : the joule losses in the secondary winding, seen from the primary

X_2' : the leakage flux of the secondary, seen from the primary

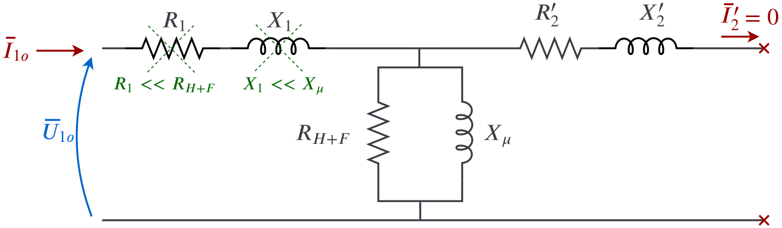
R_{H+F} : the magnetic losses in the core such as hysteresis ($\propto f$) and eddy current ($\propto f^2$)

X_μ : the magnetizing reactance

In practice, transformers are build in order to minimize the losses and the undesired effects, which leads to

$R_1 << R_{H+F} \quad ; \quad R_2' << R_{H+F} \quad ; \quad X_1 << X_\mu \quad ; \quad X_2' << X_\mu$

Open circuit test

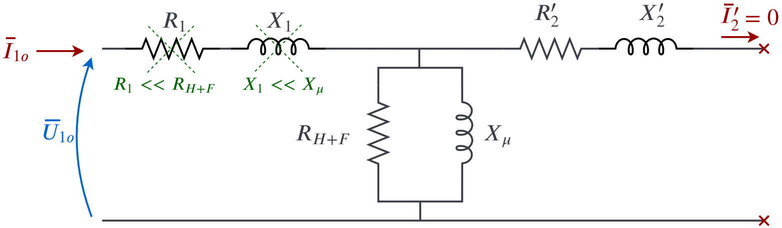


$$S_{1o} = Y^* \, U_{1o}^2 = \left(\frac{1}{R_{H+F}} + j \, \frac{1}{X_\mu} \right) U_{1o}^2 \tag{9}$$

$$P_{1o} = \;\; G \;\; U_{1o}^2 = \frac{U_{1o}^2}{R_{H+F}} \tag{10}$$

$$Q_{1o} = -B \, U_{1o}^2 = \frac{U_{1o}^2}{X_\mu} \tag{11}$$

Open circuit test

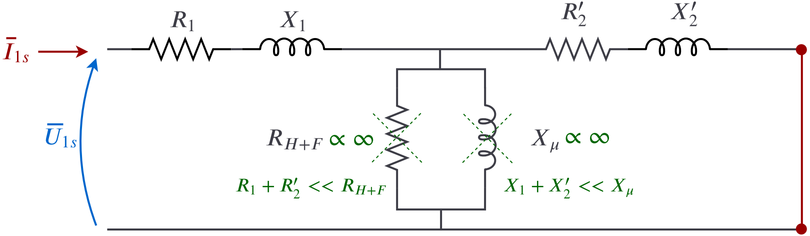


$$R_{H+F} = \frac{U_{1o}^2}{P_{1o}} \tag{12}$$

$$X_\mu = \frac{U_{1o}^2}{Q_{1o}} \tag{13}$$

where P_{1o} and Q_{1o} are the active and reactive powers consumed during the open circuit test.

