# Electromagnetic Energy Conversion
# ELEC0431

## Exercise session 6:
## Synchronous machines

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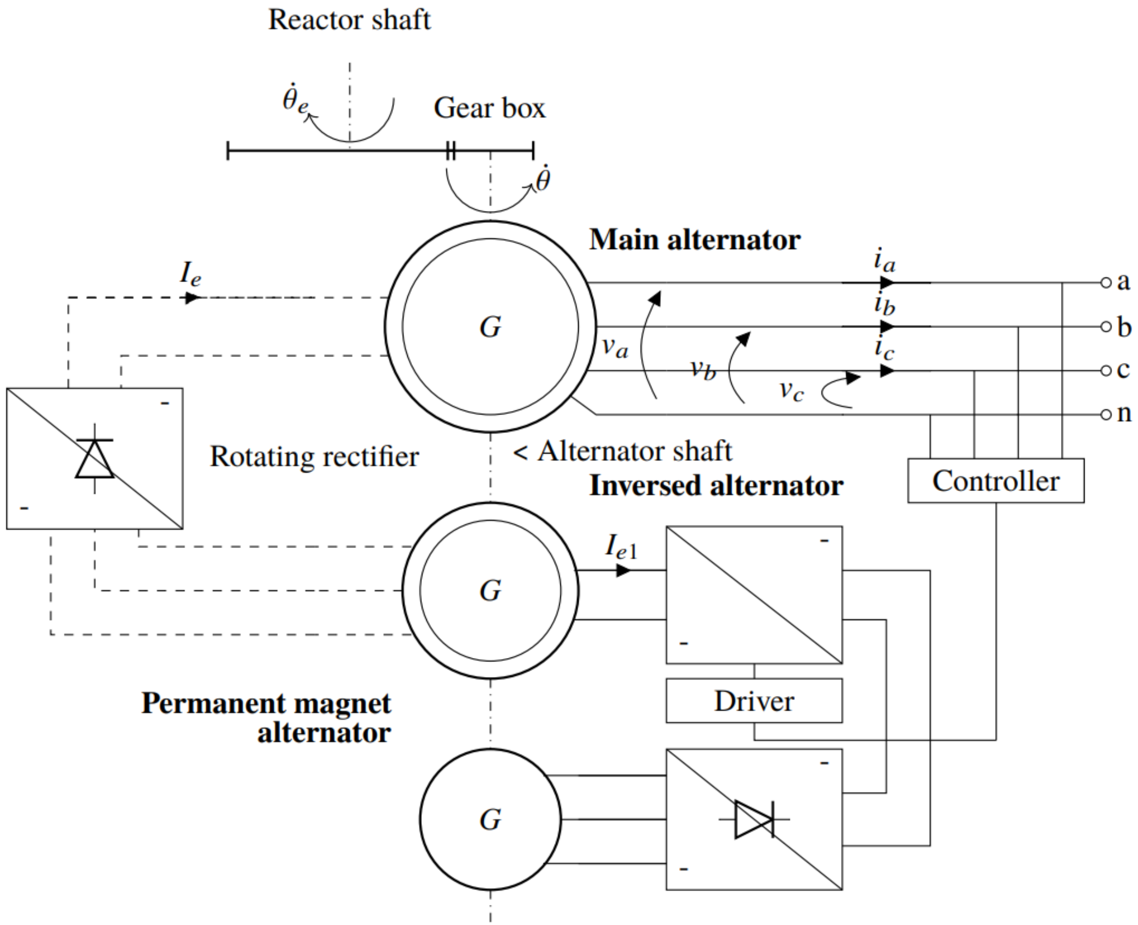
# Exercise 17: Constant air gap alternator

Several alternators are used on airplanes which, coupled to the reactors, feed all the necessary onboard electrical grids. Those alternators are characterized by a higher generated voltage and current frequency compared to alternators coupled to 50 Hz or 60 Hz electrical grids. Moreover, due the variable speed of the airplane reactors, the delivered frequency is not constant. The considered constant air gap three-phase alternator and its rotor winding are coupled following a star shape. Magnetic leakage, saturation, hysteresis and Eddy currents will be neglected. For a rotating speed of the alternator shaft of $\dot{\theta} = 11\,100\ RPM$, the frequency of the delivered voltages and currents is $f = 370\ Hz$ for a nominal apparent power $S_n = 150\ kVA$ and a direct voltage of RMS value $V_n = 115\ V$. The rotation speed of the reactor $\dot{\theta}_e$ varies from $4160\ RPM$ to $9000\ RPM$. The alternator is therefore coupled to the reactor through a gear box of ratio

