Electromagnetic Energy Conversion
ELEC0431

Exercise session 4:
Three-phase transformers

28 February 2025

Florent Purnode (florent.purnode@uliege.be)

Montefiore Institute, Department of Electrical Engineering and Computer Science,
University of Liège, Belgium

Reminder: Laboratories – Schedule and groups

To create the lab schedule, you are required to fill **by group** the doodle https://framadate.org/LabsELEC0431:

- By group of **4 students**, select **6** available time slots
  (**you may be given random sessions if less than 6 time slots were selected**).
- In the space provided for names, write the **STUDENT ID NUMBERS** of all **4 MEMBERS** of the group
  (for example: "s161514, s171856, s164442, s179088").
- A time slot can be selected by maximum six groups, **do not delay in completing this Doodle**.
- In addition to your selected time slots, your schedule could include laboratory sessions on the 18/04, 25/04, 9/05 and 16/05 mornings (Friday mornings in place of the traditional classes).

IF AND ONLY IF it is impossible for you to create a group of four students meeting the requirements, send me an email (florent.purnode@uliege.be) without delay.

> Make sure to complete the doodle by **23:59 on Friday, February 28td** (**that's tonight!**)
> (Students who would not have given their availabilities by this time will be given random time slots).

Quick reminder: Laboratories are **mandatory**
(In case of unexcused absence, an absence grade will be given for the entire course).

Reminder: Test on phasors

- The two first exercise sessions are dedicated to the introduction of phasors.

  Phasors will be used throughout the class. **Being able to work with them effortlessly is a must!**

- A **test**, focusing on the material seen during the two first exercise sessions is scheduled on **Friday 7th of March**,

  (that's next week!) at 9:00 am.

  It takes place in the usual classroom (B37 auditorium 02).

  It is a quick test of 30 minutes (we insist on the fact you should be able to manipulate phasors effortlessly).

  To take the test, you will need your calculator, ruler and protractor.

- At each exercise sessions, homeworks are available to prepare yourself for the test and the exam.

  The last-year test and past exams are also available on the webpage.

- The test accounts for 5 % of the final grade (**5 % for the test, 15 % for the laboratories ➔ 80 % for the exam**).

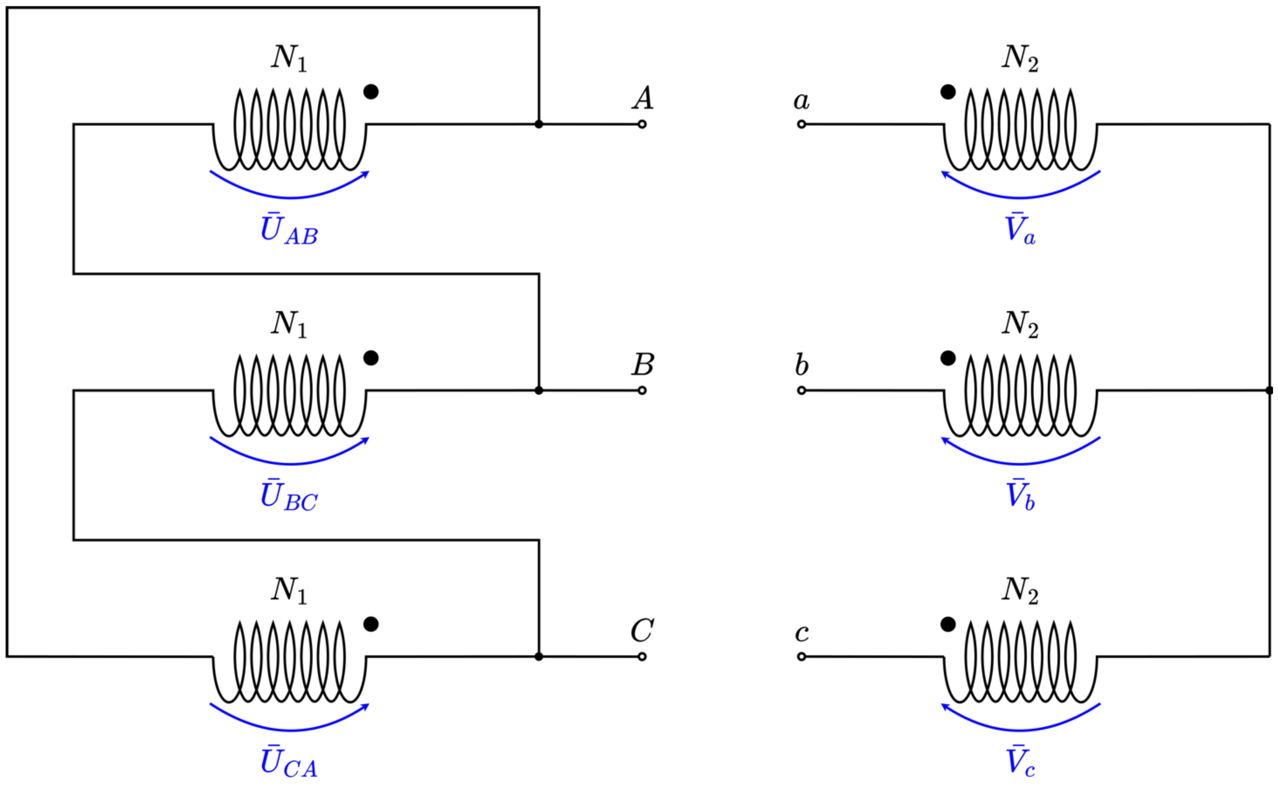
- The test will be followed by its correction and a normal exercise session.

