



# Electromagnetic Energy Conversion

## ELEC0431

### Exercise session 5: Synchronous machines

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- Three-phase synchronous generators (alternators)
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# Three-phase synchronous motors

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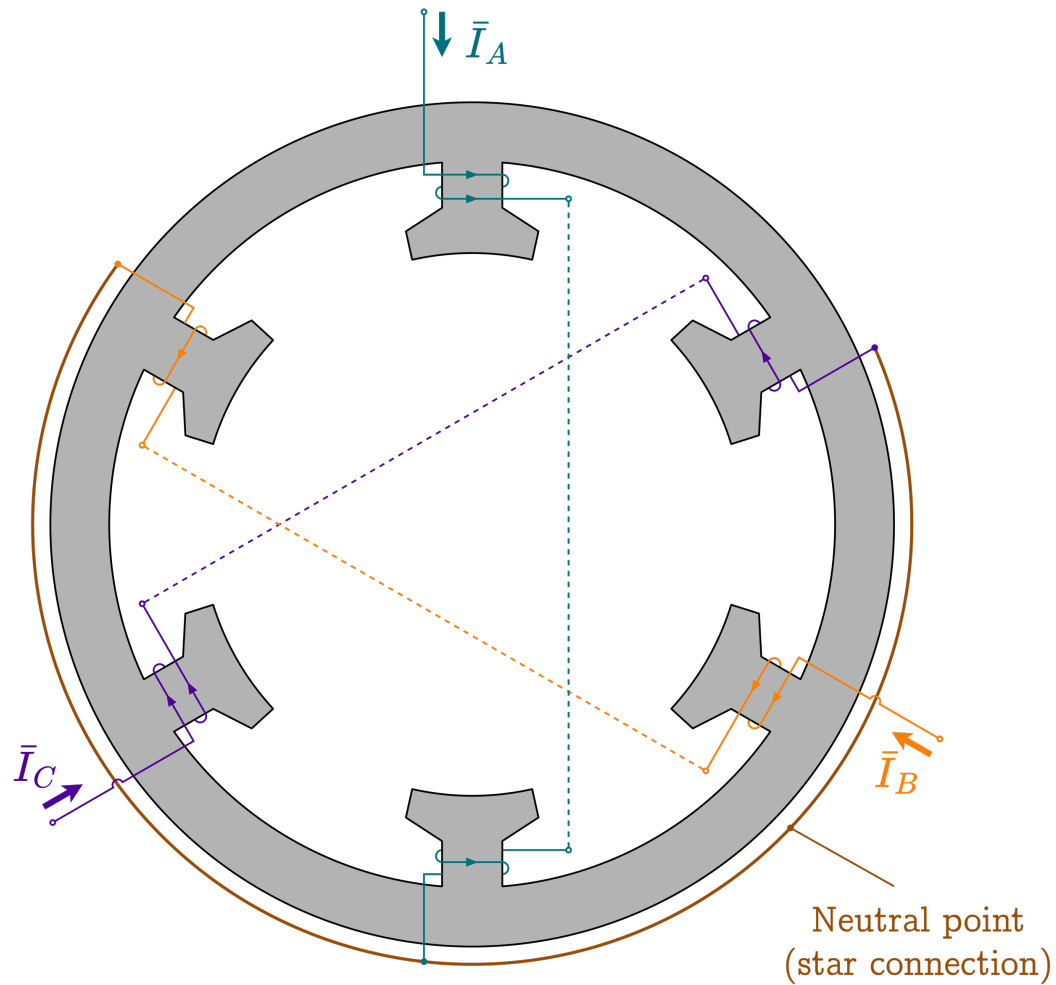
Stators in three-phase motors

