Electromagnetic Energy Conversion ELEC0431

Exercise session 3: Magnetic circuits and transformers

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In this class…

- Laboratory sessions
- Reminder magnetic circuits
- Exercise 8
- Reminder Transformers
- Exercise 10

Laboratory sessions

General information

Schedule and groups

Lab general information

- There are **four laboratories** to help you understand the theoretical and practical concepts:
  - Transformers (mono-phase and three-phase)
  - Synchronous machines
  - Asynchronous machines
  - DC machines
- Each laboratory lasts at most 4 hours. They take place in the "pyramid" (building in front of the cafeteria of Montéfiore).
- You will be guided by student monitors and a manual during the laboratories. The manual will soon be available on the webpage.
- It is strongly advised to get prepared before a laboratory by checking the corresponding part in the manual.
- There is no report to give back. However, **you will have to answer individually a quick evaluation** at the end of each laboratory.
- The evaluation focuses exclusively on the concepts seen during the laboratory. You can be asked to:
  - Explain how a particular measurement was performed during the laboratory,
  - Solve a problem encountered during the laboratory,
  - Explain a theoretical concept seen during the laboratory,
  - Etc.
- Each test represents 3.75 % of your final grade. That is **15 % of the grade** for the four laboratories.
- Laboratories are **mandatory** **(In the event of an unexcused absence, an absence grade will be given for the entire course).**

Schedule and groups

To create the laboratory schedule, you are required to fulfil the doodle “Group-of-4_ELEC0431_lab_schedule” by group of four students.

- Create a group of four students.
- Fulfil the doodle **with at least six available time slots** (you may be given a random session if less than **six** time slots were selected).
- In the space provided for names, write the student ID numbers of the four members (for example: “s181514, s201856, s214442, s219088”).
- In the space provided for emails, write the email (which will be contacted In case of issues with the schedule) of one group member.

A time slot can be selected by maximum six groups, **do not delay in completing this Doodle**.

In case it is impossible for you to fulfil at least six of the remaining time slots with your group, try to create another group.

If and only if it is really impossible for you to create a group of four students meeting the requirements, you can fulfil the second doodle “Uncomplete_groups_ELEC0431_lab_schedule” by group of three, two or alone.

- Create a group with a maximum of members.
- Fulfil the doodle **with at least ten available time slots** (you may be given a random session if less than **ten** time slots were selected).
- In the space provided for names, write the student ID numbers of every member (for example: “s181514, s201856, s214442” if you are three).
- In the space provided for emails, write the email (which will be contacted In case of issues with the schedule) of one group member.

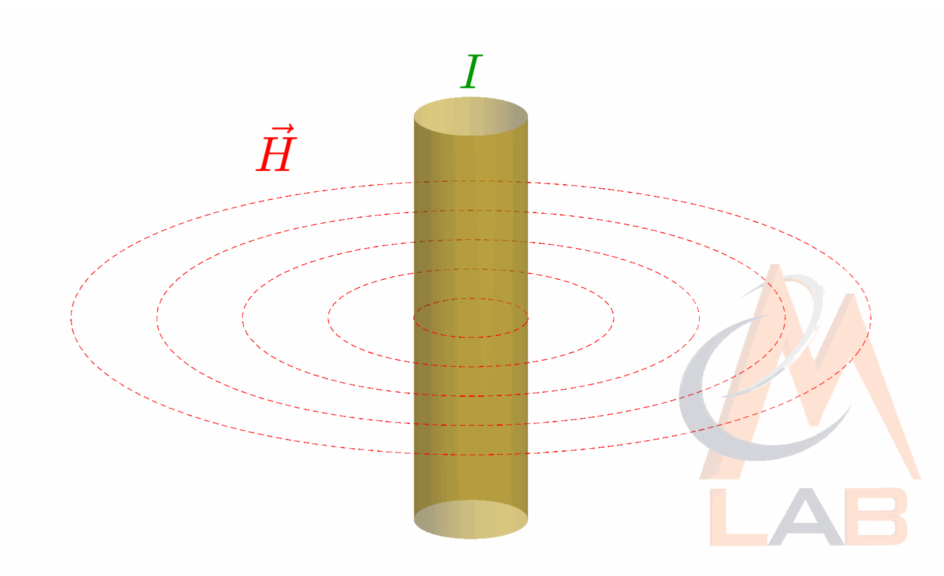
Make sure to complete one of these two Doodles by **23:59 on Friday, March 3rd**. Students who would not have given their availabilities by this day will be given random time slots.

Magnetic circuits

Ampere's law and magnetomotive force
Magnetic flux
Material magnetic constitutive law
Reluctance and magnetic circuit
From reluctance to inductance
Exercise 8

