Electromagnetic Energy Conversion ELEC0431

Exercise session 3: Magnetic circuits and transformers

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Florent Purnode (florent.purnode@uliege.be) – Nicolas Davister (ndavister@uliege.be)

Montefiore Institute, Department of Electrical Engineering and Computer Science,
University of Liège, Belgium

Lab sessions

Doodle and groups

Doodle and groups

Make groups of 4 students and choose at least 6 slots available in the doodle:

https://doodle.com/poll/u2i2bqsdam537wyy?utm_source=poll&utm_medium=link

Complete this doodle by indicating the names of all participants:

$$Firstname_1\ LASTNAME_1\ - \cdots - Firstname_4\ LASTNAME_4$$

In case your group is not complete, send me a mail: florent.purnode@uliege.be
and **Do Not fill the doodle alone**.

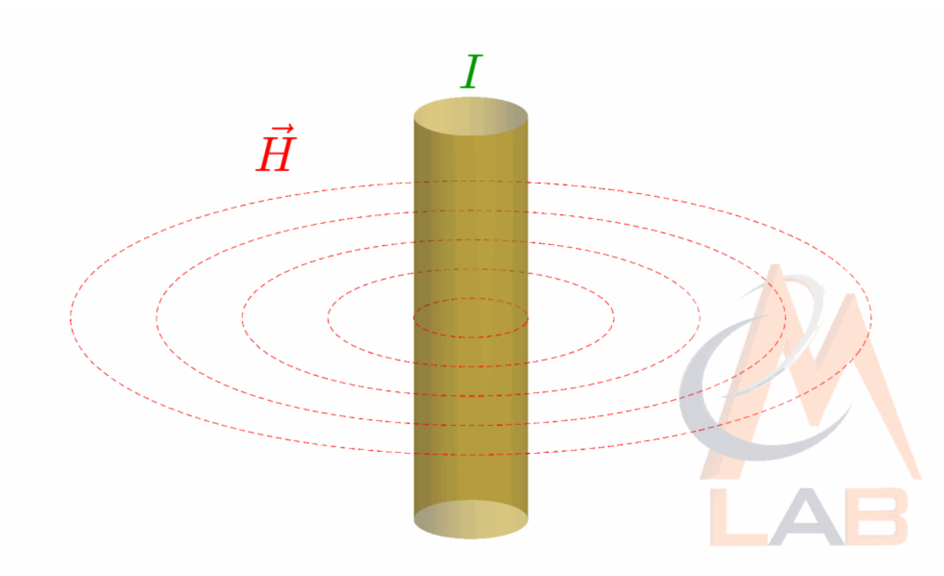
Send your response before next class 04/03/2022 !

Reminders

Reluctances and magnetic circuits

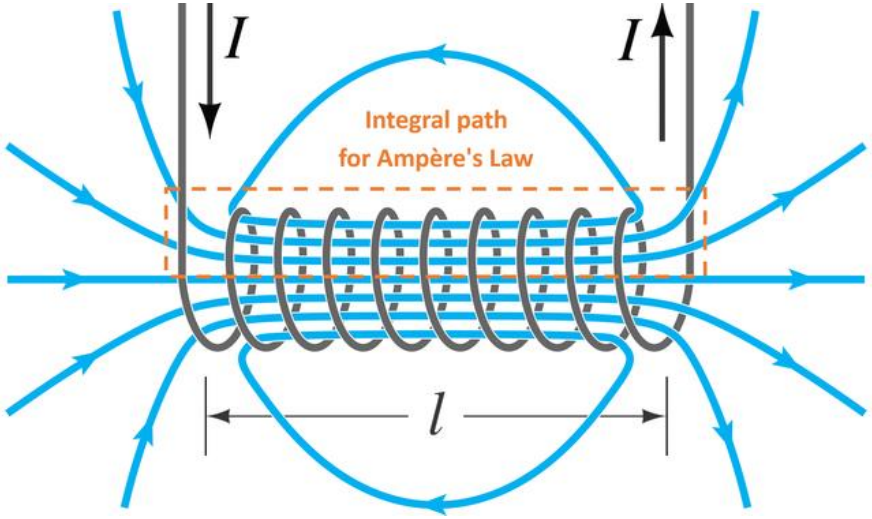
Ampere's law and magnetomotive force

Single wire



$$\oint_C \vec{H} \cdot d\vec{l} = I$$

Solenoid

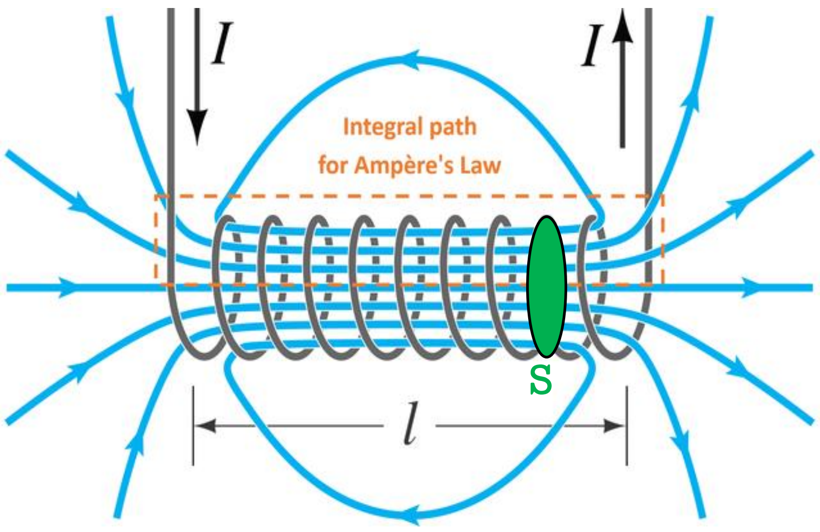


$$\oint_C \vec{H} \cdot d\vec{l} = n\,I \Rightarrow H\,l = n\,I = \mathcal{F}$$

n = the number of turns

$\mathcal{F}$ = the magnetomotive force

Magnetic flux and constitutive law



The magnetic flux ϕ is the quantity of magnetic induction $\vec{B}$ crossing a surface S:

$$\phi = \int_S \vec{B} \cdot \vec{n} \, dS$$

Which can be simplified into: $\boxed{\phi = B\,S}$

(Considering $\vec{B}$ constant over S)

Magnetic constitutive law:

$$\vec{B} = \mu \, \vec{H}$$

$$\mu = \mu_0 \, \mu_r$$

with $\mu_0 = 4\pi 10^{-7}\mathrm{H/m}$ is the permeability of vacuum and μ_r is the relative permeability.

