Electromagnetic Energy Conversion
ELEC0431

Exercise session 9:
DC machines

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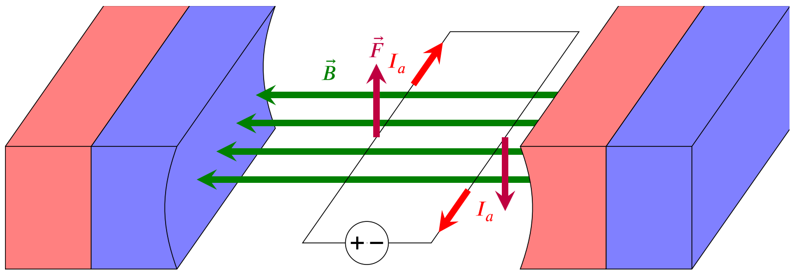
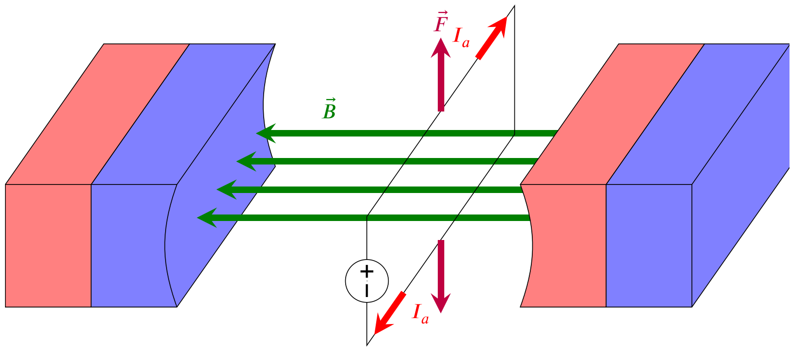
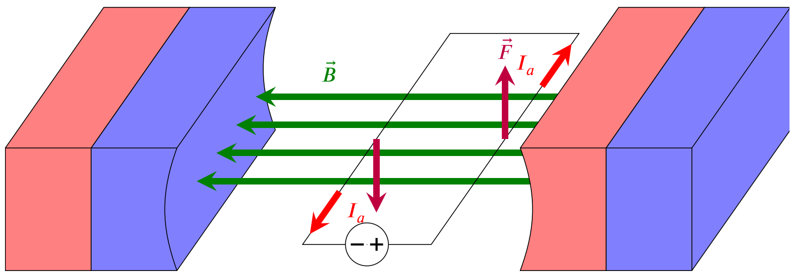
Current loop in a steady magnetic field





