



# Electromagnetic Energy Conversion

## ELEC0431

### Exercise session 9: DC machines

5 April 2024

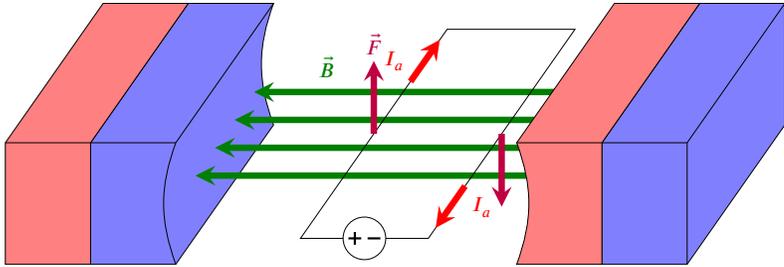
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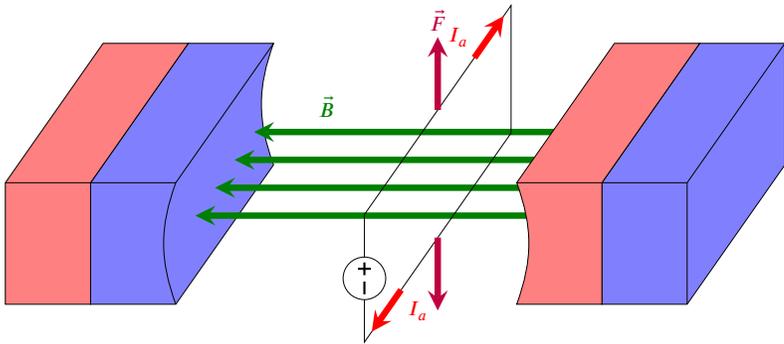
# In this class...

- Torque on a current loop in a steady magnetic field
- The basic DC motor
- DC motors
- DC generators
- The different types of excitation
- Exercise 14 & 15

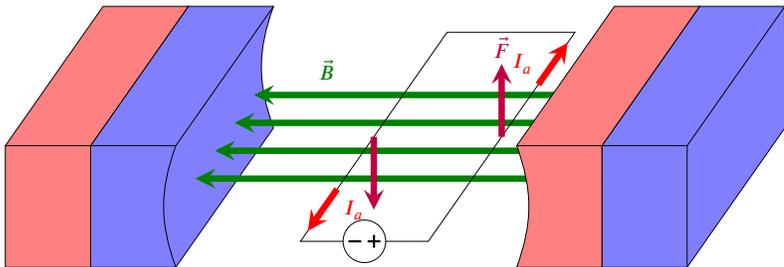
# Torque on a current loop in a steady magnetic field



We consider a closed rectangular loop of wire, which can rotate about its axis of symmetry. It is placed within a constant magnetic field perpendicular to this axis.

