Electromagnetic Energy Conversion ELEC0431

Exercise session 4: Three-phase transformers

4 March 2022

Florent Purnode (florent.purnode@uliege.be) – Nicolas Davister (ndavister@uliege.be)

Montefiore Institute, Department of Electrical Engineering and Computer Science,
University of Liège, Belgium

Lab sessions

Doodle and groups

Doodle and groups

Reminder:

Make groups of 4 students and choose at least 6 slots available in the doodle:

https://doodle.com/poll/u2i2bqsdam537wyy?utm_source=poll&utm_medium=link

Complete this doodle by indicating the names of all participants:

$$Firstname_1\ LASTNAME_1\ - \cdots - Firstname_4\ LASTNAME_4$$

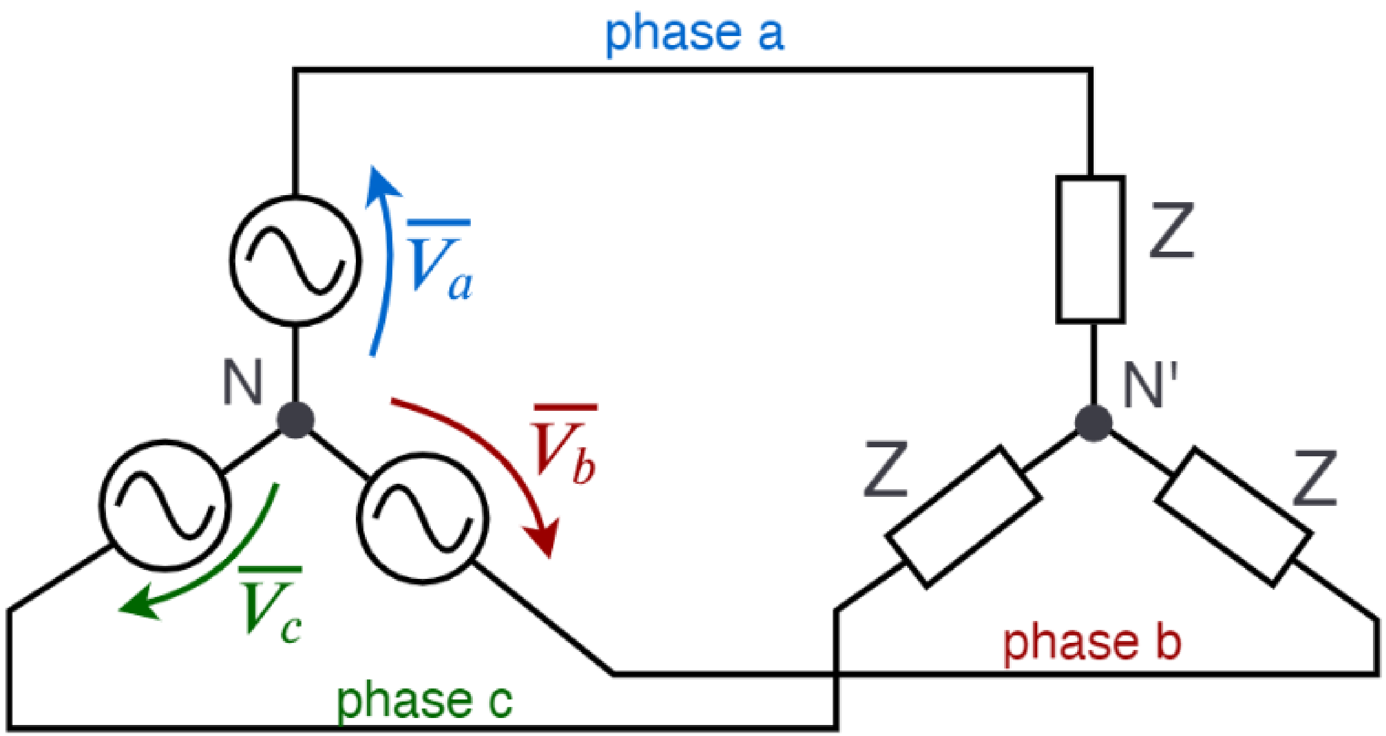
In case your group is not complete, send me a mail: florent.purnode@uliege.be and Do Not fill the doodle alone.

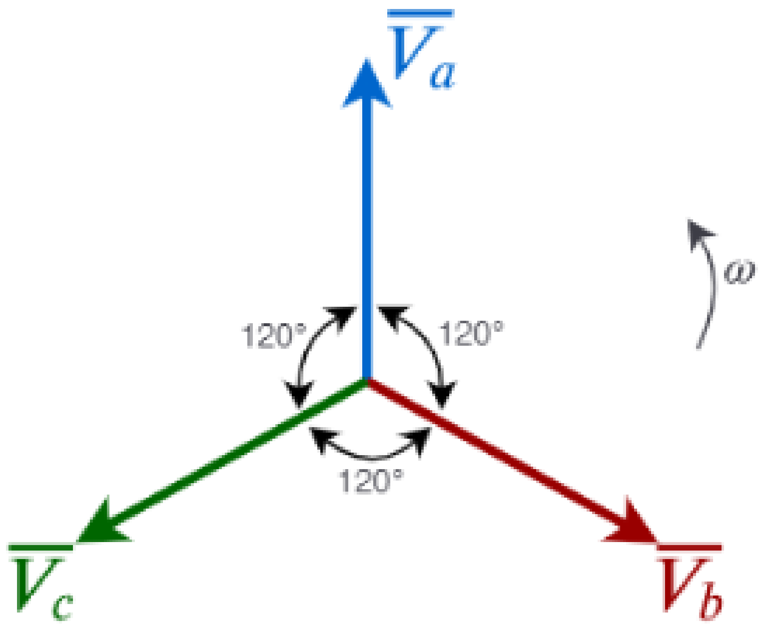
Deadline for answering the doodle is Monday March 7.
If the deadline is not met, groups will be made randomly.

