Electromagnetic Energy Conversion ELEC0431

Exercise session 4: Three-phase transformers

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Reminder: Test on phasors

- The two first exercise sessions are dedicated to the introduction of phasors.

  Phasors will be used throughout the class. Being able to work with them effortlessly is a must!

- A **test**, focusing on the material seen during the two first exercise sessions is scheduled on **Friday 6th of March**, at 9:00 am.

  It takes place in the usual classroom (B37 auditorium 02).

  It is a quick test of 30 minutes (we insist on the fact you should be able to manipulate phasors effortlessly).

  To take the test, you will need your calculator, ruler and protractor (“rapporteur” in French).

- At each exercise sessions, homeworks are available to prepare yourself for the test and the exam.

  A past test and past exams are also available on the webpage.

- The test accounts for 5 % of the final grade.

! Next week !

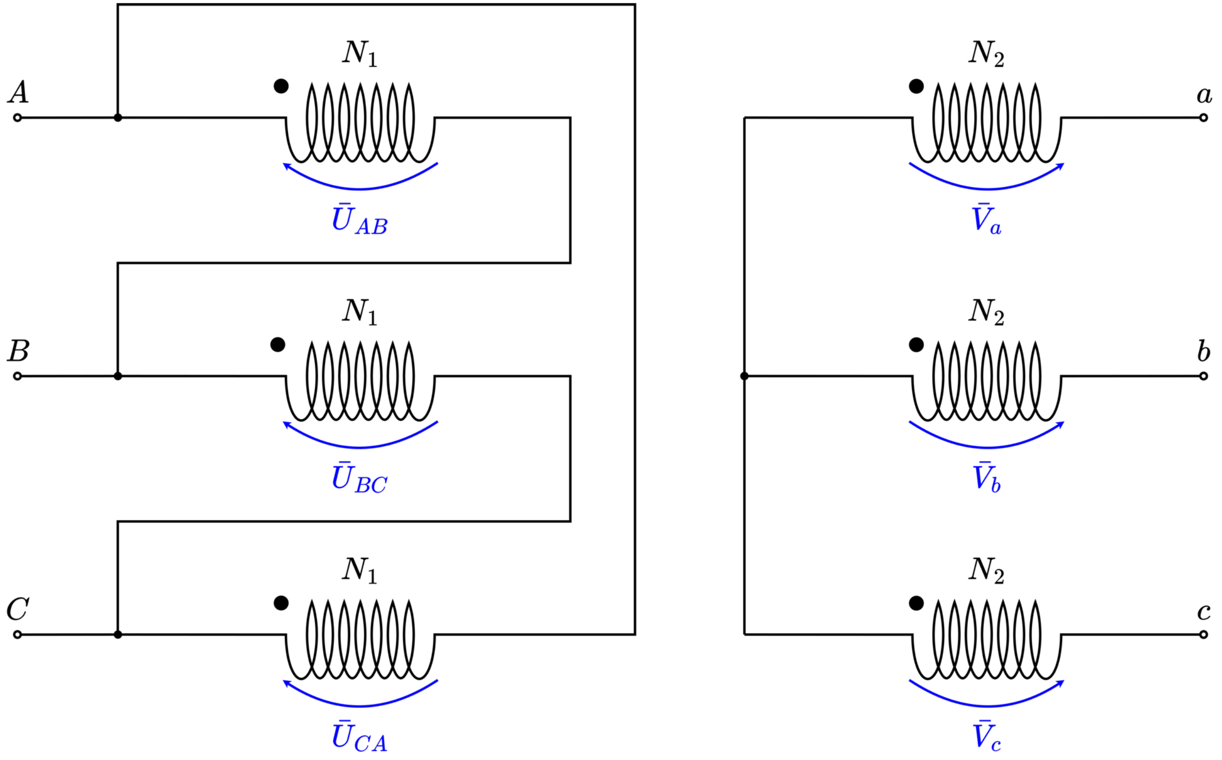
- The test will be followed by its correction and a normal exercise session.

In this class...