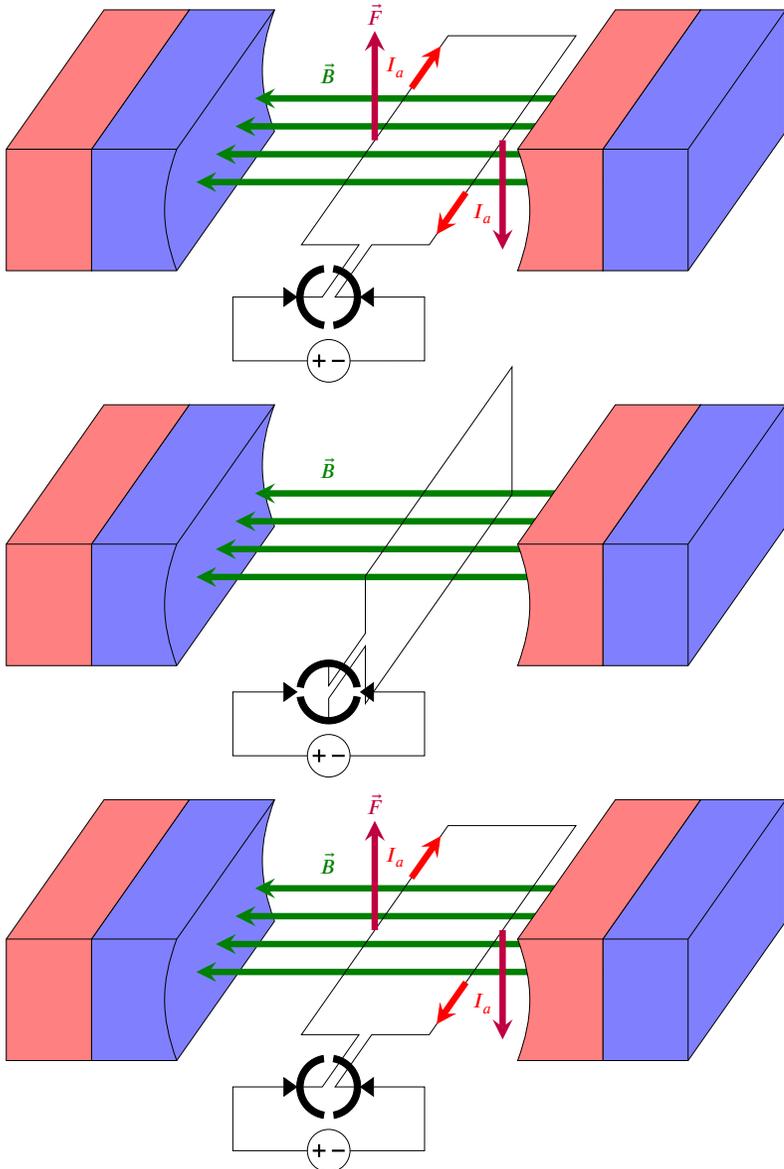


When a current  $I_a$  flows in the loop, the Laplace force ( $\vec{F} = q\vec{v} \wedge \vec{B}$ ) applies on it and create a torque (except when the field lines cross the loop perpendicularly).



As the loop rotates, the torque direction flips whenever the field lines cross the loop perpendicularly, as the current direction remained unchanged seen from the loop. It results in an oscillatory motion.

# The basic DC motor

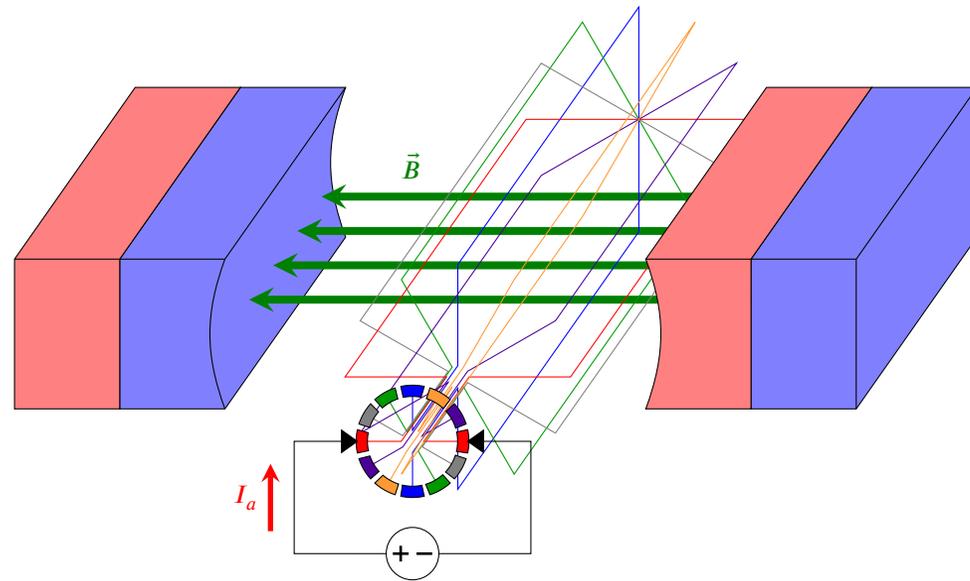


To ensure the loop keeps turning in the same direction, the direction of the current is mechanically inverted every half turn → The current in the wires is AC but the input current is DC.

