Conversion: one unique principle

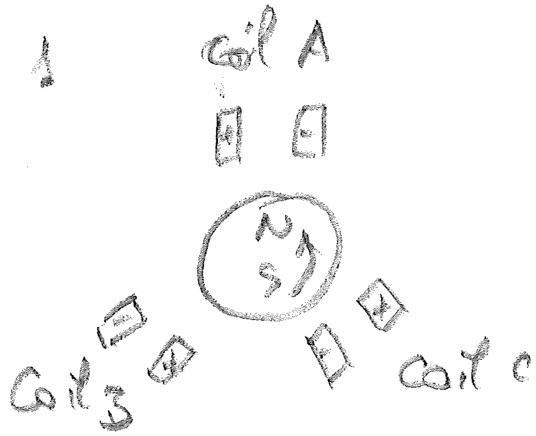
Transfer power from one "coil" to another one : create a <u>varying</u> flux with the first coil and harvest the <u>induced</u> voltage with the second coil.

<u>Transformer</u> | <u>Synchronous machine</u>

AC current at primary =>

Rotation of a DC magnetized rotor

$p = 1$



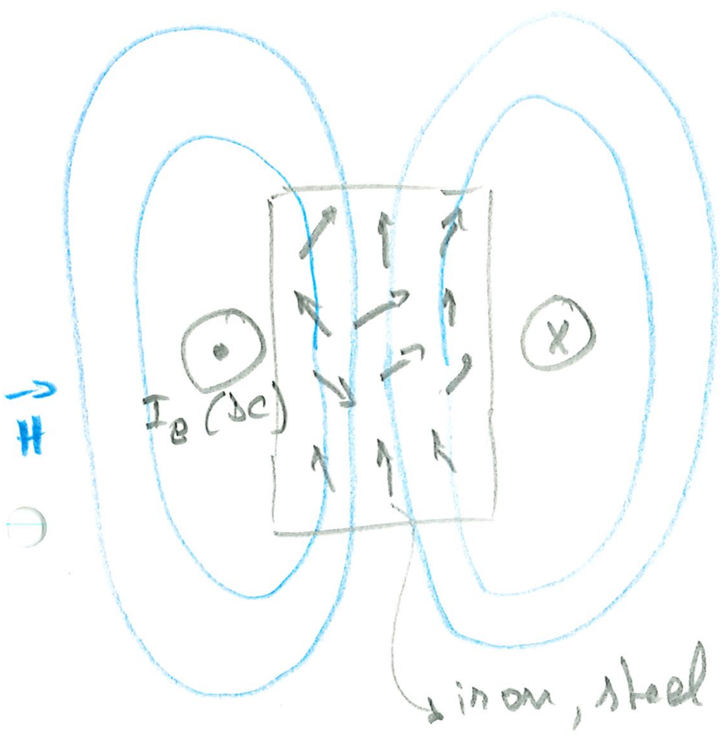
Slide à left

⚠ at least 3 coils per pole pair

<u>Several</u> stator coils regularly disposed over the circumference => poly-phase currents are induced