Magnetic field in three-phase motors

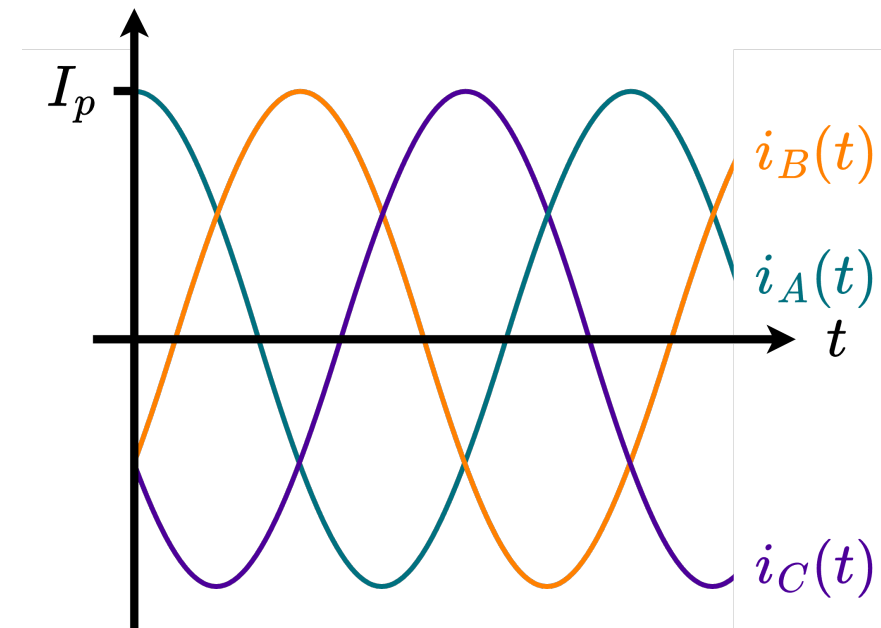
Three-phase synchronous motors

# Stators in three-phase motors

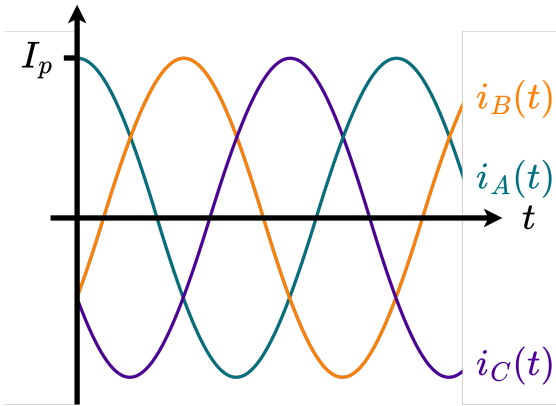
Let's consider a simple stator for a three-phase motor connected in a star configuration:



It is powered by a balanced three-phase electrical grid so that the currents  $\bar{I}_A$ ,  $\bar{I}_B$  and  $\bar{I}_C$  are out of phase by  $120^\circ$  and have equal peak amplitude  $I_p$ .



# Magnetic field in three-phase motors



At  $t = 0$ :