Reminders

Three-phase systems

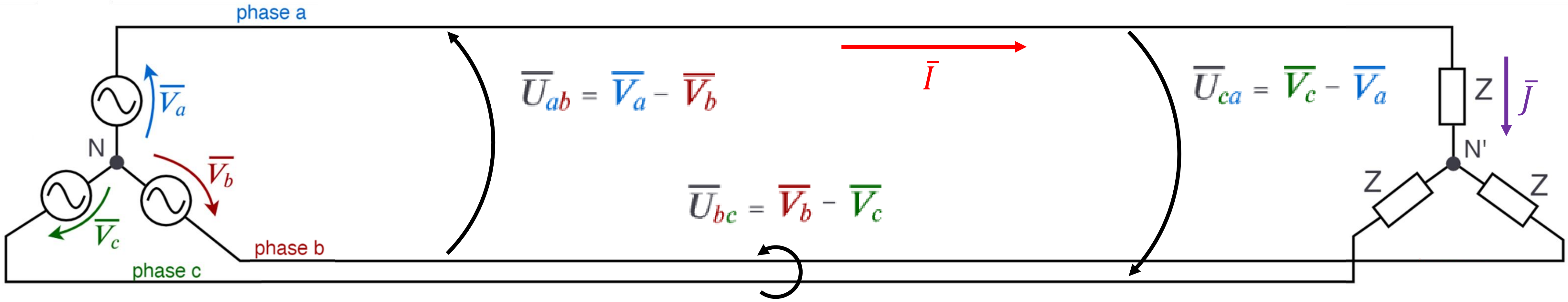
Three-phase systems





The network is balanced ➔ The loads Z are identical.
$\bar{V}_a$, $\bar{V}_b$ and $\bar{V}_c$ have the same amplitude and are dephased by 120°.

Three-phase systems

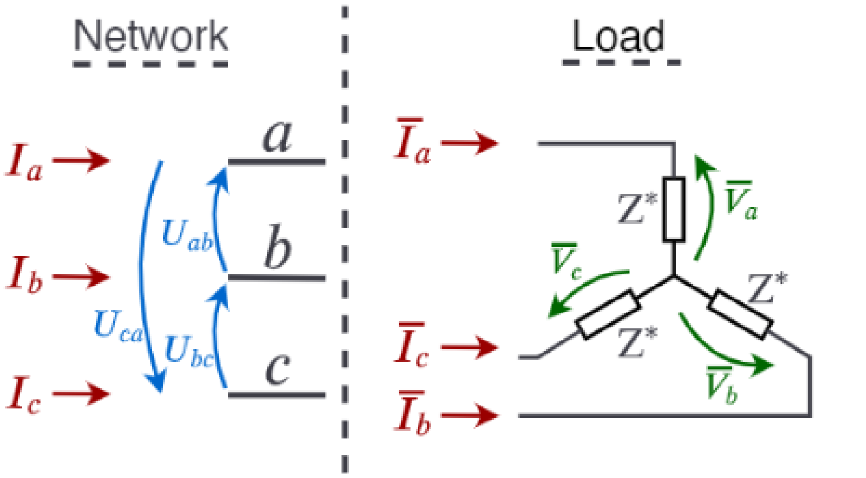


- V: Phase voltage = voltage across one load / source
- U: Line voltage = voltage measured between two different phases
- J: Phase current = current flowing through one load / source
- I: Line current = current flowing through one line

Star and triangle connections

Three-phase loads can be arranged either in star (Y) or in triangle (Δ)

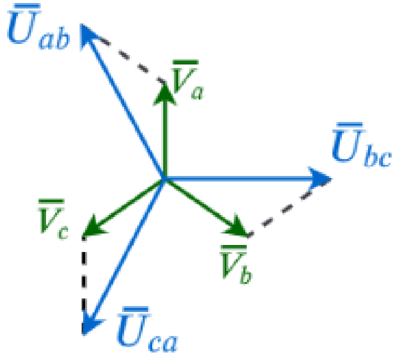
Y ➔ line current $\bar{I}$ = phase current $\bar{J}$



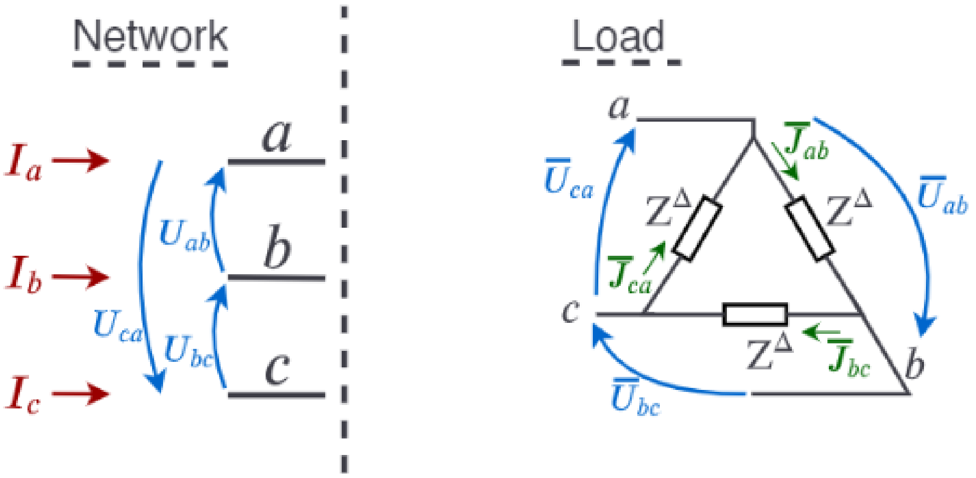
$$V_a = \frac{U_{ab}}{\sqrt{3}}$$

phase voltage

line voltage



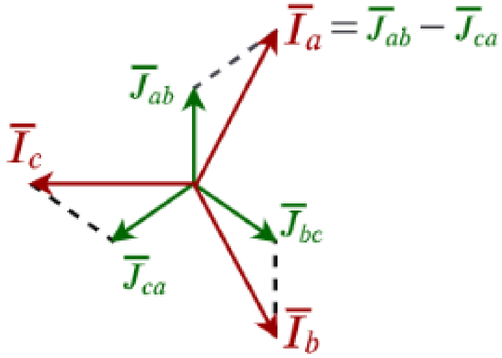
Δ ➔ phase voltage $\bar{V}$ = line voltage $\bar{U}$