Reluctance and magnetic circuit analogy

$\mathcal{F} = H\,l = n\,I$ $\qquad\qquad$ $\phi = B\,S$ $\qquad\qquad$ $B = \mu\,H$

$$\mathcal{F} = \frac{1}{\mu}\frac{l}{S}\,\phi$$

$$\boxed{\mathcal{F} = \mathcal{R}\,\phi}$$

Magnetomotive force $\qquad$ Reluctance $\qquad$ Magnetic flux

One **parallel** can be made **with Ohm's law** $\boldsymbol{V = R\,I}$. Furthermore, resistance and reactance share similar expressions :

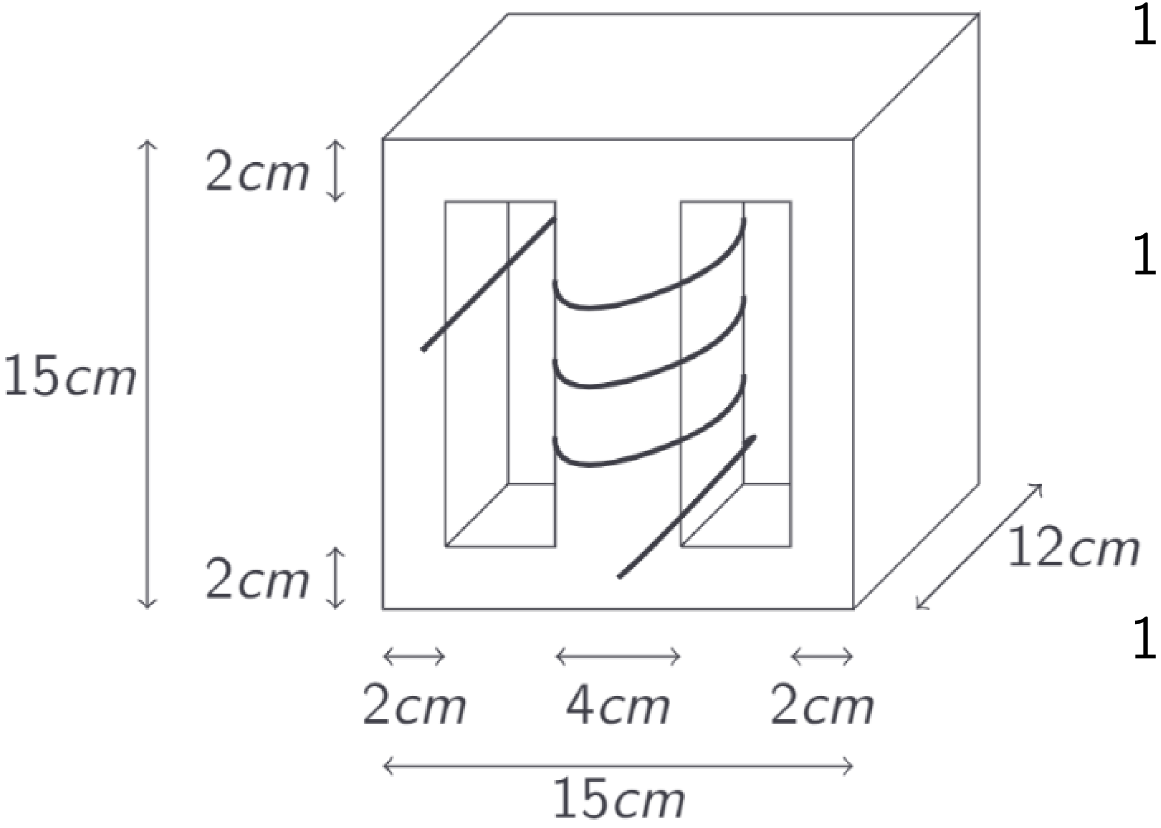
$$R = \frac{1}{\sigma}\frac{l}{S}\,\text{(Pouillet's law)} \qquad \& \qquad \mathcal{R} = \frac{1}{\mu}\frac{l}{S}.$$

Exercices

Exercise 8: Reluctance computation

Exercise 8: Reluctance computation

Consider an inductor made of an iron core and 60 turns winding, wounded around the central leg:



1. Draw an equivalent magnetic circuit of the inductor.

1. Compute the total reluctance of this circuit, considering relative permeability μ_r of 1500 for the iron. Deduce the inductance from it.

