Synchronous machines

**Synchronous generator (alternator):** transforms mechanical energy into electric energy; designed to generate sinusoidal voltages and currents; used in most power plants, for car alternators, etc.

**Synchronous motor:** transforms electric energy into mechanical energy; used for high-power applications (ships, original TGV...)

**Rotor (inductor):** 2p poles with excitation windings carrying DC current; non-laminated magnetic material

**Stator:** polyphase (e.g. 3-phase) winding in slots; laminated magnetic material

\[ \theta = \omega / p \]
No-load characteristic

Evolution of the voltage $E_v$ in a stator phase vs. intensity of the excitation current $I_e$, for a given rotation speed and with no generated stator current.

- The rotor winding, carrying the DC current $I_e$ and rotating at speed $\omega/p$, produces in the airgap a sliding m.m.f. $F_e$ (as seen from the stator).
- $F_e$ generates a magnetic flux density $B_r$ (with the same phase) in the airgap, which induces sinusoidal e.m.f.s $E_v$ in the stator windings, with a phase lag of $\pi/2$.

\[ E_v = f(I_e) \text{ with } \begin{cases} \text{speed} & \dot{\theta} = \text{constant} \\ I = 0 \end{cases} \]

\[ E_v = k_E \dot{\theta} \Phi_v(I_e) \]

Magnetic flux produced by the inductor and seen by the stator winding.

Non linearity with hysteresis.

Synchronous machines
Vector diagram with load

Diagram of magnetomotive forces and magnetic flux densities

(1) The rotor winding, carrying the DC current $I_e$ and rotating at speed $\omega/p$, produces in the airgap a sliding m.m.f. $F_e$ (as seen from the stator).

(2) The polyphase current $I$ in the stator winding produces a sliding m.m.f. $F_I$ (in phase with $I$).

(3) The resulting m.m.f. is $F_r = F_e + F_I$.

(4) $F_r$ generates a magnetic flux density $B_r$ (with the same phase) in the airgap, which induces sinusoidal e.m.f.s in the stator windings, with a phase lag of $\pi/2$. 

$F_e = \gamma I_e$ and $F_I = \delta I$
Vector diagram with load

Stator leakage flux

(see by the stator but not coming from the rotor)

\[ e_{\lambda}(t) = -\lambda \partial_t i_1 - \lambda_m \partial_t i_2 - \lambda_m \partial_t i_3 = -(\lambda - \lambda_m) \partial_t i_1 \]

because \( i_1 + i_2 + i_3 = 0 \)

Slot leakage flux

End-winding leakage flux

Diagrams

\[ \bar{U} = \bar{E}_r - jX_f \bar{I} - R \bar{I} \]

Resistance of stator winding

Total e.m.f.

\[ \bar{E}_r = \bar{E}_t - jX_f \bar{I} \]

E.m.f. induced by leakage flux

\[ \bar{E}_\lambda = -jX_f \bar{I} \]

with \( X_f = \omega (\lambda - \lambda_m) \)

Stator leakage reactance

Synchronous machines

\[ \bar{E}_r = \gamma \bar{I}_e \]

and \( \bar{F}_r = \delta \bar{I} \)

\[ \bar{F}_e = \gamma \bar{I}_e \]

\[ \bar{I}_r = \bar{F}_r = \bar{I}_e + \frac{\delta}{\gamma} \bar{I} \]

Equivalent excitation current

\( E_r \equiv \text{no-load e.m.f. produced by } I_r \)
‘Which excitation current $I_e$ should one impose in the synchronous machine to reach the functioning point corresponding to a given voltage $U$ and current $I$ in the stator, with a phase shift of $\phi$ between $U$ and $I$?’

\[
E_r = U + R I + j X_f I
\]
**Reaction**

**Demagnetizing reaction**

The m.m.f. is smaller than the no-load m.m.f. \((I_r < I_e)\)

**Magnetizing reaction**

The m.m.f. is larger than the no-load m.m.f. \((I_r > I_e)\)

Inductive behaviour of the load
(I lagging behind U)

Capacitive behaviour of the load
(I in front of U)
Zero power factor characteristic

Evolution of the stator voltage $U$ as a function of the excitation current $I_e$, for a given rotation speed and stator current, with a zero power factor

$U = f(I_e)$ with

\[
\begin{align*}
\text{speed } \dot{\theta} &= \text{constant} \\
I &= \text{constant} \\
\cos \phi &= 0 \ (\phi = \pm \pi/2)
\end{align*}
\]

Example: $\phi = \pi/2$

\[
I_e \approx I_r + \frac{\delta}{\gamma} I
\]

\[
U \approx E_r - X_f I
\]
Short-circuit characteristic

Evolution of the stator current as a function of the excitation current $I_e$, for a given rotation speed and with the stator windings in short-circuit

\[ E_r = U + R \bar{I} + j X_f \bar{I} \]

with $U = 0$

\[ I = f(I_e) \quad \text{with} \quad \begin{cases} \text{speed } \dot{\theta} = \text{constant} \\ U = 0 \end{cases} \]

\[ I \approx \frac{\beta}{X_f + \beta \frac{\delta}{\gamma}} I_e \]