In this configuration, the torque is not constant:

- It is zero when the field lines cross the loop perpendicularly.
- It is maximum when the field lines are parallel to the surface defined by the loop.

To smoothen the torque, add more wires!

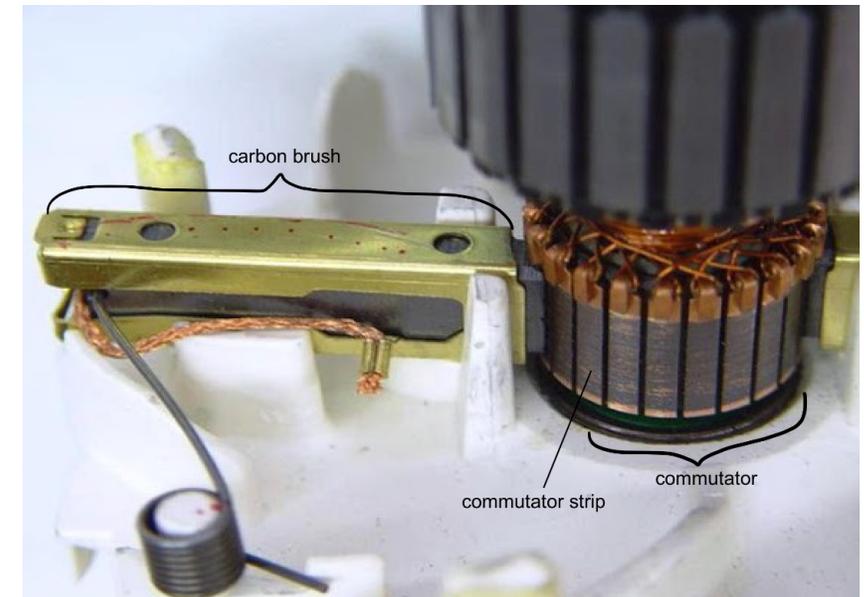
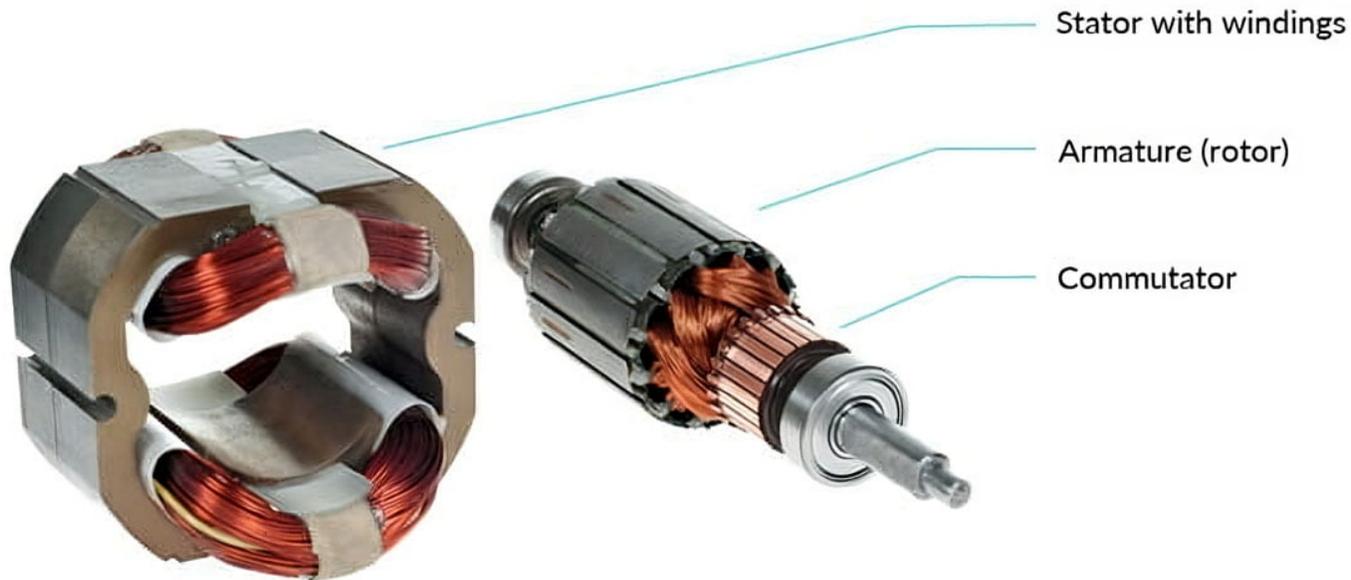