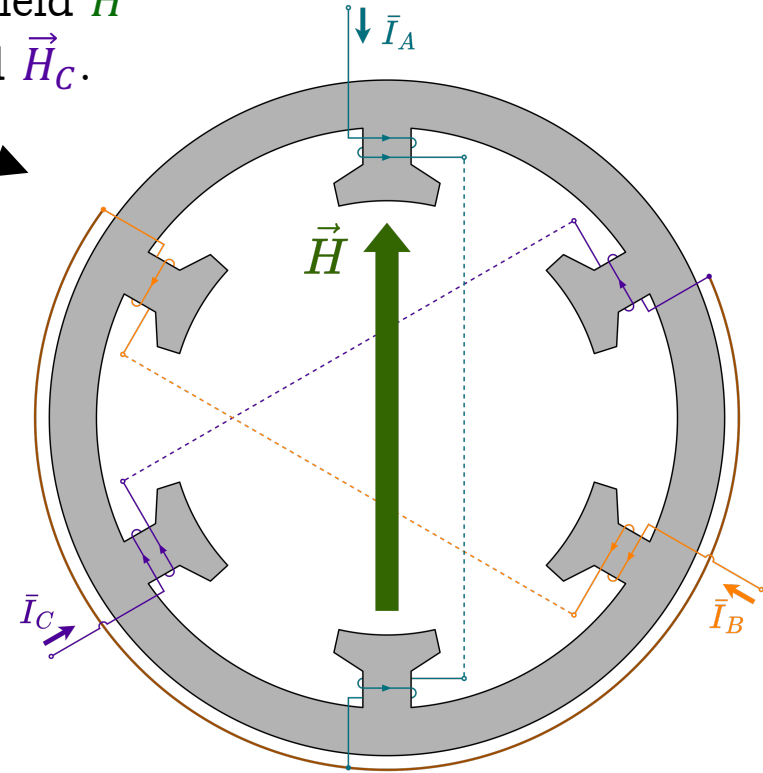
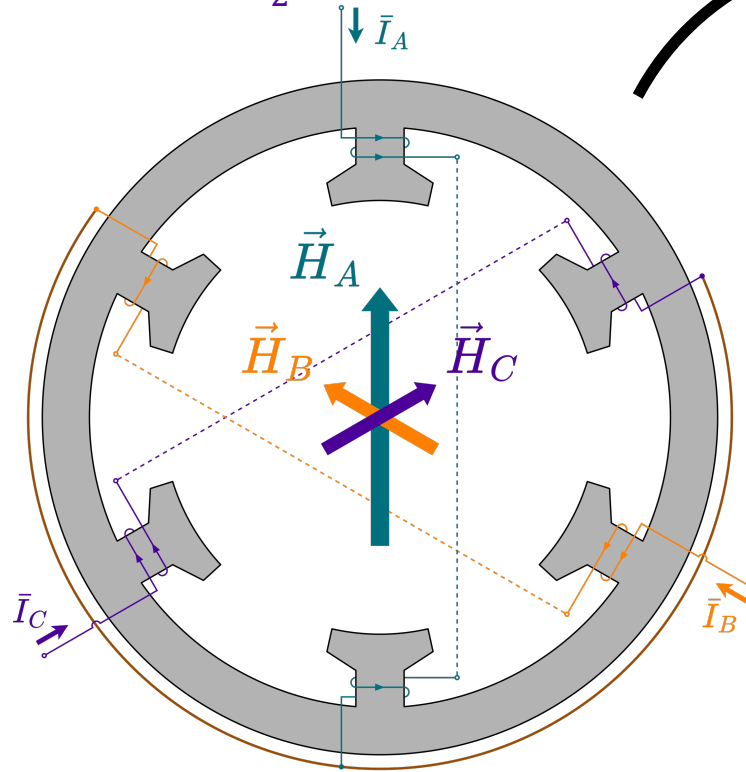
- $i_A(0) = I_p$
- $i_B(0) = -\frac{I_p}{2}$
- $i_C(0) = -\frac{I_p}{2}$

The resulting magnetic field  $\vec{H}$  is the sum of  $\vec{H}_A$ ,  $\vec{H}_B$  and  $\vec{H}_C$ .

By ampere's law, we know these currents will generate magnetic fields.

