ELEC0431 : Exercise session 4

Three-Phase transformers

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Exercises

Exercises of session 4

Wooclap code : NSATKE

Exercise 9 : three-phase transformer

The three-phase transformer, described by the normalized scheme, is connected to a balanced three-phase network of composed voltages u_{AB}, u_{BC}, u_{CA} of RMS voltage U_1 on the primary side, whereas on the secondary side, a three-phase balanced system of composed voltages u_{ab}, u_{bc}, u_{ca} of RMS voltage U_2 is obtained.

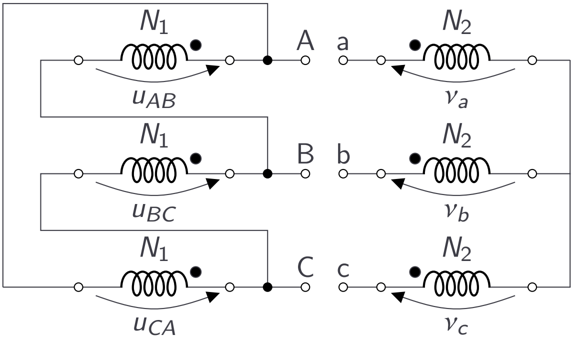


Figure: Three-phase transformer.

Exercise 9 : three-phase transformer

The line current intensities in the primary and secondary windings are respectively denoted I_1 and I_2. The transformer has the following characteristics:

- Apparent nominal power S_n = 250 kVA;
- Composed primary winding RMS voltages U_{1n} = 5.2 kV;
- Nominal frequency f_n = 50 Hz;

and ferromagnetic losses are neglected. To characterize the transformer two tests have been performed:

- Using open secondary windings, the transformer generates a composed voltage of RMS value U_{2o} = 400 V at each secondary winding, for an applied composed nominal voltage of RMS value U_{1n};
- Using short-circuited secondary windings, a composed voltage of RMS value U_{1s} = 600 V is applied at each primary winding for a total primary power P =7.35 kW, producing line current of RMS intensity I_{2s} = 350 A.

Exercise 9 : three-phase transformer

1. Calculate the transformer ratio n so that it is greater than 1;

Exercise 9 : three-phase transformer - solution

The transformer ratio is

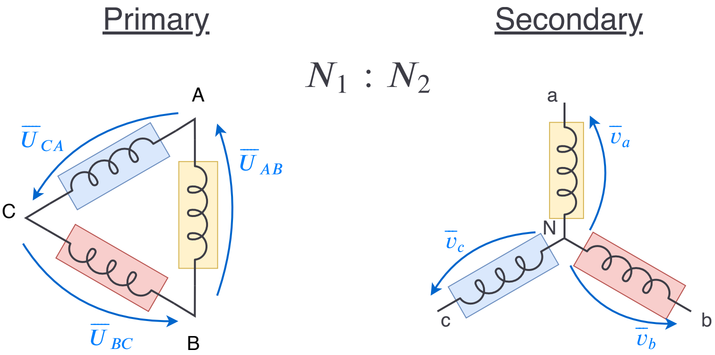
$$\boxed{n = \frac{U_{AB}}{U_{ab}} = \frac{U_{1n}}{U_{2o}} = \frac{5\ 200}{400} = 13}$$

Exercise 9 : three-phase transformer

2. For the first test condition (open secondary windings), draw a Fresnel diagram including the primary composed voltages u_{AB}, u_{BC}, u_{CA}, the direct secondary voltage v_a, v_b, v_c and the secondary composed voltages u_{ab}, u_{bc}, u_{ca};

Exercise 9 : three-phase transformer - solution

The primary windings are connected in Δ (delta) configuration while secondary windings are connected in star.



N_1 and N_2 are respectively the number of turns in the primary and the secondary.

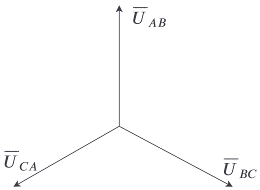
Exercise 9 : three-phase transformer - solution

$$\overline{v}_a = \frac{N_2}{N_1} \cdot \overline{U}_{AB}$$

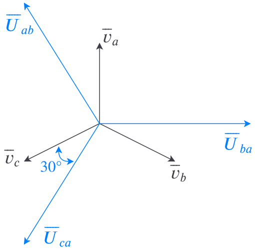
$$\overline{v}_b = \frac{N_2}{N_1} \cdot \overline{U}_{BC}$$

$$\overline{v}_c = \frac{N_2}{N_1} \cdot \overline{U}_{CA}$$

Primary



Secondary



3. Express and compute the column ratio $n_c = \frac{N_1}{N_2}$ according to n;

The column ratio can be obtained from

$$n_c = \frac{N_1}{N_2} = \frac{U_{AB}}{v_a} \qquad (1)$$

As

$$n = \frac{U_{AB}}{\sqrt{3}\; v_a}$$

$$\boxed{n_c = \sqrt{3}\; n = 22.517}$$

Exercise 9 : three-phase transformer

4. Given that the transformer is composed of 3 cores of section $A_c = 5\ \text{dm}^2$, and that the magnetic field amplitude is $B_m = 1.2\ \text{T}$, compute the number of turns N_1 of each primary winding and deduce the value of the number of turns of each winding N_2;