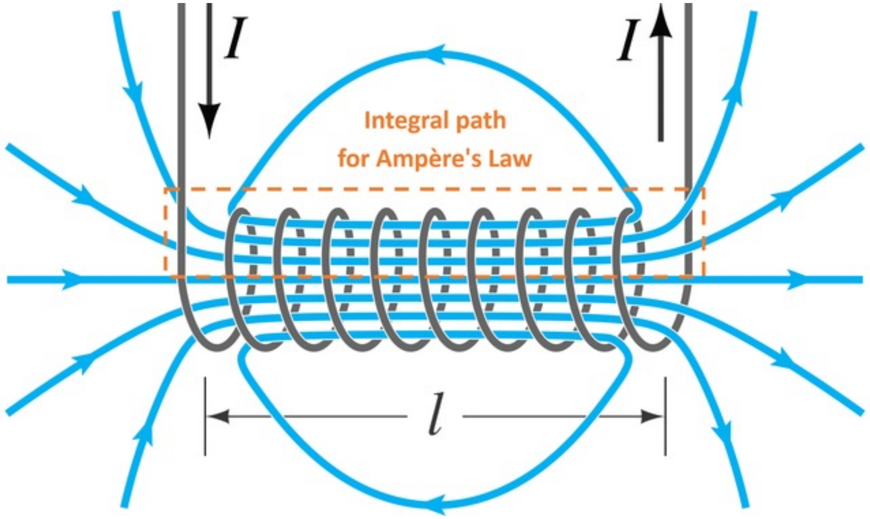
Ampere's law and magnetomotive force

Single wire



$$\oint_C \vec{H} \cdot d\vec{l} = I$$

Solenoid

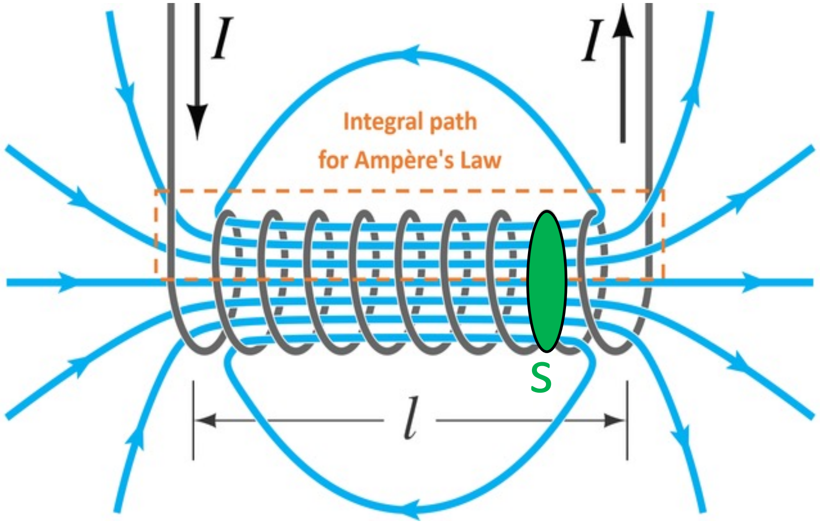


$$\oint_C \vec{H} \cdot d\vec{l} = n\,I \Rightarrow \; H\,l = n\,I = \; \mathcal{F}$$

n = the number of turns

$\mathcal{F}$ = the magnetomotive force

Magnetic flux



The magnetic flux ϕ is the quantity of magnetic induction $\vec{B}$ crossing a surface S:

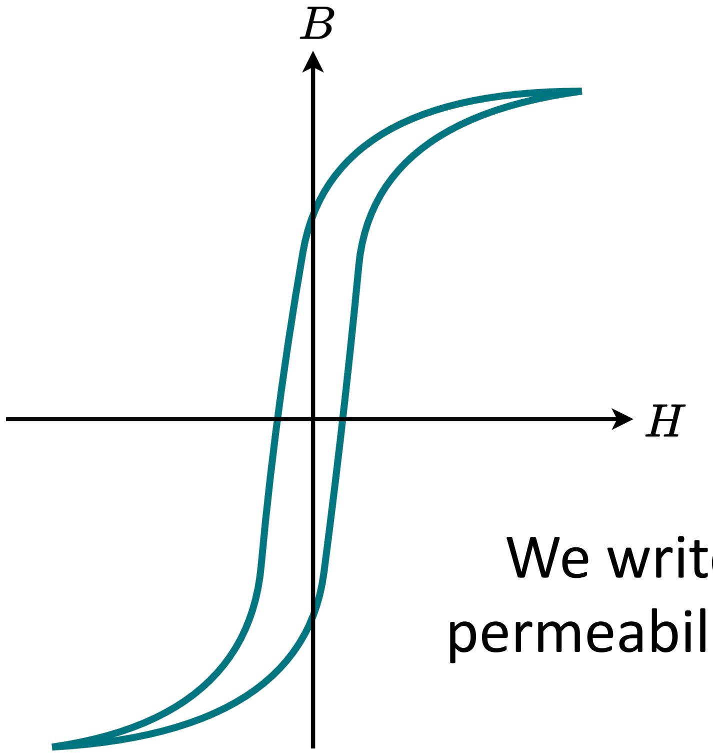
$$\phi = \int_S \vec{B} \cdot \vec{n}\, dS$$

And considering $\vec{B}$ uniform over S:

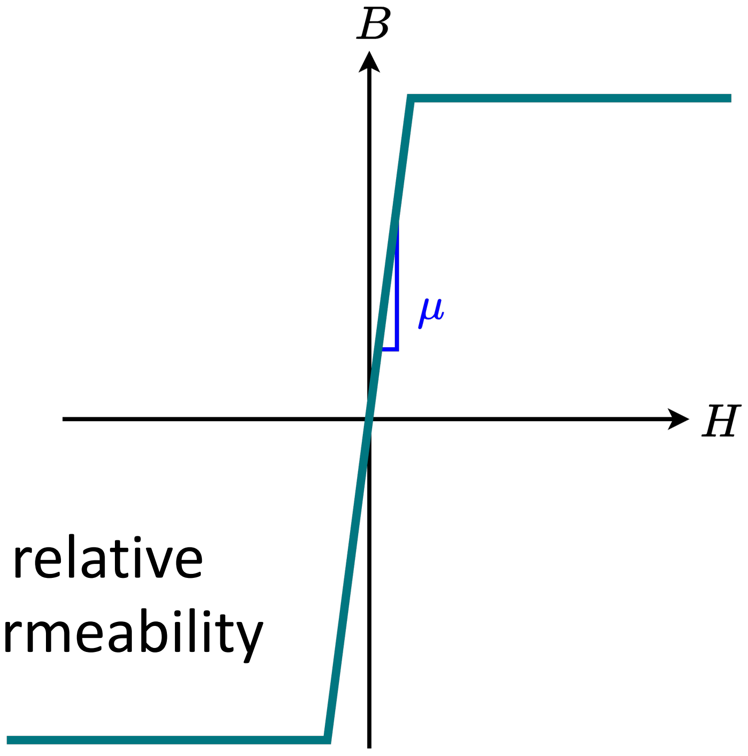
$$\boxed{\phi = B\, S}$$

Material magnetic constitutive law

In a ferromagnetic materials, the relationship between B and H is non-trivial. It exhibits saturation and hysteresis (B is a function of H and of its history).