In this class…

- Exercise 8

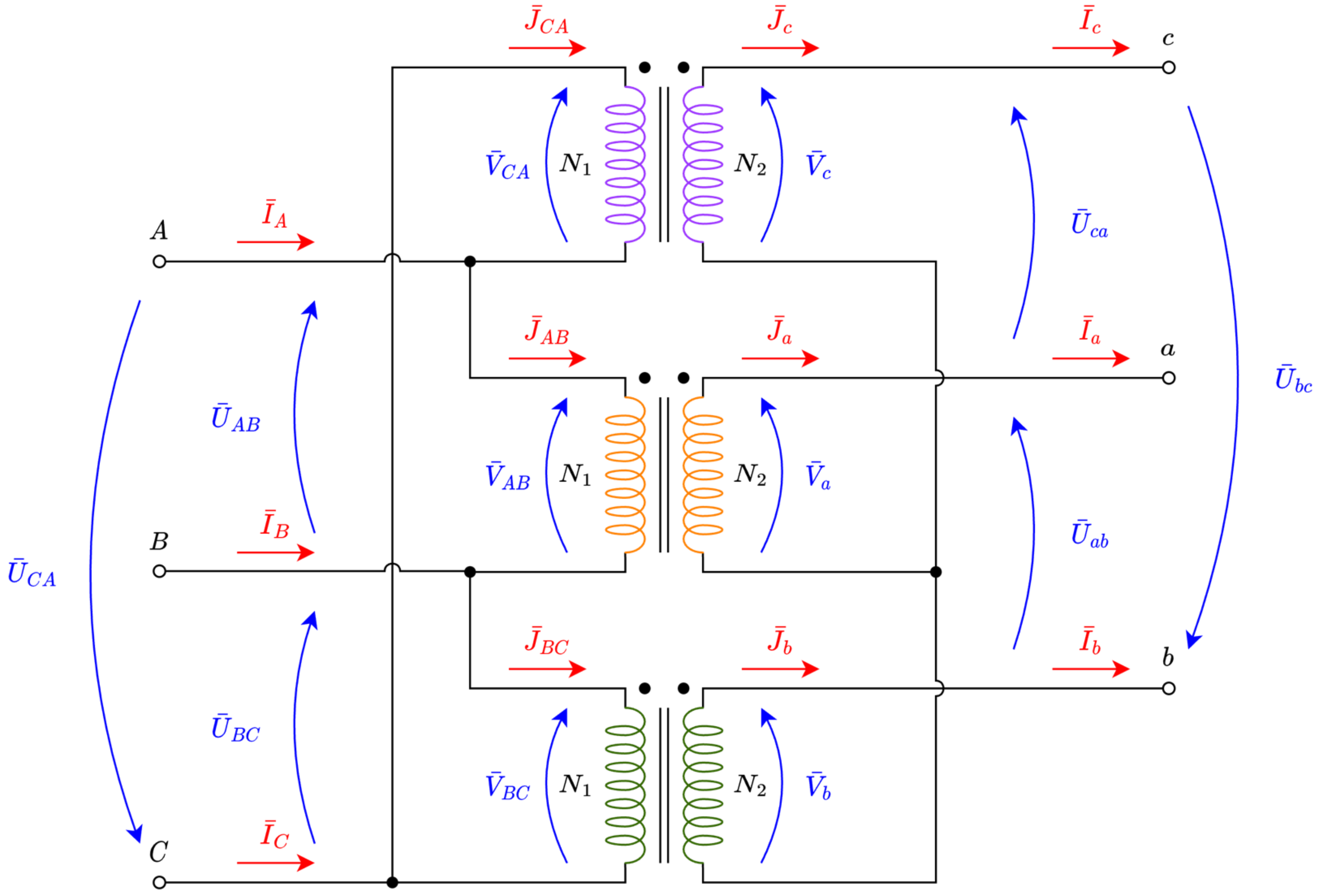
Exercise 8

Three-phase power transformers are commonly used to adapt power line voltages and to provide some galvanic insulation between two parts of an electrical grid. The three-phase transformer, described by the normalized scheme hereunder, is connected to a balanced three-phase network of line voltages $\overline{U}_{AB}$, $\overline{U}_{BC}$ and $\overline{U}_{CA}$ of RMS voltage U_1 on the primary side, whereas on the secondary side, a three-phase balanced system of line voltages $\overline{U}_{ab}$, $\overline{U}_{bc}$ and $\overline{U}_{ca}$ of RMS