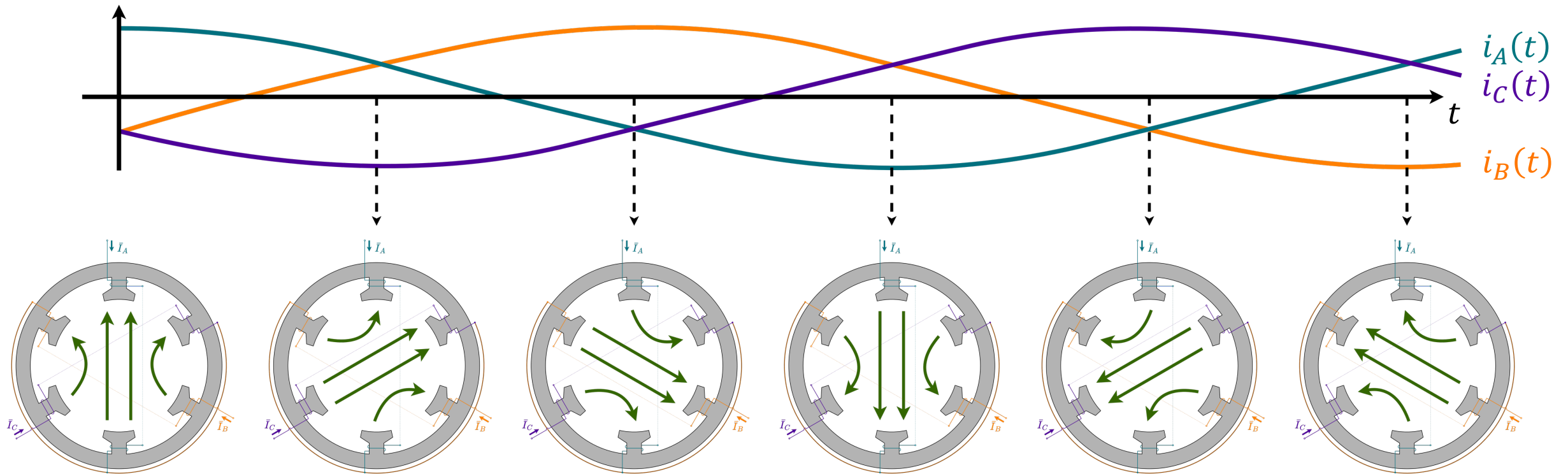
As a first approximation, we consider that

$i_A$  generates  $\vec{H}_A$ ,  
 $i_B$  generates  $\vec{H}_B$  and  
 $i_C$  generates  $\vec{H}_C$ .



How does  $\vec{H}$  varies with time ?

