ELEC0431
Electromagnetic Energy Conversion

Principles of Electromagnetism ⇨ **Various electrical devices**

University of Liège – Academic Year 2025-2026

Electromagnetic fields

Maxwell's equations

$$\mathrm{curl}\ \mathbf{h} = \mathbf{j} + \partial_t\, \mathbf{d}$$

$$\mathrm{curl}\ \mathbf{e} = -\, \partial_t\, \mathbf{b}$$

$$\mathrm{div}\ \mathbf{b} = 0$$

$$\mathrm{div}\ \mathbf{d} = \rho_v$$

Ampère-Maxwell's equation

Faraday's equation

Conservation equations

Principles of Electromagnetism

Physical fields

$\mathbf{h}$	magnetic field (A/m)	$\mathbf{e}$	electric field (V/m)
$\mathbf{b}$	magnetic flux density (T)	$\mathbf{d}$	electric displacement (C/m^2)
$\mathbf{j}$	current density (A/m^2)	ρ_v	charge density (C/m^3)

Lorentz force

Interaction of electromagnetic fields with a point charge moving at speed $\mathbf{v}$

$$\mathbf{F} = q\,(\mathbf{e} + \mathbf{v} \times \mathbf{b}) \quad \text{(N)}$$

For a conductor (electrically neutral, only negative charges moving)

$$\mathbf{f} = \mathbf{j} \times \mathbf{b} \quad \text{(N/m}^3\text{)}$$

Laplace force

Electromagnetic power

Poynting vector

$$\boldsymbol{s} = \boldsymbol{e} \times \boldsymbol{h}$$

Power exchanged with a volume (interior normal)

$$P = \oint_{\partial V} \boldsymbol{s} \cdot \boldsymbol{n} \, ds = -\int_V \operatorname{div} \boldsymbol{s} \, dv = \int_V p \, dv$$ (W)

Power density

$$p = -\operatorname{div} \boldsymbol{e} \times \boldsymbol{h} = -\boldsymbol{h} \cdot \operatorname{curl} \boldsymbol{e} + \boldsymbol{e} \cdot \operatorname{curl} \boldsymbol{h}$$ (W/m^3)

$$\Rightarrow p = \boldsymbol{h} \cdot \partial_t \boldsymbol{b} + \boldsymbol{e} \cdot \boldsymbol{j} + \boldsymbol{e} \cdot \partial_t \boldsymbol{d}$$

Material constitutive laws

Constitutive laws

$$\mathbf{b} = \mu\,\mathbf{h}$$

$$\mathbf{d} = \varepsilon\,\mathbf{e}$$

$$\mathbf{j} = \sigma\,\mathbf{e}$$

Magnetic law

Dielectric law

Ohm's law

Material characteristics

μ magnetic permeability (H/m)

ε dielectric permittivity (F/m)

σ electrical conductivity (Ω^{-1} m^{-1})

Constant (linear materials)

Function of the fields (nonlinear materials)

Tensorial (anisotropic materials)

Function of temperature, mechanical stress, ...

Magnetic constitutive law

$$\boxed{\mathbf{b} = \mu\, \mathbf{h}} \qquad \boxed{\mu = \mu_r\, \mu_0}$$

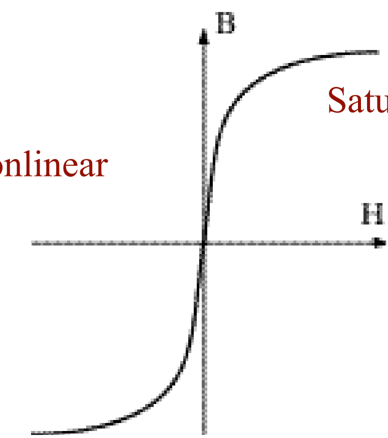
μ_r relative magnetic permeability
μ_0 magnetic permeability of vacuum (H/m)

- **Diamagnetic and paramagnetic materials**
  - **Linear materials** $\mu_r \approx 1$ (silver, copper, aluminum)
- **Ferromagnetic materials**
  - **Nonlinear materials** $\mu_r >> 1,\ \mu_r = \mu_r(h)$ (steel, iron)

b-h law

Nonlinear

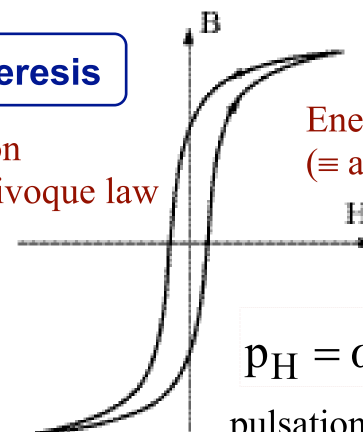
Saturation



Hysteresis

Non
univoque law

Energy dissipation
(≡ area of the cycle)