- Exercise 8

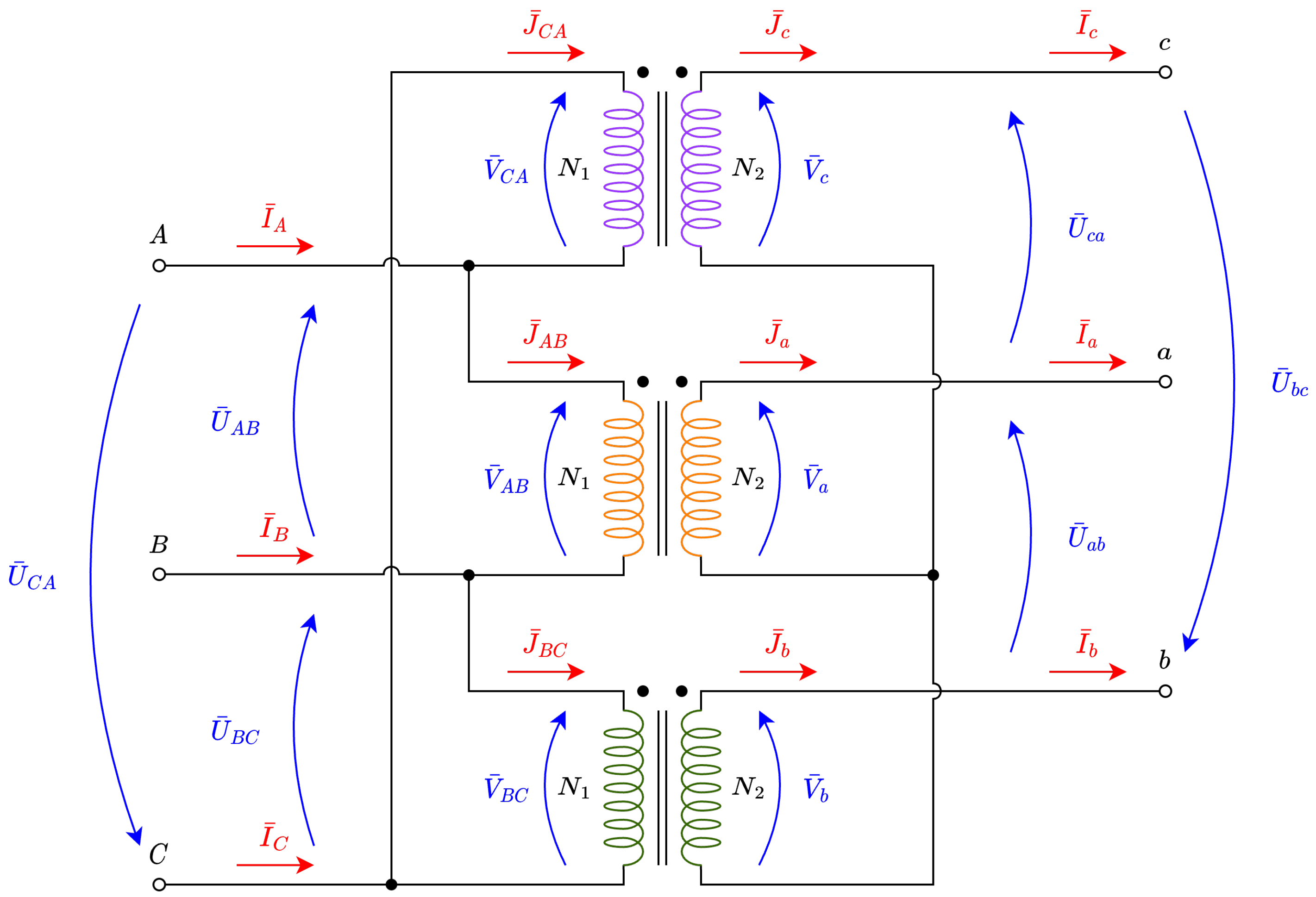
Exercise 8

Three-phase power transformers are commonly used to adapt power line voltages and to provide some galvanic insulation between two parts of an electrical grid. The three-phase transformer, described by the normalized scheme hereunder, is connected to a balanced three-phase network of line voltages $\overline{U}_{AB}$, $\overline{U}_{BC}$ and $\overline{U}_{CA}$ of RMS voltage U_1 on the primary side, whereas on the secondary side, a three-phase balanced system of line voltages $\overline{U}_{ab}$, $\overline{U}_{bc}$ and $\overline{U}_{ca}$ of RMS voltage U_2 is obtained. The line current intensities in the primary and secondary windings are respectively denoted I_1 and I_2.