voltage U_2 is obtained. The line current intensities in the primary and secondary windings are respectively denoted I_1 and I_2.



Exercise 8

Another way to draw the circuit:



- Each winding at the primary is linked to a winding at the secondary.
- When the system is balanced, one can solve it by considering only one phase.
- We define:
  - The column ratio n_c. That is the ratio between primary and secondary **phase** voltages/currents:

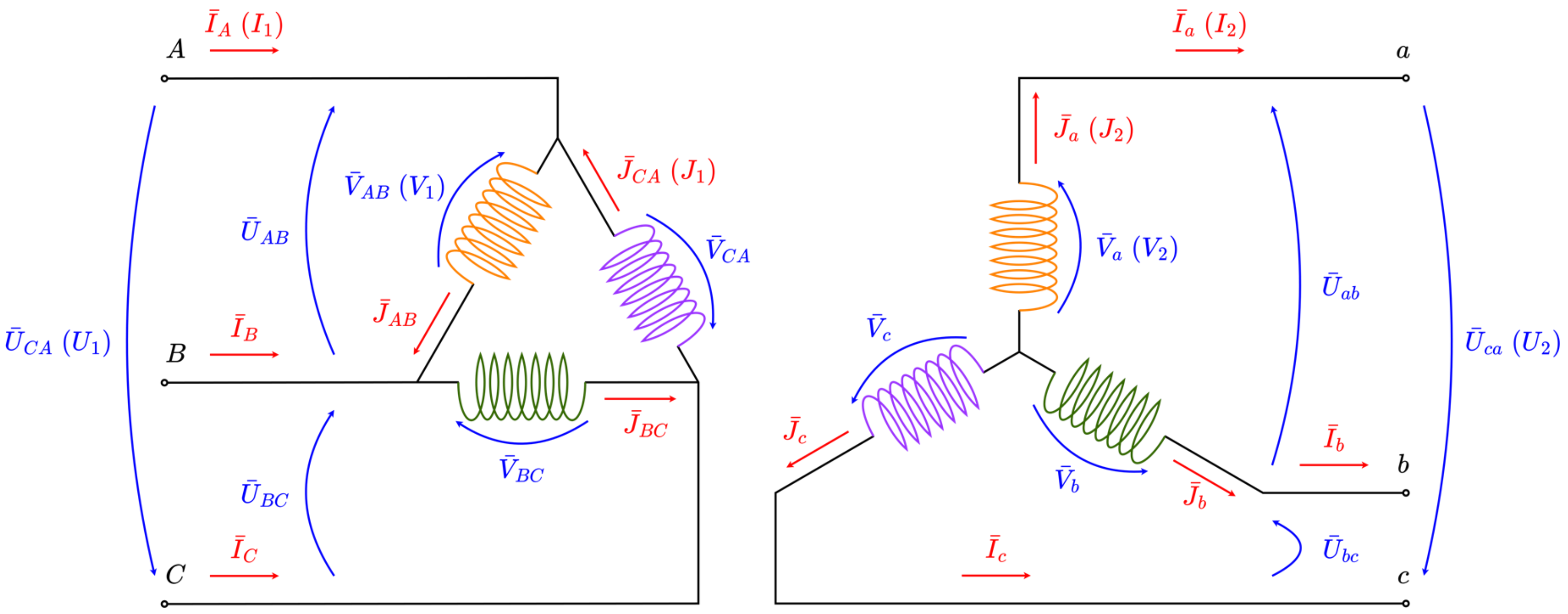
$$n_c = \frac{V_2}{V_1} = \frac{J_1}{J_2} = \frac{N_2}{N_1} \quad \text{or} \quad n_c = \frac{V_1}{V_2} = \frac{J_2}{J_1} = \frac{N_1}{N_2}.$$