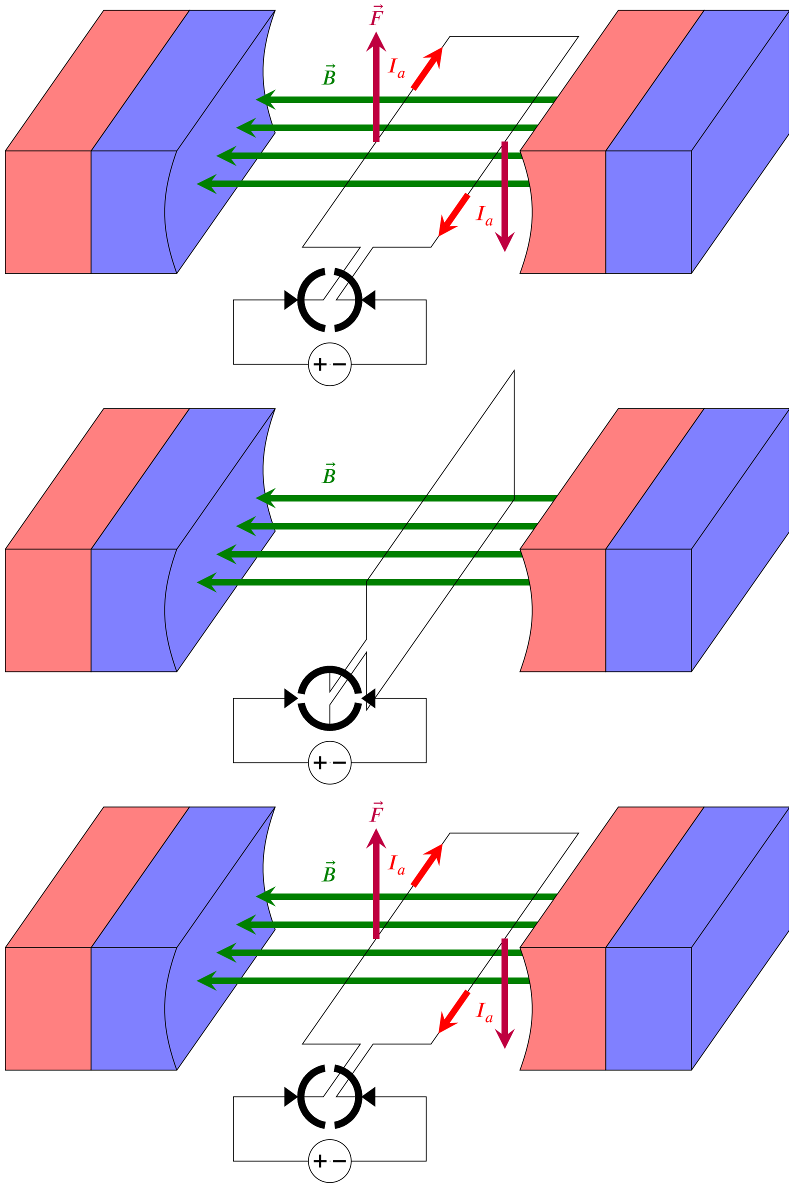


We consider a closed rectangular loop of wire, which can rotate around an axis of symmetry. It is placed within a constant magnetic field perpendicular to this axis.

When a current I_a flows in the loop, the Laplace force ($\vec{F} = q\vec{v} \wedge \vec{B}$) applies on it and create a torque (except when the field lines cross the loop perpendicularly).

As the loop rotates, the torque direction flips whenever the field lines cross the loop perpendicularly, as the current direction remained unchanged seen from the loop. It results in an oscillatory motion.

DC motor working principle

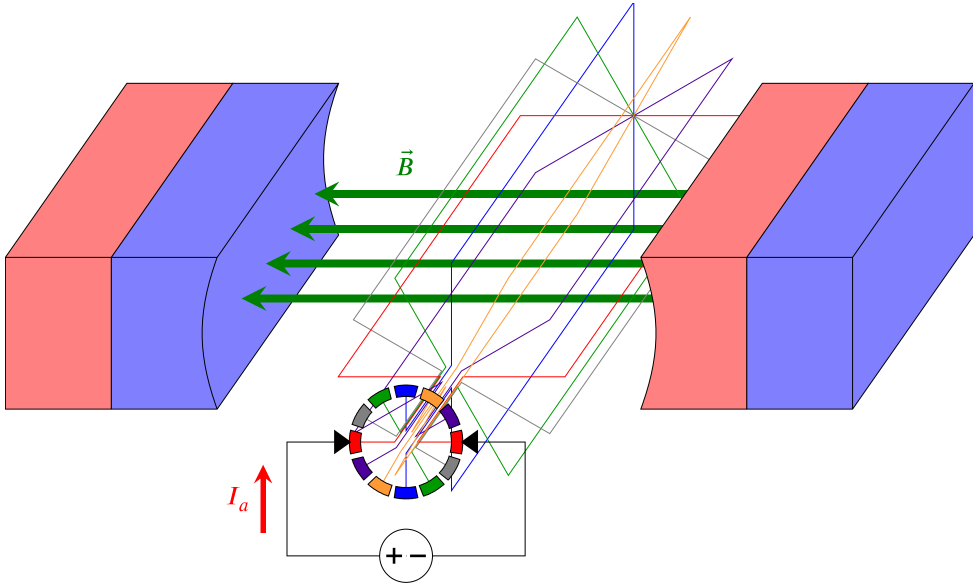


To ensure the loop keeps turning in the same direction, the direction of the current is mechanically inversed every half turn ➔ The current in the wires is AC but the input current is DC.

In this configuration, the torque is not constant:

- It is zero when the field lines cross the loop perpendicularly.
- It is maximum when the field lines are parallel to the surface defined by the loop.

To smoothen the torque, add more wires!