Exercise 8

Another way to draw the circuit:



- Each winding at the primary is linked to a winding at the secondary.
- When the system is balanced, one can solve it by considering only one phase.
- We define:
  - ➢ The column ratio n_c. That is the ratio between primary and secondary **phase** voltages/currents:

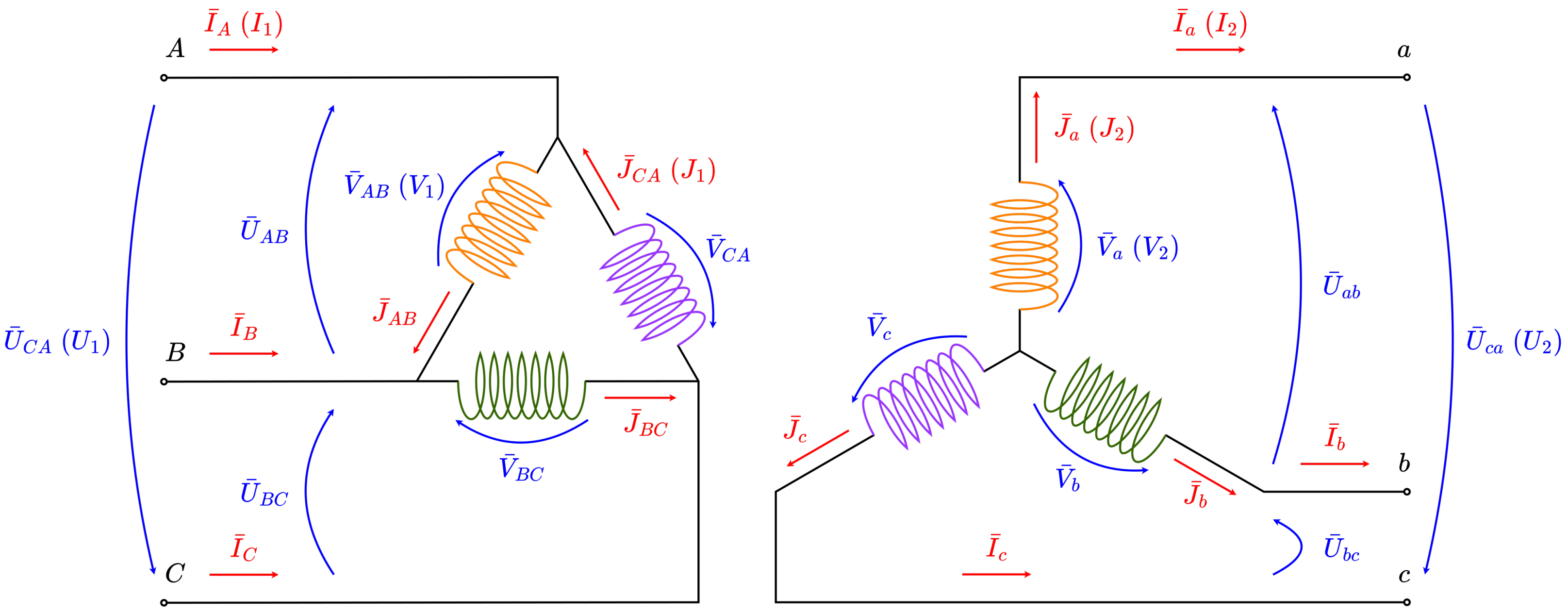
$$n_c = \frac{V_2}{V_1} = \frac{J_1}{J_2} = \frac{N_2}{N_1} \quad \text{or} \quad n_c = \frac{V_1}{V_2} = \frac{J_2}{J_1} = \frac{N_1}{N_2}.$$

  - ➢ The transformer ratio n. That is the ratio between primary and secondary **line** voltages/currents:

$$n = \frac{U_2}{U_1} = \frac{I_1}{I_2} \quad \text{or} \quad n = \frac{U_1}{U_2} = \frac{I_2}{I_1}.$$

Exercise 8

Another way to draw the circuit:



One can pass from the column ratio n_c to the transformer ratio n (and inversely) if the primary and secondary configurations (delta or star configurations) are known.

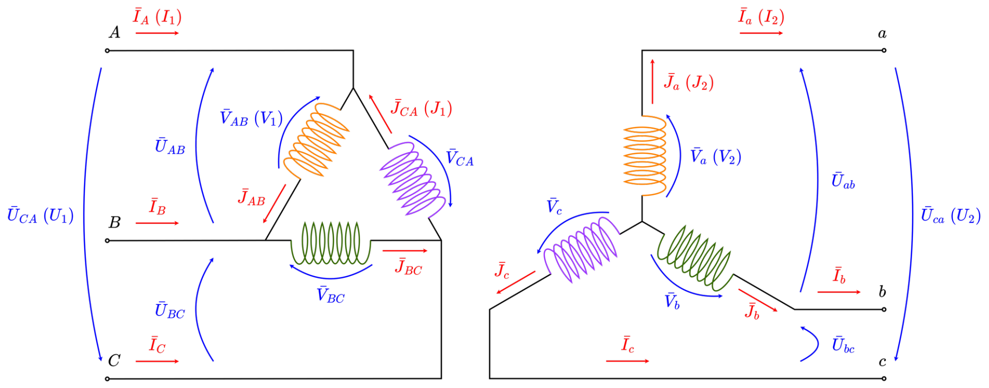
Here, with a primary delta configuration and secondary star configuration (defining $n_c = \frac{N_2}{N_1}$):

$$n_c = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{U_2/\sqrt{3}}{U_1} = \frac{n}{\sqrt{3}}$$