Steinmetz formula

$$p_H = \omega\, k_h\, b_{max}^{\nu} \quad (W/m^3)$$

pulsation ω, max. flux density b_{max}
coeficients k_h and ν ($1.5 < \nu < 1.8$)

Electromagnetic models

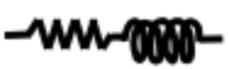
All governed by Maxwell's equations

- **Electrostatics**
  - Distribution of electric field due to static charges and levels of electric potential

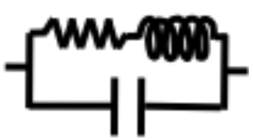
- **Electrokinetics**
  - Distribution of stationary electric current in conductors

- **Electrodynamics (or electroquasistatics, EQS)**
  - Distribution of electric field and currents in materials (both conductprs and insulators)

- **Magnetostatics**
  - Distribution of stationary magnetic field due to magnets and stationary currents



- **Magnetodynamics (or magnetoquasistatics, MQS)**
  - Distribution of magnetic field and eddy currents due to moving magnets and time-dependent currents



- **Wave propagation**
  - Electromagnetic wave propagation

MQS: quasi-stationnary approximation

$$\boxed{\mathrm{curl}\ \mathbf{h} = \mathbf{j} + \partial_t\ \mathbf{d}}$$

Small dimensions compared to wavelength

Conduction current density **>>>** **Displacement current density**

$$\boxed{\mathrm{curl}\ \mathbf{h} = \mathbf{j}}$$

Ampère's law

Applications

Electrotechnical devices (motors, generators, power transformers, ...)
Usually, frequencies up to several 100's of kHz

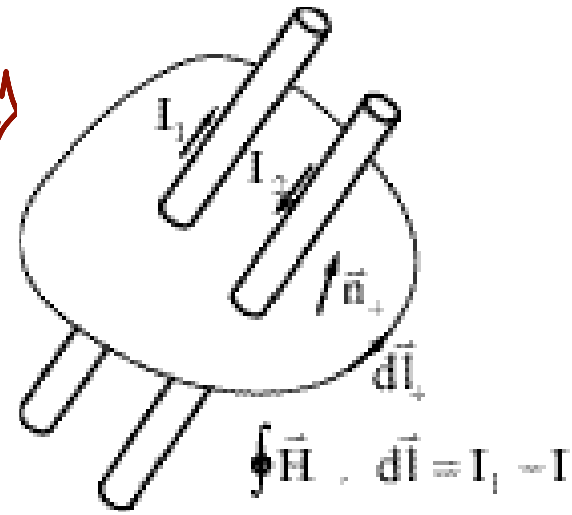
Ampère's law

$$\text{curl}\,\mathbf{h} = \mathbf{j}$$

Ampère's law

$$\oint \mathbf{h} \cdot d\mathbf{l} = I$$

The circulation of the magnetic field along a closed contour is equal to the algebraic sum of the currents crossing any surface bounded by this contour



$$\text{div}\,\mathbf{j} = 0$$

Conservation of the current

$$\oint \mathbf{j} \cdot \mathbf{n}\, ds = 0$$

The sum of the currents arriving at a given point is zero

Faraday's law

$$\boxed{\mathrm{curl}\ \mathbf{e} = -\,\partial_t\,\mathbf{b}}$$

Faraday's law

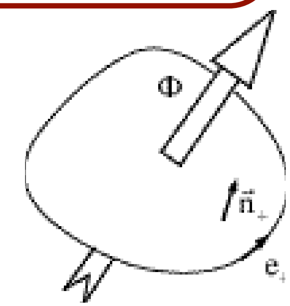
$$\boxed{\oint \mathbf{e}\cdot d\mathbf{l} = -\partial_t \Phi}$$

e.m.f

Any variation (time, movement or deformation) of the magnetic flux density embraced by a circuit (open or closed) gives rise to an electromotive force (e.m.f.) ...

Lenz' law

... which, when this circuit is closed, gives rise to currents generating magnetic flux density opposing these variations



Movement, velocity v

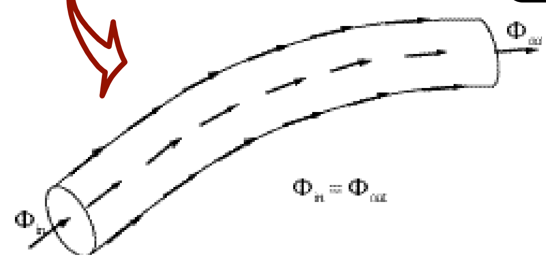
$$\boxed{\mathbf{e} = \mathbf{v} \times \mathbf{b}}$$

$$\boxed{\mathrm{div}\ \mathbf{b} = 0}$$

Conservation of the magnetic flux

$$\boxed{\oint \mathbf{b}\cdot\mathbf{n}\, ds = 0}$$

Magnetic flux lines are closed



Faraday's law – Eddy currents

$$\boxed{\mathrm{curl}\ \mathbf{e} = -\,\partial_t\,\mathbf{b}}$$

Faraday's law

$$\boxed{\oint \mathbf{e} \cdot d\mathbf{l} = -\partial_t \Phi}$$

In a massive conductor subject to time-varying magnetic field, e.m.f.s appear that give rise to currents