$$J_{ab} = \frac{I_a}{\sqrt{3}}$$

phase current

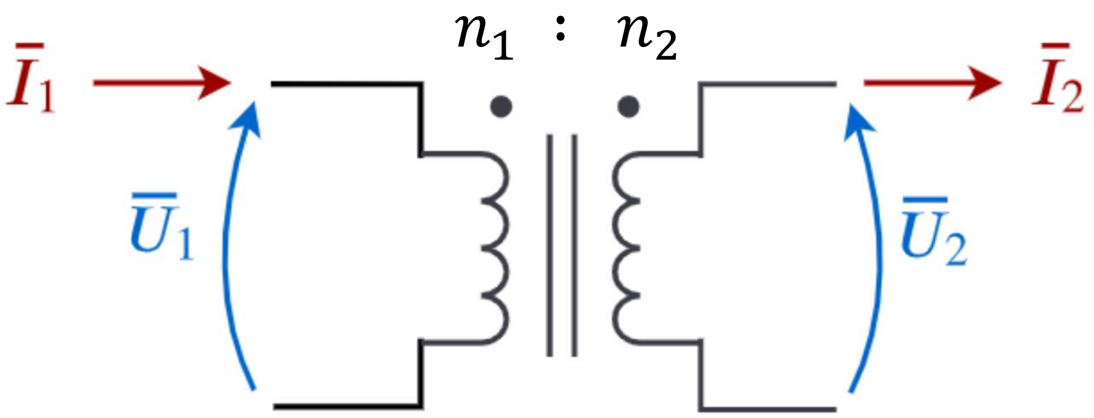
line current



Reminders

The transformer

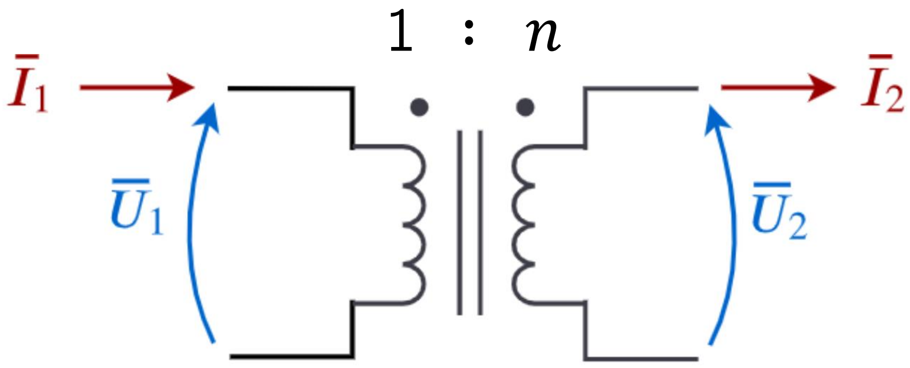
The ideal transformer



$$\boxed{n_1\,\bar{I}_1 = n_2\,\bar{I}_2}$$

$$\boxed{n_2\,\bar{U}_1 = n_1\,\bar{U}_2}$$

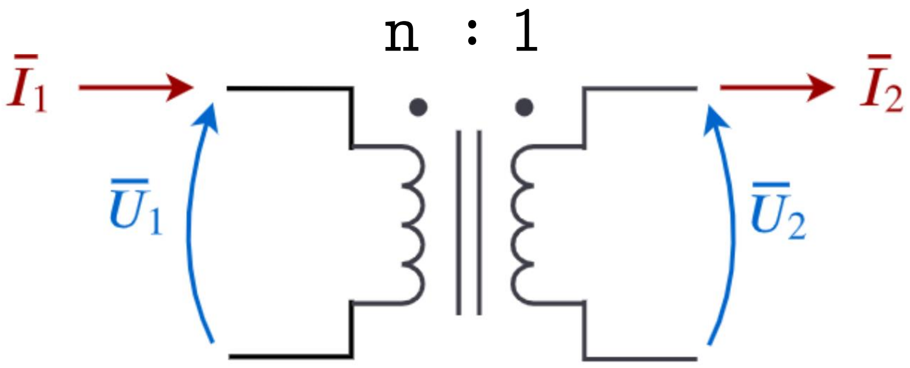
The transformer ratio n



$$n := n_2 \,/\, n_1$$

$$\bar{I}_1 = n\,\bar{I}_2$$

$$\bar{U}_2 = n\,\bar{U}_1$$

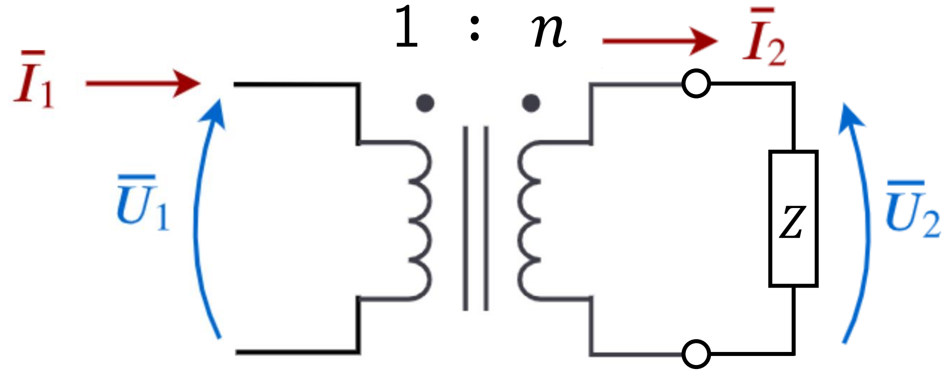


$$n := n_1 \,/\, n_2$$

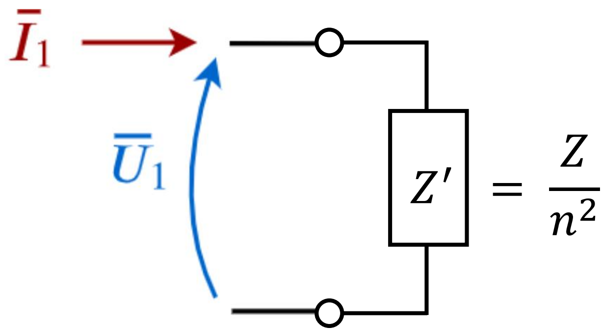