In practice and for simplicity,
B is often taken proportional to H
as long as saturation is not reached.



We write $B = \mu H = \mu_r \mu_0 H$ where μ_r is the relative permeability and $\mu_0 = 4\pi 10^{-7}\ H/m$ is the permeability of vacuum.

Reluctance and magnetic circuit

$\mathcal{F} = H\, l = n\, I$ $\phi = B\, S$ $B = \mu\, H$

$$\mathcal{F} = \frac{1}{\mu} \frac{l}{S}\, \phi$$

$$\boxed{\mathcal{F} = \mathcal{R}\, \phi}$$

Magnetomotive force

Reluctance

Magnetic flux

One **parallel** can be made **with Ohm's law** $\boldsymbol{V = R\, I}$. Furthermore, resistances and reactances share similar expressions :

$$R = \frac{1}{\sigma} \frac{l}{S} \text{ (Pouillet's law)} \quad \& \quad \mathcal{R} = \frac{1}{\mu} \frac{l}{S}.$$

From reluctance to inductance

$$v(t) = L\,\frac{di(t)}{dt}$$ (Definition of inductors)

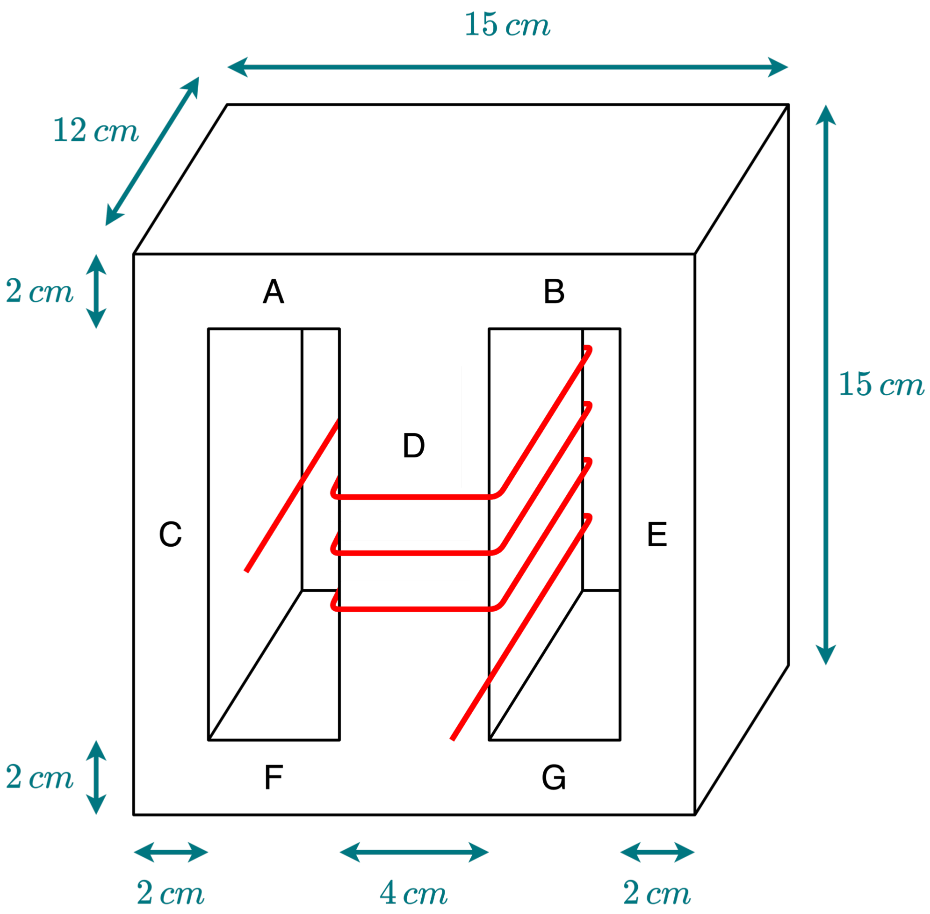
$$v(t) = n\,\frac{d\phi(t)}{dt}$$ (Lenz)

➔ $$L\,\frac{di(t)}{dt} = n\,\frac{d\phi(t)}{dt} = n\frac{d}{dt}\left(\frac{\mathcal{F}(t)}{\mathcal{R}}\right) = \frac{n^2}{\mathcal{R}}\,\frac{\mathrm{di(t)}}{\mathrm{dt}}$$

➔ $$L = \frac{n^2}{\mathcal{R}}$$

Exercise 8

Consider an inductor made of an iron core as depicted hereunder and a 60-turn winding, wounded around the central leg.



1. Draw an equivalent magnetic circuit of the inductor.

2. Compute the total reluctance of this circuit, considering a relative permeability μ_r of 1500 for the iron. Deduce the inductance from it.