1. Do the same computation as in the previous steps, but now considering a constant air gap of 0,1 mm in each leg.

Reminders

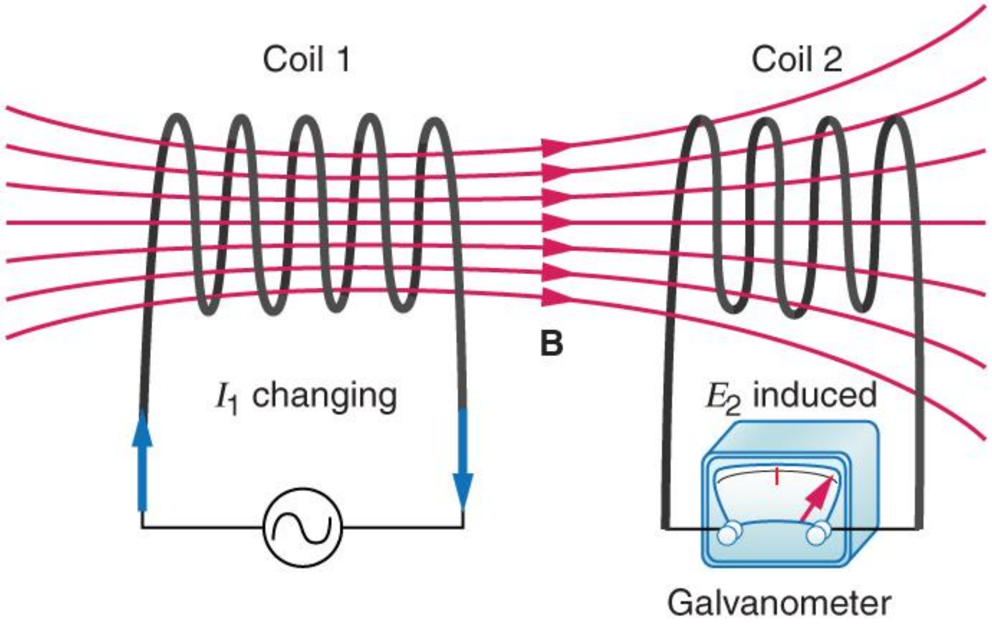
The ideal transformer

Transformer: The idea

$$\mathcal{F}_1 = n_1\, I_1$$

$$\mathcal{R}_1 = \frac{1}{\mu_0}\frac{l_1}{S_1}$$

$$\phi_1 = n_1\, I_1 \frac{\mu_0\, S_1}{l_1}$$



$$\phi_2 = \phi_1 \quad \text{(assuming perfect coupling)}$$

$$\mathcal{R}_2 = \frac{1}{\mu_0}\frac{l_2}{S_2}$$

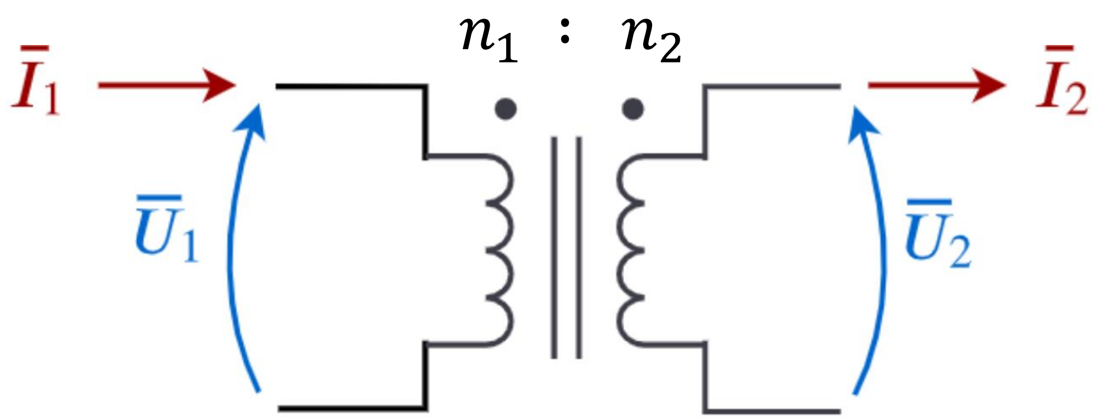
$$\mathcal{F}_2 = n_2\, I_2 = \frac{1}{\mu_0}\frac{l_2}{S_2}\,\phi_1$$

➔ $n_2\, I_2 = \frac{l_2}{\mu_0\, S_2}\, n_1\, I_1 \frac{\mu_0\, S_1}{l_1}$ and in case the dimensions are the same:

$$\boxed{n_1\, I_1 = n_2\, I_2}$$

Input and output power are the same ➔ $V_1 I_1 = V_2 I_2$ ➔ $\boxed{\dfrac{V_1}{V_2} = \dfrac{n_1}{n_2}}$

The ideal transformer



$$\boxed{n_1\,\bar{I}_1 = n_2\,\bar{I}_2}$$

$$\boxed{n_2\,\bar{U}_1 = n_1\,\bar{U}_2}$$

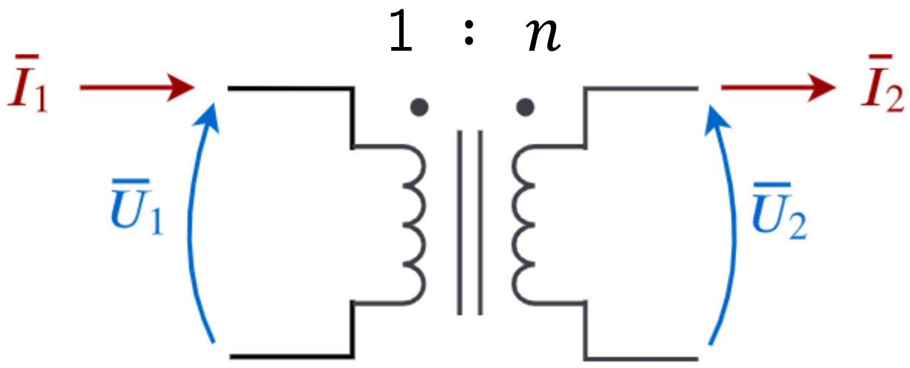
The complex power is conserved in the ideal transformer:

$$S_1 = S_2$$

$$\bar{U}_1\,\bar{I}_1^* = \bar{U}_2\,\bar{I}_2^*$$

$$P_1 + j\,Q_1 = P_2 + j\,Q_2$$

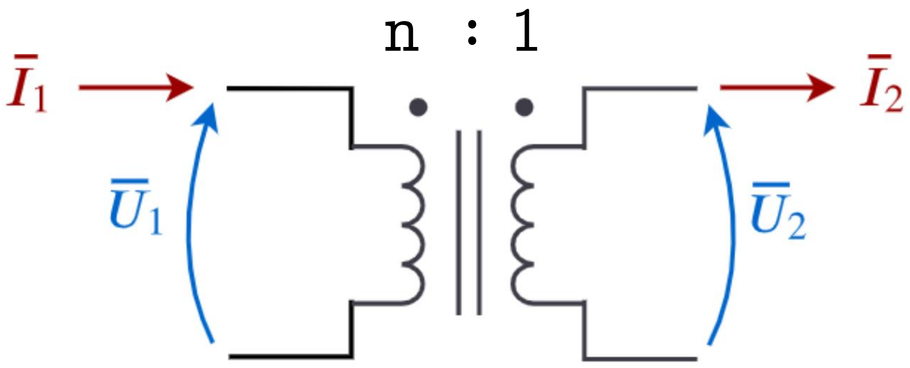
The transformer ratio n



$$n := n_2 / n_1$$

$$\bar{I}_1 = n\,\bar{I}_2$$

$$\bar{U}_2 = n\,\bar{U}_1$$

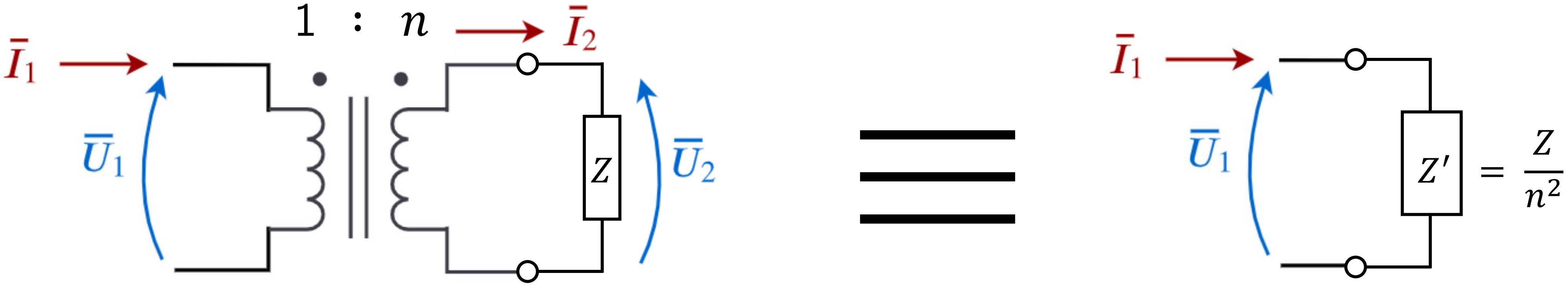


$$n := n_1 / n_2$$

$$\bar{I}_2 = n\,\bar{I}_1$$

$$\bar{U}_1 = n\,\bar{U}_2$$

Shifting impedances



$$\bar{I}_1 = n\,\bar{I}_2 \quad \& \quad \bar{U}_2 = n\,\bar{U}_1$$

The impedance can be shifted from the secondary to the primary:

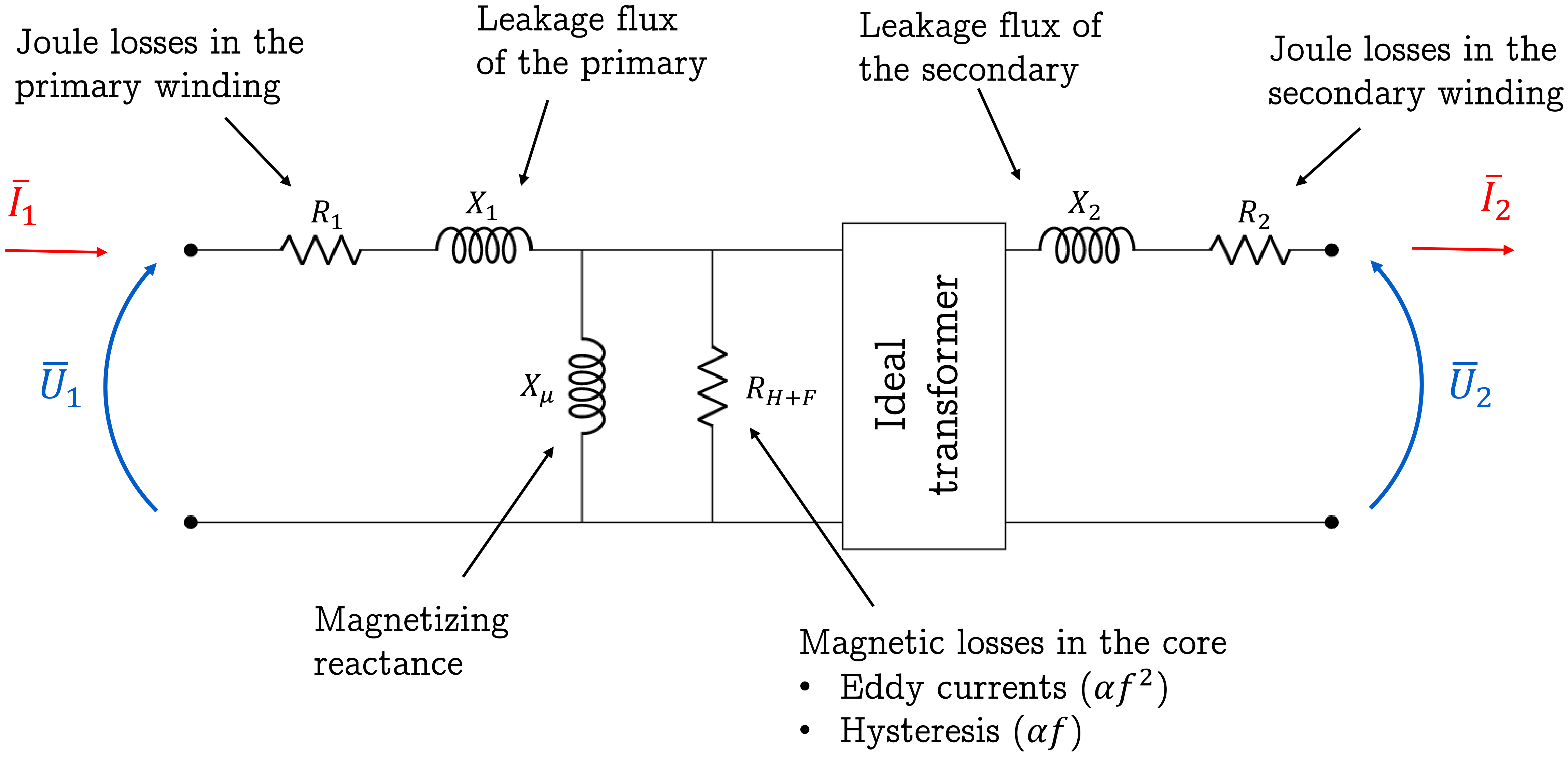
Seen from the secondary, the impedance is $Z = \frac{\bar{U}_2}{\bar{I}_2}$