$$\bar{I}_2 = n\,\bar{I}_1$$

$$\bar{U}_1 = n\,\bar{U}_2$$

Shifting impedances



$\equiv$



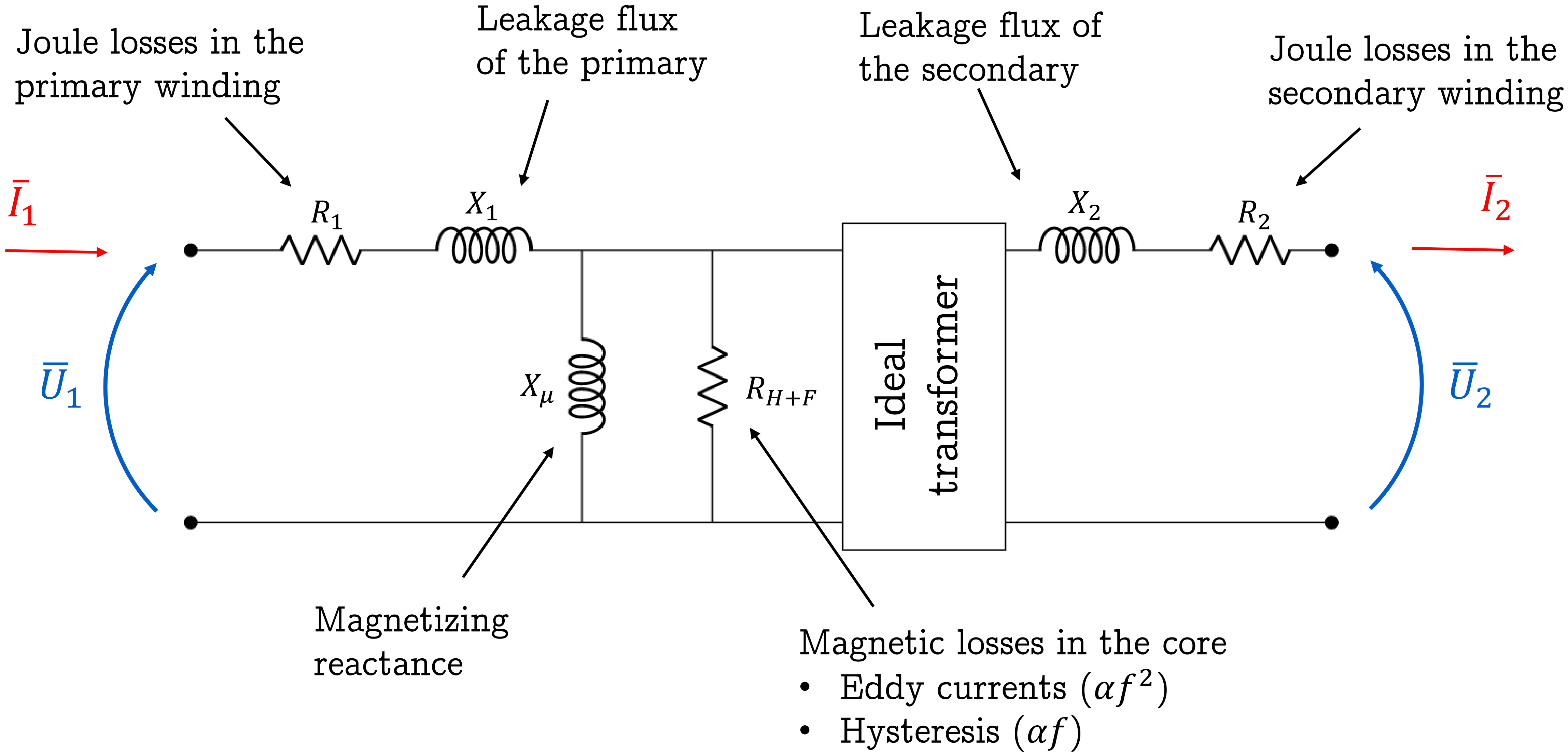
$$\bar{I}_1 = n\,\bar{I}_2 \quad \& \quad \bar{U}_2 = n\,\bar{U}_1$$

The impedance can be shifted from the secondary to the primary:

Seen from the secondary, the impedance is $Z = \frac{\bar{U}_2}{\bar{I}_2}$

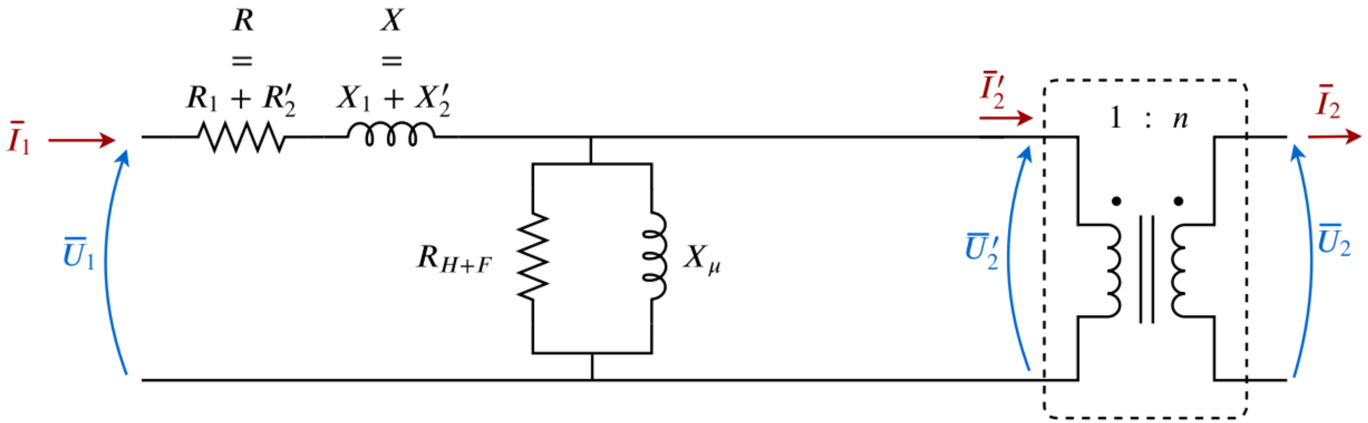
Seen from the primary, the impedance is $Z' = \frac{\bar{U}_1}{\bar{I}_1} = \frac{\bar{U}_2/n}{n\,\bar{I}_2} = \frac{1}{n^2}\frac{\bar{U}_2}{\bar{I}_2} = \frac{Z}{n^2}$