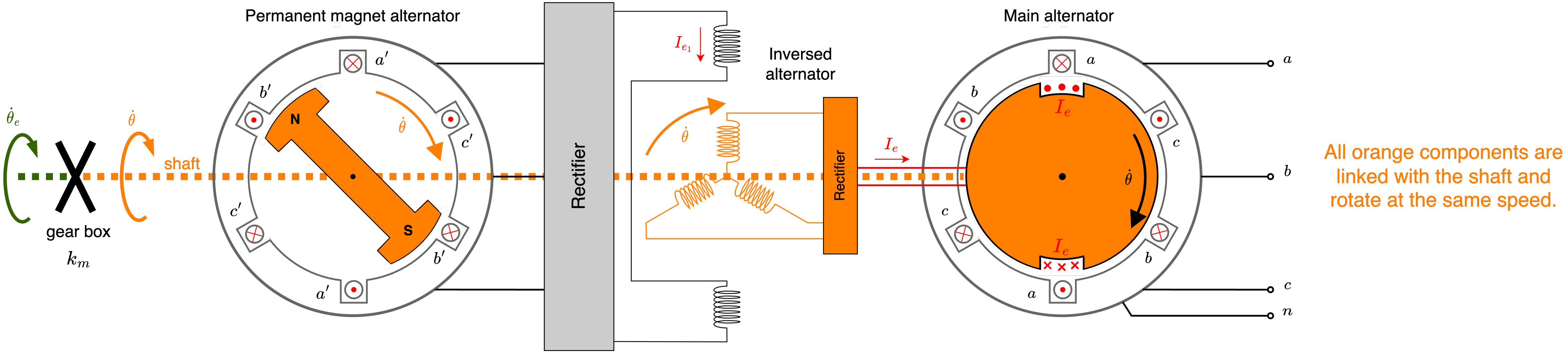
$$k_m = \frac{\dot{\theta}}{\dot{\theta}_e} = 2.67.$$

The excitation of the alternator is controlled such that the output voltage of the alternator is 115 V (direct voltage or 200 V for the composed voltage). This excitation consists of an inversed alternator coupled with a permanent magnet alternator



# Exercise 17: Constant air gap alternator

1. Explain how the excitation system works. What are the main advantages of such a system?




1) At startup, the airplane reactors start rotating at a speed $\dot{\theta}_e$, making the shaft of the synchronous machines rotate at a speed $k_e\dot{\theta}$ using a gear box.

2) The shaft makes the permanent magnet of the $1^{st}$ generator turn. It generates a three-phase current which is rectified to a DC current $I_{e1}$, used as excitation current for the $2^{nd}$ generator.

3) The current $I_{e1}$ is used at the stator of an inversed alternator. The rotor, driven by the shaft, generates a three-phase current which is rectified to a DC current $I_e$, used as excitation current of the main alternator.

**+**

The first permanent magnet alternator ensures an autonomous start.

There is no use of any brush. It makes the system more reliable and safer (no sparks).

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2. Express the frequency $f$ of the generated voltages and currents with respect to the rotation speed of the reactor $\dot{\theta}_e$, the gear box ratio $k_m$ and the number of pairs of poles of the alternator $p$.
3. Deduce the number of pair of poles, as well as the minimal and maximal values $f_{min}$, $f_{max}$ of the generated voltages and currents.
4. For an airplane, justify the relevance of a system working at a variable frequency in the targeted range.
5. Calculate the nominal RMS current $I_{sn}$ of the line currents of the alternator.
6. The flux generated by one pole is:

$$\phi(t) = \phi_m \cos\left(p(\dot{\theta}t - \theta_0)\right)$$

   where $\phi_m$ is the maximum flux amplitude, $p$ the number of pairs of poles, $\dot{\theta}$ the speed of rotation, $t$ the time variable and $\theta_0$ the initial angular position of the rotor. Express the electromotive force $e_s$ induced in a single turn of the rotor with respect to $\phi_m$, $f$, $t$ and $\theta_0$. Deduce the RMS value $E_s$ of $e_s(t)$ with respect to $\phi_m$ and $f$.
7. The RMS value $E$ of the induced electromotive force in a phase is $E = k_b\, N_s\, E_s$ where $k_b = 0.85$ is the coil factor and $N_s = 16$ is the number of turns per phase. The magnetic circuit is built using laminations allowing to reach a maximal magnetic field corresponding to a flux amplitude $\phi_{m0} = 6.84\ mWb$ and a current $I_{e0} = 2.95\ A$.
   Express the RMS value $E$ of the electromotive force induced in each phase with respect to $k_b$, $N_s$, $\phi_{m0}$, $I_{e0}$, $I_e$ and $f$.
   Plot $E$ with respect to $I_e$ in the range between $f_{min}$ and $f_{max}$ and conclude.

