Synchronous machines

**Synchronous generator (alternator):** transforms mechanical energy into electric energy; designed to generate sinusoidal voltages and currents; used in most power plants, for car alternators, etc.

**Synchronous motor:** transforms electric energy into mechanical energy; used for high-power applications (ships, original TGV...)

**Rotor (inductor):** 2p poles with excitation windings carrying DC current; non-laminated magnetic material

**Stator:** polyphase (e.g. 3-phase) winding in slots; laminated magnetic material

\[ \theta = \omega/p \]
No-load characteristic

Evolution of the voltage $E_v$ in a stator phase vs. intensity of the excitation current $I_e$, for a given rotation speed and with no generated stator current

- The rotor winding, carrying the DC current $I_e$ and rotating at speed $\omega/p$, produces in the airgap a sliding m.m.f. $F_e$ (as seen from the stator).
- $F_e$ generates a magnetic flux density $B_r$ (with the same phase) in the airgap, which induces sinusoidal e.m.f.s $E_v$ in the stator windings, with a phase lag of $\pi/2$.

$$E_v = f(I_e) \quad \text{with} \quad \begin{cases} \text{speed} & \dot{\theta} = \text{constant} \\ I = 0 \end{cases}$$

$$E_v = k_E \dot{\theta} \Phi_v(I_e)$$

Magnetic flux produced by the inductor and seen by the stator winding

Non linearity with hysteresis

Synchronous machines
(1) The rotor winding, carrying the DC current $I_e$ and rotating at speed $\omega/p$, produces in the airgap a sliding m.m.f. $F_e$ (as seen from the stator).

(2) The polyphase current $I$ in the stator winding produces a sliding m.m.f. $F_I$ (in phase with $I$).

(3) The resulting m.m.f. is $F_r = F_e + F_I$.

(4) $F_r$ generates a magnetic flux density $B_r$ (with the same phase) in the airgap, which induces sinusoidal e.m.f.s in the stator windings, with a phase lag of $\pi/2$. 

$$F_e = \gamma I_e \quad \text{and} \quad F_I = \delta I$$
Vector diagram with load

**Stator leakage flux**

(see by the stator but not coming from the rotor)

\[ e_\lambda(t) = -\lambda \partial_t i_1 - \lambda_m \partial_t i_2 - \lambda_m \partial_t i_3 = -\left(\lambda - \lambda_m\right) \partial_t i_1 \]

because \( i_1 + i_2 + i_3 = 0 \)

\[ \bar{E}_\lambda = -j X_f \bar{I} \]

with \( X_f = \omega \left(\lambda - \lambda_m\right) \)

**End-winding leakage flux**

**Slot leakage flux**

**Diagrams**

\[ \bar{U} = \bar{E}_r - j X_f \bar{I} - R \bar{I} \]

Resistance of stator winding

\[ \bar{F}_e = \gamma \bar{I}_e \]

and \( \bar{F}_f = \delta \bar{I} \)

\[ \bar{F}_r = \gamma \bar{I}_e + \delta \bar{I} \]

\[ \bar{I}_r = \frac{\bar{F}_r}{\gamma} = \bar{I}_e + \frac{\delta}{\gamma} \bar{I} \]

Equivalent excitation current

\( E_r \equiv \text{no-load e.m.f. produced by } I_r \)

Synchronous machines
‘Which excitation current $I_e$ should one impose in the synchronous machine to reach the operating point corresponding to a given voltage $U$ and current $I$ in the stator, with a phase shift of $\varphi$ between $U$ and $I$?’

$E_r \equiv$ no-load e.m.f produced by the equivalent current $I_r$

$$\bar{E}_r = \bar{U} + R \bar{I} + jX_f \bar{I}$$
Reaction

**Demagnetizing reaction**

The m.m.f. is smaller than the no-load m.m.f. \((I_r < I_e)\)

Inductive behaviour of the load
(I lagging behind U)

**Magnetizing reaction**

The m.m.f. is larger than the no-load m.m.f. \((I_r > I_e)\)

Capacitive behaviour of the load
(I in front of U)
Zero power factor characteristic

Evolution of the stator voltage $U$ as a function of the excitation current $I_e$, for a given rotation speed and stator current, with a zero power factor

$$\begin{align*}
I_e &\approx I_r + \frac{\delta}{\gamma} I \\
U &\approx E_r - X_f I
\end{align*}$$

Example: $\varphi = \pi/2$
Short-circuit characteristic

Evolution of the stator current as a function of the excitation current $I_e$, for a given rotation speed and with the stator windings in short-circuit

\[ E_r = U + R \bar{I} + j X_f \bar{I} \]

\[ I = f(I_e) \] with \[ \begin{cases} \text{speed } \dot{\theta} = \text{constant} \\ U = 0 \end{cases} \]

\[ I_e \approx I_r + \frac{\delta}{\gamma} I \]

\[ E_r = \beta I_r \]