Practical transformer



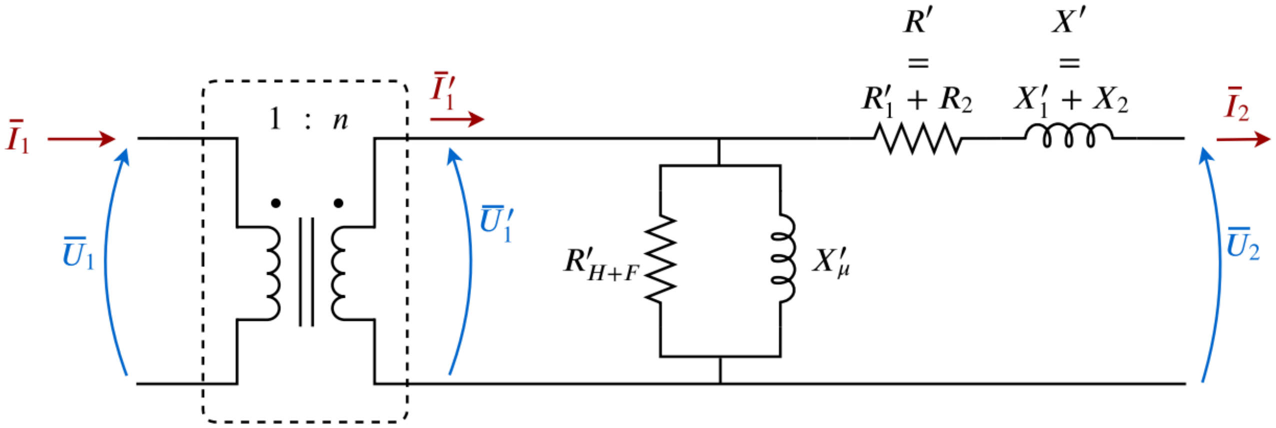
Simplified model

Under the approximations $R_2' \ll R_{H+F}$ and $X_2' \ll X_\mu$, the primary and secondary components can be gathered together.

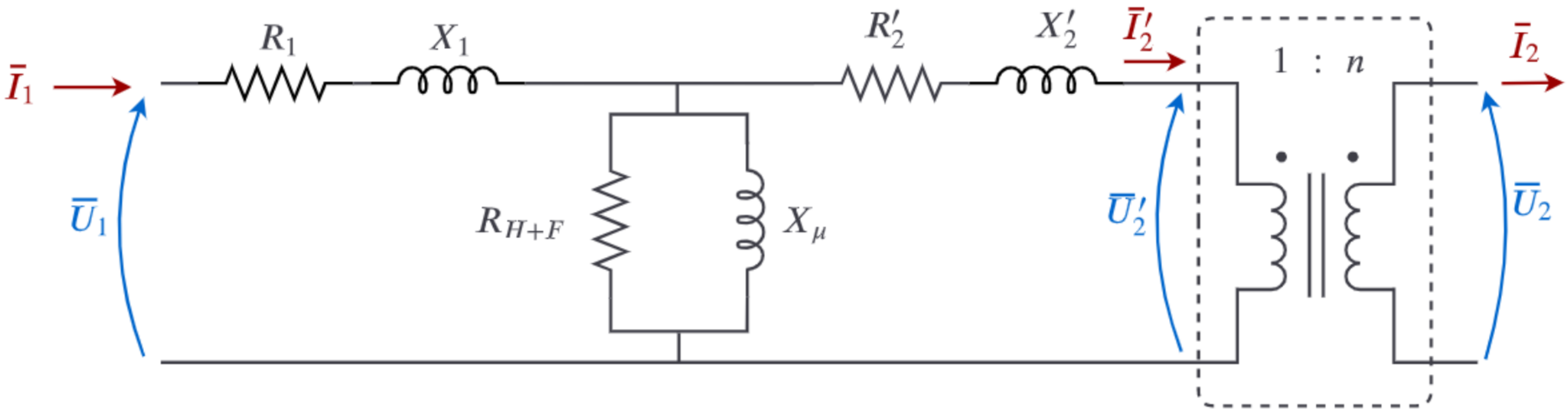


← Gathered to the primary

Gathered to the secondary →



Measuring components



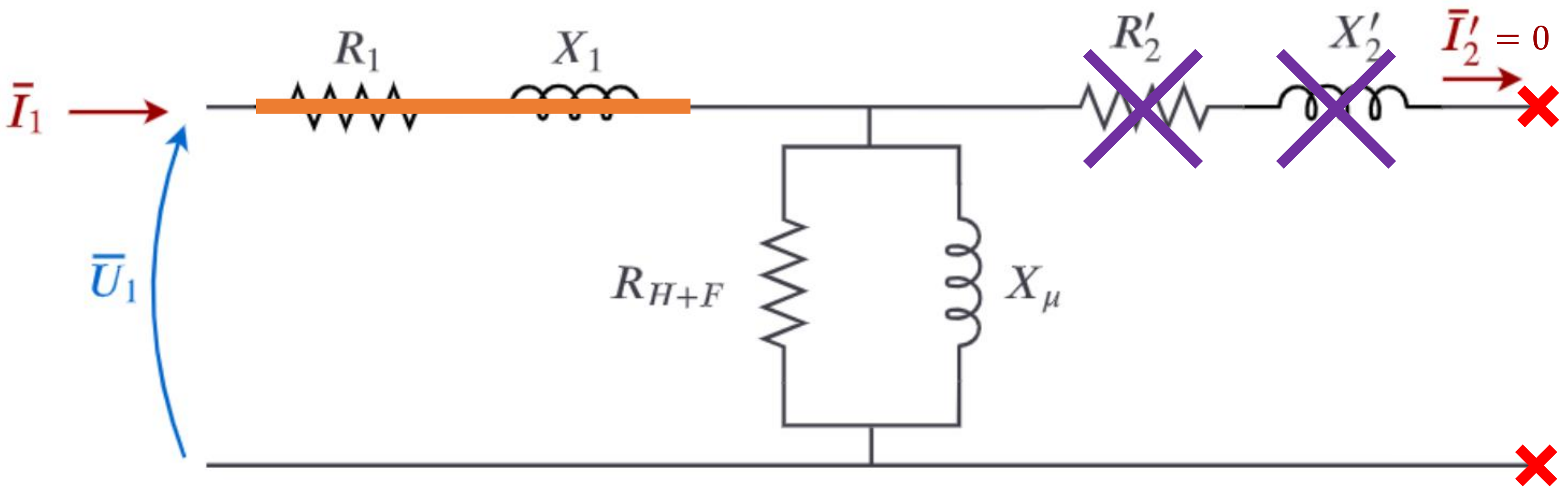
➔ How to measure R_1, X_1, etc. ?