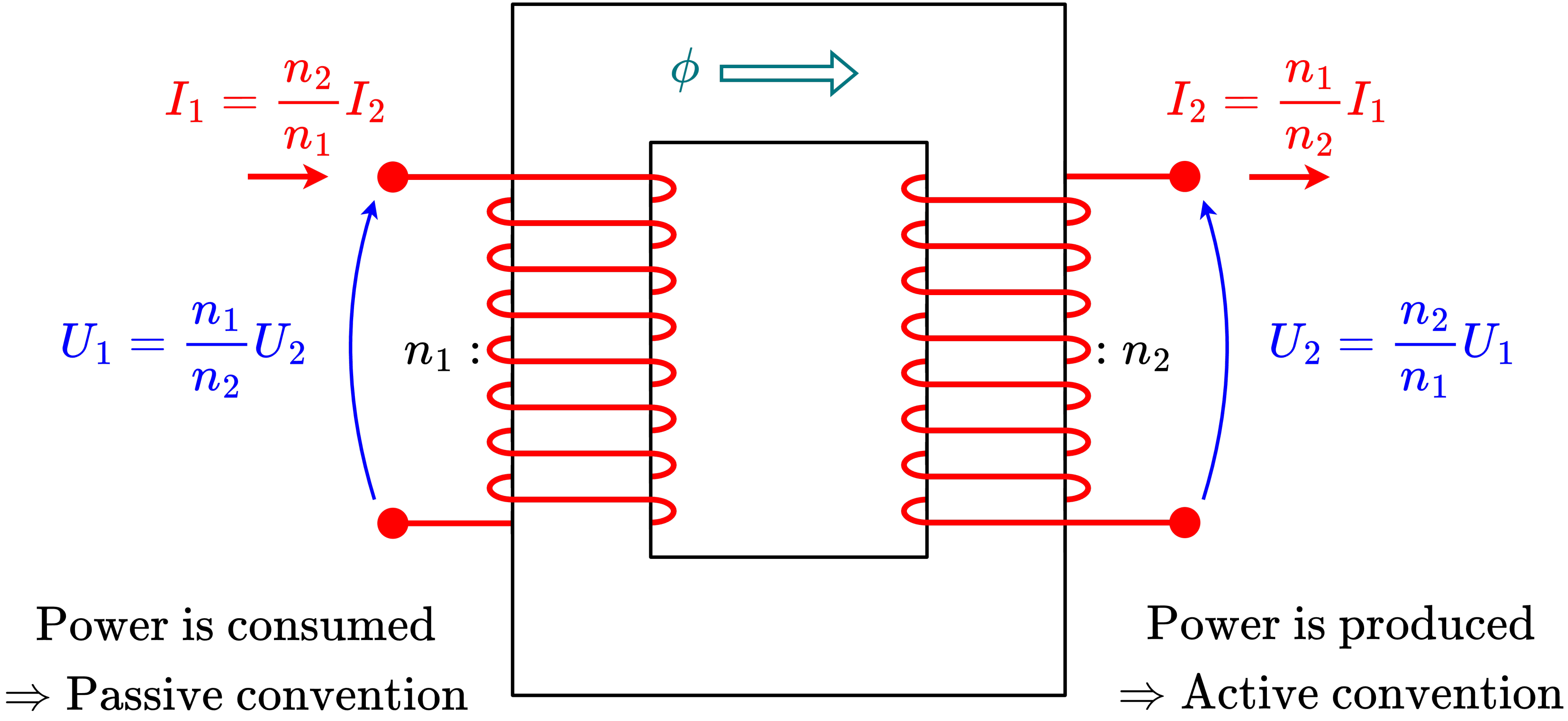
3. Do the same computation as in the previous steps, but now considering a constant air gap of $0.1\ mm$ in each leg.

Transformers

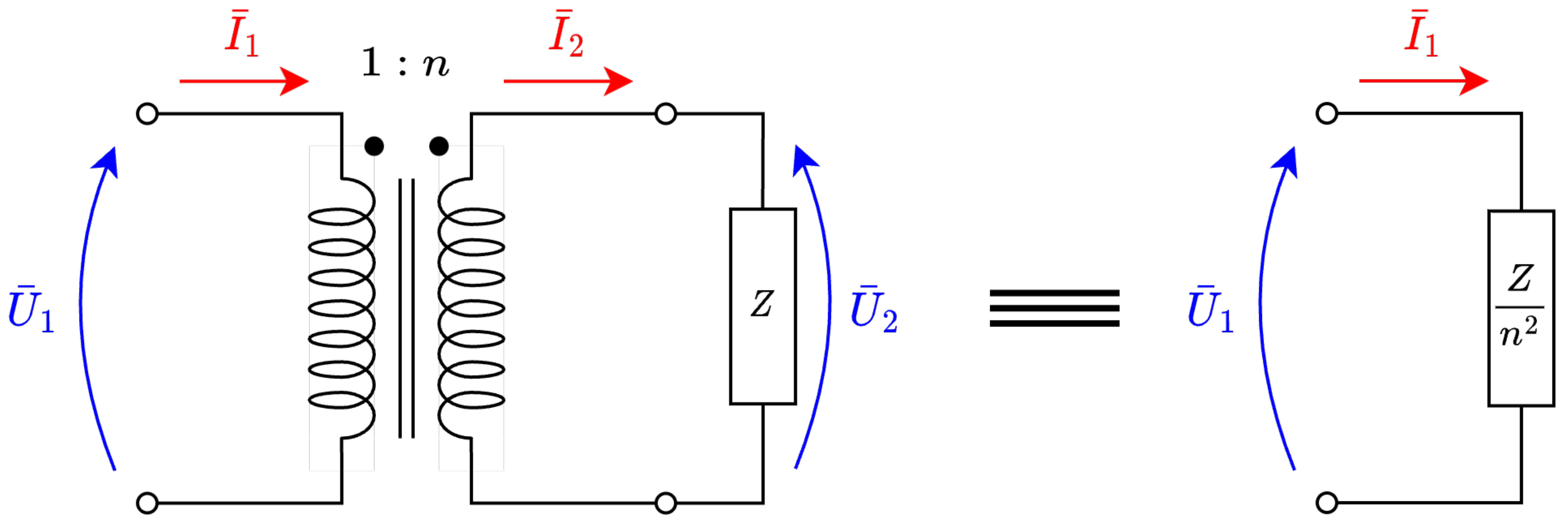
The ideal transformer
Shifting impedances
The real transformer
Open-circuit test
Short-circuit test
Exercise 10

The ideal transformer

A transformer consists of two or more coils wrapped around a magnetic core, used to increase or decrease an AC voltage.



