Electromagnetic Energy Conversion
ELEC0431

Exercise session 3:
Magnetic circuits and transformers

21 February 2025

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Laboratories – Schedule and groups

To create the lab schedule, you are required to fill **by group** the doodle https://framadate.org/LabsELEC0431:

- By group of **4 students**, select **6** available time slots
  (**you may be given random sessions if less than 6 time slots were selected**).
- In the space provided for names, write the **STUDENT ID NUMBERS** of all **4 MEMBERS** of the group
  (for example: "s161514, s171856, s164442, s179088").
- A time slot can be selected by maximum six groups, **do not delay in completing this Doodle**.
- In addition to your selected time slots, your schedule could include laboratory sessions on the 18/04, 25/04, 9/05 and 16/05 mornings (Friday mornings in place of the traditional classes).

IF AND ONLY IF it is impossible for you to create a group of four students meeting the requirements, send me an email (florent.purnode@uliege.be) without delay.

Make sure to complete the doodle by **23:59 on Friday, February 28td**,
(Students who would not have given their availabilities by this time will be given random time slots).

Quick reminder: Laboratories are **mandatory**
(In case of unexcused absence, an absence grade will be given for the entire course).

In this class…

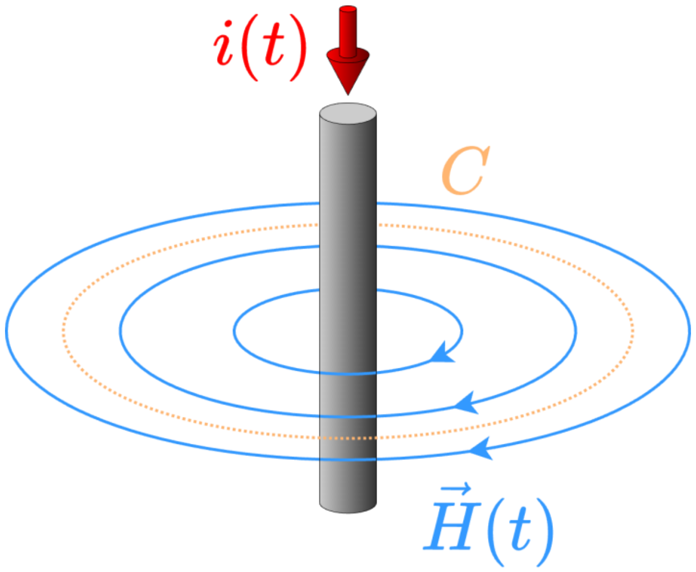
- Magnetic circuits
- Exercise 6
- Transformers
- Exercise 7

Magnetic circuits

Ampere's law and magnetomotive force
Magnetic permeability and magnetic flux
Ferromagnetic materials
Reluctance and magnetic circuit
Exercise 6

Ampere's law and magnetomotive force

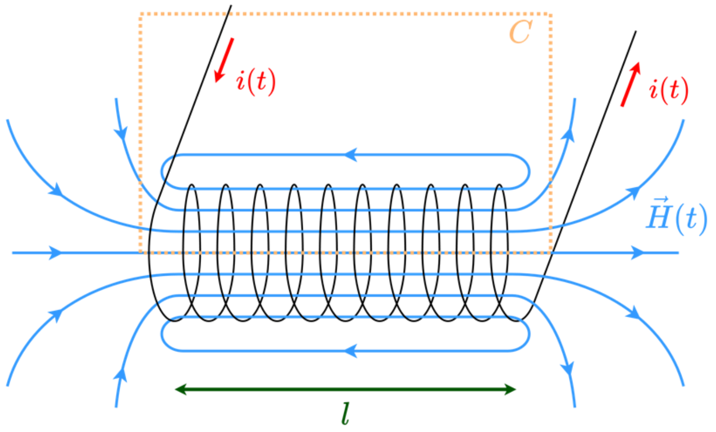
A current flowing into a wire generates a magnetic field H.



Ampere's law relates the magnetic field $\vec{H}(t)$ circulating around a closed loop C to the current $i(t)$ passing through that loop:

$$\oint_C \vec{H}(t) \cdot d\vec{l} = i(t)$$

A solenoid is a coil of wires.



Considering N turns, the magnetic field $\vec{H}(t)$ generated is N times the magnetic field for a single wire:

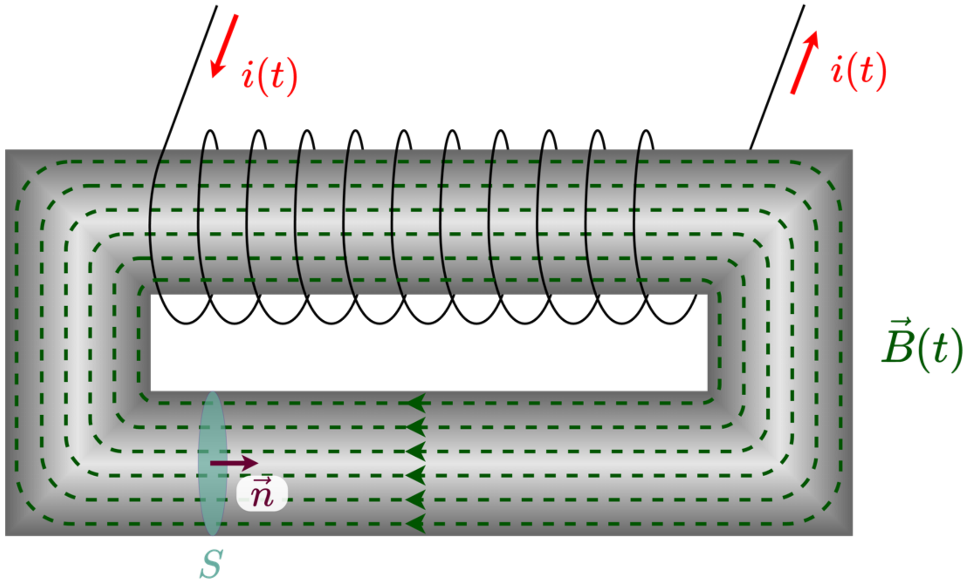
$$\oint_C \vec{H}(t) \cdot d\vec{l} = N\, i(t)$$

Inside of the coil, considering RMS values, it simplifies to:

$$H\, l = N\, I = \mathcal{F}$$

$\mathcal{F}$ is the **magnetomotive force**.