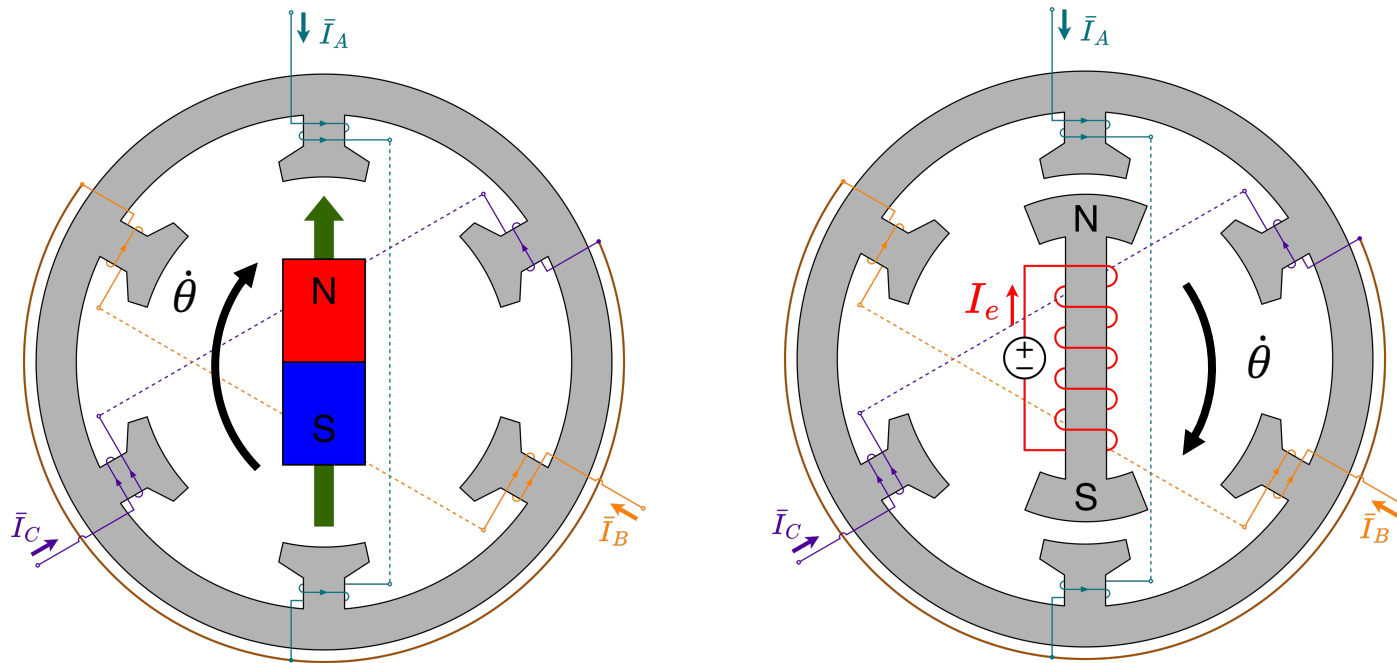
# Magnetic field in three-phase motors



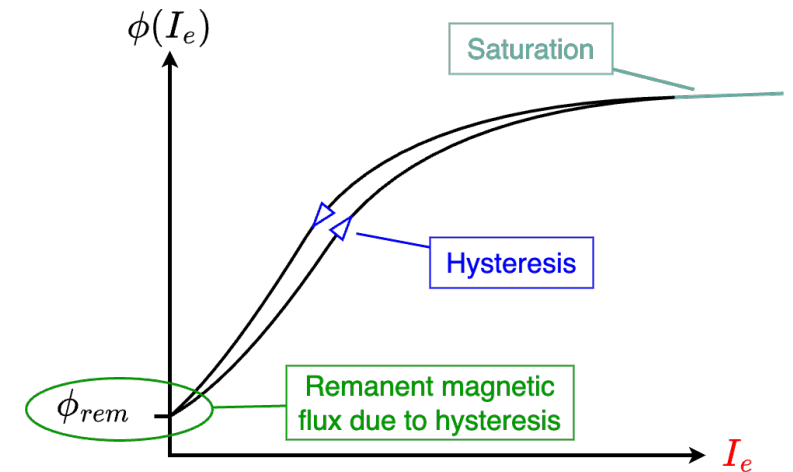
The magnetic field generated in the stator is constant in amplitude and rotates at the frequency of the three-phase power source.

# Three-phase synchronous motors

One can put a permanent magnet or an electromagnet in the stator. In both cases, they will align with the magnetic field and rotate at the same frequency as the three-phase power source → Synchronous motor



Note that electromagnets use ferromagnetic materials exhibiting hysteresis and saturation:



# Three-phase synchronous generators (alternators)

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Induced emf  $\bar{E}_v$

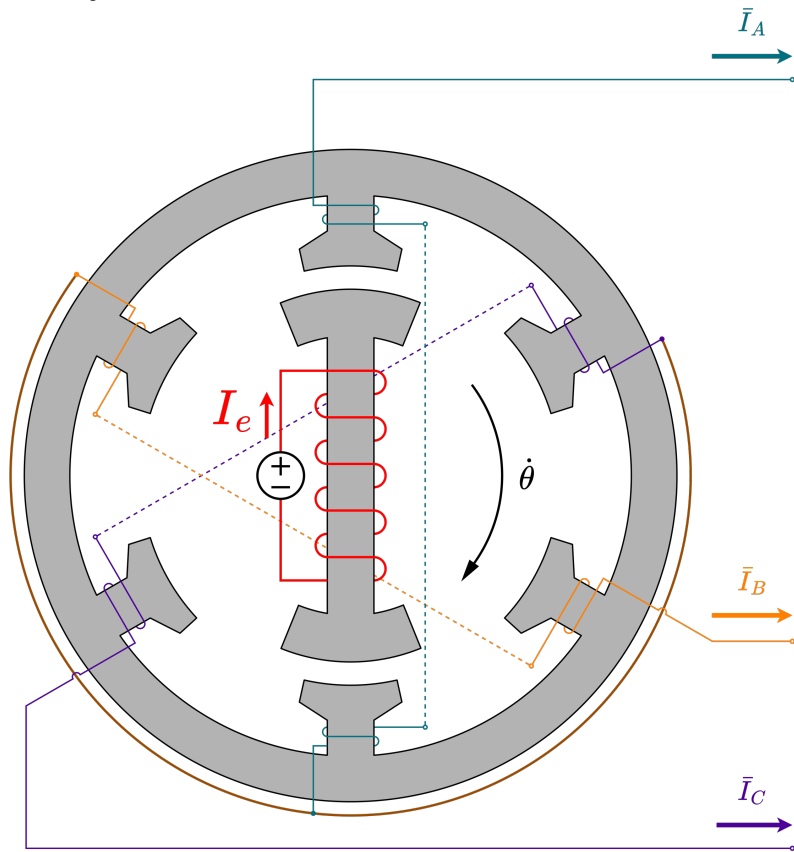
Behn-Eschenburg's model

Number of pairs of poles

Exercise 9

# Induced emf $\bar{E}_v$

Three-phase synchronous generators and three-phase synchronous motors are built similarly.



An electromagnet generates a flux  $\phi(I_e)$ , rotating with the shaft of the machine.

As the magnetic flux perceived by the stator windings varies, it produces an emf, whose amplitude is proportional to  $\frac{d\phi(I_e)}{dt}$ .

Since it rotates at a speed  $\dot{\theta}$ ,  $\frac{d\phi(I_e)}{dt}$  is proportional to  $\dot{\theta} \phi(I_e)$ .

The RMS amplitude  $E_v$  of the produced emf is thus given by:

$$E_v = k_e \dot{\theta} \phi(I_e),$$

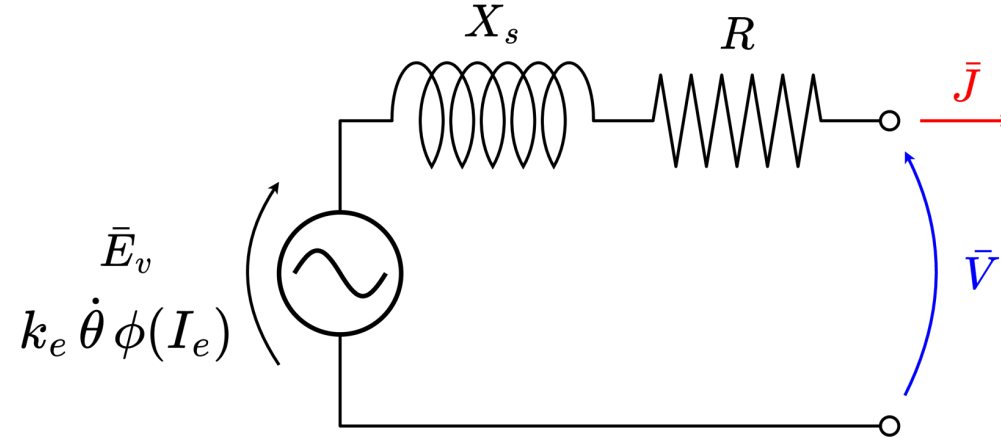
with  $k_e$  a scaling factor to determine.

# Behn-Eschenburg's model

The emf  $\bar{E}_v$  generated in one phase is not directly equal to the corresponding output phase voltage  $\bar{V}$ . This is due to:

- the stator leakage fluxes,
- the armature reaction,
- the wire resistance.