# DC motors

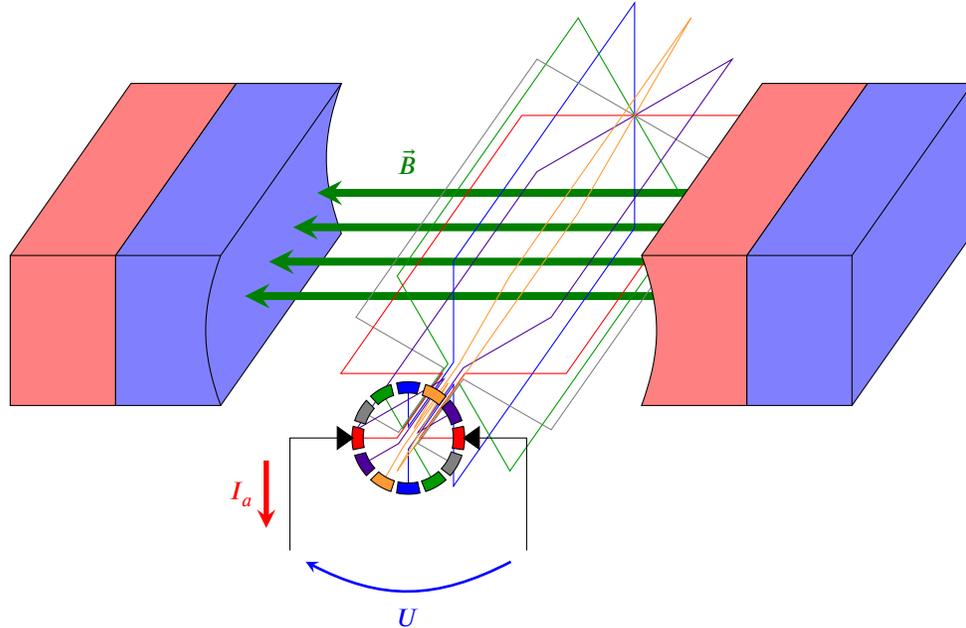
The steady magnetic field can be generated by an electromagnet. It is driven by an excitation current  $I_e$ :

- In DC machines,  $I_e$  flows in the stator windings.
- In synchronous and asynchronous machines,  $I_e$  flows in the rotor windings.

The rotor is called the armature, a current  $I_a$  flows in it.



# DC generators



The DC **generator** is very similar to the DC **motor**.

The armature (rotor) is placed within a magnetic field, generated by the stator.

As the armature is made to rotate, it perceives a varying magnetic flux inducing an emf.

The amplitude  $E$  of this emf increases with the speed of rotation  $\dot{\theta}$  and with the amplitude of the flux  $\phi$  (itself function of the excitation current in case the magnetic field is generated by an electromagnet)

$$\rightarrow E_v = k_e \dot{\theta} \phi(I_e) \quad (\text{same as for synchronous machines})$$

The commutator and the brushes ensure these emfs are converted into a DC current  $I_a$ .

Note that the armature current  $I_a$  also produces a magnetic field  $\Delta\Phi(I_a)$  which reduces the emf.