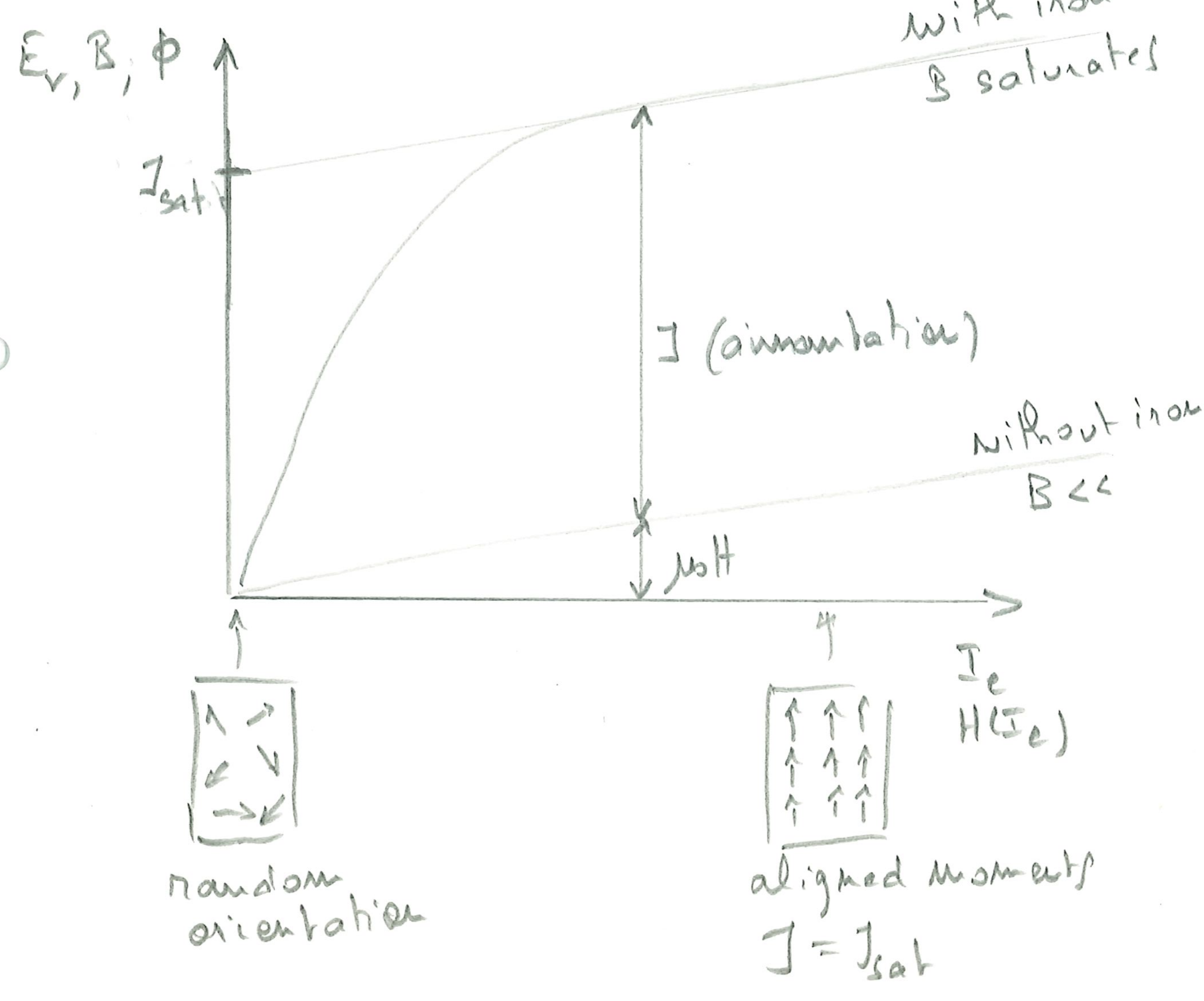
Electrical machine have always the same number of pole pairs at the stator and at the rotor.

Slide 2: Magnetic saturation

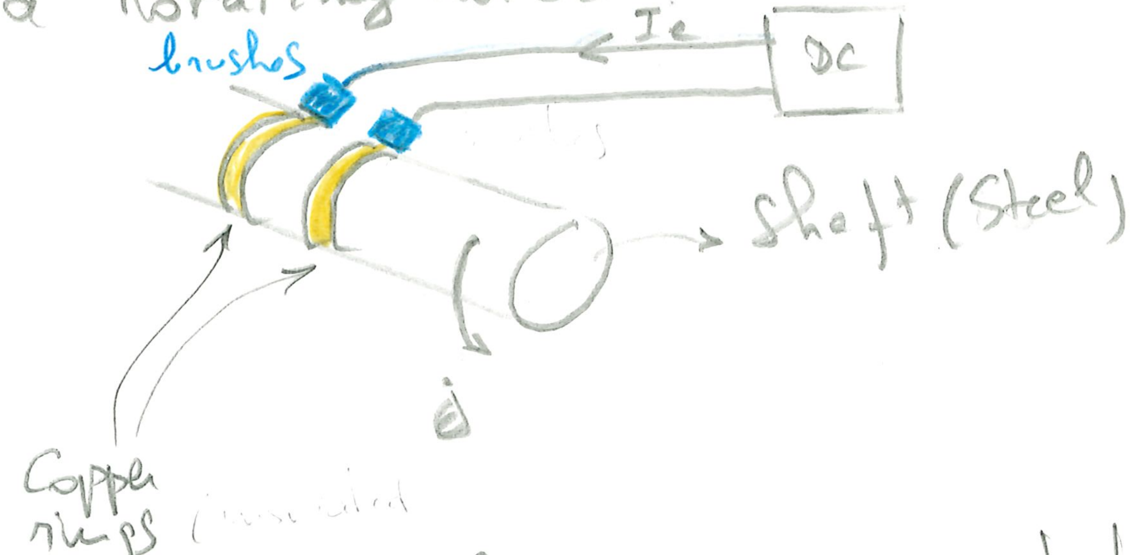


microscopic magnetic moments tend to align with the magnetic field $\vec{H}(I_e)$

$$B = \mu_0 H + J$$



How can one inject a current in a rotating rotor ?