  - The transformer ratio n. That is the ratio between primary and secondary **line** voltages/currents:

$$n = \frac{U_2}{U_1} = \frac{I_1}{I_2} \quad \text{or} \quad n = \frac{U_1}{U_2} = \frac{I_2}{I_1}.$$

Exercise 8

Another way to draw the circuit:



One can pass from the column ratio n_c to the transformer ratio n (and inversely) if the primary and secondary configurations (delta or star configurations) are known.

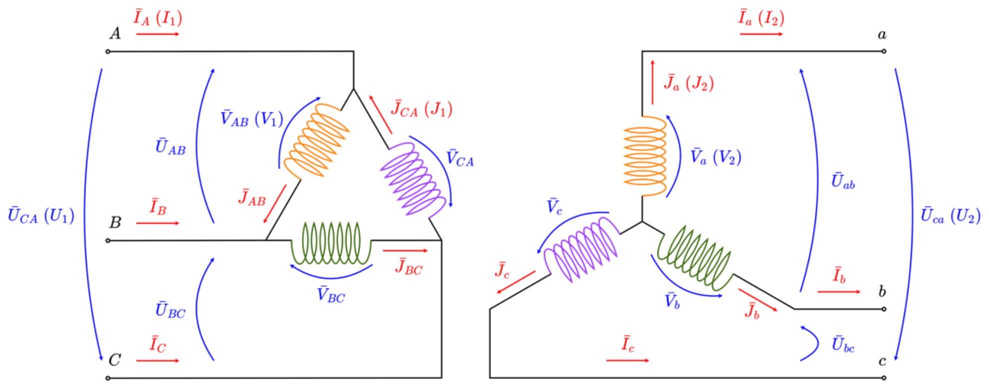
Here, with a primary delta configuration and secondary star configuration (defining $n_c = \frac{N_2}{N_1}$):

$$n_c = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{U_2/\sqrt{3}}{U_1} = \frac{n}{\sqrt{3}}$$

Exercise 8

Transformer characteristics:

- Apparent nominal power $|S_n| = 250$ kVA
- Line primary-winding nominal RMS voltages $U_{1n} = 5.2$ kV
- Nominal frequency $f_n = 50$ Hz