Magnetic permeability and magnetic flux



The magnetic permeability μ links Magnetic flux $\vec{H}$ and magnetic flux density $\vec{B}$:

$$\vec{B} = \mu\vec{H} = \mu_0\mu_r\vec{H}$$

- $\mu_0 = 4\pi 10^{-7} H/m$ is the permeability of vacuum
- μ_r is the relative permeability. It varies from one material to the other ($\mu_r = 1$ for air).

The magnetic flux $\phi(t)$ is the quantity of magnetic flux density $\vec{B}(t)$ crossing a surface S:

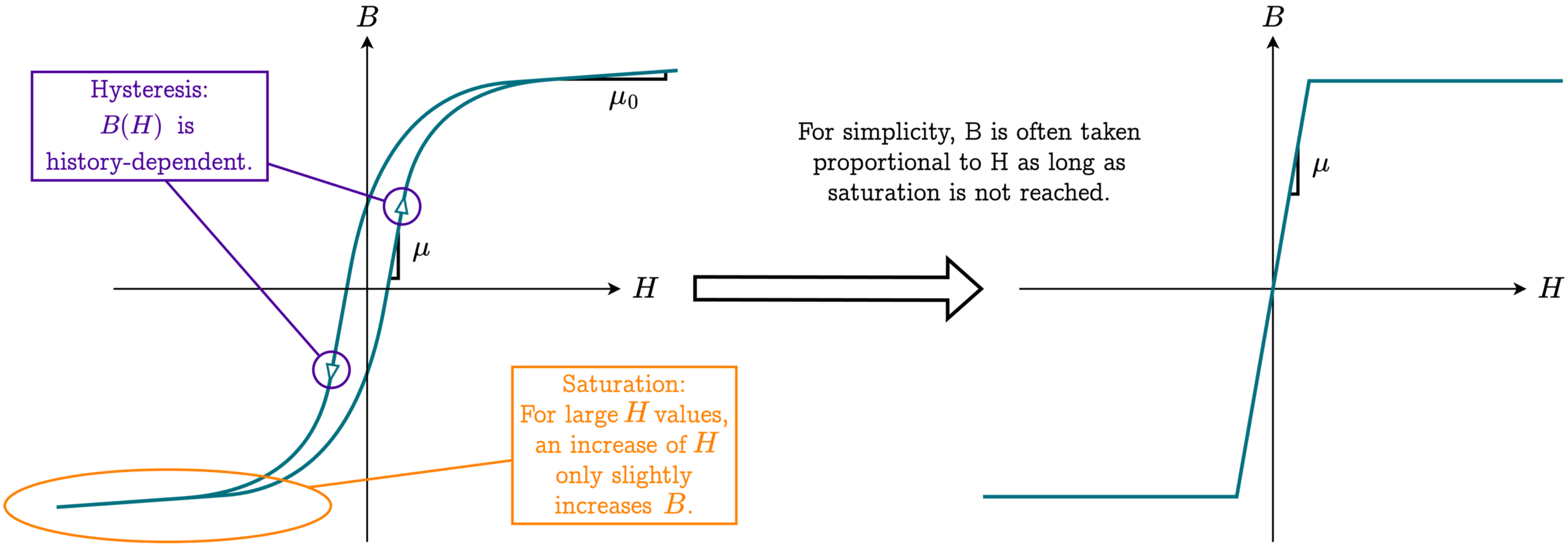
$$\phi(t) = \int_S \vec{B}(t) \cdot \vec{n}\ ds.$$

With $\vec{B}(t)$ uniform over S and considering the RMS values:

$$\phi = B\ S = \mu\ H\ S$$

Ferromagnetic materials

Ferromagnetic materials have a large magnetic permeability μ. For this reason, they are often used to handle high magnetic fluxes. They however exhibit hysteresis and saturation:



Reluctance and magnetic circuit

From Ampere's law (slide 5): $Hl = NI = \mathcal{F}$

From magnetic constitutive law (slide 6): $\phi = \mu HS$

$$\mathcal{F} = N\,I = \frac{l}{\mu\,S}\phi = \mathcal{R}\,\phi$$

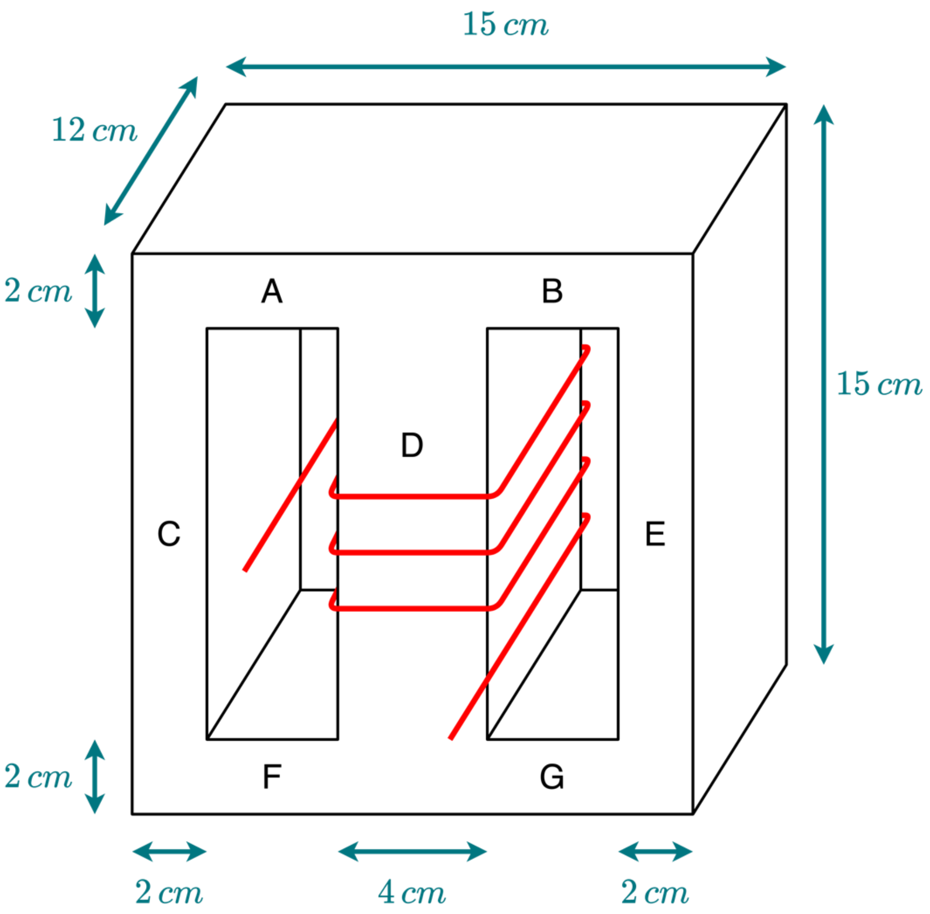
Magnetomotive force — Reluctance — Magnetic flux

The relation linking magnetomotive force, reluctance and magnetic flux is similar to the Ohm's law linking voltage, resistance and current:

Ohm's law: $V = R\,I$	Magnetic circuit equation: $\mathcal{F} = \mathcal{R}\,\phi$
Pouillet's law: $R = \frac{l}{\sigma\,S}$	Magnetic reluctance formula: $\mathcal{R} = \frac{l}{\mu\,S}$
σ is the conductivity $\left[\frac{S}{m}\right]$	μ is the permeability $\left[\frac{H}{m}\right]$

Exercise 6

Consider an inductor made of an iron core as depicted hereunder and a 60-turn winding, wounded around the central leg.



1. Draw an equivalent magnetic circuit of the inductor.

2. Compute the total reluctance of this circuit, considering a relative permeability μ_r of 1500 for the iron. Deduce the inductance from it.

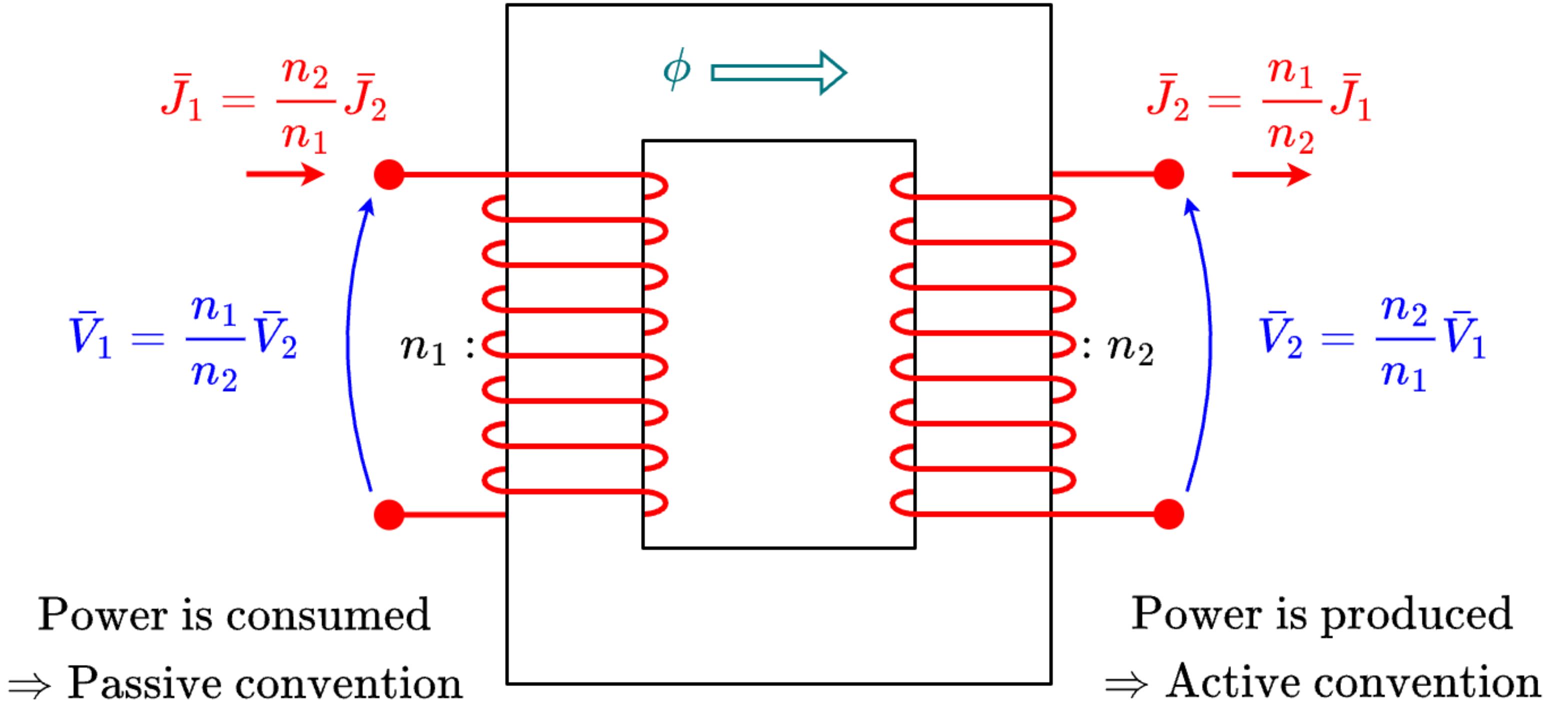
3. Do the same computation as in the previous steps, but now considering a constant air gap of $0.1\ mm$ in each leg.