Eddy (or induced) currents

Heating by Joule effect
(degrades efficiency)

Reduction of the global magnetic flux *(Lenz's law) (degrades material efficiency)*

Laminated magnetic materials

Stacks of thin magnetic sheets, parallel to the magnetic flux density and electrically isolated

For thin sheets, eddy current losses:

$$p_F = \frac{\omega^2\, e_t^2\, \sigma}{16}\, b_{max}^2 \qquad (\mathrm{W/m^3})$$

pulsation ω, sheet thickness e_t,
electrical conductivity σ,
max. magnetic flux density b_{max}

Skin effect

$$\mathrm{curl}\ \mathbf{e} = -\,\partial_t\,\mathbf{b}$$

Faraday's law

$\Rightarrow$ *Skin effect*

$$\delta = \sqrt{\frac{2}{\omega\,\sigma\,\mu}} \quad (\mathrm{m})$$

The skin depth δ *characterizes the depth in the material at which the current (and the magnetic field) tend to concentrate.*
Increasing the frequency leads to smaller δ*, which leads to currents concentrated closer to the surface of the conductor.*

ω pulsation (rad/m)
σ electrical conductivity ($\Omega^{-1}\ \mathrm{m}^{-1}$)
μ magnetic permeability (H/m)

Magnetic circuits

Produced by electric currents (e.g. in windings) or magnets

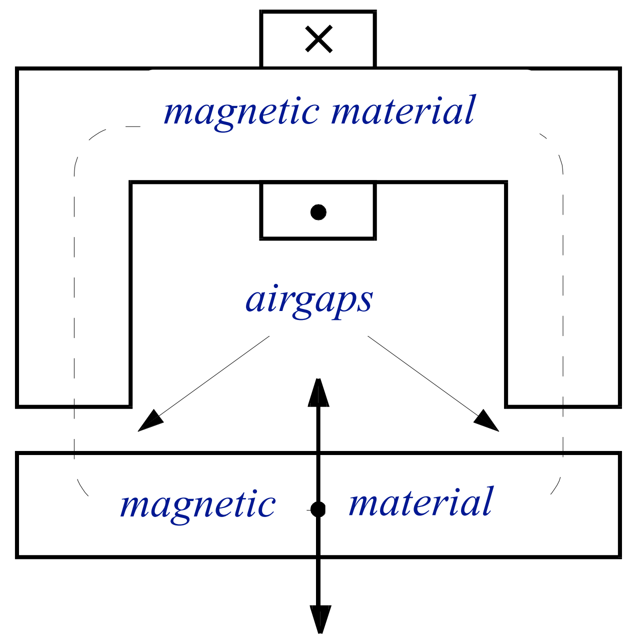
Magnetic field

through which the transfer of conversion of energy is carried out (e.g. between windings for electrical energy)

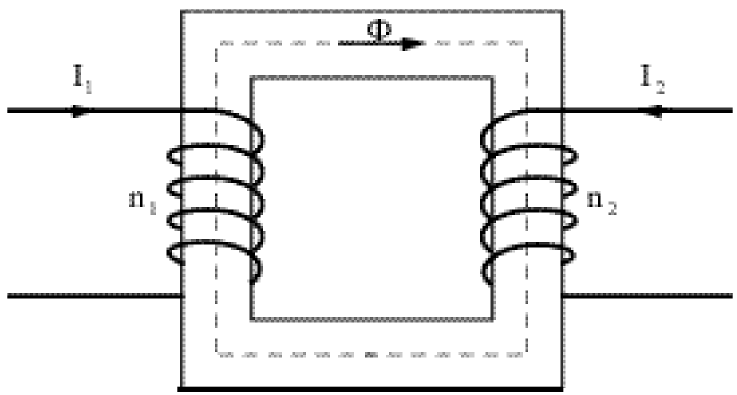
Interest in high magnetic coupling (good magnetic link)

Magnetic circuits with magnetic materials to channel the magnetic flux density

with airgaps (e.g. separating moving parts)



Ideal magnetic circuit



neutral fiber length of the circuit

Magnetomotive forces (m.m.f.)