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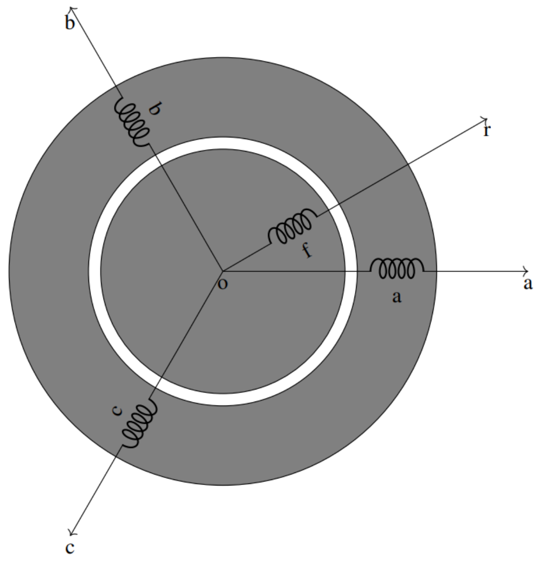
The stator of the machine is composed of three-phase windings whose phases are noted $a$, $b$ and $c$, while the rotor is composed of a inductor winding noted $f$. Each phase has an impedance composed of a resistance $R_s$, a self inductance $\lambda$ (also noted $L_s$) and a mutual inductance $\lambda_m$ (also noted $M_s$) with respect to each other phase.

The mutual inductances between each phase and the inductor phase have a sinusoidal pulsation with respect to the rotation angle $\theta$:

$$\lambda_{m,af} = \lambda_{m,sf} \cos(p\theta)$$

$$\lambda_{m,bf} = \lambda_{m,sf} \cos\left(p\left(\theta - \frac{2\pi}{3}\right)\right)$$

$$\lambda_{m,cf} = \lambda_{m,sf} \cos\left(p\left(\theta + \frac{2\pi}{3}\right)\right)$$




8. Express the total fluxes $\Psi_a$, $\Psi_b$ and $\Psi_c$ crossing the phase windings $a$, $b$ and $c$ with respect to the flowing current intensities $i_a$, $i_b$ and $i_c$, the excitation current intensity $I_e$, the self inductance $\lambda$, the stator mutual inductance $\lambda_m$, the mutual inductance between the stator and the rotor $\lambda_{m,sf}$ and the angle $p\theta$.

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9. Express the voltages $v_a$, $v_b$ and $v_c$ across the phase windings $a$, $b$ and $c$ with respect to $i_a$, $i_b$, $i_c$, the total flux derivatives of $\Psi_a$, $\Psi_b$ and $\Psi_c$ and $R_s$.

10. Show that the direct voltages of the stator can be written:

$$v_a = e_a - R_s\, i_a - \lambda_f \frac{di_a(t)}{dt}$$

$$v_b = e_b - R_s\, i_b - \lambda_f \frac{di_b(t)}{dt}$$

$$v_c = e_c - R_s\, i_c - \lambda_f \frac{di_c(t)}{dt}$$

Express the electromotive forces $e_a$, $e_b$ and $e_c$ with respect to $\lambda_{m,sf}$, $I_e$, $\omega$, $t$, $p$ and $\theta_0$. Express also their common RMS value $E$ with respect to $\lambda_{m,sf}$, $I_e$ and $\omega$. Explain the significance of $\lambda_f$ reexpress it in terms of $\lambda$ and $\lambda_m$.

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The single-phase equivalent model of Behn-Eschenburg is now considered with $R_s = 0.4\ m\Omega$.
To characterize the alternator two tests have been performed:

- Using open stator windings, at the speed of rotation $\dot{\theta} = 11\ 100$ RPM, the RMS direct voltage values have been measured with respect to the RMS current intensity $I_e$ flowing through the inductor.

- Using short-circuited stator windings, at the speed of rotation $\dot{\theta} = 11\ 100$ RPM, the RMS current intensity $I_c$ have been measured with respect to the RMS current intensity $I_e$ flowing through the inductor.

| $I_e$ [A] | $E$ [V] | $I_s$ [A] |
|---|---|---|
| 0.4 | 21.2 | 94.8 |
| 0.8 | 42.2 | 190 |
| 1.2 | 63.6 | 284 |
| 1.6 | 84.8 | 379 |
| 2 | 106 | 474 |
| 2.4 | 122 | 569 |
| 3 | 137 | 670 |
| 3.6 | 143 | 770 |
| 4.2 | 145 | 860 |
| 4.8 | 147 | 948 |
| 5.4 | 148 | 1040 |

11. Knowing that $E = \alpha\ \omega\ I_e$, compute the value of the coefficient $\alpha$ for $I_e = 0.4\ A$, $3\ A$ and $5.4\ A$.