Exercise 8

Transformer characteristics:

- Apparent nominal power $|S_n| = 250$ kVA
- Line primary-winding nominal RMS voltages $U_{1n} = 5.2$ kV
- Nominal frequency $f_n = 50$ Hz



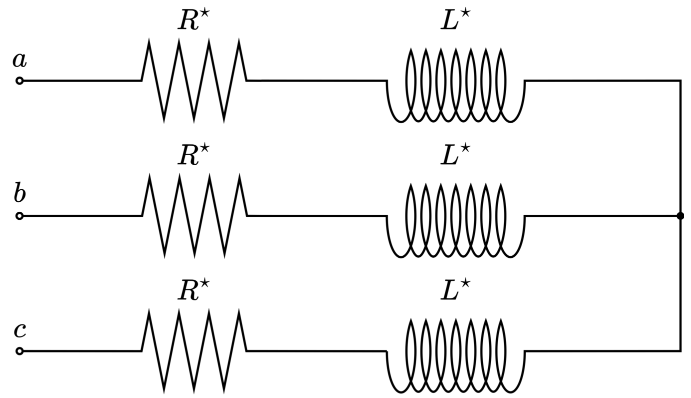
Two tests have been performed to characterize the transformer:

- With open secondary windings, $U_{2o} = 400$ V & $U_{1o} = U_{1n}$.
- With short-circuited secondary windings, $U_{1s} = 600$ V, $P_{3\varphi} = 7.35$ kW & $I_{2s} = 350$ A.

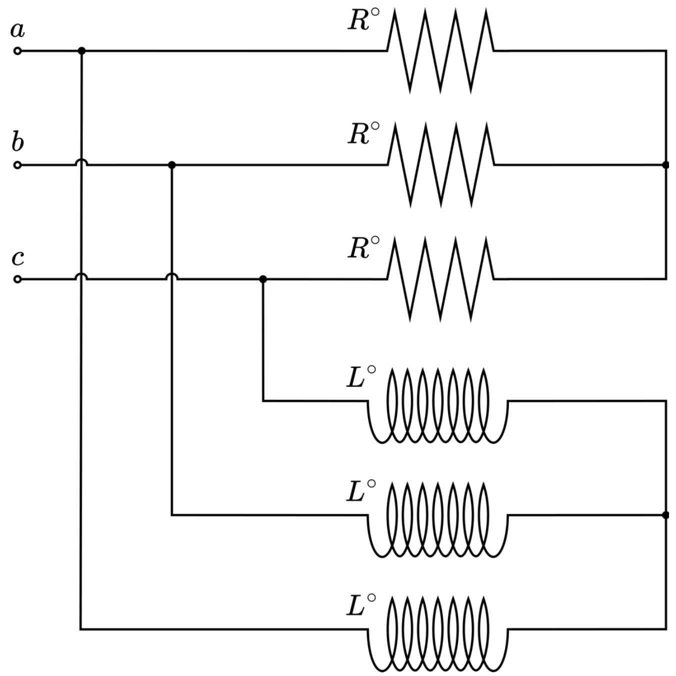
1. Calculate the transformer ratio n so that it is greater than 1.
2. For the first test condition (open circuit), draw a phasor diagram including the primary line voltages, the secondary phase voltages and the secondary line voltages.
3. Express and compute the column ratio n_c according to the transformer ratio n.
4. The transformer is composed of 3 cores of section $A_c = 5\, dm^2$ and the magnetic field amplitude is $B_m = 1.2$ T. Compute the number of turns of each primary winding and the number of turns of each secondary winding.
5. Draw a single-phase equivalent model of the transformer, with impedances gathered and moved to the secondary windings, and calculate the resistance R' and the reactance X'.

Exercise 8

The nominal regime is now considered by applying the line nominal voltage U_{1n} at the primary windings and connecting a three-phase balanced load on the secondary side (detailed here on right). Each branch is composed of a resistor of value $R^{\star} = 554\,m\Omega$ in series with a coil of value $L^{\star} = 3.05\,mH$.



