Electromagnetic Energy Conversion ELEC0431

Exercise session 6: Synchronous machines

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Exercise 10: Constant air gap alternator

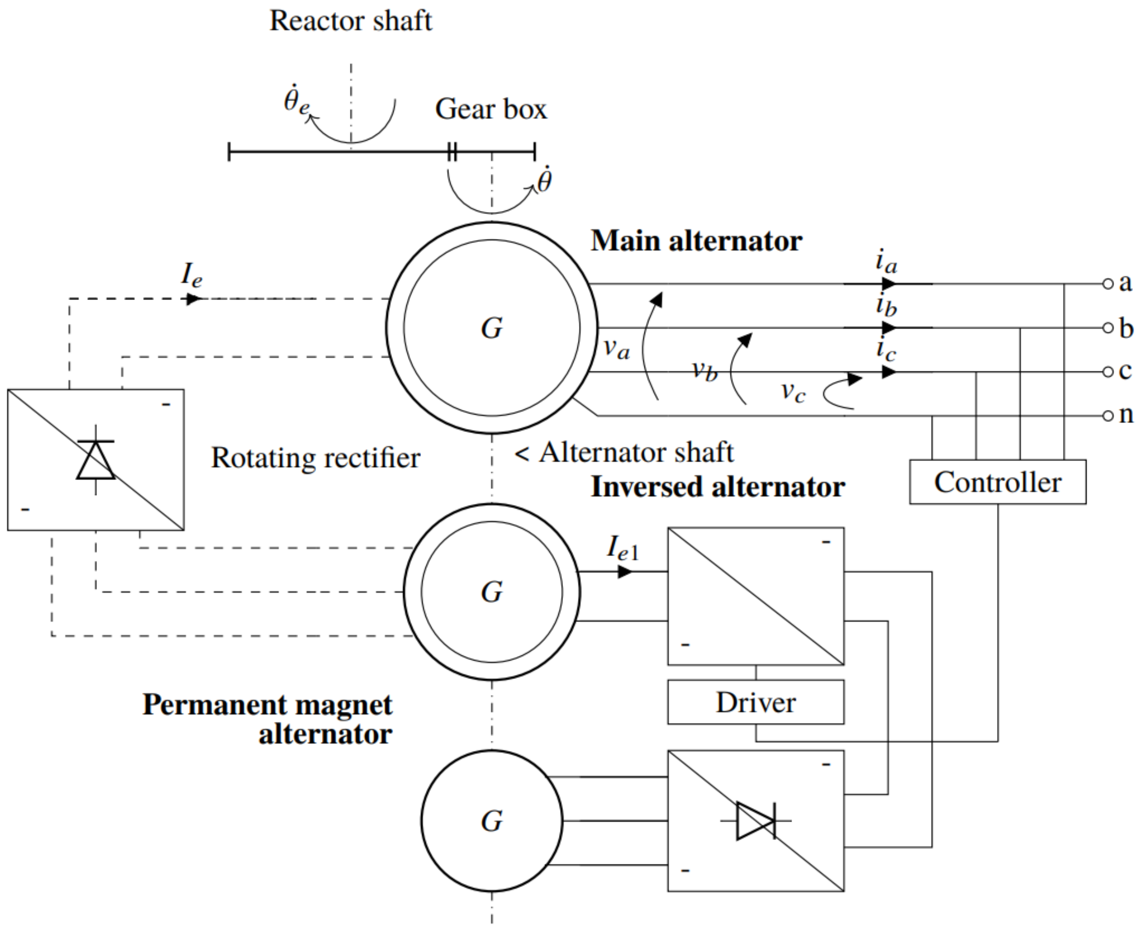
Several alternators are used on airplanes which, coupled to the reactors, feed all the necessary onboard electrical grids. Those alternators are characterized by a higher generated voltage-and-current frequency compared to alternators coupled to 50 Hz or 60 Hz electrical grids. Moreover, due the variable speed of the airplane reactors, the delivered frequency is not constant.

In nominal regime, the main-alternator shaft rotates at a speed $\dot{\theta}_n = 11\,100$ RPM, the frequency of the delivered voltages and currents is $f = 370\,Hz$, the nominal apparent power is $|S_n| = 150$ kVA and the RMS phase voltage is $V_n = 115$ V.