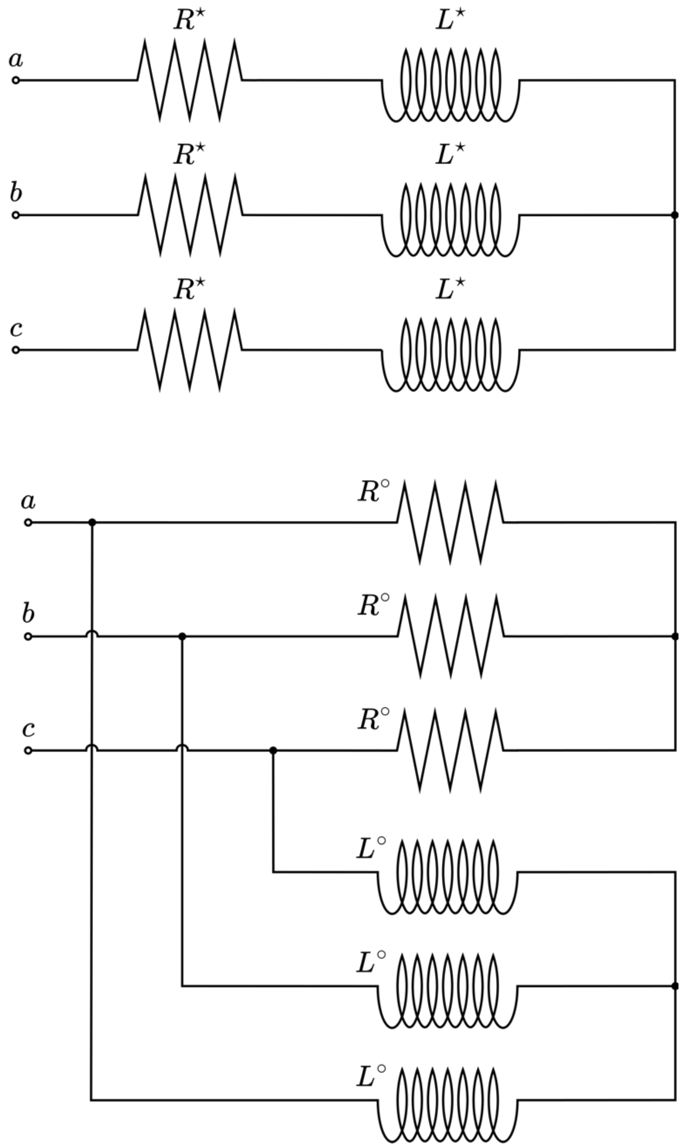
Two tests have been performed to characterize the transformer:

- With open secondary windings, $U_{2o} = 400$ V & $U_{1o} = U_{1n}$.
- With short-circuited secondary windings, $U_{1s} = 600$ V, $P_{3\varphi} = 7.35$ kW & $I_{2s} = 350$ A.

1. Calculate the transformer ratio n so that it is greater than 1.
2. For the first test condition (open circuit), draw a phasor diagram including the primary line voltages, the secondary phase voltages and the secondary line voltages.
3. Express and compute the column ratio n_c according to the transformer ratio n.
4. The transformer is composed of 3 cores of section $A_c = 5\ dm^2$ and the magnetic field amplitude is $B_m = 1.2$ T. Compute the number of turns of each primary winding and the number of turns of each secondary winding.
5. Draw a single-phase equivalent model of the transformer, with impedances gathered and moved to the secondary windings, and calculate the resistance R' and the reactance X'.

Exercise 8

The nominal regime is now considered by applying the line nominal voltage U_{1n} at the primary windings and connecting a three-phase balanced load on the secondary side (detailed here on right). Each branch is composed of a resistor of value $R^\star = 554\ m\Omega$ in series with a coil of value $L^\star = 3.05\ mH$.



6. Calculate the power factor $\cos(\varphi_2)$ of this load.
7. Considering the single-phase equivalent model of point 5., draw a phasor diagram showing the phase voltages $\bar{V}_1$ and $\bar{V}_2$ and the phase current $\bar{J}_2$.
8. Deduce the line current I_2 and the line voltage U_2.
9. Compute the active power P_2 flowing from the transformer to the load.
10. Considering no iron loss, find the transformer efficiency η.
11. Another load is used, compute the value of the resistance R° and the inductance L° such that this load is equivalent to the one detailed previously.