6. Calculate the power factor $\cos(\varphi_2)$ of this load.
7. Considering the single-phase equivalent model of point 5., draw a phasor diagram showing the phase voltages $\bar{V}_1$ and $\bar{V}_2$ and the phase current $\bar{J}_2$.
8. Deduce the line current I_2 and the line voltage U_2.
9. Compute the active power P_2 flowing from the transformer to the load.
10. Considering no iron loss, find the transformer efficiency η.
11. Another load is used, compute the value of the resistance R° and the inductance L° such that this load is equivalent to the one detailed previously.



Homework 17

In Belgium, most of the railways are powered using DC 3 kV voltage. High speed train lines are however supplied with AC 25 kV 50 Hz (single-phase) voltage, requiring the use of high-power single-phase transformers. In this exercise, such a transformer is considered. A nominal RMS voltage of $U_{1n} = 25$ kV with nominal frequency $f = 50$ Hz is supplied to the primary winding with an apparent power $|S_n| = 5.6$ MVA.

To characterize the transformer two tests have been performed:

- Using open secondary winding with the nominal voltage applied to the primary, the transformer generates a voltage $U_{2_o} = 1.36$ kV at the secondary winding, for a current drawn at the primary $I_{1_o} = 1.25$ A, and a consumed active power $P_{1_o} = 6.8$ kW.
- Using short-circuited secondary winding, the transformer consumes an active power $P_{1_s} = 25$ kW, considering that a reduced voltage of 10.1 % of U_{1n} was applied to the primary winding to maintain the secondary winding current to its nominal value I_{2n}.

1. Calculate the transformer ratio n.
2. Determine the nominal RMS secondary current I_{2n} and primary current I_{1n} (assuming an ideal transformer).
3. Compute the power factor $\cos\varphi_{1_o}$ for the first test (open secondary winding) and deduce the phase shift φ_{1_o} of the current at the primary winding with respect to the primary winding voltage.
4. Give the reactive power Q_{1_o} for the first test (open secondary winding).

Homework 17 – Cont'd

5. Considering the model of a transformer with impedances moved and gathered at the secondary, calculate the resistance R'_{H+F} and the magnetizing inductance L'_{μ}.
6. For the second test (shorted secondary winding), compute the primary winding voltage U_{1s}, and calculate the values of the resistance R' and of the inductance L' of the equivalent model.
7. Compare the R' and $X' = 2\pi f L'$, and propose a simplified version of the transformer equivalent model.

The nominal regime is now considered by applying the nominal voltage U_{1n} at the primary winding and connecting a load at the secondary winding, drawing an RMS current $I_2 = 4.097$ kA with a power factor $\cos\varphi_2$, the current being ahead on the voltage. The current $\bar{I}_2$ in the secondary winding is aimed to be in phase with the voltage $\bar{U}_1$ of the primary winding.

8. Draw a phasor diagram showing the secondary voltage $\bar{U}_2$, the primary voltage seen from the secondary $\bar{U}'_1$, the secondary current $\bar{J}_2$, and the voltage drop across X'.
9. Compute the phase shift φ_2 of the current $\bar{I}_2$ with respect to $\bar{U}_2$, and deduce the load power factor $\cos\varphi_2$.
10. Compute the RMS voltage value U_2 appearing at the secondary winding.

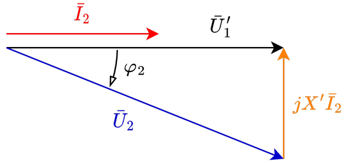
Homework 17 – Cont'd

11. Compute the reactive power Q_2 drawn by the load at the secondary winding, and the reactive power Q_1 at the primary winding.

12. Compute the active power P_2 drawn by the load at the secondary winding, and the active power P_1 at the primary winding. Do not neglect the resistance R'.

13. Compute the transformer efficiency η.

Answers:

1. $n = 18.38$
2. $I_{2n} = 4117.12$ A, $I_{1n} = 224$ A
3. $\cos\varphi_{1o} = 0.2176$, $\varphi_{1o} = 77.43°$
4. $Q_{1o} = 30\,501$ var
5. $R'_{H+F} = 272\ \Omega$, $X'_\mu = 60.64\ \Omega$, $L'_\mu = 193$ mH
6. $U_{1s} = 2525$ V, $R' = 1.475\ m\Omega$, $X' = 33.34\ m\Omega$, $L' = 106.12\ \mu$H
7. $R' \ll X' \rightarrow R'$ can be neglected
8. 

9. $\varphi_2 = -5.73°$, $\cos\varphi_2 = 0.995$
10. $U_2 = 1367$ V
11. $Q_2 = -559\,168$ var, $Q_1 = 30\,967$ var
12. $P_2 = 5\,572.6$ kW, $P_1 = 5\,604.2$ kW
13. $\eta = 99.43$ %