Seen from the primary, the impedance is $Z' = \frac{\bar{U}_1}{\bar{I}_1} = \frac{\bar{U}_2/n}{n\,\bar{I}_2} = \frac{1}{n^2}\frac{\bar{U}_2}{\bar{I}_2} = \frac{Z}{n^2}$

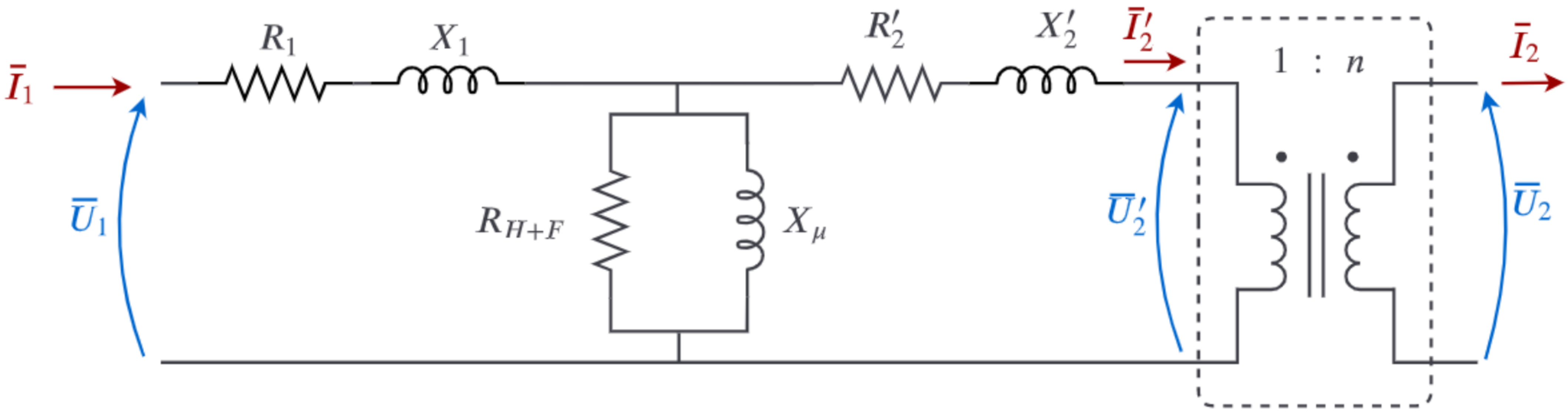
Reminders

The practical transformer

Practical transformer – What's missing ?



Measuring components



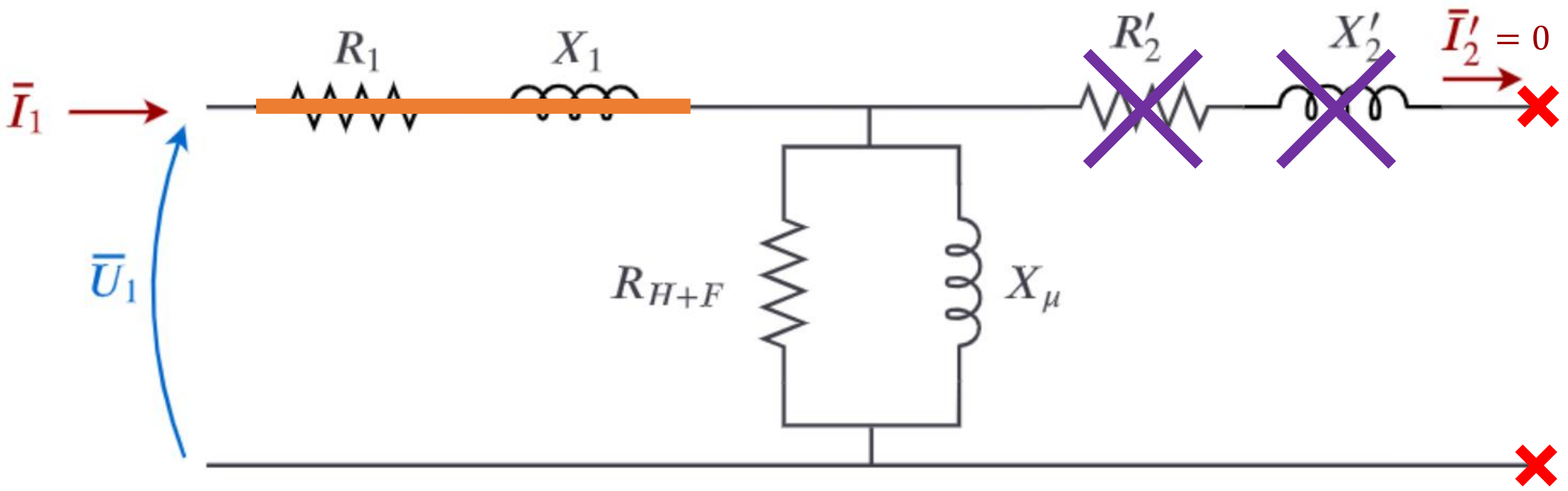
➔ How to measure R_1, X_1, etc. ?