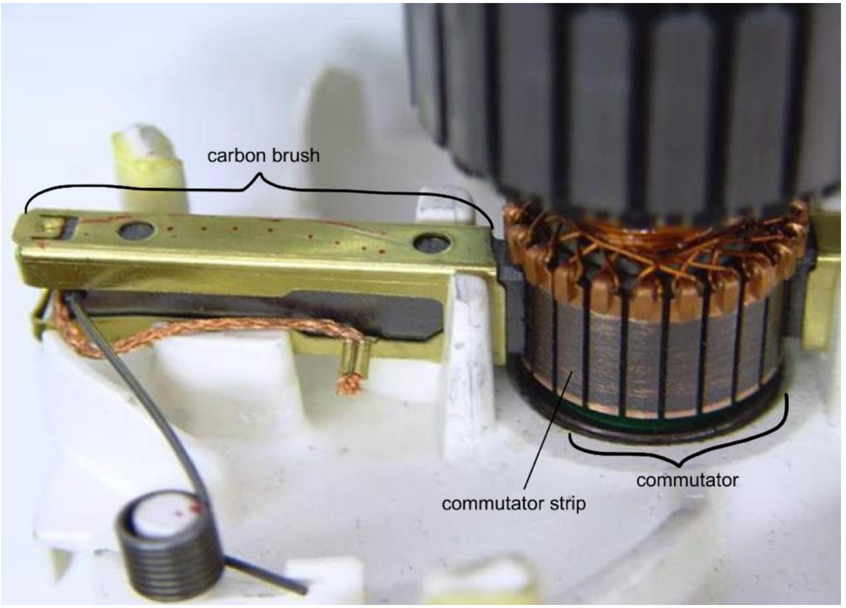


DC motors

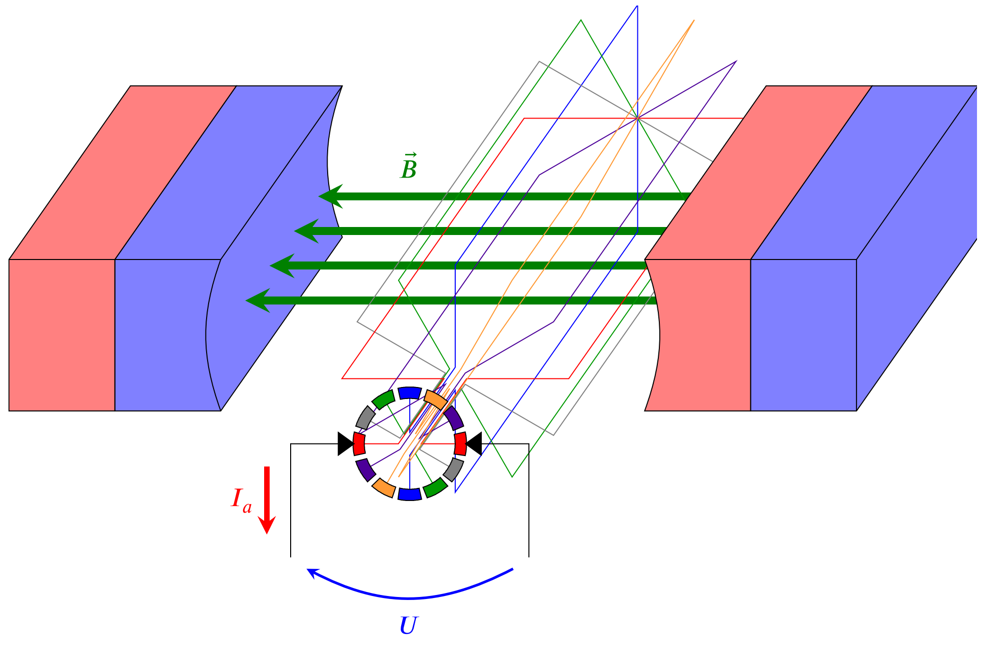
The steady magnetic field can be generated by an electromagnet. It is driven by an excitation current I_e:

- In DC machines, I_e flows in the **stator** windings.
- In synchronous and asynchronous machines, I_e flows in the **rotor** windings.

The rotor is called the armature, a current I_a flows in it.



DC generators



The DC **generator** is very similar to the DC **motor**.

The armature (rotor) is placed within a magnetic field generated by the stator (permanent magnet or electromagnet).