In the rotor, the rings are connected to the excitation windings

At which speed should the rotor rotate to induce 50 Hz voltages ?

$$1 \text{ rpm} = \frac{2\pi \text{ rad}}{60 \text{ s}}$$

$$\omega = 2\pi f$$

$$\omega = p\dot{\theta}$$

$$\dot{\theta} = \frac{\omega}{p} = \frac{2\pi f}{p} \text{ (rad/s)}$$

$$= \frac{2\pi f}{p} \frac{60}{2\pi} \text{(rpm)} = \frac{f\,60}{p} \text{ rpm}$$

for $f = 50$ Hz, $\dot{\theta} = \frac{3000}{p}$ rpm

PRIMARY

Rotor (excitation)

DC + rotation

$\rightarrow \hat{\mathcal{F}}_e = \gamma I_e$ phasor

Synchronous machine — regarded as a transformer

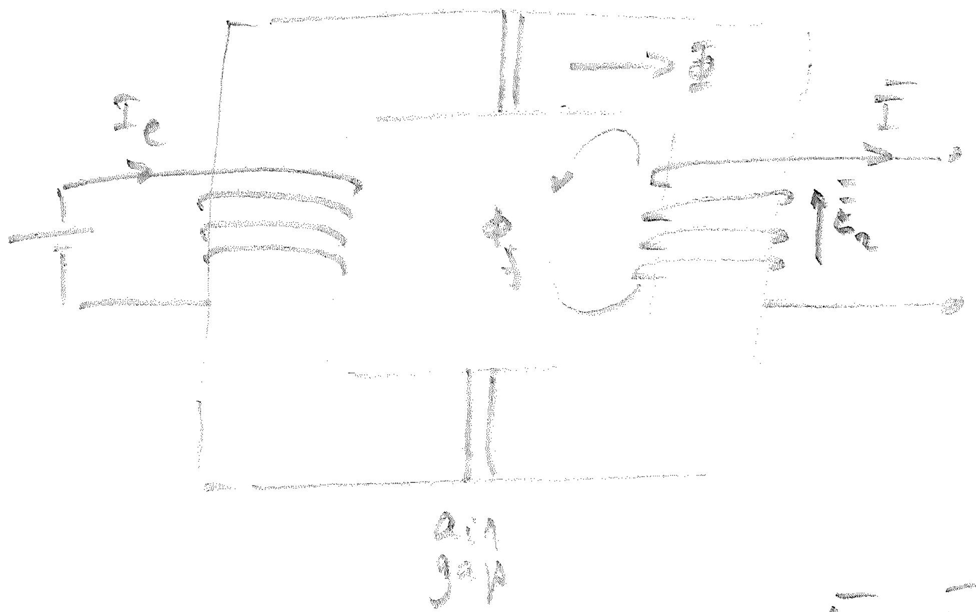
SECONDARY

One Stator phase

AC $\rightarrow \hat{\mathcal{F}}_I = \delta \bar{I}$ phasor

Transfo slide 2:

$$\begin{cases} v_2 = -R_2 i_2 - \ell_2 \partial_t i_2 + e_2 \\ e_2 = + n_2 \partial_t \Phi \end{cases}$$



$$\bar{V} = -R\bar{I} - jX_l\bar{I} + \bar{E}_2$$

$$\bar{E}_2 \propto -j\omega\bar{\Phi}$$

active convention because alternator = generator

$\bar{\Phi}$ is due to both I_e and $\bar{I} \rightarrow \bar{\Phi} = \dfrac{\hat{\mathcal{F}}_e + \hat{\mathcal{F}}_I}{\mathcal{R}}$ (magnetic circuit)

γ, δ play the role of the numbers of turns ($\hat{\mathcal{F}} = nI$)