The rotation speed of the reactor $\dot{\theta}_e$ varies from 4160 RPM to 9000 RPM. The alternator is coupled to the reactor through a gear box of ratio

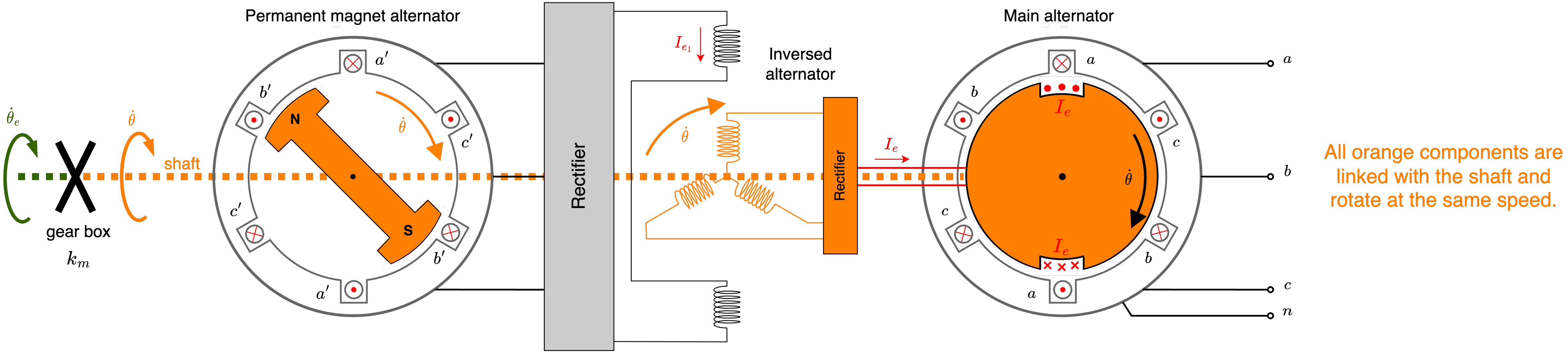
$$k_m = \frac{\dot{\theta}}{\dot{\theta}_e} = 2.67.$$

The excitation of the alternator is controlled such that the output phase voltage of the alternator is 115 V (200 V line voltage). This excitation consists of an inversed alternator coupled with a permanent magnet alternator.



Exercise 10: Constant air gap alternator

1. Explain how the excitation system works. What are the main advantages of such a system?



1) At startup, the airplane reactor starts rotating at a speed $\dot{\theta}_e$, making the shaft of the alternator rotate at a speed $\dot{\theta} = k_m\, \dot{\theta}_e$ using a gear box.

2) The shaft activates a permanent-magnet alternator, generating three-phase currents.

3) The three-phase currents are rectified and used for the excitation of the inversed alternator (I_{e1}).

4) The shaft activates the rotor of the inversed alternator, generating three-phase currents.

5) The three-phase currents are rectified and used for the excitation of the main alternator (I_e).

➕

The first permanent magnet alternator ensures an autonomous start.

There is no use of any brush. It makes the system more reliable and safer (no sparks).

Exercise 10: Constant air gap alternator

2. Express the frequency f of the generated voltages and currents with respect to the rotation speed of the reactor $\dot{\theta}_e$, the gear box ratio k_m and the number of pairs of poles of the alternator p.
3. Deduce the number of pair of poles, and the minimal and maximal values f_{min}, f_{max} of the generated voltages and currents.
4. For an airplane, justify the relevance of a system working at a higher frequency.
5. Compute the nominal RMS line current I_n (the main alternator is connected in star configuration).
6. The flux generated by one pole is:

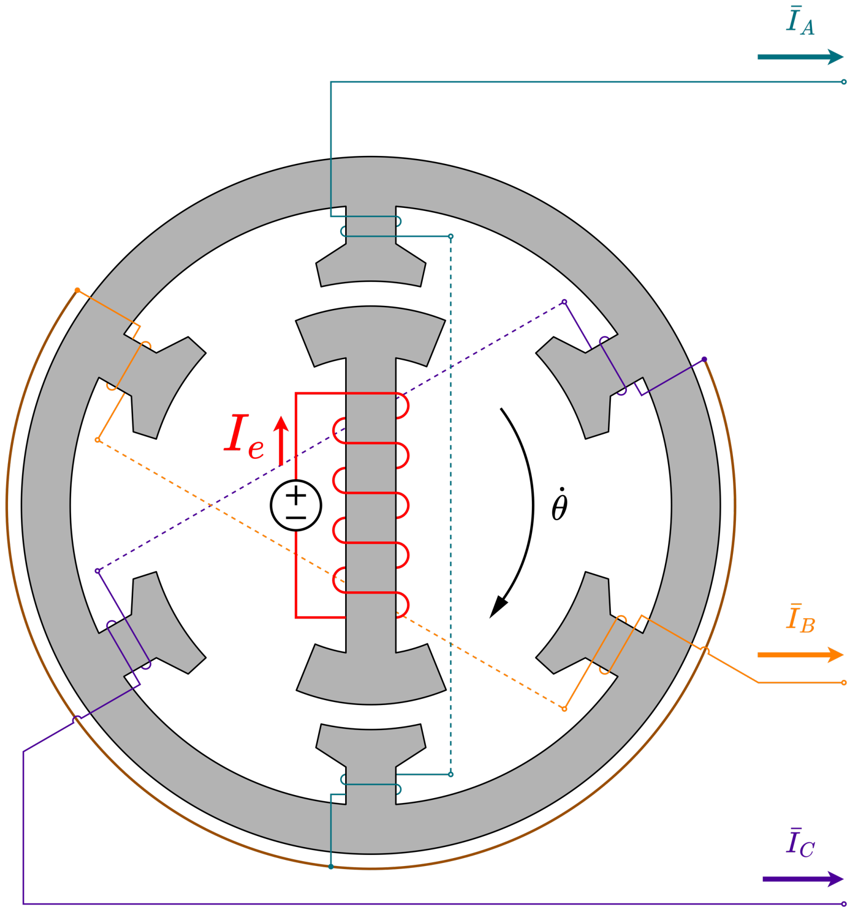
$$\phi(t) = \phi_m \cos\left(p(\dot{\theta}t - \theta_0)\right)$$

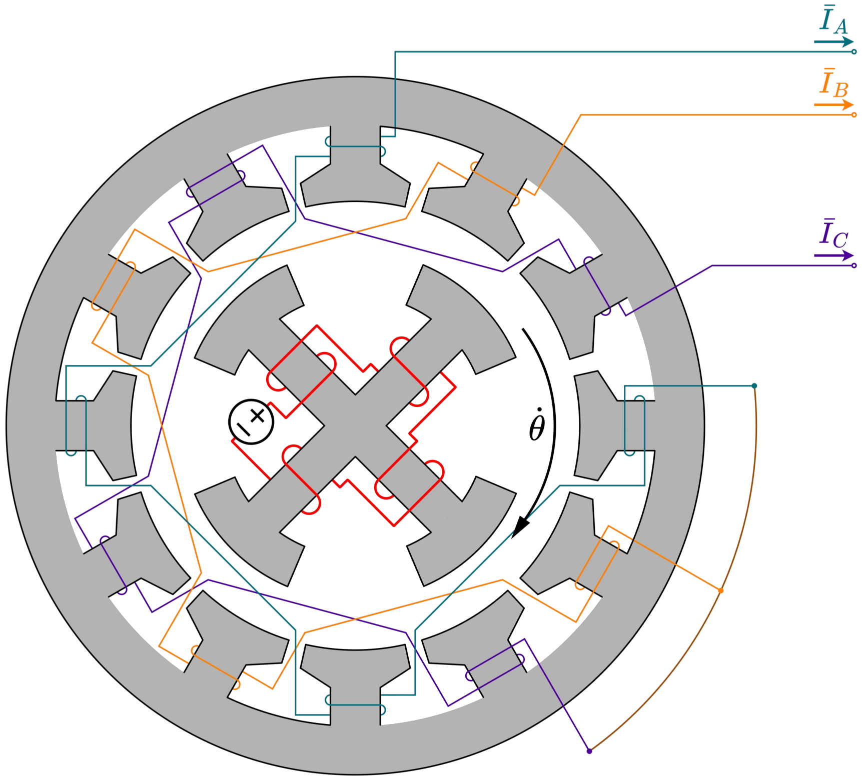
   where ϕ_m is the maximum flux amplitude, p the number of pairs of poles, $\dot{\theta}$ the speed of rotation, t the time variable and θ_0 the initial angular position of the rotor. Express the electromotive force $e_s(t)$ induced in a single turn of the rotor with respect to ϕ_m, f, t, p and θ_0. Deduce the RMS value E_s of $e_s(t)$ with respect to ϕ_m and f.