Non saturated machine

Synchronous machines
When the magnetic materials are not saturated, the combined effect of the reaction and of stator leakage fluxes can be taken into account thanks to a single parameter: the synchronous reactance $X_s$.

$$\frac{E_v}{I_e} = \frac{E_r}{I_r} = \text{constant}$$

Equal angles OAB and A’OB’

Similar triangles OAB and OA’B’

A, B and C colinear

$$E_v = U + R I + j X_s I$$

Synchronous reactance

**Behn-Eschenburg’s method – Synchronous reactance $X_s$**
Experimental determination of $X_s$

Behn-Eschenburg’s method – Synchronous reactance $X_s$

$U = 0 \implies \bar{E}_v = (R + j X_s) \bar{I}_{cc}$

$\implies R + j X_s = \frac{\bar{E}_v}{\bar{I}_{cc}}$ with $R \ll X_s$

$X_s \approx \frac{E_v(I_e)}{I_{cc}(I_e)}$

Approximation when magnetic materials are saturated!

Synchronous machines
Exterior characteristic

Alternator exterior characteristic

Evolution of the voltage $U$ on a given stator phase as a function of the current $I_{in}$ in this phase, when the alternator drives a load characterized by a constant power factor, at constant speed and excitation.

$U = f(I)$ with:

- speed $\dot{\theta} = \text{constant}$
- $I_e = \text{constant}$
- $\cos \phi = \text{constant}$
Network connection

Need for interconnection of electric power plants

Economical organization of power production + Stability of the network despite local defects

Synchronization of an alternator on an ideal (infinitely powerful) AC network

The current should be zero when the connection is made → 4 conditions

1. same pulsation \( \omega \) (correct rotation speed)
2. same amplitudes for \( E_v \) and \( U \) (adjusting \( I_e \))
3. no phase shift between \( E_v \) and \( U \)
4. identical phase ordering (in a 3-phase system)

Synchronous machines
Behaviour with load

Electromagnetic power

Active electric power

\[ P_{\text{elm}} \approx P = 3U I \cos \phi \]

Torque

\[ C = \frac{P}{\omega / p} = \frac{3p}{\omega} U I \cos \phi \]

The variations of the rotor mechanical angle \( \Delta \delta_{\text{mec}} \) are proportional to the variations of the internal (electric) angle \( \Delta \delta_{\text{int}} \)

\[ \Delta \delta_{\text{mec}} = \frac{\Delta \delta_{\text{int}}}{p} \]

After network synchronization, there is no exchanged current. Then:

- If mechanical power is provided to the alternator, \( E_v \) gets ahead of \( U \) \( \Rightarrow \delta_{\text{int}} \) increases (until the equilibrium of the electromagnetic and mechanical torques)
- If a braking torque is applied to the alternator, \( E_v \) gets behind \( U \) \( \Rightarrow \delta_{\text{int}} \) decreases (negative torque)
Behaviour with load

Static stability

\[ \delta_{\text{int}} < 0 \quad \text{and} \quad \delta_{\text{int}} > 0 \]

Increasing the mechanical (breaking) torque leads to an increase of the absolute value of \( \delta_{\text{int}} \), and thus to a decrease in the absolute value of \( C_{\text{elm}} \).  

\( \Rightarrow \) unstable

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\( \Rightarrow \) unstable

Internal angle

Synchronous machines

Unstable

Unstable

Unstable

Unstable

The equilibrium is reached when the two torques are equal.

Increasing the mechanical (breaking) torque leads to an increase of \( \delta_{\text{int}} \) and thus of \( C_{\text{elm}} \).  

\( \Rightarrow \) stable.

The equilibrium is reached when the two torques are equal.
Behaviour with load

Power diagram

\[ MB = X_s I \cos \phi = \frac{X_s}{3} \frac{3}{U} I \cos \phi = \frac{X_s}{3} P = \frac{X_s}{3} P_{elm} \]

Active power \( P \)

\[ O'B = X_s I \sin \phi = \frac{X_s}{3} \frac{3}{U} I \sin \phi = \frac{X_s}{3} Q \]

Reactive power \( Q \)

Alternator

Motor

Synchronous machines
V-curves (Mordey curves)

Evolution of the stator current \( I \) as a function of the excitation current \( I_e \) of a synchronous machine connected to an ideal network, at constant active power.

![Diagram]

- **Constant active power**
- **under-excited machine**
- **Over-excited machine**

\[ \phi < 0 \quad \text{...} \quad \phi > 0 \]

Network = inductive load
Machine = capacitif system

Static stability limit

Synchronous machines