As the armature is made to rotate, it perceives a varying magnetic flux inducing an emf.

The amplitude of this emf increases with the speed of rotation $\dot{\theta}$ and with the amplitude of the flux ϕ (itself function of the excitation current in case the magnetic field is generated by an electromagnet)

➔ $E_v = k_e \, \dot{\theta} \, \phi(I_e)$ (same as for synchronous machines)

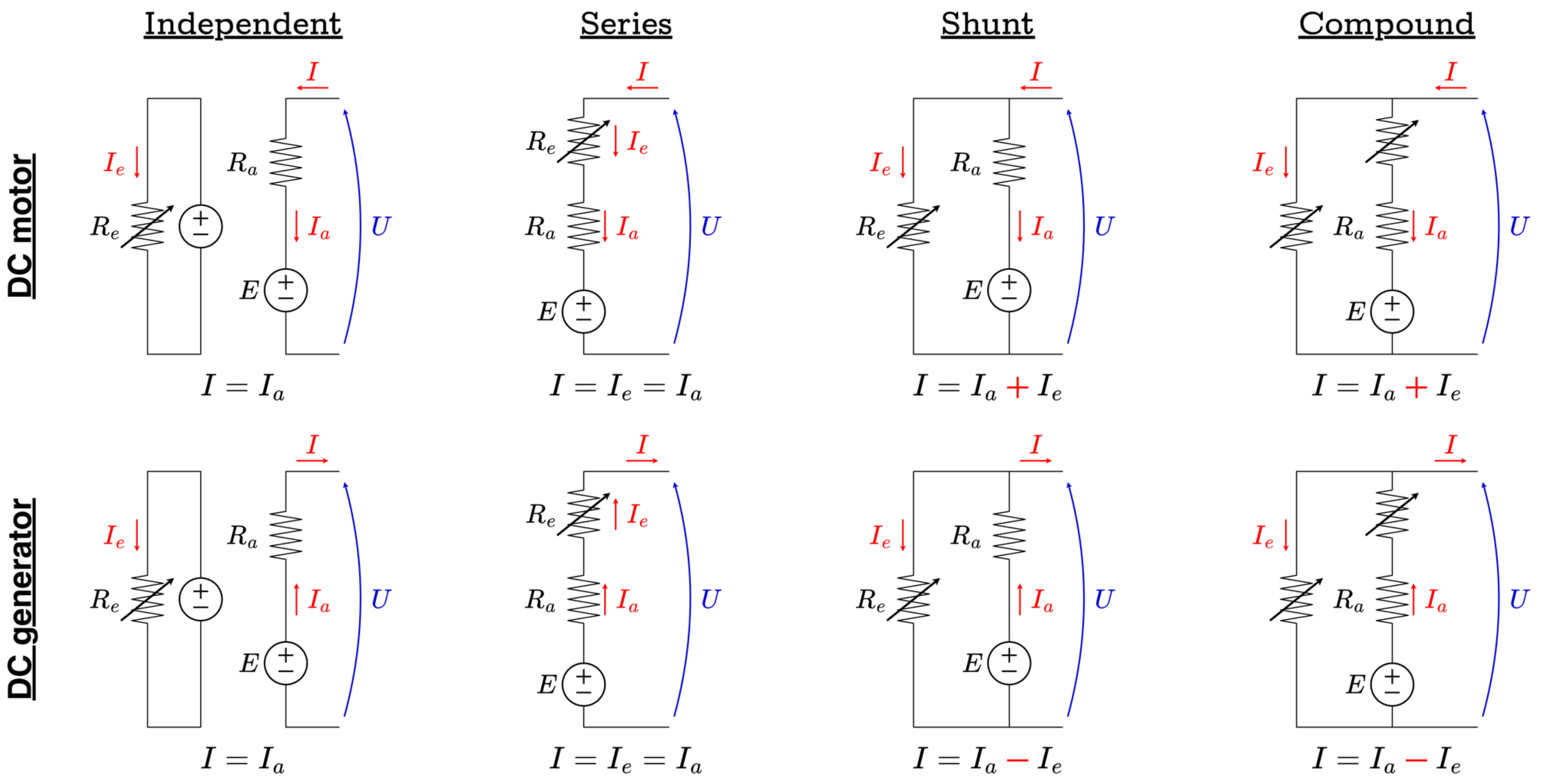
The commutator and the brushes ensure these emfs are converted into a DC voltage.