# Exercise 17: Constant air gap alternator

12. Plot the open stator windings curve, $E$ with respect to $I_e$, for $f_{min} = 370\ Hz$ and $f_{max} = 770\ Hz$.

| $I_e$ [A] | $E$ [V] | $I_s$ [A] |
|---|---|---|
| 0.4 | 21.2 | 94.8 |
| 0.8 | 42.2 | 190 |
| 1.2 | 63.6 | 284 |
| 1.6 | 84.8 | 379 |
| 2 | 106 | 474 |
| 2.4 | 122 | 569 |
| 3 | 137 | 670 |
| 3.6 | 143 | 770 |
| 4.2 | 145 | 860 |
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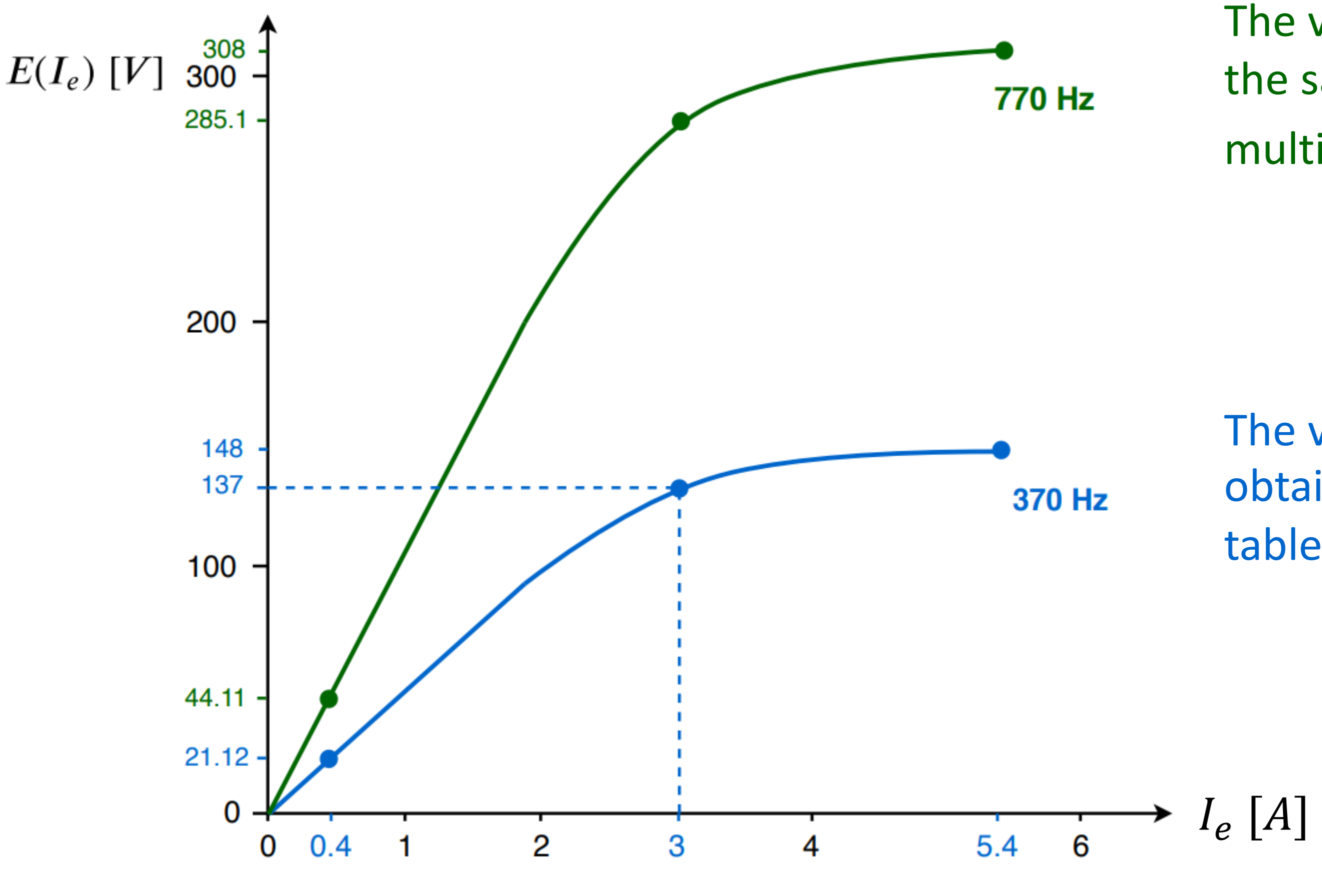


The values for $770\ Hz$ are the same as for $370\ Hz$ but multiplied by the factor $\frac{770}{370}$.