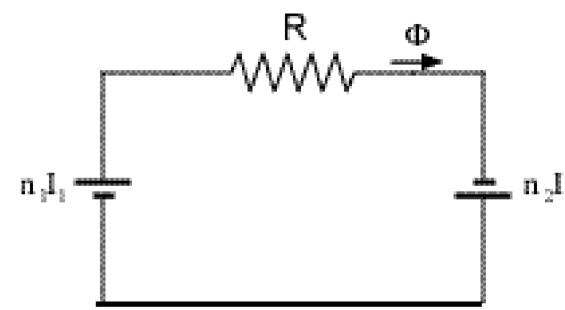
$$\oint \mathbf{h} \cdot d\mathbf{l} = h\,\ell = n_1 I_1 + n_2 I_2$$

section of the circuit

$$\Phi = b\,S = \mu\, h\, S = (n_1 I_1 + n_2 I_2)\frac{\mu S}{\ell} = \frac{n_1 I_1 + n_2 I_2}{R}$$

Reluctance of the circuit

$$R = \frac{\ell}{\mu S}$$



$$\Phi_{t1} = n_1 \Phi = \frac{n_1^2}{R} I_1 + \frac{n_1 n_2}{R} I_2 = L_1 I_1 + M_{12} I_2$$

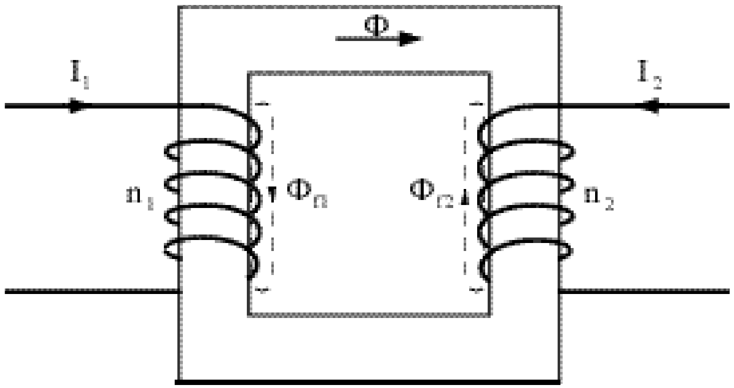
$$\Phi_{t2} = n_2 \Phi = \frac{n_1 n_2}{R} I_1 + \frac{n_2^2}{R} I_2 = M_{21} I_1 + L_2 I_2$$

Inductances

$$L_1 = \frac{n_1^2}{R}, \quad L_2 = \frac{n_2^2}{R}, \quad M = M_{12} = M_{21} = \frac{n_1 n_2}{R}$$

Perfect magnetic coupling $\quad L_1 L_2 = M^2$

Real magnetic circuit



$$\Phi = \frac{n_1 I_1 + n_2 I_2}{R}$$ Useful flux

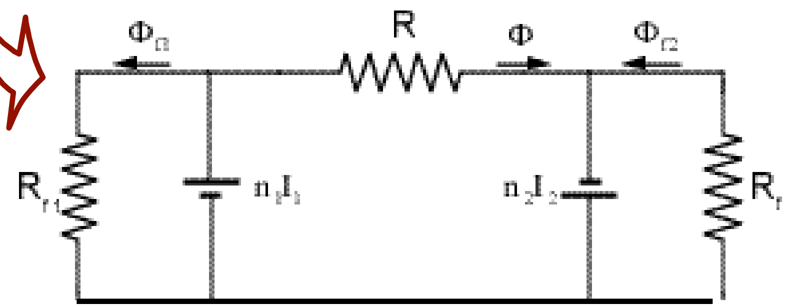
Leakage flux

$$\Phi_{f1} = \frac{n_1 I_1}{R_{f1}} \quad \text{et} \quad \Phi_{f2} = \frac{n_2 I_2}{R_{f2}}$$

Leakage reluctances

$$\Phi_{t1} = n_1\left(\Phi + \Phi_{f1}\right) = \left(\frac{n_1^2}{R} + \frac{n_1^2}{R_{f1}}\right) I_1 + \frac{n_1 n_2}{R} I_2 = L_1 I_1 + M_{12} I_2$$

$$\Phi_{t2} = n_2\left(\Phi + \Phi_{f2}\right) = \frac{n_1 n_2}{R} I_1 + \left(\frac{n_2^2}{R} + \frac{n_2^2}{R_{f2}}\right) I_2 = M_{21} I_1 + L_2 I_2$$



Inductances

$$L_1 = \frac{n_1^2}{R} + \frac{n_1^2}{R_{f1}}, \quad L_2 = \frac{n_2^2}{R} + \frac{n_2^2}{R_{f2}}, \quad M = M_{12} = M_{21} = \frac{n_1 n_2}{R}$$

Non-ideal magnetic coupling $L_1 L_2 \geq M^2$