- Open circuit test
- Short circuit test

Note that, in practice, transformers are build in order to minimize the losses. Thus:

$$R_1, R_2' \ll R_{H+F} \quad \& \quad X_1, X_2' \ll X_\mu$$

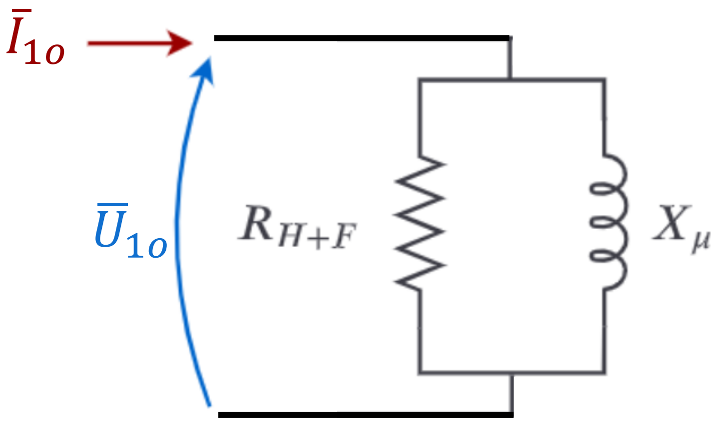
Open circuit test



- First simplification: $\bar{I}'_2 = 0$ ➔ No current in R'_2 and in X'_2.
- Second simplification: $R_1 \ll R_{H+F}$ & $X_1 \ll X_\mu$ ➔ R_1 and X_1 can be ignored.

⇨ Only R_{H+F} and X_μ remain.

Open circuit test



$$Z = \left(\frac{1}{R_{H+F}} + \frac{1}{j\,X_{\mu}}\right)^{-1}$$

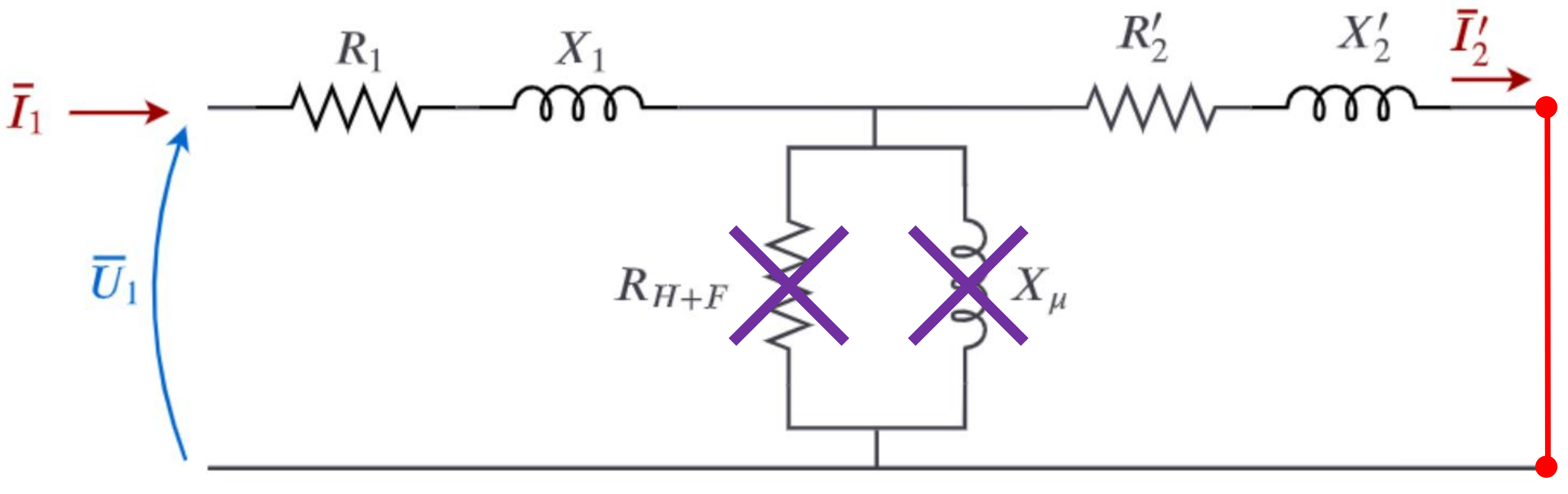
$$\bar{U}_{1o} = Z\,\bar{I}_{1o} \Rightarrow \bar{I}_{1o} = \frac{\bar{U}_{1o}}{Z} = \left(\frac{1}{R_{H+F}} + \frac{1}{j\,X_{\mu}}\right)\bar{U}_{1o}$$

$$S_{1o} = \bar{U}_{1o}\,\bar{I}_{1o}^{*} = \bar{U}_{1o}\left(\frac{1}{R_{H+F}} + \frac{1}{-j\,X_{\mu}}\right)\bar{U}_{1o}^{*} = \frac{U_{1o}^{2}}{R_{H+F}} + j\,\frac{U_{1o}^{2}}{X_{\mu}}$$

$$\Rightarrow \quad P_{1o} = \frac{U_{1o}^{2}}{R_{H+F}} \quad \& \quad Q_{1o} = \frac{U_{1o}^{2}}{X_{\mu}} \quad \Rightarrow \quad \boxed{R_{H+F} = \frac{U_{1o}^{2}}{P_{1o}} \quad \& \quad X_{\mu} = \frac{U_{1o}^{2}}{Q_{1o}}}$$

Where P_{1o} and Q_{1o} are the active and reactive powers consumed during the open circuit test.