Short circuit test

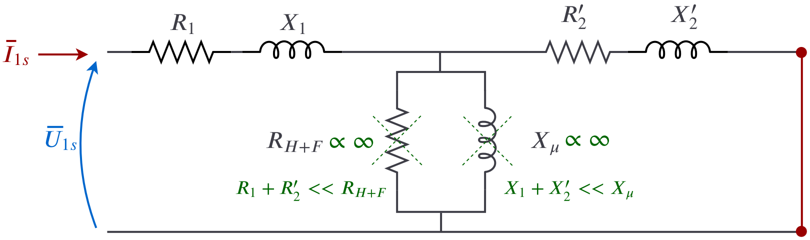


$$S_{1s} = Z\, I_{1s}^2 \tag{14}$$

$$P_{1s} = R\, I_{1s}^2 = (R_1 + R'_2)\, I_{1s}^2 \tag{15}$$

$$Q_{1s} = X\, I_{1s}^2 = (X_1 + X'_2)\, I_{1s}^2 \tag{16}$$

Short circuit test



$$\boxed{R_1 + R'_2 = \frac{P_{1s}}{I_{1s}^2}} \quad (17)$$

$$\boxed{X_1 + X'_2 = \frac{Q_{1s}}{I_{1s}^2}} \quad (18)$$

where P_{1s} and Q_{1s} are the active and reactive powers consumed during the short circuit test.

Exercises

Exercises of session 3

Wooclap code : UYIFUP

Exercise 7 : reluctance computation

Consider an inductor made of an iron core (as described in Figure 1) and a 60 turns winding, wounded around the central leg.

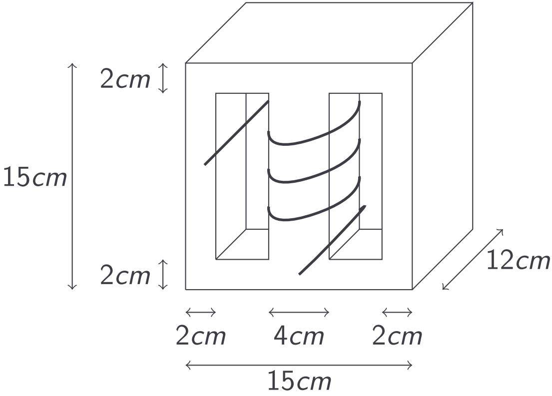


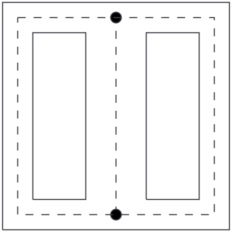
Figure: Magnetic circuit of the inductor

Exercise 7 : reluctance computation

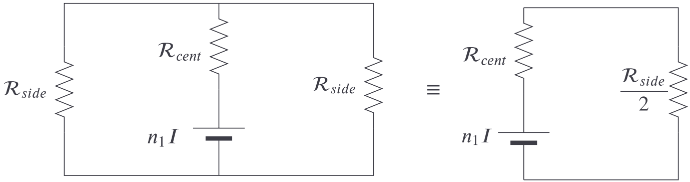
1. Draw an equivalent magnetic circuit of the inductor;
2. Compute the total reluctance of this circuit, considering a relative permeability μ_r of 1500 for the iron. Deduce the inductance from it;
3. Do the same computation as in the previous steps, but now considering a constant air gap of $0.1mm$ in each leg;

Exercise 7 : reluctance computation - solution

The mean path used by the magnetic flux can be represented as



1. Decomposing the reluctance circuit into $\mathcal{R}_{side}$ and $\mathcal{R}_{cent}$



Exercise 7 : reluctance computation - solution

2.

$$\mathcal{R}_{side} = \frac{1}{\mu} \cdot \frac{l_{side}}{S_{side}} = \frac{1}{\mu_0 \cdot \mu_r} \cdot \frac{l_{side}}{S_{side}}$$
$$= \frac{1}{4\pi \cdot 10^{-7} \cdot 1500} \cdot \frac{2 \cdot (0.075 - \frac{0.02}{2}) + 0.13}{0.02 \cdot 0.12}$$
$$= 57\ 472 \quad \mathrm{H}^{-1}$$

$$\mathcal{R}_{cent} = \frac{1}{\mu} \cdot \frac{l_{cent}}{S_{cent}} = \frac{1}{\mu_0 \cdot \mu_r} \cdot \frac{l_{cent}}{S_{cent}}$$
$$= \frac{1}{4\pi \cdot 10^{-7} \cdot 1500} \cdot \frac{0.13}{0.04 \cdot 0.12}$$
$$= 14\ 638 \quad \mathrm{H}^{-1}$$