- Open circuit test
- Short circuit test

Note that, in practice, transformers are build in order to minimize the losses. Thus:

$$R_1, R'_2 \ll R_{H+F} \quad \& \quad X_1, X'_2 \ll X_\mu$$

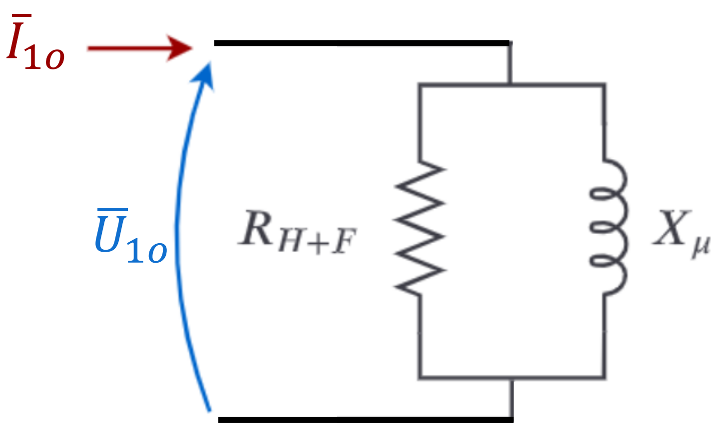
Open circuit test



- First simplification: $\bar{I}'_2 = 0$ ➔ No current in R'_2 and in X'_2.
- Second simplification: $R_1 \ll R_{H+F}$ & $X_1 \ll X_\mu$ ➔ R_1 and X_1 can be ignored.

⇨ Only R_{H+F} and X_μ remain.

Open circuit test



$$Z = \left(\frac{1}{R_{H+F}} + \frac{1}{j\,X_\mu}\right)^{-1}$$

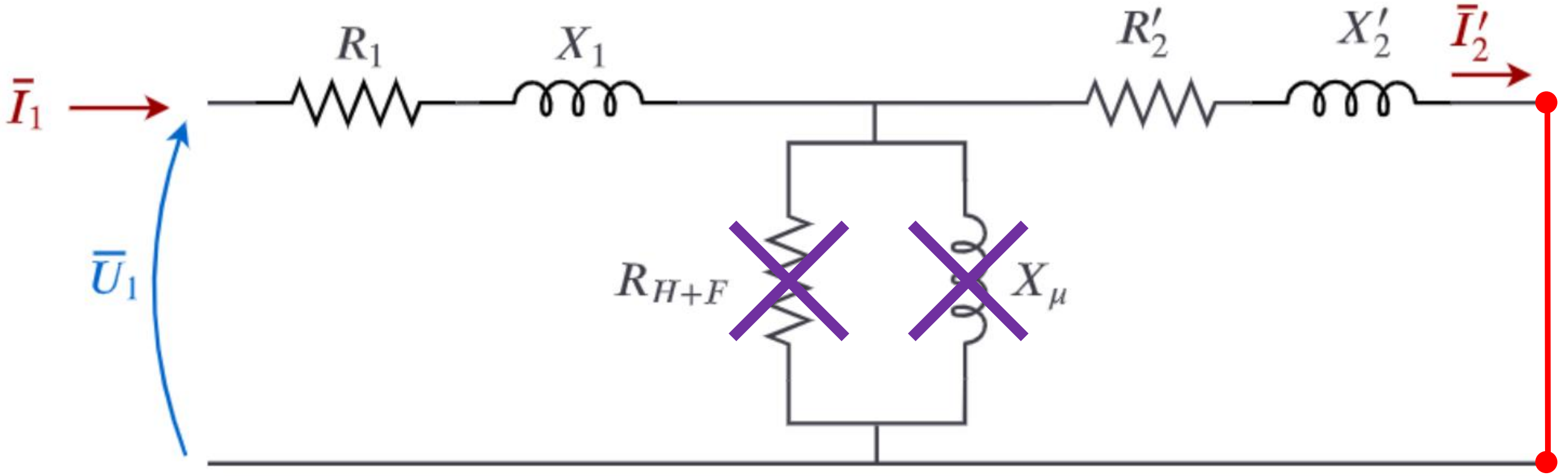
$$\bar{U}_{1o} = Z\,\bar{I}_{1o} \Rightarrow \bar{I}_{1o} = \frac{\bar{U}_{1o}}{Z} = \left(\frac{1}{R_{H+F}} + \frac{1}{j\,X_\mu}\right)\bar{U}_{1o}$$

$$S_{1o} = \bar{U}_{1o}\,\bar{I}_{1o}^{*} = \bar{U}_{1o}\left(\frac{1}{R_{H+F}} + \frac{1}{-j\,X_\mu}\right)\bar{U}_{1o}^{*} = \frac{U_{1o}^2}{R_{H+F}} + j\,\frac{U_{1o}^2}{X_\mu}$$

$$\Rightarrow \quad P_{1o} = \frac{U_{1o}^2}{R_{H+F}} \quad \& \quad Q_{1o} = \frac{U_{1o}^2}{X_\mu} \quad \Rightarrow \quad \boxed{R_{H+F} = \frac{U_{1o}^2}{P_{1o}} \quad \& \quad X_\mu = \frac{U_{1o}^2}{Q_{1o}}}$$

Where P_{1o} and Q_{1o} are the active and reactive powers consumed during the open circuit test.