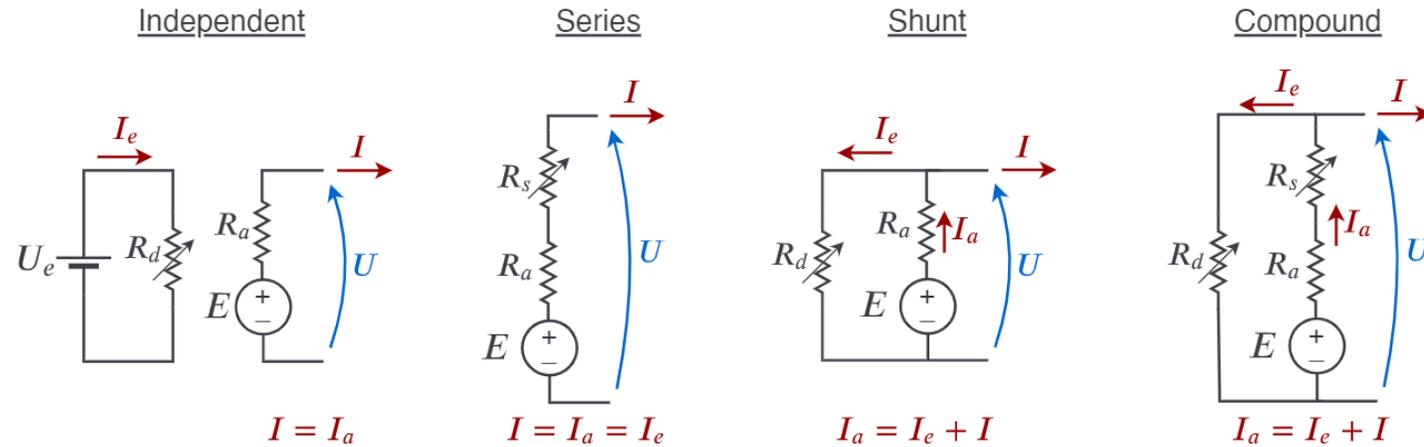
$$\rightarrow E = E_v - k_e \dot{\theta} \Delta\Phi(I_a) = k_e \dot{\theta} (\phi(I_e) - \Delta\Phi(I_a)).$$

This is called the armature reaction.

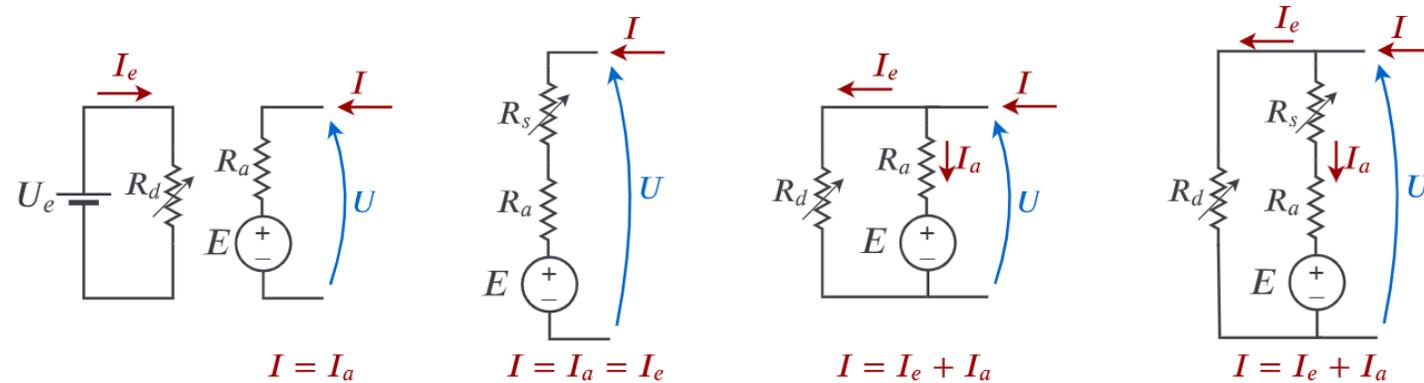
# The different types of excitation

When using electromagnets, different winding connections are possible:

DC machine  
as a **generator**:



DC machine  
as a **motor**:



From generator to motor, only  $I$  and  $I_a$  change of direction.