Shifting impedances



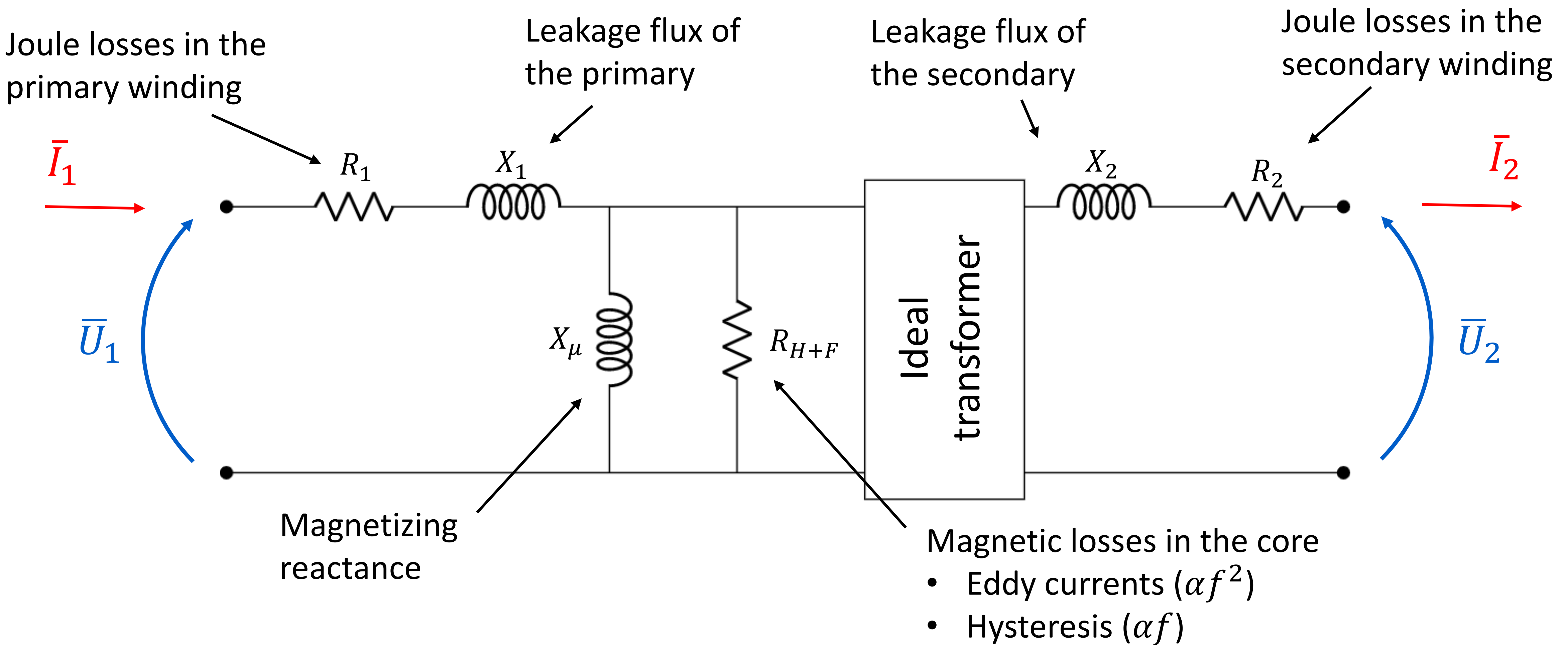
$$\bar{I}_1 = n\,\bar{I}_2$$

$$\bar{U}_2 = n\,\bar{U}_1$$

Seen from the secondary, the impedance is $Z = \frac{\bar{U}_2}{\bar{I}_2}$

Seen from the primary, the impedance is $Z' = \frac{\bar{U}_1}{\bar{I}_1} = \frac{\bar{U}_2/n}{n\,\bar{I}_2} = \frac{1}{n^2}\frac{\bar{U}_2}{\bar{I}_2} = \frac{Z}{n^2}$

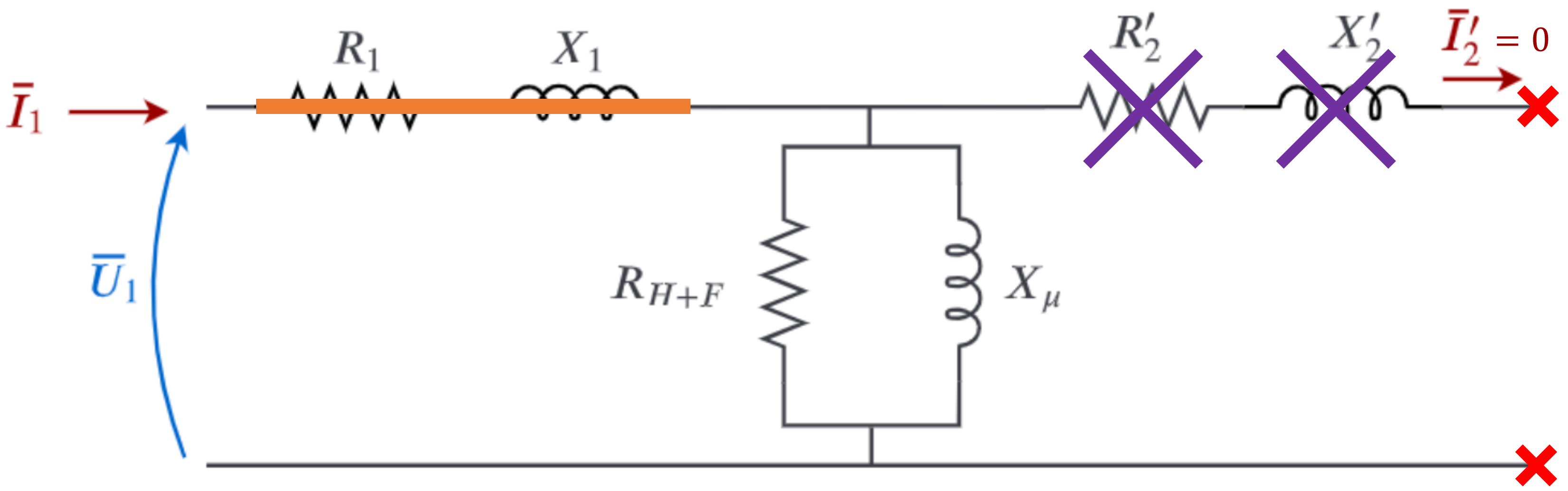
The real transformer



Note that, in practice, transformers are build in order to minimize the losses. Thus:

$$R_1, R_2' \ll R_{H+F} \quad \& \quad X_1, X_2' \ll X_\mu$$

Open circuit test

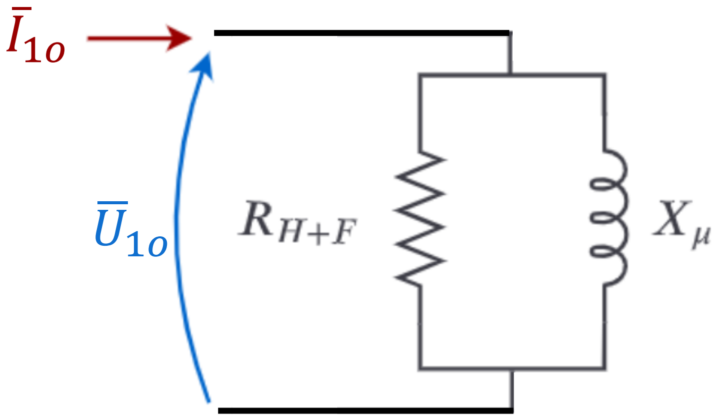


- First simplification: $\bar{I}_2' = 0$ ➔ No current in R_2' and in X_2'.
- Second simplification: $R_1 \ll R_{H+F}$ & $X_1 \ll X_\mu$

  ➔ The voltage drop at R_1 and X_1 is negligeable ➔ R_1 and X_1 can be ignored.

⇨ Only R_{H+F} and X_μ remain.

Open circuit test



$$Z = \left(\frac{1}{R_{H+F}} + \frac{1}{j\,X_{\mu}}\right)^{-1}$$

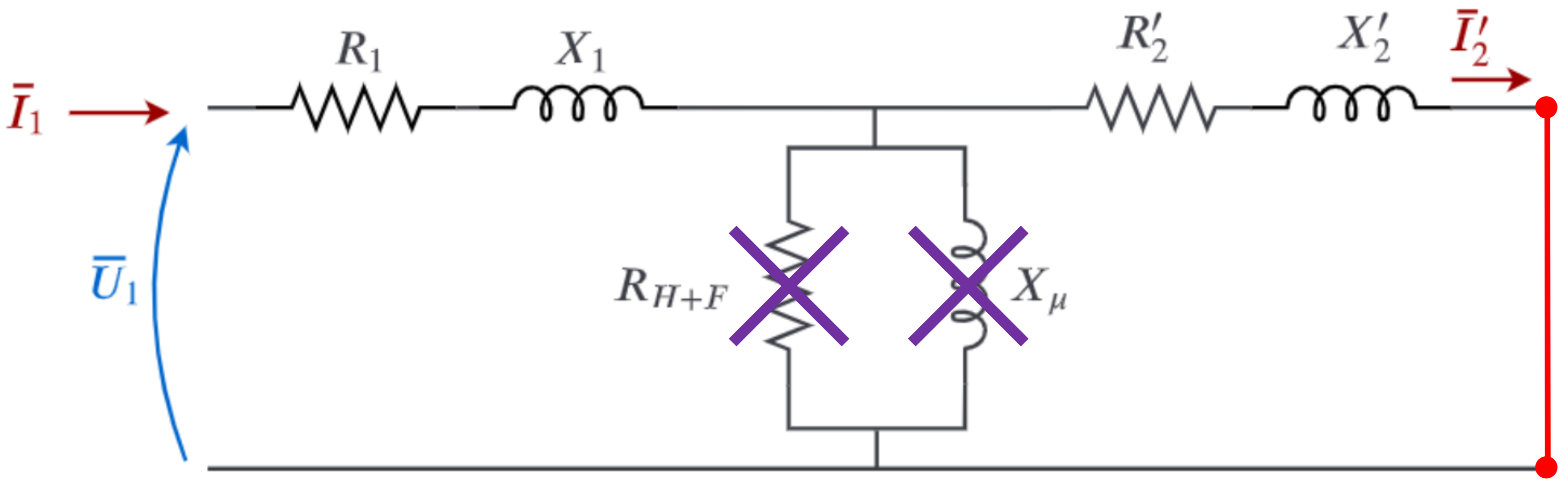
$$\bar{U}_{1o} = Z\,\bar{I}_{1o} \Rightarrow \bar{I}_{1o} = \frac{\bar{U}_{1o}}{Z} = \left(\frac{1}{R_{H+F}} + \frac{1}{j\,X_{\mu}}\right)\bar{U}_{1o}$$

$$S_{1o} = \bar{U}_{1o}\,\bar{I}_{1o}^{*} = \bar{U}_{1o}\left(\frac{1}{R_{H+F}} + \frac{1}{-j\,X_{\mu}}\right)\bar{U}_{1o}^{*} = \frac{U_{1o}^{2}}{R_{H+F}} + j\,\frac{U_{1o}^{2}}{X_{\mu}}$$

$$\Rightarrow \quad P_{1o} = \frac{U_{1o}^{2}}{R_{H+F}} \quad \& \quad Q_{1o} = \frac{U_{1o}^{2}}{X_{\mu}} \quad \Rightarrow \quad \boxed{R_{H+F} = \frac{U_{1o}^{2}}{P_{1o}} \quad \& \quad X_{\mu} = \frac{U_{1o}^{2}}{Q_{1o}}}$$

Where P_{1o} and Q_{1o} are the active and reactive powers consumed during the open circuit test.