Exercise 9 : three-phase transformer - solution

The maximum magnetic flux in the transformer as

$$\phi_m = B_m \, A_c \tag{2}$$

and according to Lenz law,

$$\begin{aligned} U_1(t) &= -N_1 \, \frac{d}{dt}(\phi_1(t)) \\ U_{AB} \, \sqrt{2} \, \cos(\omega \, t) &= -N_1 \, \frac{d}{dt}(\phi_m \, \cos(\omega \, t + \frac{\pi}{2})) \\ U_{AB} \, \sqrt{2} \, \cos(2\pi \, f \, t) &= N_1 \, 2\pi \, f \, \phi_m \, \cos(2\pi \, f \, t) \end{aligned}$$

where U_{AB} is the RMS value of the voltage and B_m is the peak value of the magnetic induction.

Exercise 9 : three-phase transformer - solution

$$\boxed{N_1 = \frac{\sqrt{2}\,U_{AB}}{2\pi\, f\ B_m\ A_c} = \frac{\sqrt{2}\cdot 5200}{2\pi\cdot 50\cdot 1.2\cdot 0.05} = 390} \tag{3}$$

Then, as the column ratio n_c is equal to $\frac{N_1}{N_2}$,

$$\boxed{N_2 = \frac{N_1}{n_c} = \frac{390}{22.517} = 17} \tag{4}$$

Remark that

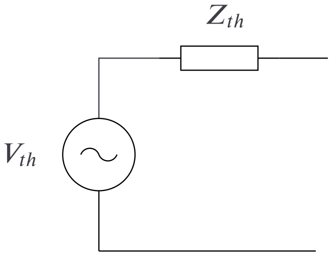
$$B_m = \frac{\sqrt{2}\ U}{2\pi\ f\ N\ A_c} \tag{5}$$

the magnetic induction is directly proportional to the voltage and is inversely proportional to the frequency, number of turns and the cross-section area.

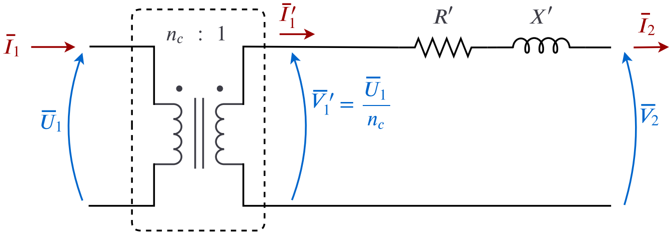
5. Using a simple single-phase equivalent model (leak resistance and inductance moved to the secondary windings), provide the Thévenin's model seen from a secondary winding and calculate the resistance R_{eq} (R') and the reactance X_{eq} (X') of this model;

Exercise 9 : three-phase transformer - solution

Considering the Thévenin equivalent in which, V_{th} is equal to the voltage at no load of the circuit while Z_{th} is the circuit impedance when all voltage sources are shorted.



And considering the transformer equivalent circuit.



By definition of the Thevenin equivalent circuit,

$$\boxed{V_{th} = V_2 = \frac{U_2}{\sqrt{3}} = 230.94 \quad \text{V}} \tag{6}$$

$$Z_{th} = R' + jX'$$

where R' and X' are obtained from the short circuit measurements :

$$\boxed{R' = \frac{P_{2s}}{I_{2s}^2} = \frac{\frac{7350}{3}}{350^2} = 20 \quad m\Omega} \tag{7}$$

$$U_{2s} = \frac{U_{1s}}{n} = \frac{600}{13} = 46.15 \quad V$$

$$S_{2s} = V_{2S}\ I_{2s} = \frac{U_{2S}}{\sqrt{3}}\ I_{2s} = 9\ 326.26 \quad VA$$

$$\rightarrow Q_{2s} = \sqrt{S_{2s}^2 - P_{2s}^2} = 8\ 998.7 \quad var$$

which finally leads to

$$\boxed{X' = \frac{Q_{2s}}{I_{2s}^2} = 73.4 \quad m\Omega} \tag{8}$$

Exercise 9 : three-phase transformer

The nominal regime is now considered by applying the composed nominal voltage U_{1n} at the primary windings and connecting a three-phase balanced load on the secondary side (detailed in 2). Each branch is composed of a resistor of value R^* =554 mΩ in series with a coil of value $L^* = 3.05$ mH.

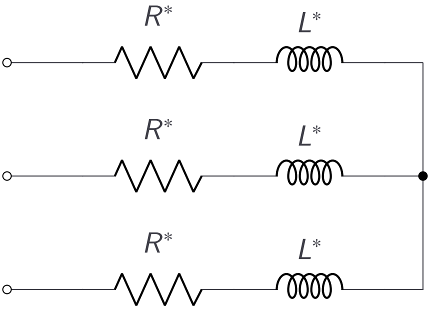
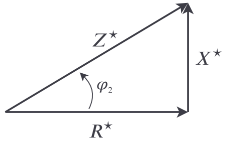


Figure: First load connected to the secondary side of the three-phase transformer.