7. The RMS value E of the induced electromotive force in a phase is $E = k_b\, N_s\, E_s$ where $k_b = 0.85$ is the coil factor and $N_s = 16$ is the number of turns per phase. The magnetic circuit is built using laminations allowing to reach a maximal magnetic field corresponding to a flux amplitude $\phi_{m0} = 6.84\, mWb$ and a current $I_{e0} = 2.95\, A$.

   Assuming the ferromagnetic materials remain unsaturated and neglecting hysteresis and Eddy currents, express the RMS value E of the electromotive force induced in each phase with respect to k_b, N_s, ϕ_{m0}, I_{e0}, I_e and f.

Reminder ex session 5: Number of pairs of poles





The number of pairs of poles p links the speed of rotation $\dot{\theta}$ to the pulsation ω of the currents and voltages:

$$\dot{\theta} = \frac{\omega}{p}$$

Each phase has two poles
➔ One pair of poles ($p = 1$)

Each phase has four poles
➔ Two pairs of poles ($p = 2$)

Exercise 10: Constant air gap alternator

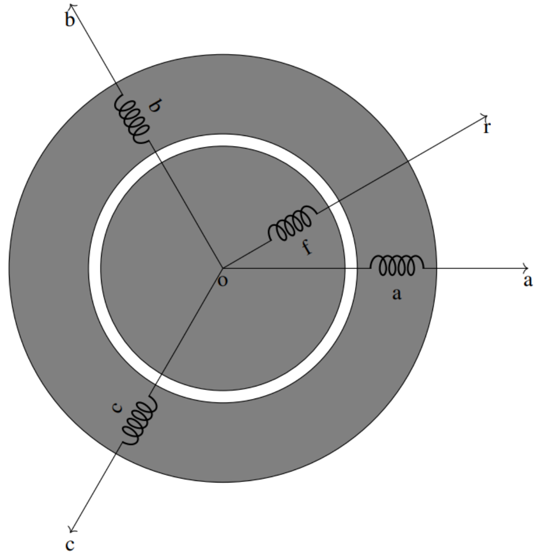
The stator of the alternator is composed of three-phase windings whose phases are noted a, b and c, while the rotor is composed of an inductor winding noted f. Each phase has an impedance composed of a resistance R_s, a self inductance λ (also noted L_s) and a mutual inductance λ_m (also noted M_s) with respect to each other phase.

The mutual inductances between each phase and the inductor phase have a sinusoidal pulsation with respect to the rotation angle θ:

$$\lambda_{m,af} = \lambda_{m,sf}\cos(p\theta)$$

$$\lambda_{m,bf} = \lambda_{m,sf}\cos\left(p\left(\theta - \frac{2\pi}{3}\right)\right)$$

$$\lambda_{m,cf} = \lambda_{m,sf}\cos\left(p\left(\theta + \frac{2\pi}{3}\right)\right)$$



8. Express the total fluxes Ψ_a, Ψ_b and Ψ_c crossing the phase windings a, b and c with respect to the flowing current intensities i_a, i_b and i_c, the excitation current intensity I_e, the self inductance λ, the stator mutual inductance λ_m, the mutual inductance between the stator and the rotor $\lambda_{m,sf}$ and the angle $p\theta$.

Exercise 10: Constant air gap alternator

9. Express the voltages v_a, v_b and v_c across the phase windings a, b and c with respect to i_a, i_b, i_c, the total flux derivatives of Ψ_a, Ψ_b and Ψ_c and R_s.

10. Show that the direct voltages of the stator can be written:

$$v_a = e_a - R_s\, i_a - \lambda_f \frac{di_a(t)}{dt}$$

$$v_b = e_b - R_s\, i_b - \lambda_f \frac{di_b(t)}{dt}$$

$$v_c = e_c - R_s\, i_c - \lambda_f \frac{di_c(t)}{dt}$$

Express the electromotive forces e_a, e_b and e_c with respect to $\lambda_{m,sf}$, I_e, $\dot{\theta}$, t and p.