# Exercise 14: Brushed DC motor

The motor of a hammer drill has the following characteristics:

- independent excitation DC motor
- Nominal power  $P_n = 800 \text{ W}$
- Nominal speed of rotation  $\dot{\theta}_n = 1500 \text{ RPM}$
- Nominal input voltage  $U_n = 220 \text{ V}$
- Nominal rotor current intensity  $I_n = 4.6 \text{ A}$
- Nominal stator current intensity  $I_{en} = 0.35 \text{ A}$

Using two different excitation currents  $I_e$ , the electromotive force has been determined for different rotation speeds:

$I_{e1} = 0.35 \text{ A}$ :

$n$ [RPM]	$E$ [V]
1670	240
1510	220
1380	200
1040	150
820	120
510	75
110	20
0	0

$I_{e2} = 0.2 \text{ A}$ :

$n$ [RPM]	$E$ [V]
1800	186
1450	150
1150	120
850	90
560	60
260	30
0	0

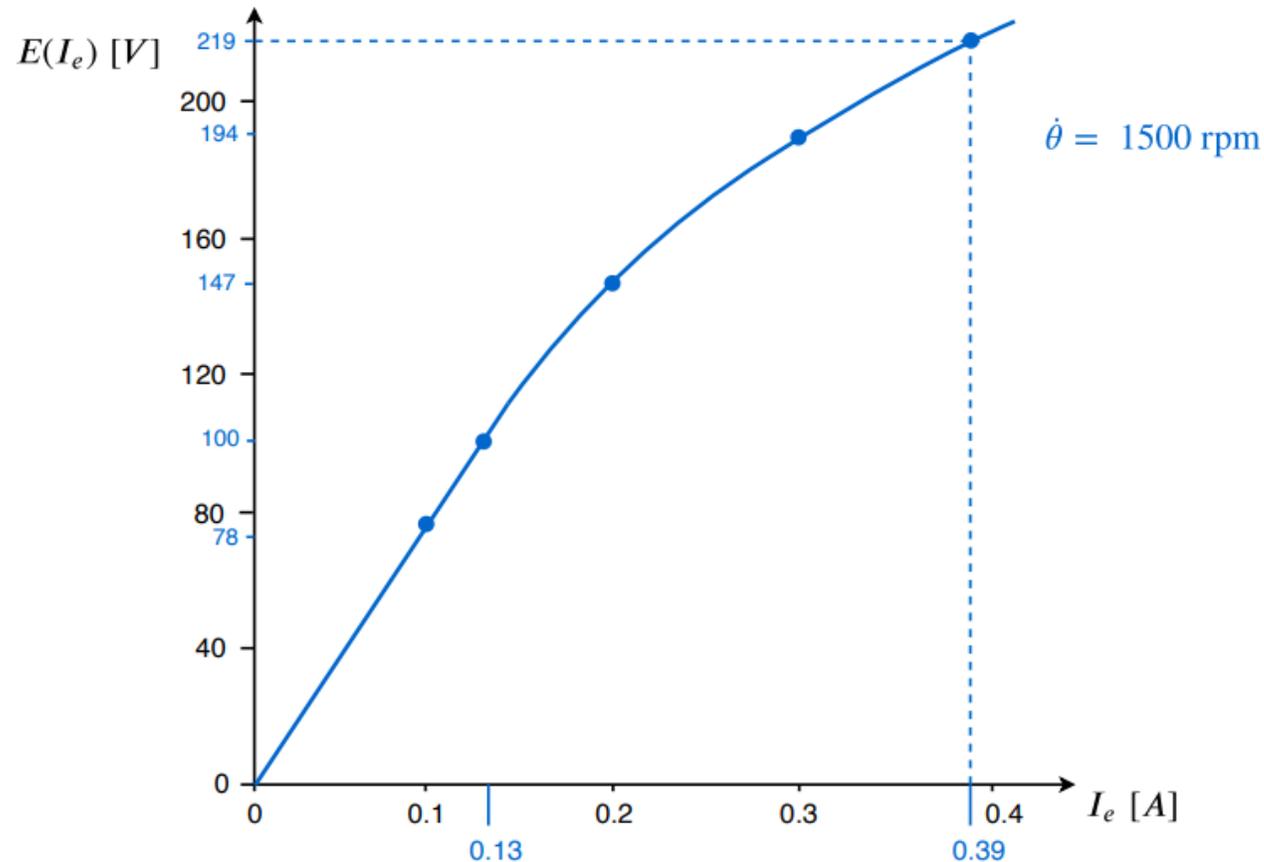
1. Plot  $E$  with respect to  $\dot{\theta}$  for  $I_{e1}$  and  $I_{e2}$  and justify the shape of the curves.
2. Show that the flux  $\Phi$  is not proportional to the excitation current intensity  $I_e$ .