Homework 17

In Belgium, most of the railways are powered using DC 3 kV voltage. High speed train lines are however supplied with AC 25 kV 50 Hz (single-phase) voltage, requiring the use of high-power single-phase transformers. In this exercise, such a transformer is considered. A nominal RMS voltage of $U_{1n} = 25$ kV with nominal frequency $f = 50$ Hz is supplied to the primary winding with an apparent power $|S_n| = 5.6$ MVA.

To characterize the transformer two tests have been performed:

- Using open secondary winding with the nominal voltage applied to the primary, the transformer generates a voltage $U_{2o} = 1.36$ kV at the secondary winding, for a current drawn at the primary $I_{1o} = 1.25$ A, and a consumed active power $P_{1o} = 6.8$ kW.
- Using short-circuited secondary winding, the transformer consumes an active power $P_{1s} = 25$ kW, considering that a reduced voltage of 10.1 % of U_{1n} was applied to the primary winding to maintain the secondary winding current to its nominal value I_{2n}.

1. Calculate the transformer ratio n.
2. Determine the nominal RMS secondary current I_{2n} and primary current I_{1n} (assuming an ideal transformer).
3. Compute the power factor $\cos\varphi_{1o}$ for the first test (open secondary winding) and deduce the phase shift φ_{1o} of the current at the primary winding with respect to the primary winding voltage.
4. Give the reactive power Q_{1o} for the first test (open secondary winding).

Homework 17 – Cont'd

5. Considering the model of a transformer with impedances moved and gathered at the secondary, calculate the resistance R'_{H+F} and the magnetizing inductance L'_{μ}.
6. For the second test (shorted secondary winding), compute the primary winding voltage U_{1s}, and calculate the values of the resistance R' and of the inductance L' of the equivalent model.
7. Compare the R' and $X' = 2\pi f L'$, and propose a simplified version of the transformer equivalent model.

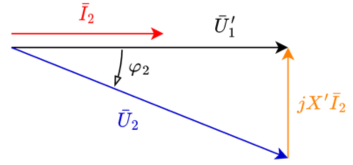
The nominal regime is now considered by applying the nominal voltage U_{1n} at the primary winding and connecting a load at the secondary winding, drawing an RMS current $I_2 = 4.097$ kA with a power factor $\cos\varphi_2$, the current being ahead on the voltage. The current $\bar{I}_2$ in the secondary winding is aimed to be in phase with the voltage $\bar{U}_1$ of the primary winding.

8. Draw a phasor diagram showing the secondary voltage $\bar{U}_2$, the primary voltage seen from the secondary $\bar{U}'_1$, the secondary current $\bar{J}_2$, and the voltage drop across X'.
9. Compute the phase shift φ_2 of the current $\bar{I}_2$ with respect to $\bar{U}_2$, and deduce the load power factor $\cos\varphi_2$.
10. Compute the RMS voltage value U_2 appearing at the secondary winding.

Homework 17 – Cont'd

11. Compute the reactive power Q_2 drawn by the load at the secondary winding, and the reactive power Q_1 at the primary winding.
12. Compute the active power P_2 drawn by the load at the secondary winding, and the active power P_1 at the primary winding. Do not neglect the resistance R'.
13. Compute the transformer efficiency η.

Answers:

1. n = 18.38
2. $I_{2n} = 4117.12$ A, $I_{1n} = 224$ A
3. $\cos\varphi_{1o} = 0.2176$, $\varphi_{1o} = 77.43°$
4. $Q_{1o} = 30\,501$ var
5. $R'_{H+F} = 272\,\Omega$, $X'_\mu = 60.64\,\Omega$, $L'_\mu = 193$ mH
6. $U_{1s} = 2525$ V, $R' = 1.475\,m\Omega$, $X' = 33.34\,m\Omega$, $L' = 106.12\,\mu$H
7. $R' \ll X' \rightarrow R'$ can be neglected
8. 

9. $\varphi_2 = -5.73°$, $\cos\varphi_2 = 0.995$
10. $U_2 = 1367$ V
11. $Q_2 = -559\,168$ var, $Q_1 = 30\,967$ var
12. $P_2 = 5\,572.6$ kW, $P_1 = 5\,604.2$ kW
13. $\eta = 99.43$ %