Note that the armature current I_a also produces a magnetic field $\Delta\Phi(I_a)$ which reduces the emf (armature reaction).

➔ $E = E_v - k_e \, \dot{\theta} \, \Delta\Phi(I_a) = k_e \, \dot{\theta} \, \big(\phi(I_e) - \Delta\Phi(I_a)\big)$.

DC machine configurations

When using electromagnets, different winding connections are possible:



From generator to motor, only I and I_a change of direction.

Exercise 13: Brushed DC motor

A hammer drill is made up of a DC motor in independent configuration. It has the following nominal characteristics:

- Output power $P_n = 800$ W
- Speed of rotation $\dot{\theta}_n = 1500$ RPM
- Input voltage $U_n = 220$ V
- Armature current $I_n = 4.6$ A
- Excitation current $I_{e,n} = 0.35$ A

The total emf E on the motor brushes has been measured as a function of the speed of rotation, for two different excitation currents:

$I_{e,1} = I_{e,n} = 0.35$ A:

$\dot{\theta}$ [RPM]	0	110	510	820	1040	1380	1510	1670
E [V]	0	20	75	120	150	200	220	240

$I_{e,2} = 0.2$ A:

$\dot{\theta}$ [RPM]	0	260	560	850	1150	1450	1800
E [V]	0	30	60	90	120	150	186

1. Plot E with respect to the speed of rotation $\dot{\theta}$, for $I_{e,1}$ and $I_{e,2}$. Justify the shape of the curves.
2. Is the magnetic flux ϕ proportional to the excitation current intensity I_e?
3. At nominal speed of rotation, E has been measured for different I_e's. Plot $E(I_e)$ and justify the shape of the curve.

I_e [A]	0.07	0.08	0.1	0.11	0.13	0.14	0.17	0.18	0.2	0.22	0.24	0.26	0.28	0.3	0.31	0.33	0.35	0.39
E [V]	56	65	78	87	100	107	130	137	147	158	168	179	188	194	198	204	210	219

Exercise 13: Brushed DC motor

The input voltage U and input current I have been measured for different excitation currents, keeping the motor at its nominal speed of rotation and without any load:

I_e [A]	0.1	0.15	0.2	0.25	0.3	0.35	0.4
U [V]	85	120	147	179	198	211	221
I [A]	0.92	0.66	0.56	0.48	0.45	0.44	0.43

4. Using an equivalent model of the motor with a stator resistance $R_e = 512.1\ \Omega$ and armature resistance $R_a = 4.6\ \Omega$, plot the collective losses p_c (i.e. ferromagnetic + mechanical losses) with respect to I_e.
5. Determine the mechanical losses p_m at the nominal speed of rotation.

The drill is connected to a DC voltage source of 212 V. When drilling a hole, it remains at its nominal speed of rotation and draws a current I of 3 A.

6. Calculate the electromotive force and deduce the value of the excitation current.
7. Compute the shaft output power P_{out}.
8. Deduce the resistive torque C_r induced by the drilling process.

Exercise 14: Brushed DC motor with series excitation

Consider a brushed DC motor with its excitation in series. This motor is fed by a constant voltage source $U = 220\,V$. To simplify the study, the armature and inductor resistances, the collective losses and the armature reaction are neglected.

1. Show that the electromagnetic torque is proportional to the square of the consumed current.
2. Show that the electromagnetic torque is inversely proportional to the square of the rotational speed of the motor.
3. Deduce that there is a runaway of the motor at no load.
4. According to sub-question 2, one can write that:

   $$C_{elm} = \frac{a}{\dot{\theta}^2}$$

   where C_{elm} corresponds to the electromagnetic torque of the motor in Nm, $\dot{\theta}$ is the speed of rotation in RPM and a is the constant to be determined. The nameplate of the machine indicates a nominal voltage of $220\,V$, a nominal speed of rotation of $1200\,RPM$ and a nominal current of $7.8\,A$. Deduce the value of the constant a.
5. Draw the mechanical characteristics of C_{elm} with respect to the speed of rotation.
6. The motor drives a hoist whose resistive torque is constant: $C_r = 10\,Nm$. Deduce the rotation speed.