# Exercise 14: Brushed DC motor

Maintaining the nominal speed of rotation, the electromotive force is measured for different excitation currents  $I_e$ :

3. Plot  $E$  with respect to  $I_e$  and justify the shape of the curves.

$I_e$ [A]	$E$ [V]
0.39	219
0.35	210
0.33	204
0.31	198
0.3	194
0.28	188
0.26	179
0.24	168
0.22	158
0.2	147
0.18	137
0.17	130
0.14	107
0.13	100
0.11	87
0.1	78
0.08	65
0.07	56



The curve behaves as a  $B(H)$  curve. It is linear for low excitation current  $I_e$ . At some point when further increasing  $I_e$ , saturation occurs.

# Exercise 14: Brushed DC motor

Some measurements have allowed to quantify the stator resistance  $R_e = 512.1 \Omega$  and the armature resistance  $R_a = 4.6 \Omega$ .

4. Draw the equivalent model of the motor.

An unloaded test at constant nominal speed has been performed to measure the voltage across the rotor  $U$  and the current drawn in the rotor  $I$  for different excitation currents value  $I_e$ .



$I_e$ [A]	$U$ [V]	$I$ [A]
0.4	222	0.43
0.35	213	0.44
0.3	198	0.45
0.25	176	0.48
0.2	151	0.56
0.15	120	0.66
0.1	85	0.92

5. Plot the collective (i.e. ferromagnetic plus mechanical) losses  $p_c$  with respect to  $I_e$ .
6. For the linear part of the curve, determine the mechanical losses  $p_m$  at the nominal speed of rotation.

A hole is drilled using the drill. The nominal speed of rotation remains constant while the rotor draws a current  $I_0$  of 3 A when a voltage  $U_0 = 212 V$  is measured on the rotor terminals.

7. Calculate the electromotive force and deduce the value of the excitation current  $I_{e0}$ .
8. Compute the shaft output power  $P_{mec}$ .
9. Deduce the resistive torque  $C_r$  induced by the drilling process.

# Exercise 15: Brushed DC motor with series excitation

Consider a brushed DC motor with its excitation in series. This motor is fed by a constant voltage source  $U = 220\text{ V}$ . To simplify the study, the armature and inductor resistances, the collective losses and the armature reaction are neglected.

1. Show that the electromagnetic torque is proportional to the square of the consumed current.
2. Show that the electromagnetic torque is inversely proportional to the square of the rotational speed of the motor.
3. Deduce that there is a runaway of the motor at no load.
4. According to sub-question 2, one can write that:

$$C_{elm} = \frac{a}{\dot{\theta}^2}$$

where  $C_{elm}$  corresponds to the electromagnetic torque of the motor in  $Nm$ ,  $\dot{\theta}$  is the speed of rotation in  $RPM$  and  $a$  is the constant to be determined. The nameplate of the machine indicates a nominal voltage of  $220\text{ V}$ , a nominal speed of rotation of  $1200\text{ RPM}$  and a nominal current of  $7.8\text{ A}$ . Deduce the value of the constant  $a$  (In the following, we will take the value of  $a = 20 \cdot 10^6\text{ Nm RPM}^2$ ).

5. Draw the mechanical characteristics of  $C_{elm}$  with respect to the speed of rotation.
6. The motor drives a hoist whose resistive torque is constant:  $C_{elm} = 10\text{ Nm}$ . Deduce the rotation speed.