$\dfrac{\delta}{\gamma}$ plays ——— the transformation ratio $\dfrac{n_2}{n_1}$

Terminology

Excitation I_e =
magnetisation of the machine

I = armature current
= armature reaction

d-axis = quantites that are in phase with the phasor $\bar{I}_e$

q-axis = quantities that are in quadrature (at an angle $\pm \frac{\pi}{2}$) with the phasor $\bar{I}_e$

<u>Slide 3</u> $\propto$ is proportional to

$\bar{E}_n \propto -j\omega \bar{\phi}$ $\quad \bar{\phi} = \dfrac{\bar{F}_e + \bar{F}_I}{\mathcal{R}}$

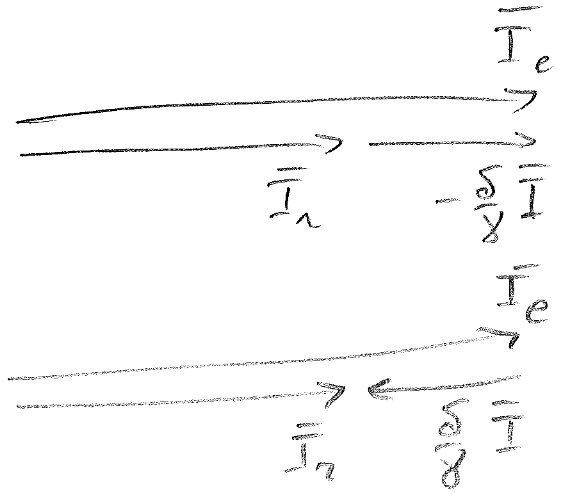
with $\bar{\phi}$ the <u>resultant</u> ϕ embraced by the turns of one phase of the stator windings (the three phases are assumed to be balanced)

$$\bar{E}_n \propto \omega \frac{\bar{F}_e + \bar{F}_I}{\mathcal{R}} \propto \frac{\omega}{\mathcal{R}}(\gamma \bar{I}_e + \delta \bar{I}) = \frac{\omega}{\mathcal{R}} \gamma \bar{I}_n$$

$$\bar{E}_v \propto \omega \frac{\bar{F}_e}{\mathcal{R}} \propto \frac{\omega}{\mathcal{R}}(\gamma \bar{I}_e) = \frac{\omega}{\mathcal{R}} \gamma \bar{I}_e$$

$$\Rightarrow \frac{\bar{E}_n}{\bar{I}_n} = \frac{\bar{E}_v}{\bar{I}_e} = \beta$$ (used slide 8)

Slide 7



$$\bar I_e = \bar I_r - \frac{\delta}{\gamma}\bar I$$

By projection of the phasors

$$|\bar I_e| = |\bar I_r| + |\tfrac{\gamma}{\delta}\bar I|$$

$$\Rightarrow \quad I_e = I_r + \frac{\gamma}{\delta} I$$

with a sign change with respect to the initial phasor identity

Zero power factor characteristic:

$$U(I_e) = E_r(I_r) - X_f I \qquad \varphi = \frac{\pi}{2}$$

$$= \underbrace{E_r\left(I_e - \frac{\delta}{\gamma} I\right)}_{\text{Shift to the right}} \underbrace{- X_f I}_{\text{Shift to the bottom}}$$

with respect to the characteristic for $\bar I = 0$

Stability

Spring: $F_{spring} = -kx = -\partial_x\left(\frac{kx^2}{2}\right)$

It derives from a potential

$$F_{resultant} = F - kx = -\partial_x\left(-Fx + \frac{kx^2}{2}\right)$$
$$= -\partial_x \psi$$

$\psi = \frac{kx^2}{2} - Fx$ is the potential from which $F_{resultant}$ derives.

Stability $\Longrightarrow$ $\partial_x^2 \psi > 0$

always true in this case ($k > 0$)

Alternator: $x \equiv \delta_{int}$

$$C_{elm} = \frac{3p}{\omega X_s} U E_v \sin x \quad \text{resistant torque}$$

$$C_{resultant} = C - \frac{3p}{\omega X_s} U E_v \sin x$$
$$= -\partial_x\left(-Cx - \frac{3p}{\omega X_s} U E_v \cos x\right) = -\partial_x \psi$$

Stability $\Longrightarrow$ $\partial_x^2 \psi > 0$

$\Longrightarrow$ $\frac{3p}{\omega X_s} U E_v \cos x > 0$

$\Longrightarrow$ $x = \delta_{int} \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ (Slide 14)