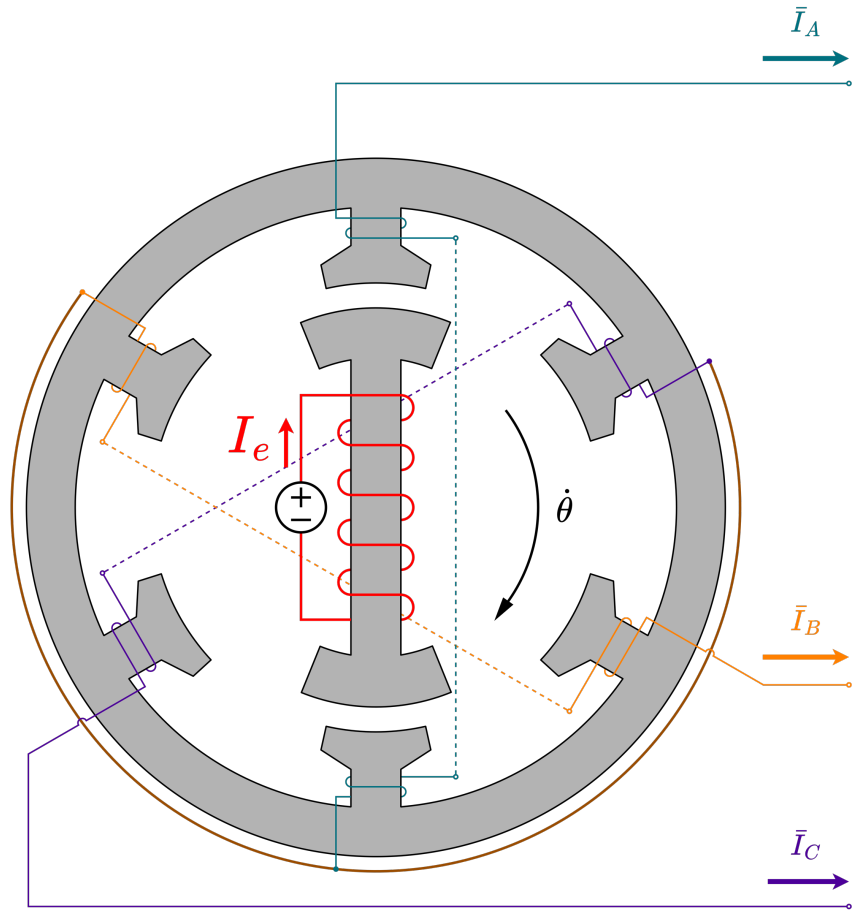
When the ferromagnetic materials are not saturated, we can account for these effects using the Behn-Eschenburg's model:



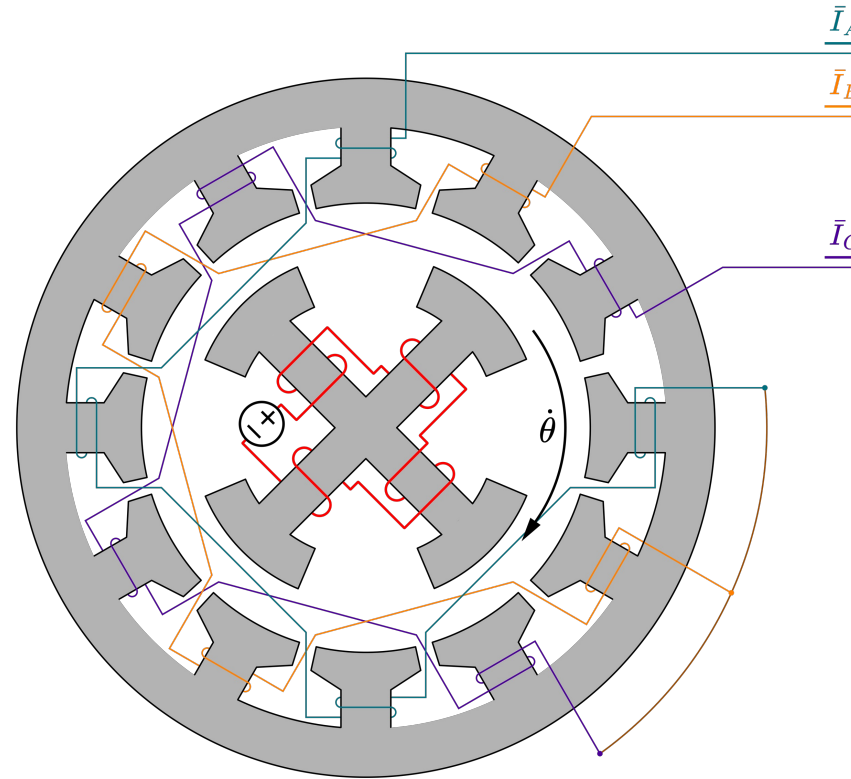
Note:

- Often,  $X_s \gg R$ .
- Without any load,  $\bar{J} = 0$  and  $\bar{E}_v = \bar{V} \rightarrow E_v$  is the no-load emf.

# Number of pairs of poles



Each phase has two poles  
→ One pair of poles ( $p = 1$ )



Each phase has four poles  
→ Two pairs of poles ( $p = 2$ )

The number of pairs of poles  $p$  links the speed of rotation  $\dot{\theta}$  to the pulsation  $\omega$  of the currents and voltages:

$$\dot{\theta} = \frac{\omega}{p}$$

# Exercise 9: Three-phase turbo-alternator

Turbo-alternators are alternators coupled to turbines allowing to convert the mechanical power of a moving fluid (steam or liquid) to electrical power. In this exercise the turbo-alternator has the following nominal characteristics:

- Power  $P_n = 600 \text{ MW}$
- Frequency  $f_n = 50 \text{ Hz}$
- Speed of rotation  $\dot{\theta}_n = 3000 \text{ RPM}$
- Power factor  $\cos \phi_n = 0.9$
- Line voltages  $U_n = 20 \text{ kV}$
- Ferromagnetic losses  $p_f = 543 \text{ kW}$
- Mechanical losses  $p_m = 1.35 \text{ MW}$
- Rotor resistance  $R_e = 0.17 \Omega$
- Excitation system efficiency  $\eta_e = 0.92$
- Stator phase resistance  $R = 2.3 \text{ m}\Omega$ .

To characterize the turbo-alternator three tests have been performed:

- Using open stator windings, at the nominal speed of rotation  $\dot{\theta}_n$ , the RMS direct voltage values have been measured with respect to the RMS current intensity  $I_e$  flowing through the inductor (table on the right).
- When turning at the nominal speed of rotation  $\dot{\theta}_n$ , using short-circuited stator windings and an excitation current  $I_e$  of  $1.18 \text{ kA}$ , an RMS current equal to half the nominal RMS value flows in each of the stator windings.
- When turning at the nominal speed of rotation  $\dot{\theta}_n$ , connecting an inductive load and using an excitation current  $I_e$  of  $2.085 \text{ kA}$ , an RMS current equal to half the nominal RMS value flows in each of the stator windings for an RMS line voltage of  $10 \text{ kV}$ .

$I_e$ [A]	$E_v$ [kV]
400	5.2
700	9.1
963	11.5
1200	13
1450	14
1900	15

# Exercise 9: Three-phase turbo-alternator

1. Calculate the nominal RMS intensity  $I_n$  of the line currents.
2. Compute the total losses and the turbo-alternator efficiency at the nominal operating point, knowing the RMS excitation current value is  $I_e = 3.2 \text{ kA}$  and that the alternator is in star configuration.
3. Calculate the mechanical power the turbine must provide for each of the considered test.

Using Behn-Eschenburg's model with the experimental measurements:

4. Calculate the (unsaturated) synchronous reactance  $X_s$  of the turbo-alternator.
5. Draw the phasor diagram corresponding to the Behn-Eschenburg's model for the nominal operating point, assuming the current lags the voltage.
6. Compute the RMS value  $E_v$  of the synchronous electromotive force.
7. Draw the internal angle  $\delta_{int}$  and give its value.