Non saturated machine

Synchronous machines
When the magnetic materials are not saturated, the combined effect of the reaction and of stator leakage fluxes can be taken into account thanks to a single parameter: the synchronous reactance $X_s$

$$\frac{E_v}{I_e} = \frac{E_r}{I_r} = \text{constant}$$

Equal angles OAB and A’OB’

Similar triangles OAB and OA’B’

A, B and C colinear

$$E_v = U + R \overline{I} + jX_s \overline{I}$$

Synchronous reactance
Experimental determination of $X_s$

**Behn-Eschenburg’s method – Synchronous reactance $X_s$**

$U = 0 \implies \overline{E_v} = (R + jX_s) \overline{I_{cc}}$

$\implies R + jX_s = \frac{\overline{E_v}}{\overline{I_{cc}}}$

with $R << X_s$

$X_s \approx \frac{\overline{E_v}(I_e)}{\overline{I_{cc}}(I_e)}$

*Approximation when magnetic materials are saturated!*

Synchronous machines
Evolution of the voltage $U$ on a given stator phase as a function of the current $I$ in this phase, when the alternator drives a load characterized by a constant power factor, at constant speed and excitation.

\[ U = f(I) \quad \text{with:} \quad \begin{cases} \text{speed } \dot{\theta} = \text{constant} \\ I_e = \text{constant} \\ \cos \phi = \text{constant} \end{cases} \]
Network connection

Need for interconnection of electric power plants

Economical organization of power production + Stability of the network despite local defects

Synchronization of an alternator on an ideal (infinitely powerful) AC network

Large number of production units in parallel ⇒ constant voltage and frequency

The current should be zero when the connection is made → 4 conditions

1. same pulsation \( \omega \) (correct rotation speed)
2. same amplitudes for \( E_v \) and \( U \) (adjusting \( I_e \))
3. no phase shift between \( E_v \) and \( U \)
4. identical phase ordering (in a 3-phase system)

Synchronous machines
Behaviour with load

Electromagnetic power

\[ P_{\text{elm}} \approx P = 3 \, U \, I \cos \varphi \]

Torque

\[ C = \frac{P}{\omega} = \frac{3 \, p}{\omega} \, U \, I \, \cos \varphi \]

Internal angle

\[ X_s \, I \, \cos \varphi = E_v \, \sin \delta_{\text{int}} \]

\[ C = \frac{3 \, p}{\omega \, X_s} \, U \, E_v \, \sin \delta_{\text{int}} \]

The variations of the rotor mechanical angle \( \Delta \delta_{\text{mec}} \) are proportional to the variations of the internal (electric) angle \( \Delta \delta_{\text{int}} \)

\[ \Delta \delta_{\text{mec}} = \frac{\Delta \delta_{\text{int}}}{p} \]

After network synchronization, there is no exchanged current. Then:

- If mechanical power is provided to the alternator, \( E_v \) gets ahead of \( U \) \( \Rightarrow \delta_{\text{int}} \) increases (until the equilibrium of the electromagnetic and mechanical torques)
- If a braking torque is applied to the alternator, \( E_v \) gets behind \( U \) \( \Rightarrow \delta_{\text{int}} \) decreases (negative torque)
Increasing the mechanical (breaking) torque leads to an increase of the absolute value of $\delta_{\text{int}}$ and thus to a decrease in the absolute value of $C_{\text{elm}} \Rightarrow \text{unstable}$.

Increasing the mechanical torque leads to an increase of $\delta_{\text{int}}$ and thus of $C_{\text{elm}} \Rightarrow \text{stable}$.

The equilibrium is reached when the two torques are equal.
Behaviour with load

Power diagram

\[ MB = X_s \ I \cos \varphi = \frac{X_s}{3 \ U} \ U \ I \cos \varphi = \frac{X_s}{3 \ U} \ P = \frac{X_s}{3 \ U} \ P_{elm} \]

Active power \( P \)

\[ O'B = X_s \ I \sin \varphi = \frac{X_s}{3 \ U} \ U \ I \sin \varphi = \frac{X_s}{3 \ U} \ Q \]

Reactive power \( Q \)

Synchronous machines
V-curves (Mordey curves)

Evolution of the stator current $I$ as a function of the excitation current $I_e$ of a synchronous machine connected to an ideal network, at constant active power.

Constant active power

under-excited machine

Over-excited machine

"V"

$\phi < 0$ ...

$\phi > 0$

Network = inductive load
Machine = capacitif system

Synchronous machines