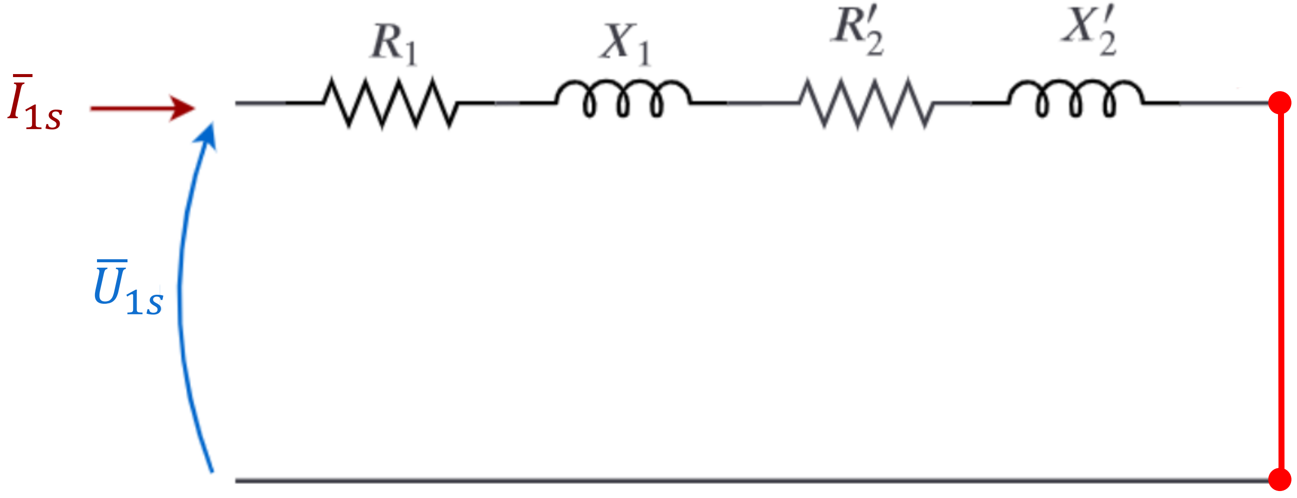
Short circuit test



Simplification: $R_1, R_2' \ll R_{H+F}$ & $X_1, X_2' \ll X_\mu$ ➔ R_{H+F} and X_μ can be ignored.

⇨ Only R_1, R_2', X_1 and X_2' remain.

Short circuit test



$$Z = R_1 + j\,X_1 + R'_2 + j\,X'_2$$

$$= (R_1 + R'_2) + j\,(X_1 + X'_2)$$

$$\bar{U}_{1s} = Z\,\bar{I}_{1s}$$

$$S = \bar{U}_{1s}\,\bar{I}^{*}_{1s} = (R_1 + R'_2)\,I^2_{1s}$$

$$+j\,(X_1 + X'_2)\,I^2_{1s}$$

$$\Rightarrow \quad P_{1s} = (R_1 + R'_2)\,I^2_{1s} \quad \& \quad Q_{1s} = (X_1 + X'_2)\,I^2_{1s}$$

$$\Rightarrow \quad \boxed{R_1 + R'_2 = \frac{P_{1s}}{I^2_{1s}} \quad \& \quad X_1 + X'_2 = \frac{Q_{1s}}{I^2_{1s}}}$$

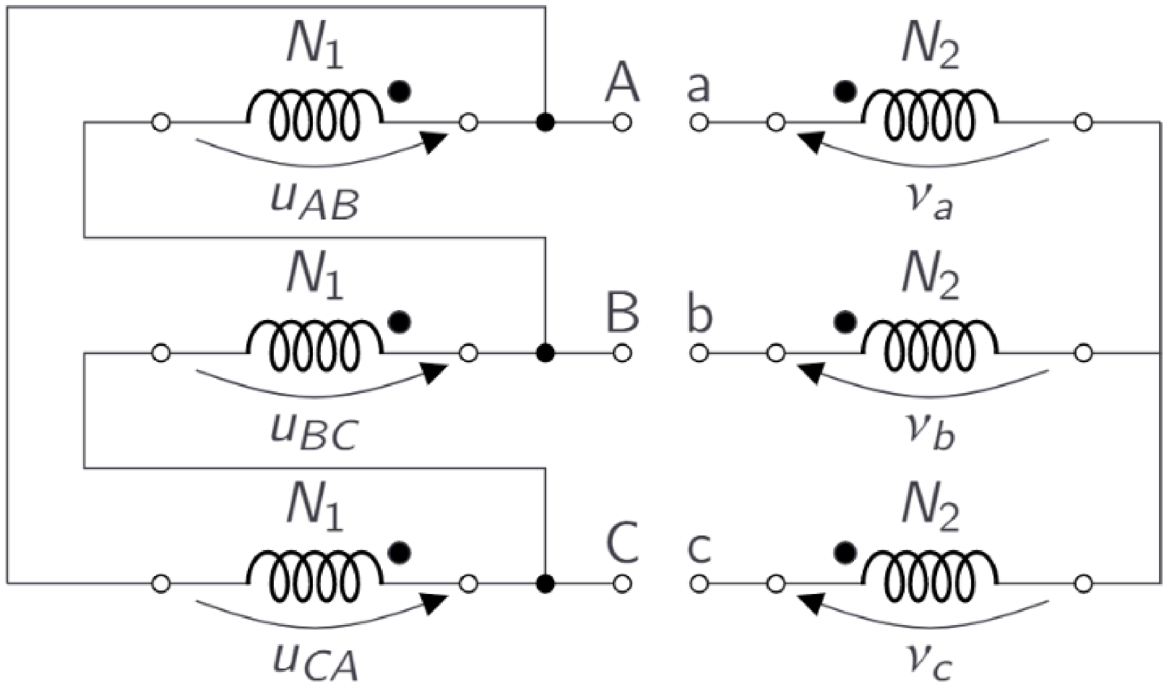
Where P_{1s} and Q_{1s} are the active and reactive powers consumed during the short circuit test.