Finally,

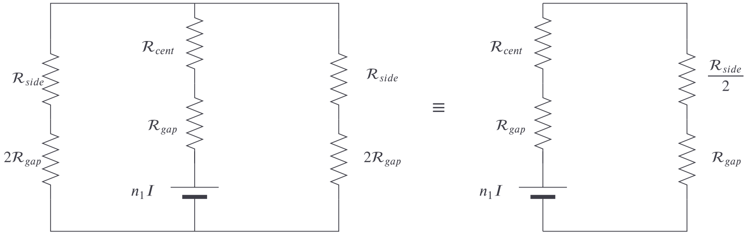
$$\boxed{\mathcal{R}_{tot} = \mathcal{R}_{cent} + \frac{\mathcal{R}_{side}}{2} = 43\ 104 \quad \mathrm{H}^{-1}}$$

Exercise 7 : reluctance computation - solution

Knowing the total reluctance, the related inductance can be deduced:

$$\boxed{L = \frac{n_1^2}{\mathcal{R}_{tot}} = 83.5 \quad \text{mH}} \tag{19}$$

3. The explanation will be the same as before but with a gap of 0.1 mm in each leg added.



Exercise 7 : reluctance computation - solution

$$\begin{aligned}\mathcal{R}_{gap} &= \frac{1}{\mu} \cdot \frac{l_{gap}}{S_{gap}} \\ &= \frac{1}{4\pi \cdot 10^{-7} \cdot 1} \cdot \frac{0.0001}{0.04 \cdot 0.12} \\ &= 16\ 579 \quad H^{-1}\end{aligned}$$

Finally, as shown on the figure,

$$\mathcal{R}_{tot} = \mathcal{R}_{cent} + \frac{\mathcal{R}_{side}}{2} + 2 \cdot \mathcal{R}_{gap} = 76\ 262 \quad H^{-1}$$

$$\boxed{L = \frac{n_1^2}{\mathcal{R}_{tot}} = 47.2 \quad mH}$$

Exercise 10: Single-phase autotransformer

When a galvanic insulation is not required, due to its better efficiency, reduced cost and smaller size, the autotransformer is an interesting alternative to the classical transformer. Autrotransformers are also known to have larger short circuit currents which is not always suitable. Two tests are performed on the transformer, illustrated in Fig. 31:

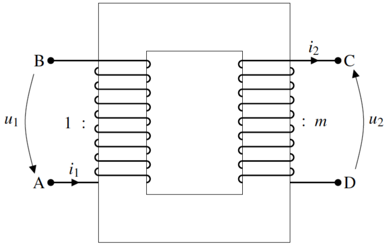


Figure 31: Single-phase transformer.

- Using open secondary winding, the transformer generates a voltage of RMS value $U_{2o} = 100\,\mathrm{V}$ at the secondary winding, for an applied voltage of RMS value $U_{1o} = 20\,\mathrm{V}$ with a drawn current intensity of RMS value $I_{1o} = 3.2\,\mathrm{A}$ and a consumed power $P_{1o} = 8\,\mathrm{W}$;
- Using short-circuited secondary winding, a voltage of RMS value $U_{1s} = 0.8\,\mathrm{V}$ for a total power of $P_{1o} = 24\,\mathrm{W}$ is measured, causing a current flow of RMS value $I_{2s} = 10\,\mathrm{A}$ through the secondary winding.

Considering a simplified equivalent model of the transformer (resistances and inductances gathered and moved to the secondary winding):

1. Calculate the transformer ratio m;
2. Calculate the resistance R'_{H+F} and the magnetizing inductance L'_{μ}, placed at the secondary of the transformer;
3. Compute the values of the resistance R' and the reactance X' corresponding to the Joule losses and the leakage reactance, placed at the secondary of the transformer.

Exercise 10 : single-phase transformer

Using the transformer connected to a load on the secondary side drawing a current of RMS value I_2=12 A with a power factor $\cos\phi_2 = 0.8$ (the current is lagging the voltage), a RMS voltage of $U_1 = 20$ V is applied to the primary winding.