Transformers

The ideal transformer
The real transformer
Shifting impedances
Open-circuit and short-circuit tests
Exercise 7

The ideal transformer

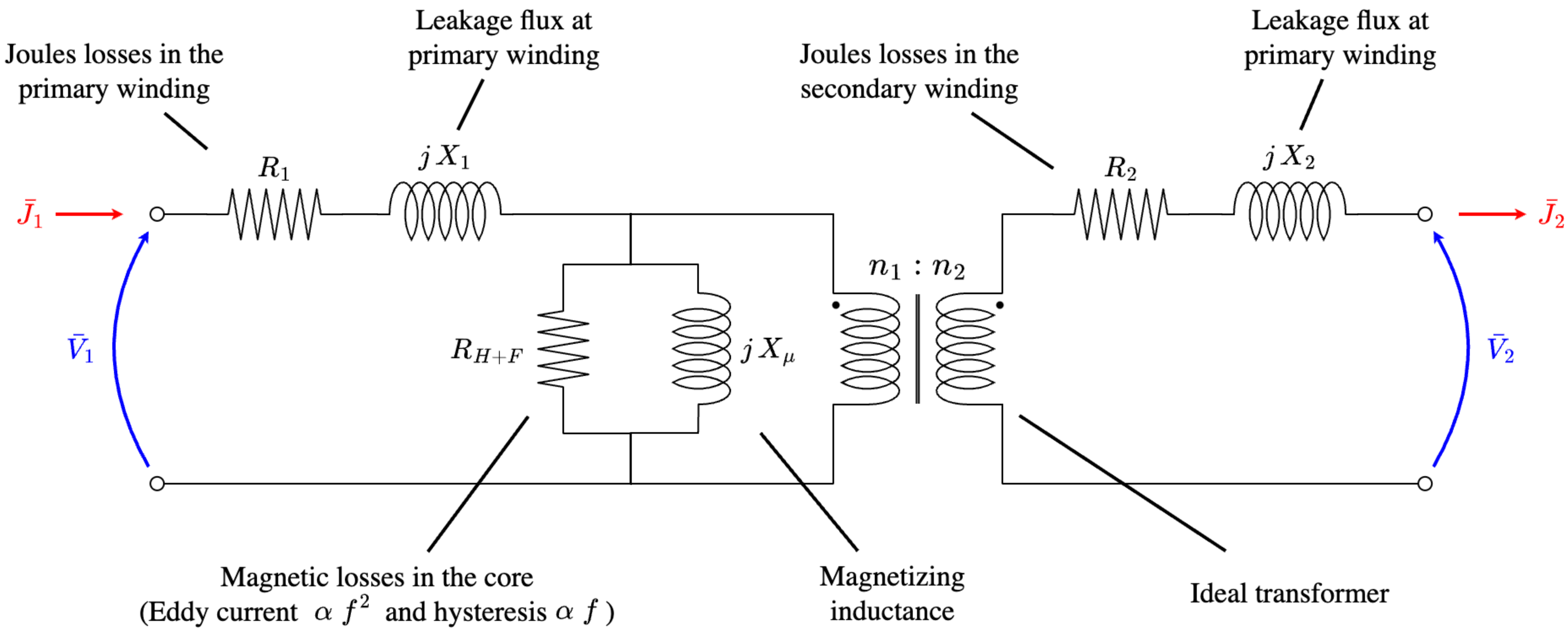
A transformer consists of two or more coils wrapped around a magnetic core, used to increase or decrease an AC voltage/current:



In an ideal transformer:

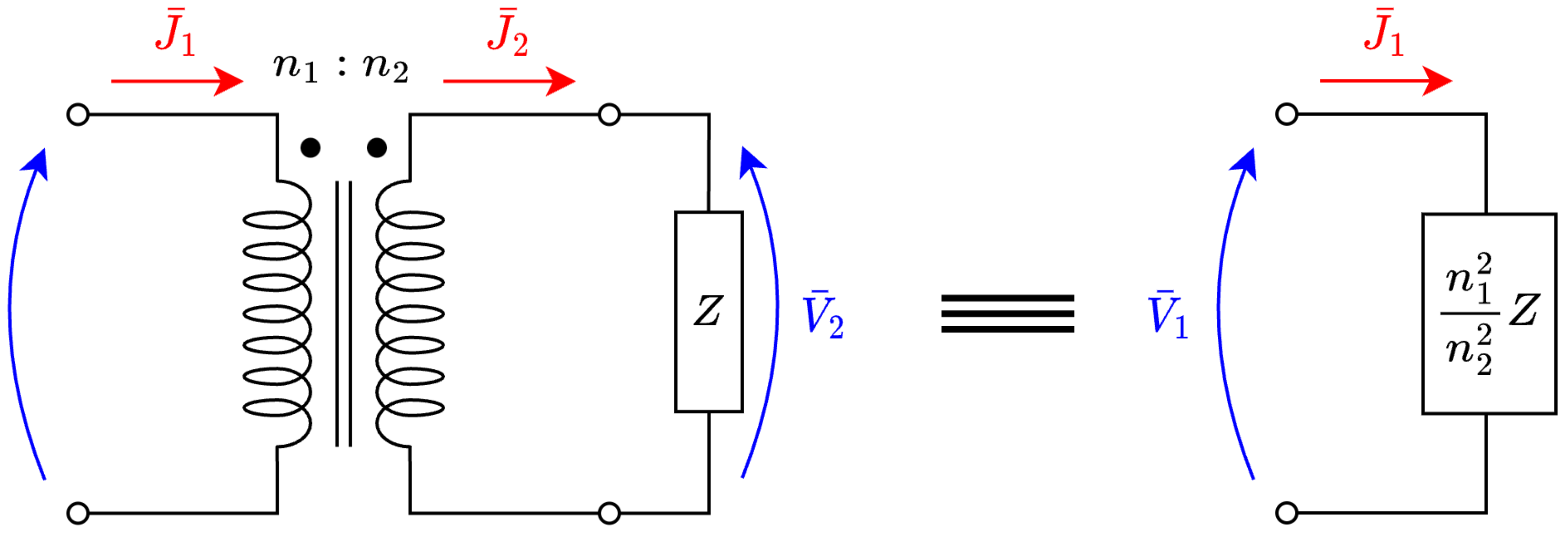
$$n_2\,\bar{V}_1 = n_1\,\bar{V}_2 \qquad \text{and} \qquad n_1\,\bar{J}_1 = n_2\,\bar{J}_2$$

The real transformer



In practice, transformers are built to minimize the losses ➔ $R_1, R_2, X_1, X_2 \ll R_{H+F},\ X_\mu$

Shifting impedances



$$n_2\, \bar{V}_1 = n_1\, \bar{V}_2$$

$$n_1\, \bar{J}_1 = n_2\, \bar{J}_2$$

Seen from the secondary, the impedance is $Z = \frac{\bar{V}_2}{\bar{J}_2}$

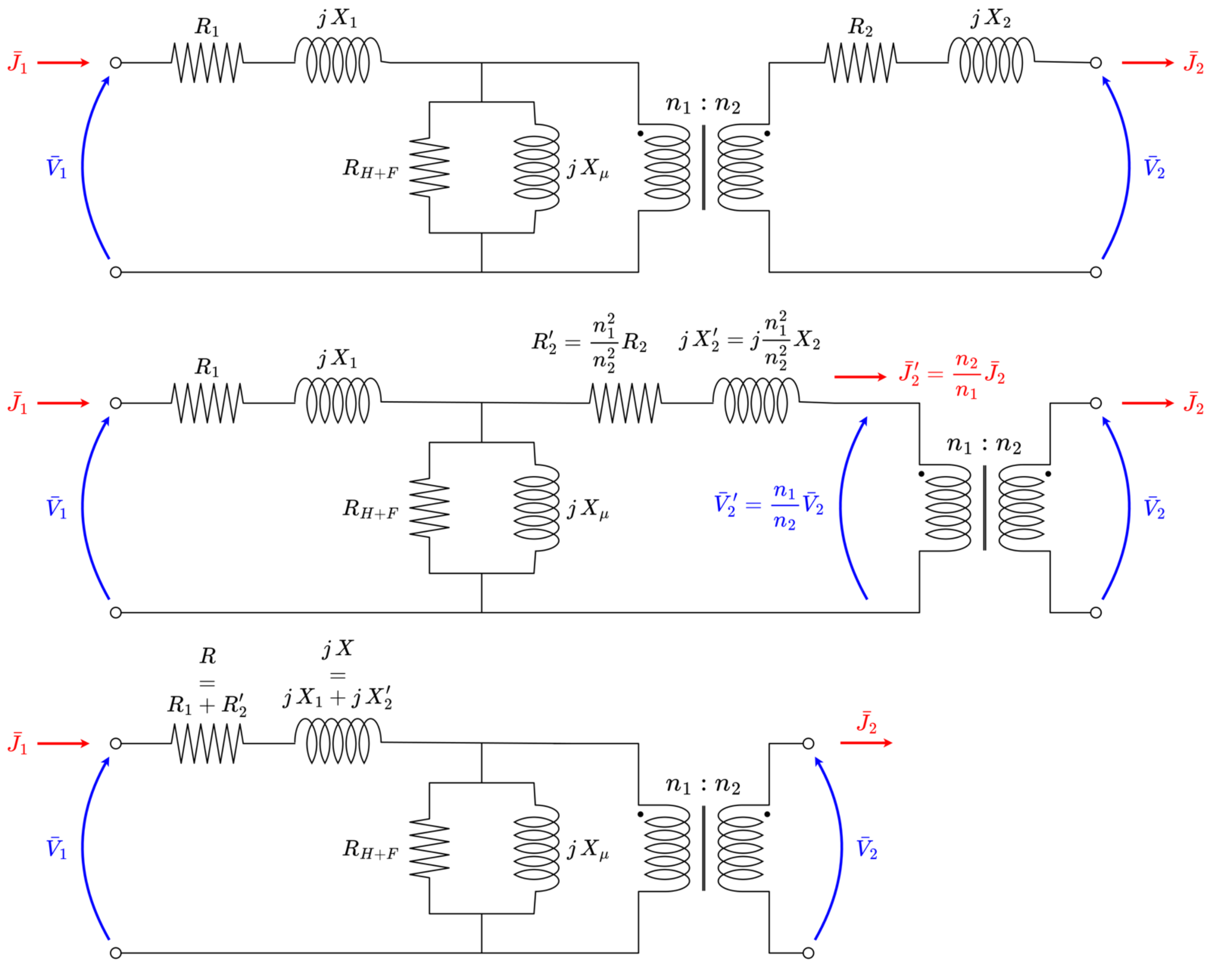
Seen from the primary, the impedance is $Z' = \frac{\bar{V}_1}{\bar{J}_1} = \frac{\bar{V}_2\left(\frac{n_1}{n_2}\right)}{\bar{J}_2\left(\frac{n_2}{n_1}\right)} = \frac{n_1^2}{n_2^2}\,\frac{\bar{V}_2}{\bar{J}_2} = \frac{n_1^2}{n_2^2}\, Z$