Exercises

Exercise 9: Three phase transformer

Exercise 9: Three phase transformer

The three-phase transformer, described by the normalized scheme, is connected to a balanced three-phase network of composed voltages u_{AB}, u_{BC}, u_{CA} of RMS voltage U_1 on the primary side, whereas on the secondary side, a three-phase balanced system of composed voltages u_{ab}, u_{bc}, u_{ca} of RMS voltage U_2 is obtained.



Exercise 9: Three phase transformer

The line current intensities in the primary and secondary windings are respectively denoted I_1 and I_2. The transformer has the following characteristics:

- Apparent nominal power $S_n = 250$ kVA;
- Composed primary winding RMS voltages $U_{1n} = 5.2$ kV;
- Nominal frequency $f_n = 50$ Hz;

and ferromagnetic losses are neglected. To characterize the transformer two tests have been performed:

- Using open secondary windings, the transformer generates a composed voltage of RMS value $U_{2o} = 400$ V at each secondary winding, for an applied composed nominal voltage of RMS value U_{1n};
- Using short-circuited secondary windings, a composed voltage of RMS value $U_{1s} = 600$ V is applied at each primary winding for a total primary power P =7.35 kW, producing line current of RMS intensity $I_{2s} = 350$ A.

Exercise 9: Three phase transformer

1. Calculate the transformer ratio n so that it is greater than 1.

    In the statement:

    - Composed primary winding RMS voltages $U_{1n} = 5{,}2\ kV$
    - Output RMS voltage in open circuit $U_{2o} = 400\ V$

2. For the first test condition (open secondary windings), draw a Fresnel diagram including the primary composed voltages u_{AB}, u_{BC}, u_{CA}, the direct secondary voltage v_a, v_b, v_c and the secondary composed voltages u_{ab}, u_{bc}, u_{ca}.

3. Express and compute the column ratio $n_c = \frac{N_1}{N_2}$ according to n.

Exercise 9: Three phase transformer

4. Given that the transformer is composed of three cores of section $A_c = 5\,dm^2$, and that the magnetic field amplitude is $B_m = 1.2\,T$, compute the number of turns N_1 of each primary winding and deduce the value of the number of turns of each winding N_2.

In the statement:

- Composed primary winding RMS voltages $U_{1n} = 5{,}2\,kV$
- Nominal frequency $f_n = 50\,Hz$

Exercise 9: Three phase transformer

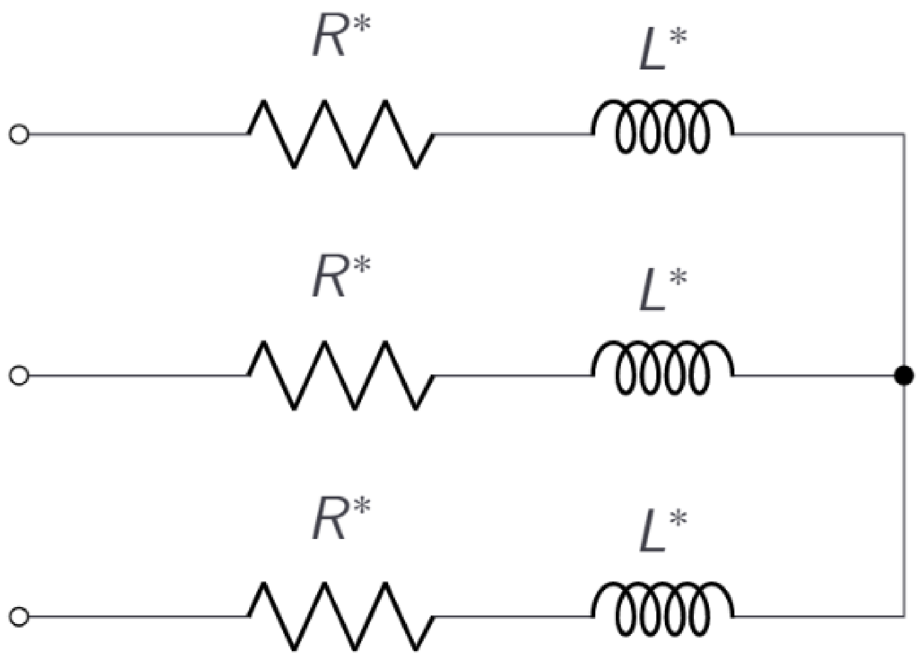
5. Using a simple single-phase equivalent model (leak resistance and inductance moved to the secondary windings), provide the Thevenin's model seen from a secondary winding and calculate the resistance $R_{eq}(R')$ and the reactance $X_{eq}(X')$ of this model.

In the statement:

- Short-circuit input voltages $U_{1s} = 600\,V$
- Short-circuit total primary power $P = 7{,}35\,kW$
- Short-circuit secondary line current $I_{2s} = 350\,A$

Exercise 9: Three phase transformer

The nominal regime is now considered by applying the composed nominal voltage U_{1n} at the primary windings and connecting a three-phase balanced load on the secondary side (detailed in 2). Each branch is composed of a resistor of value R^* =554 mΩ in series with a coil of value $L^* = 3.05$ mH.



Exercise 9: Three phase transformer

6. Calculate the power factor $\cos(\varphi_2)$ of this load.

In the statement: $R^* = 554\, m\Omega$ and $L^* = 3.05\, mH$

7. Draw the Fresnel diagram corresponding to the balanced single-phase equivalent model. Deduce the RMS values of the current intensities I_2 and the composed voltages U_2.

In the statement: Composed primary winding RMS voltages $U_{1n} = 5{,}2\, kV$

8. Compute the power P_2 flowing from the transformer to the load.

9. Calculate the transformer efficiency η.

Exercise 9: Three phase transformer

10. Another load is used. Compute the value of the resistance $R°$ and of the inductance $L°$ such that this load is equivalent to the previous one.