4. Calculate the RMS voltage U_2 appearing across the secondary winding by using a wise approximation of the voltage dropout ΔU_2 and justify that the approximation is relevant;
5. Deduce the active power P_2 provided to the load;
6. Calculate the RMS current I_1 on the primary side;
7. Compute the transformer efficiency η.

Exercise 10 : single-phase transformer - solution

1. The transformer ratio is

$$\boxed{m = \frac{U_{2o}}{U_{1o}} = \frac{100}{20} = 5}$$

2. In open circuit

$$S_{1o} = U_{1o} \cdot I_{1o} = 20 \cdot 3.2 = 64 \text{ VA} \tag{20}$$

with the reactive power

$$Q_{1o} = \sqrt{S_{1o}^2 - P_{1o}^2} = \sqrt{64^2 - 8^2} = 63.498 \text{ var} \tag{21}$$

The components seen from the primary can be obtained from

$$R_{H+F} = \frac{U_{1o}^2}{P_{1o}} = \frac{20^2}{8} = 50 \ \ \Omega \tag{22}$$

$$X_\mu = \frac{U_{1o}^2}{Q_{1o}} = \frac{20^2}{63.498} = 6.299 \ \ \Omega \tag{23}$$

$$\tag{24}$$

Exercise 10 : single-phase transformer - solution

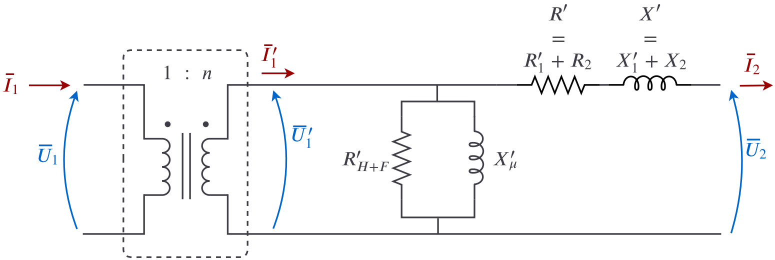


Figure: Circuit of transformer simplified to the secondary.

Exercise 10 : single-phase transformer - solution

The components can be converted from primary to secondary winding :

$$R'_{H+F} = R_{H+F}\, m^2 = \frac{U_{1o}^2}{P_{1o}}\, m^2 \tag{25}$$

$$X'_{\mu} = X_{\mu}\, m^2 = \frac{U_{1o}^2}{Q_{1o}}\, m^2 \tag{26}$$

$$R'_{H+F} = 1250\ \Omega$$
$$X'_{\mu} = 157.485\ \Omega$$
$$L'_{\mu} = 497.359\ \text{mH}$$

Exercise 10 : single-phase transformer - solution

3. In short circuit

$$I_{1s} = m\ I_{2s} = 5 \cdot 10 = 50\ \text{A} \tag{27}$$

Then, compute the apparent power

$$S_{1s} = U_{1s} \cdot I_{1s} = 0.8 \cdot 50 = 40\ \text{VA} \tag{28}$$

Then, the reactive power can be computed as

$$Q_{1s} = \sqrt{S_{1s}^2 - P_{1s}^2} = \sqrt{40^2 - 24^2} = 32\ \text{var} \tag{29}$$

The components seen from the primary can be obtained from

$$R = \frac{P_{1s}}{I_{1s}^2} = \frac{24}{50^2} = 9.6\ \ \text{m}\Omega \tag{30}$$

$$X = \frac{Q_{1s}}{I_{1s}^2} = \frac{32}{50^2} = 12.8\ \ \text{m}\Omega \tag{31}$$