The real transformer – impedances gathered at primary

One can shift the impedances from the secondary to primary side of the ideal transformer

R_2' and $j\,X_2'$ can pass on the other side of the magnetizing branch since

$$R_2', X_2' \ll R_{H+F}, X_\mu$$

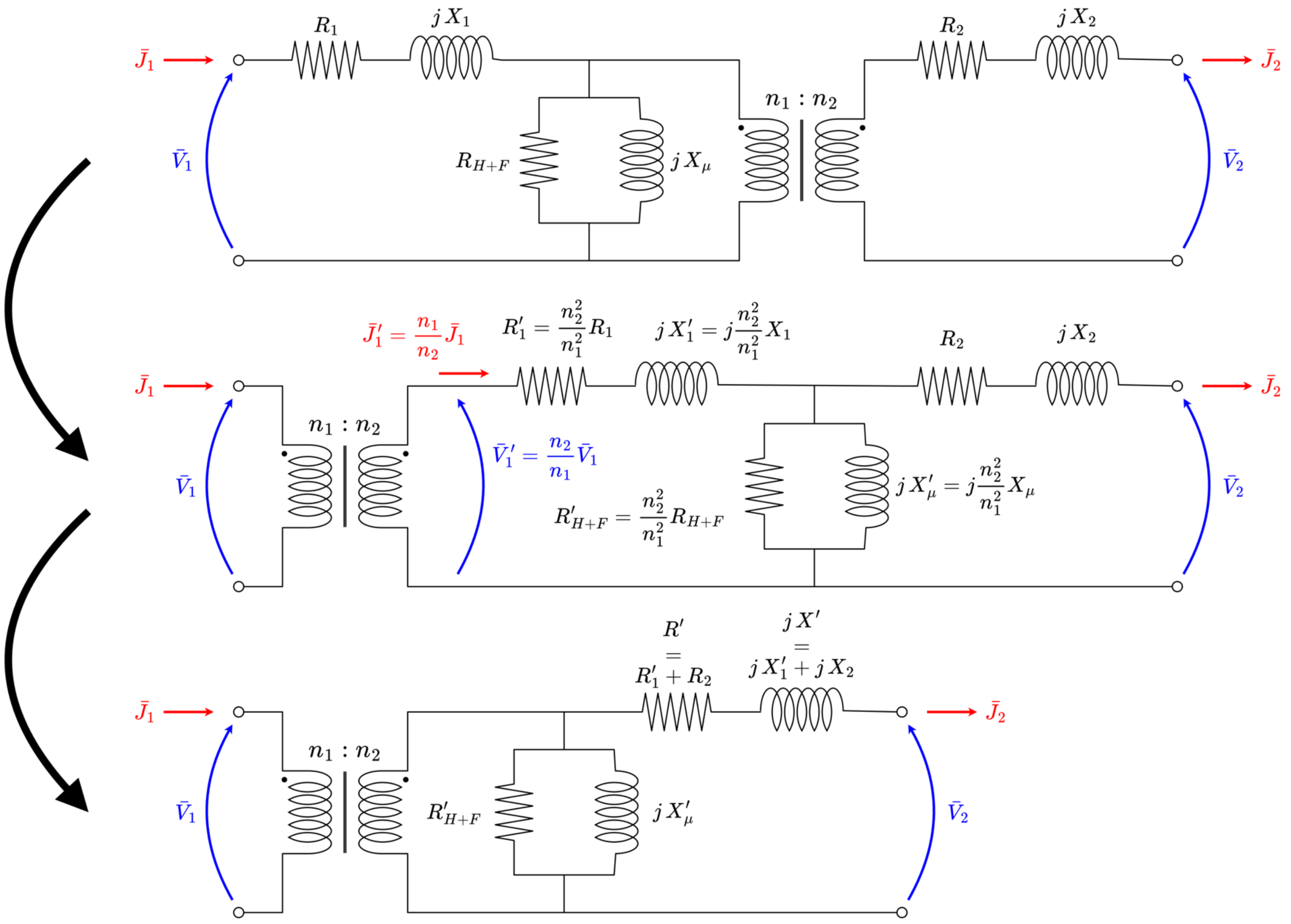


The real transformer – impedances gathered at secondary

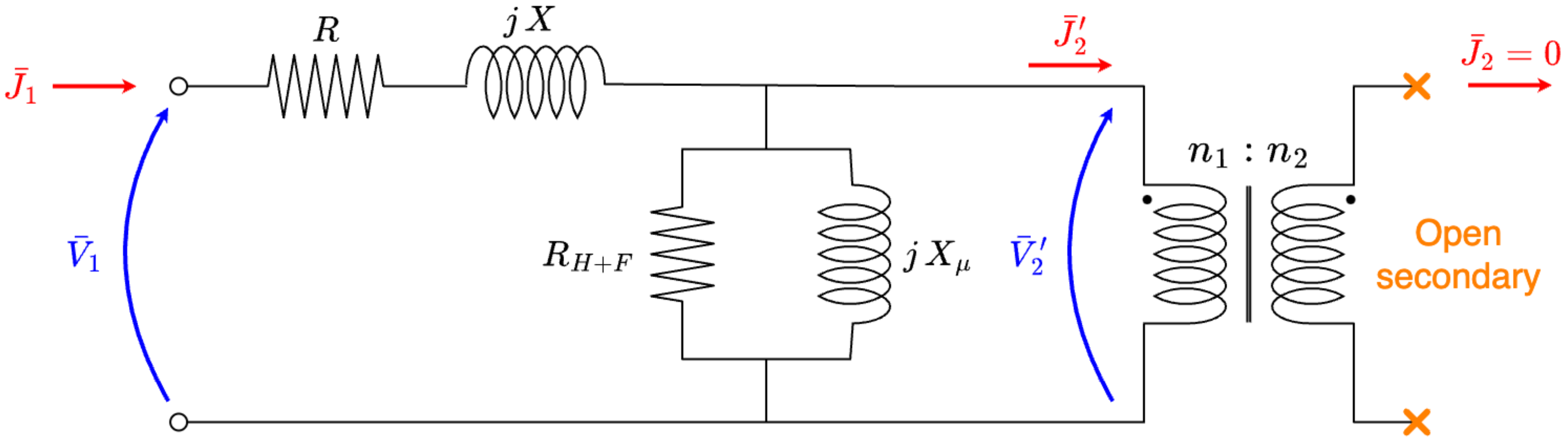