Short circuit test

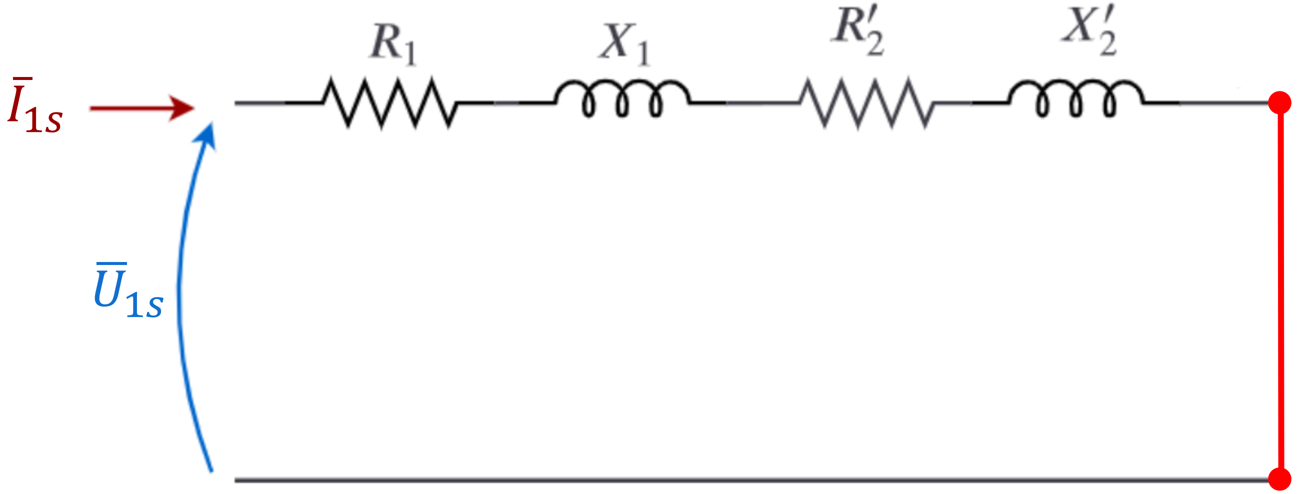


Simplification: $R_1, R_2' \ll R_{H+F}$ & $X_1, X_2' \ll X_\mu$

➔ Almost no current in the magnetizing branch ➔ R_{H+F} and X_μ can be removed.

⇨ Only R_1, R_2', X_1 and X_2' remain.

Short circuit test



$$Z = R_1 + j\,X_1 + R_2' + j\,X_2'$$
$$= (R_1 + R_2') + j\,(X_1 + X_2')$$
$$\bar{U}_{1s} = Z\,\bar{I}_{1s}$$
$$S = \bar{U}_{1s}\,\bar{I}_{1s}^{*} = (R_1 + R_2')\,I_{1s}^2 + j\,(X_1 + X_2')\,I_{1s}^2$$

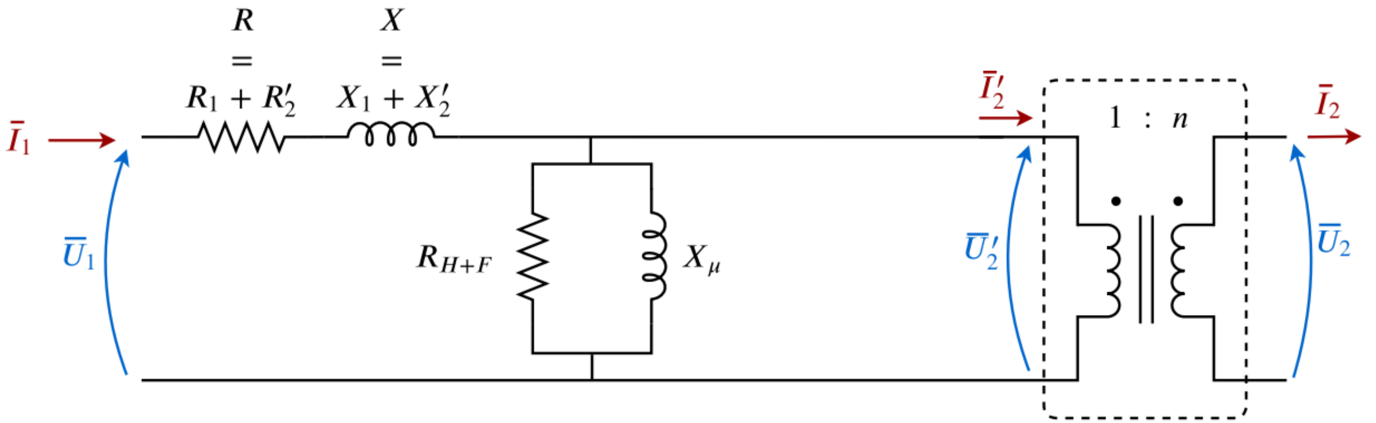
$\Rightarrow$ $P_{1s} = (R_1 + R_2')\,I_{1s}^2$ & $Q_{1s} = (X_1 + X_2')\,I_{1s}^2$

$\Rightarrow$ $$\boxed{R_1 + R_2' = \frac{P_{1s}}{I_{1s}^2} \quad \& \quad X_1 + X_2' = \frac{Q_{1s}}{I_{1s}^2}}$$

Where P_{1s} and Q_{1s} are the active and reactive powers consumed during the short circuit test.