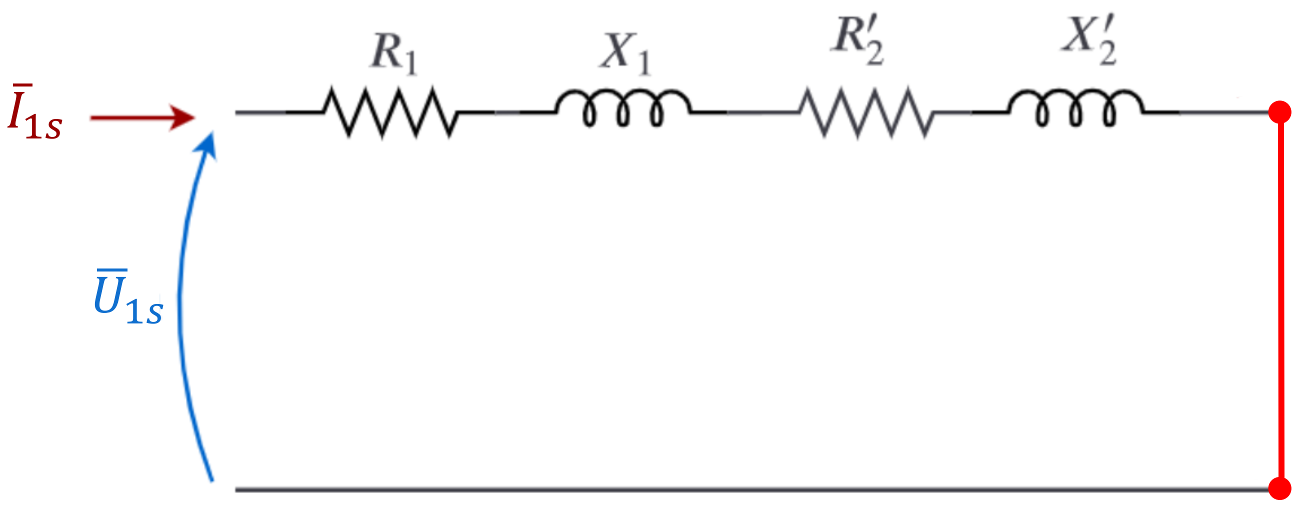
Short circuit test



Simplification: $R_1, R'_2 \ll R_{H+F}$ & $X_1, X'_2 \ll X_\mu$ ➔ R_{H+F} and X_μ can be ignored.

⇨ Only R_1, R'_2, X_1 and X'_2 remain.

Short circuit test



$$Z = R_1 + j\,X_1 + R_2' + j\,X_2'$$
$$= (R_1 + R_2') + j\,(X_1 + X_2')$$
$$\bar{U}_{1s} = Z\,\bar{I}_{1s}$$
$$S = \bar{U}_{1s}\,\bar{I}_{1s}^{*} = (R_1 + R_2')\,I_{1s}^2 + j\,(X_1 + X_2')\,I_{1s}^2$$

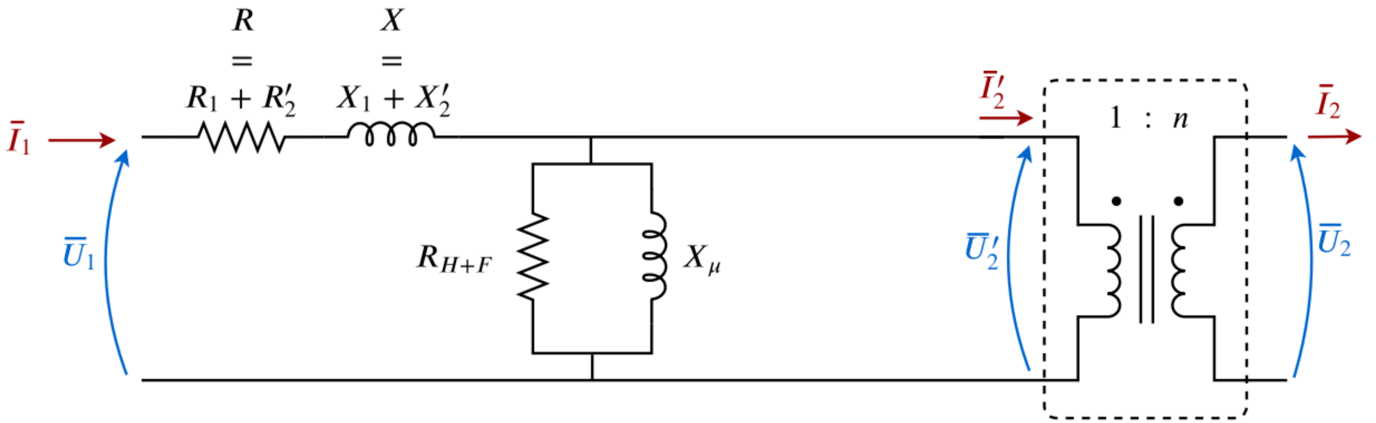
$$\Rightarrow \quad P_{1s} = (R_1 + R_2')\,I_{1s}^2 \quad \& \quad Q_{1s} = (X_1 + X_2')\,I_{1s}^2$$

$$\Rightarrow \quad \boxed{R_1 + R_2' = \frac{P_{1s}}{I_{1s}^2} \quad \& \quad X_1 + X_2' = \frac{Q_{1s}}{I_{1s}^2}}$$

Where P_{1s} and Q_{1s} are the active and reactive powers consumed during the short circuit test.