The values for $370\ Hz$ are obtained directly from the table

13. Calculate the synchronous reactance $X_s$ for the linear part of the curve.

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14. Plot the short-circuited stator windings curve, $I_s$ with respect to =$I_e$, for $f_{min} = 370\ Hz$ and $f_{max} = 770\ Hz$.

| $I_e$ [A] | $E$ [V] | $I_s$ [A] |
|---|---|---|
| 0.4 | 21.2 | 94.8 |
| 0.8 | 42.2 | 190 |
| 1.2 | 63.6 | 284 |
| 1.6 | 84.8 | 379 |
| 2 | 106 | 474 |
| 2.4 | 122 | 569 |
| 3 | 137 | 670 |
| 3.6 | 143 | 770 |
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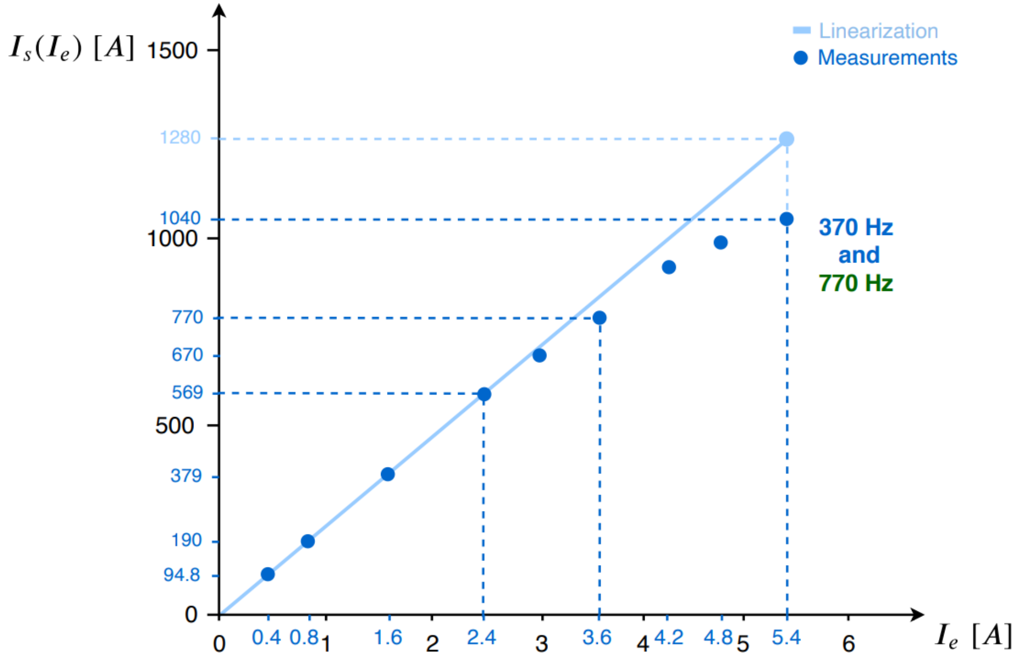


When ignoring $R_s$ compared to $X_s$, the curve $I_s(I_e)$ doesn't depend on the frequency since $X_s$ itself increases with the frequency.

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15. The alternator is connected to a star-shaped load composed of 3 resistors of value $R_L = 0.5\ \Omega$ working at a frequency $f = 500\ Hz$ for an excitation current $I_e = 2\ A$.

    a) Calculate the stator RMS current and voltage values $I_s$ and $V_s$.

    b) Sketch the Behn-Eschenburg diagram.

    c) Explain how $I_s$ and $V_s$ vary when the frequency increases.

16. Working at constant excitation current $I_e$, a balanced inductive load is now considered with a corresponding impedance $Z = R_c + j\omega L_c$ for each phase.

    a) Sketch the Behn-Eschenburg diagram for a power factor $\cos\varphi = 0.75$.

    b) Express the stator RMS voltage $V_s$ with respect to $\alpha$, $\omega$, $R_c$, $L_c$, $L_s$ and $I_e$.

    c) Express the resistive torque $C_r$ with respect to $p$, $\alpha$, $\omega$, $R_c$, $L_c$, $L_s$ and $I_e$.

    d) How does the frequency variation influence the load power factor?.