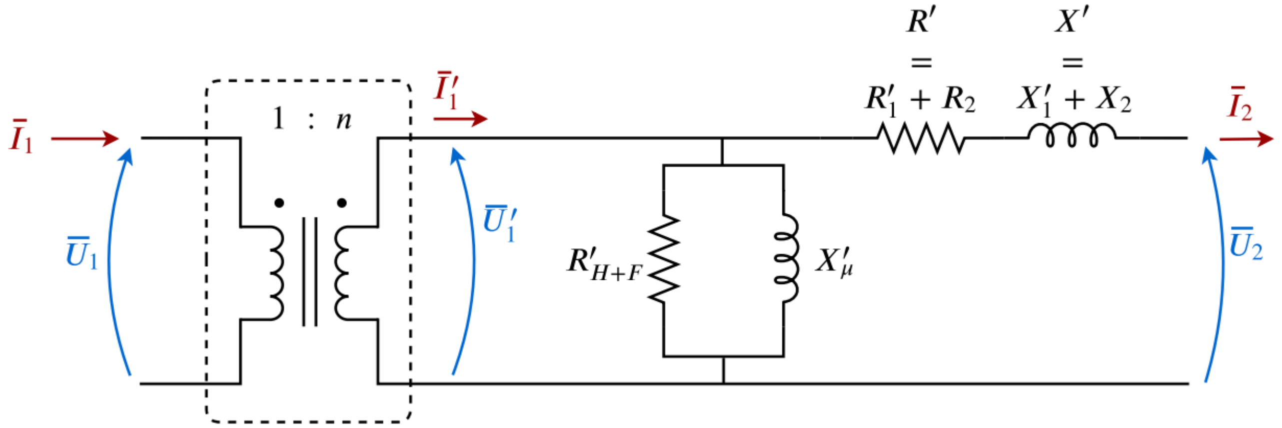
Simplified model

Under the approximations $R_2' \ll R_{H+F}$ and $X_2' \ll X_\mu$, the primary and secondary components can be gathered together.



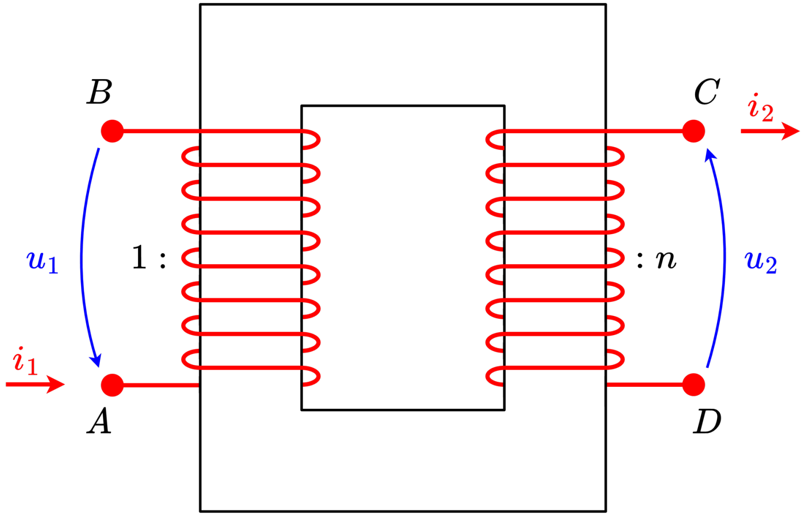
🡐 Gathered to the primary

Gathered to the secondary ➔



Exercise 10

When a galvanic insulation is not required, due to its better efficiency, reduced cost and smaller size, the autotransformer is an interesting alternative to the classical transformer. Autrotransformers are also known to have larger short circuit currents which is not always suitable. Two tests are performed on the transformer, illustrated hereunder:



- Using open secondary winding, the transformer generates a voltage of RMS value $U_{2o} = 100\ V$ at the secondary winding, for an applied voltage of RMS value $U_{1o} = 20\ V$ with a drawn current intensity of RMS value $I_{1o} = 3.2\ A$ and a consumed power $P_{1o} = 8\ W$.
- Using short-circuited secondary winding, a voltage of RMS value $U_{1s} = 0.8\ V$ for a total power of $P = 24\ W$ is measured, causing a current flow of RMS value $I_{2s} = 10\ A$ through the secondary winding.

Considering a simplified equivalent model of the transformer (resistances and inductances gathered and moved to the secondary winding):

1. Calculate the transformer ratio n.
2. Calculate the resistance $R_{H+F}^{'}$ and the magnetizing inductance $L_{\mu}^{'}$, placed at the secondary of the transformer.
3. Compute the values of the resistance R' and the reactance X' corresponding to the Joule losses and the leakage reactance, placed at the secondary of the transformer.

Exercise 10

Using the transformer connected to a load on the secondary side drawing a current of RMS value $I_2 = 12\ A$ with a power factor $\cos(\varphi) = 0.8$ (the current is lagging the voltage), a RMS voltage of $U_1 = 20\ V$ is applied to the primary winding.

4. Calculate the RMS voltage U_2 appearing across the secondary winding by using a wise approximation of the voltage dropout ΔU_2 and justify that the approximation is relevant.
5. Deduce the active power P_2 provided to the load.
6. Calculate the RMS current I_1 on the primary side.
7. Compute the transformer efficiency η.

To turn the transformer into an autotransformer, the terminals A and D are connected together, such that the primary winding is located between C and B and the secondary between C and D.

8. Compute the RMS voltage U_1' to be applied on the primary winding to reach a RMS voltage value of $U_{2o} = 100\ V$ at the terminals of the secondary winding.
9. Calculate the RMS current I_{1o}' drawn at the primary winding in the case of an open secondary winding.
10. Neglecting the open circuit magnetomotive force, provide the Thévenin's model of the secondary winding including the electromotive force as well as the impedance components namely R_s' and X_s'.

Homework