6. Calculate the power factor $\cos \varphi_2$ of this load;

Exercise 9 : three-phase transformer - solution

$$X^* = 2\pi f \; L^* = 958.2 \quad \text{m}\Omega \tag{9}$$



$$\varphi_2 = \arctan\left(\frac{X^*}{R^*}\right) = 59.96^\circ$$

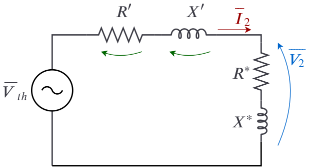
$$\boxed{\cos(\varphi_2) = 0.5005} \tag{10}$$

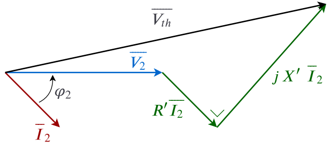
An other solution is :

$$\cos\left(\varphi_2\right) = \frac{R^*}{\sqrt{(R^*)^2 + (X^*)^2}}$$

7. Draw the Fresnel diagram corresponding to the balanced single-phase equivalent model. Deduce the RMS values of the current intensities I_2 and the composed voltages U_2;

In nominal regime, when the three-phase load is connected to the secondary of the transformer, the Fresnel diagram is as following :





From this diagram, the current $\overline{I_2}$ and voltage $\overline{V_2}$ can be calculated as

$$\overline{I_2} = \frac{V_{th}}{Z_{th} + R^* + jX^*}$$

$$\boxed{I_2 = \frac{V_{th}}{\sqrt{(R' + R^*)^2 + (X' + X^\star)^2}} = 195.7 \quad \text{A}} \qquad (11)$$

and the voltage is

$$\overline{V_2} = (R^* + jX^*)\,\overline{I_2}$$

$$\boxed{V_2 = \sqrt{(R^*)^2 + (X^*)^2}\; I_2 = 216.58 \quad \text{V}} \tag{12}$$

8. Compute the power P_2 flowing from the transformer to the load;

Exercise 9 : three-phase transformer - solution

The active power consumed by the load is

$$\boxed{P_2 = V_2\ I_2\ \cos\varphi_2 = 216.58 \cdot 195.7 \cdot 0.5 = 21.219 \quad \text{kW}} \qquad (13)$$

Exercise 9 : three-phase transformer

9. Calculate the transformer efficiency η;

Exercise 9 : three-phase transformer - solution

In order to comput the transformer efficiency, one can first compute the losses in the transformer, noted P_{loss}

$$P_{\text{loss}} = R' \ I_2^2 = 0.02 \cdot 195.7^2 = 765.97 \quad \text{W} \tag{14}$$

Then, the efficiency can be computed as

$$\boxed{\eta = \frac{P_2}{P_2 + P_{\text{loss}}} = \frac{21\ 219}{21\ 984} = 0.9651\,\%} \tag{15}$$

10. Another load is used, compute the value of the resistance R° and the inductance L° such that this load is equivalent to the one detailed in next figure.

Exercise 9 : three-phase transformer

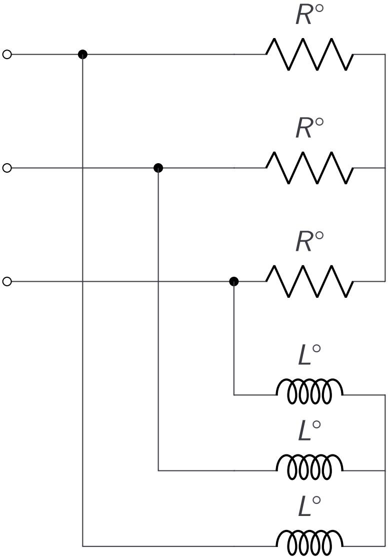


Figure: Second load connected to the secondary side of the three-phase

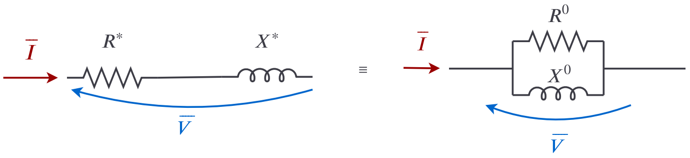


Figure: Equivalence between the two loads.

With the following equations,

$$V = \sqrt{{R^*}^2 + {X^*}^2}\, I \tag{16}$$

$$P = R^* I^2 = \frac{V^2}{R^0} \tag{17}$$

$$Q = X^* I^2 = \frac{V^2}{X^0} \tag{18}$$

one can obtain

$$R^0 = \frac{V^2}{I^2} \cdot \frac{1}{R^*} = \frac{{R^*}^2 + {X^*}^2}{R^*} \tag{19}$$

$$X^0 = \frac{V^2}{I^2} \cdot \frac{1}{X^*} = \frac{{R^*}^2 + {X^*}^2}{X^*} \tag{20}$$

(21)