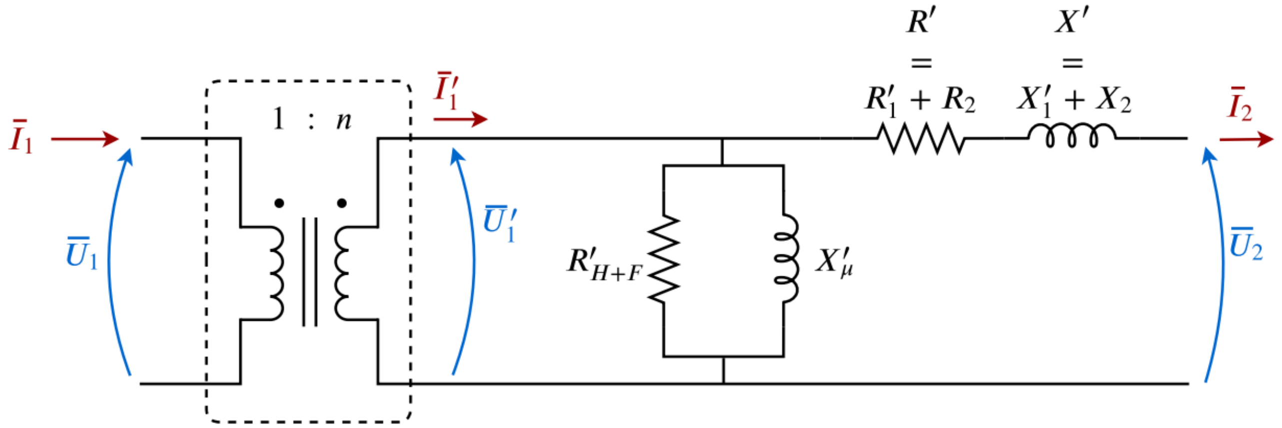
Simplified model

Under the approximations $R_2' \ll R_{H+F}$ and $X_2' \ll X_\mu$, the primary and secondary components can be gathered together.



🡐 Gathered to the primary

Gathered to the secondary 🡒

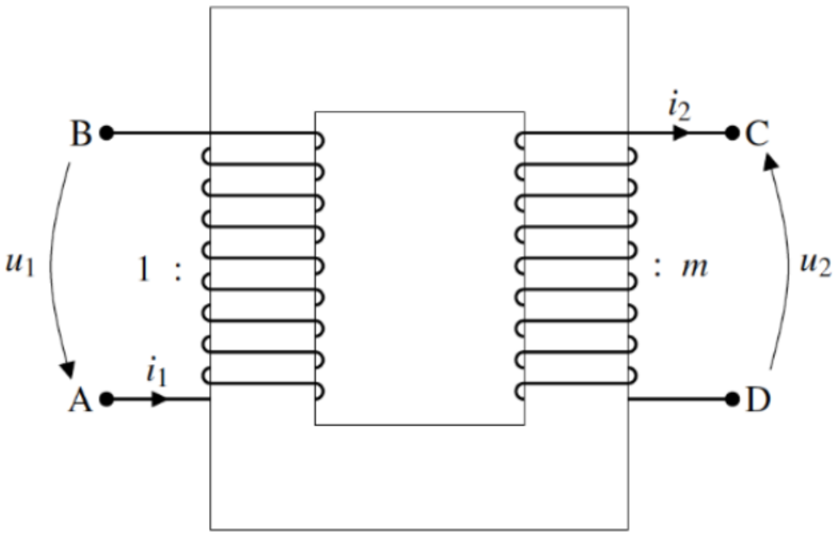


Exercices

Exercise 10: Single-phase autotransformer

Exercise 10: Single-phase autotransformer

When a galvanic insulation is not required, due to its better efficiency, reduced cost and smaller size, the autotransformer is an interesting alternative to the classical transformer. Autrotransformers are also known to have larger short circuit currents which is not always suitable. Two tests are performed on the transformer:



- Using open secondary winding, the transformer generates a voltage of RMS value $U_{2o} = 100\,\text{V}$ at the secondary winding, for an applied voltage of RMS value $U_{1o} = 20\,\text{V}$ with a drawn current intensity of RMS value $I_{1o} = 3.2\,\text{A}$ and a consumed power $P_{1o} = 8\,\text{W}$;
- Using short-circuited secondary winding, a voltage of RMS value $U_{1s} = 0.8\,\text{V}$ for a total power of $P_{1o} = 24\,\text{W}$ is measured, causing a current flow of RMS value $I_{2s} = 10\,\text{A}$ through the secondary winding.

Exercise 10: Single-phase autotransformer

Considering a simplified equivalent model of the transformer (resistances and inductances gathered and moved to the secondary winding):

1. Calculate the transformer ratio m;
2. Calculate the resistance R'_{H+F} and the magnetizing inductance L'_{μ}, placed at the secondary of the transformer;
3. Compute the values of the resistance R' and the reactance X' corresponding to the Joule losses and the leakage reactance, placed at the secondary of the transformer.

Exercise 10: Single-phase autotransformer

Using the transformer connected to a load on the secondary side drawing a current of RMS value I_2=12 A with a power factor $\cos\phi_2 = 0.8$ (the current is lagging the voltage), a RMS voltage of $U_1 = 20$ V is applied to the primary winding.

4. Calculate the RMS voltage U_2 appearing across the secondary winding by using a wise approximation of the voltage dropout ΔU_2 and justify that the approximation is relevant;
5. Deduce the active power P_2 provided to the load;
6. Calculate the RMS current I_1 on the primary side;
7. Compute the transformer efficiency η.