Exercise 10: Constant air gap alternator

The single-phase equivalent model of Behn-Eschenburg is now considered with $R = 0.4\ m\Omega$. To characterize the alternator two tests have been performed:

- Using open stator windings, at the speed of rotation $\dot{\theta} = 11\,100$ RPM, the RMS phase voltage has been measured with respect to the RMS current intensity I_e flowing through the inductor.
- Using short-circuited stator windings, at the speed of rotation $\dot{\theta} = 11\,100$ RPM, the RMS current intensity I_s has been measured with respect to the RMS current intensity I_e flowing through the inductor.

I_e [A]	E_v [V]	I_s [A]
0.4	21.2	94.8
0.8	42.2	190
1.2	63.6	284
1.6	84.8	379
2	106	474
2.4	122	569
3	137	670
3.6	143	770
4.2	145	860
4.8	147	948
5.4	148	1040

11. Plot the open stator windings curve, E_v with respect to I_e, for $f_{min} = 370\ Hz$ and $f_{max} = 770\ Hz$.
12. At nominal speed of rotation, compute the synchronous reactance X_s for the linear part of the curve. Deduce the inductance L_s.
13. Plot the short-circuited stator windings curve, I_s with respect to I_e, for $f_{min} = 370\ Hz$ and $f_{max} = 770\ Hz$.
14. The alternator is connected to a star-shaped load composed of 3 resistors of value $R_L = 0.5\ \Omega$ working at a frequency $f = 500\ Hz$ for an excitation current $I_e = 2\ A$.

    a) Calculate the stator RMS current and voltage values I and V.

    b) Sketch the Behn-Eschenburg diagram and explain how I and V vary when the frequency increases.

Homework 18

A three-phase alternator coupled in star configuration provides a line current $I_n = 200$ A under a line voltage $U_n = 400$ V at 50 Hz. The power factor is $\cos\varphi_n = 0.866$ with a resistive-inductive load. In between one phase of the stator and the neutral point, the resistance is $30\, m\Omega$ and the synchronous reactance is $750\, m\Omega$. Iron losses amounts to 6 kW. Losses in the excitation system are negligeable, as well as mechanical losses.

1. Compute the nominal output power of the alternator.
2. Compute the Joule losses in the stator.
3. Compute the efficiency of the alternator in this configuration.
4. Compute the RMS value of the internal emf under Behn-Eschenburg assumption.
5. Provide the internal load angle δ_{int}.
6. With a purely resistive load and assuming the iron losses, I_n and U_n are kept constant, compute the efficiency of the alternator.

Answers:

1. $P_n = 120$ kW
2. $p_{js} = 3.6\ kW$
3. $\eta = 92.6\ \%$
4. $E_v = 336\ V$
5. $\delta_{int} = 22.2°$
6. $\eta' = 93.5\ \%$

Homework 19

A synchronous condenser is a DC excited synchronous motor, whose rotating shaft is not connected to any mechanical load. By controlling its excitation current, using a voltage regulator, the condenser is able to generate or absorb reactive power as needed to adjust the voltage on the power grids, or to improve the power factor.

- Synchronous speed: 428 RPM for 14 poles,
- Star-shape coupling with a phase voltage $V_n = 8.95$ kV,
- Nominal intensity $I_n = 6.33$ kA,
- Apparent nominal power $S_n = 170$ MVA,
- Nominal synchronous reactance $X_s = 1.2\ \Omega$ at the nominal frequency $f_n = 50$ Hz.

1. The machine is first used as an alternator, providing a total three-phase active power $P_{3\varphi} = 100$ MW and a total three-phase reactive power $Q_{3-\varphi} = 50$ Mvar. Calculate the phase shift φ and the line current intensity I.
2. The machine is now turned into a freely spinning motor, keeping the excitation current constant and assuming $P \approx 0$ W, calculate the reactive power Q provided to the motor.

Answers:

1. $\varphi = 26.565°, I = 4.16\ kA$
2. $Q = -69.14\ Mvar$