One can shift the impedances from the primary to secondary side of the ideal transformer

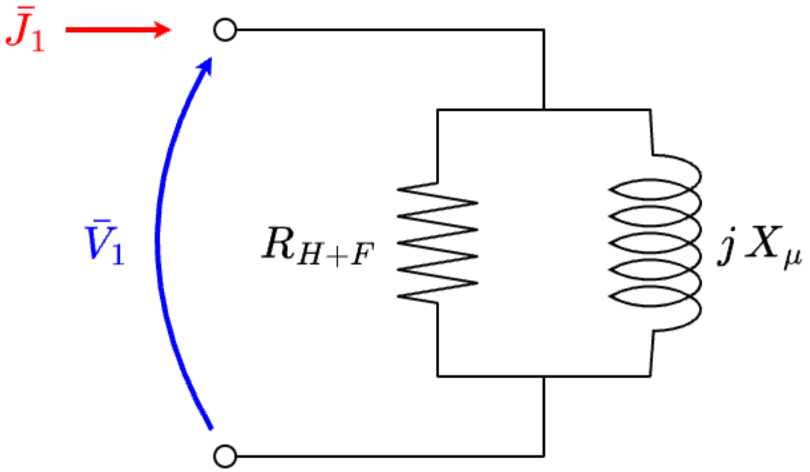
R_1' and $j\,X_1'$ can pass on the other side of the magnetizing branch since

$R_1', X_1' \ll R_{H+F}, X_\mu$



Open circuit test





$\bar{J}_2 = 0 \;\rightarrow \bar{J}_2' = 0$

$R, X \ll R_{H+F}, X_\mu$

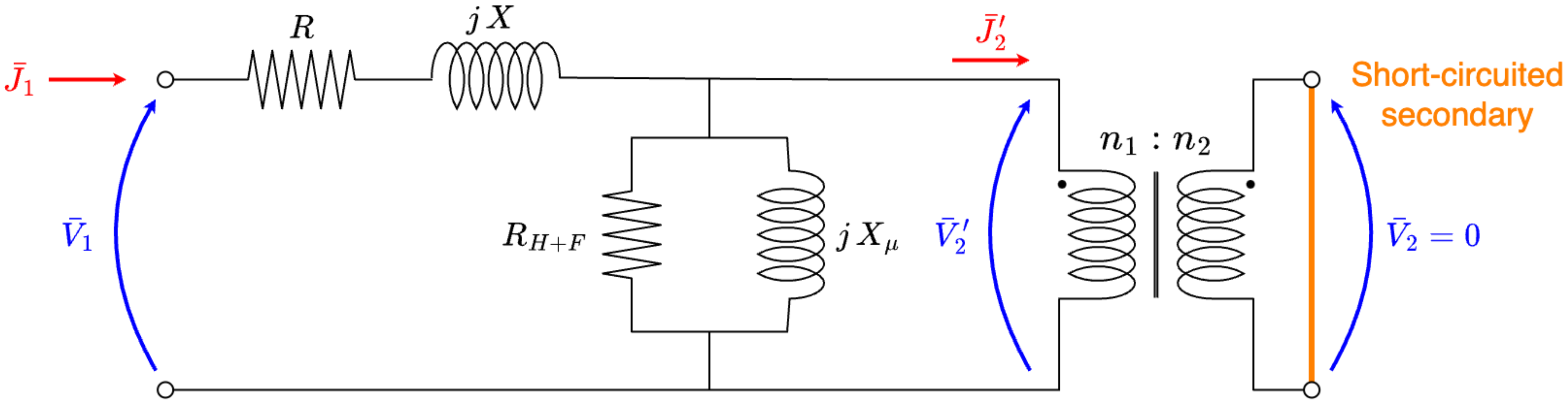
$\rightarrow \bar{V}_2' \gg (R + jX)\,\bar{J}_1$