Exercise 10 : single-phase transformer - solution

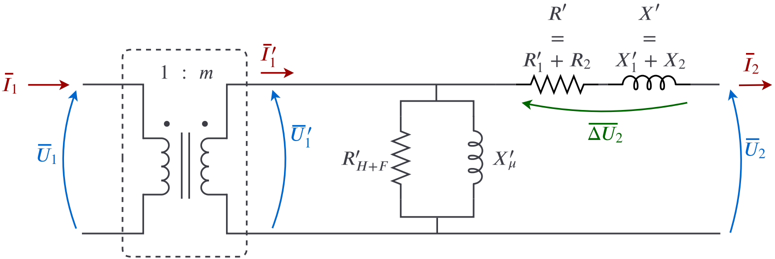
The components can be converted from primary to secondary winding :

$$R' = R\ m^2 = \frac{P_{1s}}{I_{1s}^2}\, m^2 \tag{32}$$

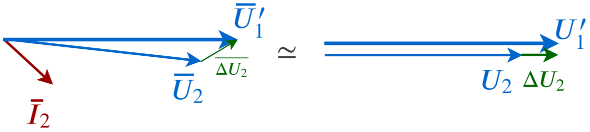
$$X' = X\ m^2 = \frac{Q_{1s}}{I_{1s}^2}\, m^2 \tag{33}$$

$$R' = 240\ \ \mathrm{m\Omega}$$
$$X' = 320\ \ \mathrm{m\Omega}$$
$$L' = 1.019\ \ \mathrm{mH}$$

4. $\overline{\Delta U_2}$ is the voltage drop at the secondary due to $\bar{I}_2$.



The approximation can be summarized on a phasor diagram as



Exercise 10 : single-phase transformer - solution

The voltage drop ΔU_2 is obtained from

$$\Delta U_2 = Z' \; I_2 = \sqrt{R'^2 + X'^2} \; I_2 = \sqrt{0.24^2 + 0.32^2} \cdot 12 = 4.8 \;\; \text{V} \tag{34}$$

Finally, the RMS value of the voltage at the secondary is

$$\boxed{U_2 = U_1' - \Delta U_2 = 100 - 4.8 = 95.2 \;\; \text{V}} \tag{35}$$

5. The active power can be directly deduced from the voltage, the current and the knowledge of the power factor.

$$\boxed{P_2 = U_2 \; I_2 \; \cos(\phi_2) = 95.2 \cdot 12 \cdot 0.8 = 913.92 \text{ W}} \tag{36}$$

6. The RMS primary current is the sum of the magnetizing current $\overline{I}_\mu$ and the secondary current $\overline{I}_2$,

$$\overline{I}'_1 = \overline{I}_\mu + \overline{I}_2 \tag{37}$$

where

$$\overline{I}_\mu = Y_\mu \, \overline{U}'_1 = \left(\frac{1}{R'_{H+F}} - j \, \frac{1}{X'_\mu} \right) \overline{U}'_1 \tag{38}$$

and

$$\overline{I}_2 = \frac{\overline{U}'_1 - \overline{U}_2}{R' + j \, X'} \simeq \frac{\Delta U_2}{R' + j \, X'} \tag{39}$$

Finally,

$$\boxed{\overline{I}'_1 = \left(\frac{1}{R'_{H+F}} - j \, \frac{1}{X'_\mu} \right) \overline{U}'_1 + \frac{\Delta U_2}{R' + j \, X'} = 7.28 - j \, 10.24} \tag{40}$$

Thus, I'_1 is 12.564 A and $\boxed{I_1 = m \, I'_1 = 5 \cdot 12.564 = 62.82 \text{ A}}$

Exercise 10 : single-phase transformer - solution

7. The efficiency can be defined based on the transformer losses and output power, such that

$$\eta = \frac{P_2}{P_1} = \frac{P_2}{P_2 + P_{\text{loss}}} \tag{41}$$

where

$$P_{\text{loss}} = R' I_2^2 + \frac{U_1'^2}{R_{H+F'}} = 0.24 \cdot 12^2 + \frac{100^2}{1250} = 42.56 \text{ W} \tag{42}$$

leading to

$$\boxed{\eta = \frac{P_2}{P_2 + P_{\text{loss}}} = \frac{913.92}{913.92 + 42.56} = 95.55\%} \tag{43}$$

Remark that here, computing the input power P_1 under the approximation made previously (ΔU_2 scalar) would not provide the correct answer.