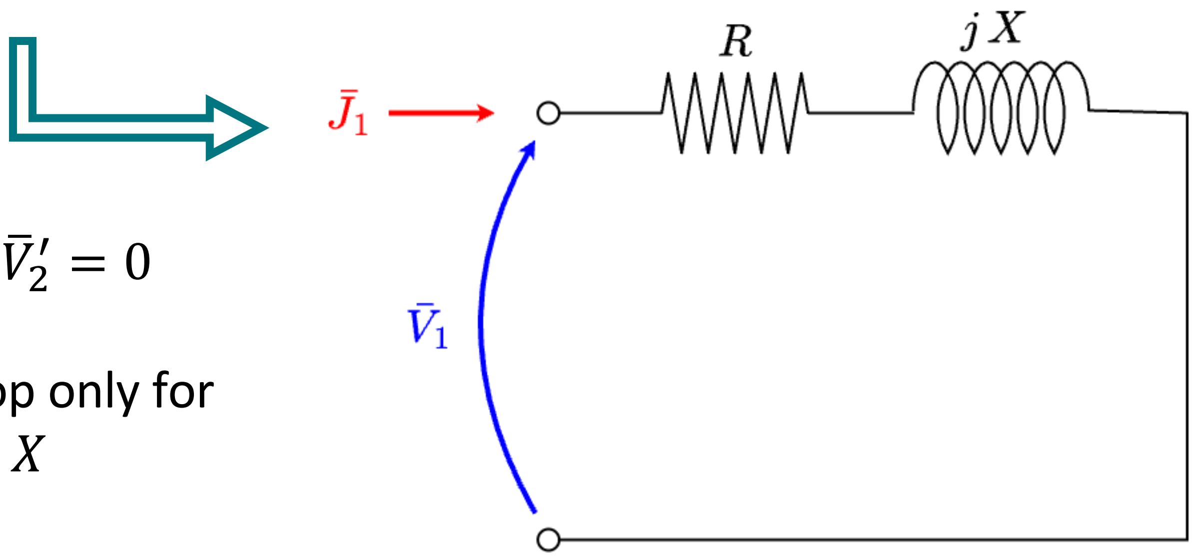
$\rightarrow$ R and X can be neglected

$$P \approx \frac{V_1^2}{R_{\mathrm{H}+F}}$$

$$Q \approx \frac{V_1^2}{X_\mu}$$

Short-circuit test





$\bar{V}_2 = 0 \;\rightarrow \bar{V}_2' = 0$

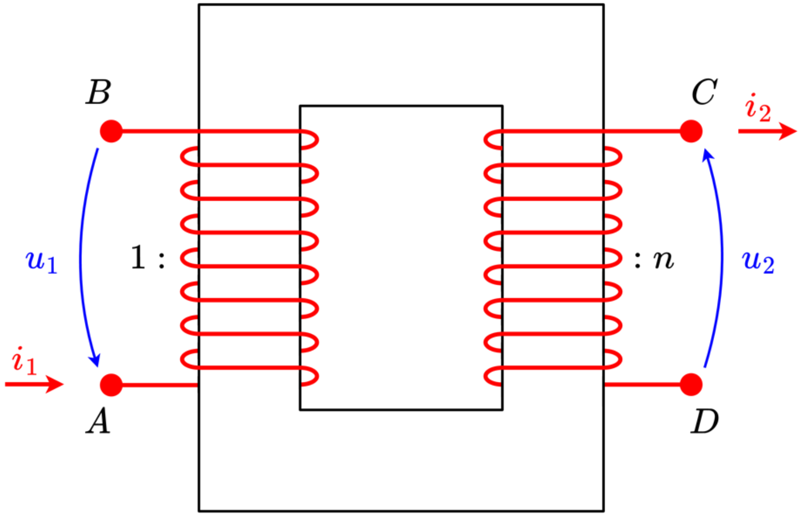
→ Voltage drop only for R and X

$P \approx R\,J_1^2$

$Q \approx X\,J_1^2$

Exercise 7

Two tests are performed on the transformer illustrated hereunder:



- Using open secondary winding, the transformer generates a voltage of RMS value $U_{2o} = 100\ V$ at the secondary winding, for an applied voltage of RMS value $U_{1o} = 20\ V$ with a drawn current intensity of RMS value $I_{1o} = 3.2\ A$ and a consumed power $P_{1o} = 8\ W$.
- Using short-circuited secondary winding, a voltage of RMS value $U_{1s} = 0.8\ V$ for a total power of $P_{1s} = 24\ W$ is measured, causing a current flow of RMS value $I_{2s} = 10\ A$ through the secondary winding.

Considering a simplified equivalent model of the transformer (resistances and inductances gathered and moved to the secondary winding) and a frequency of 50 Hz:

1. Calculate the transformer ratio n.
2. Calculate the resistance R'_{H+F} and the magnetizing inductance L'_{μ}.
3. Compute the resistance R' and the inductance L'.

Exercise 7

Using the transformer connected to a load on the secondary side drawing a current of RMS value $I_2 = 12\ A$ with a power factor $\cos(\varphi) = 0.8$ (the current is lagging the voltage), an RMS voltage $U_1 = 20\ V$ is applied to the primary winding.

4. Calculate the RMS voltage U_2 appearing across the secondary winding.
   (What wise approximation can be made here?)

5. Deduce the active power P_2 provided to the load.

6. Calculate the RMS current I_1 on the primary side.

7. Compute the transformer efficiency η.