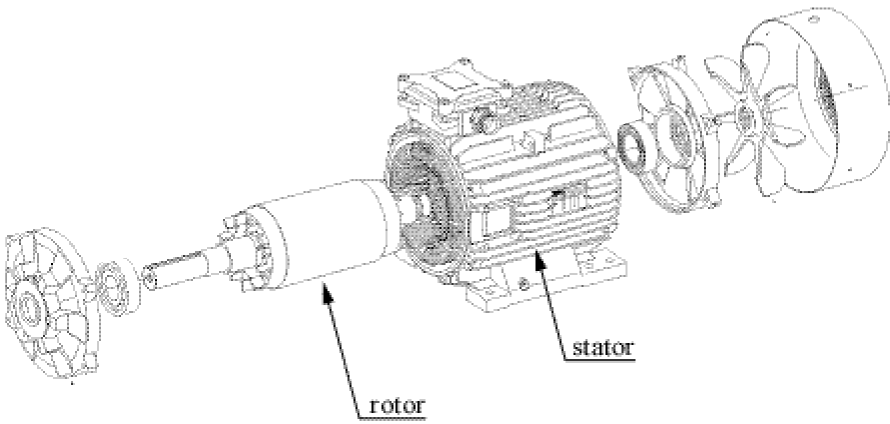
Asynchronous machines

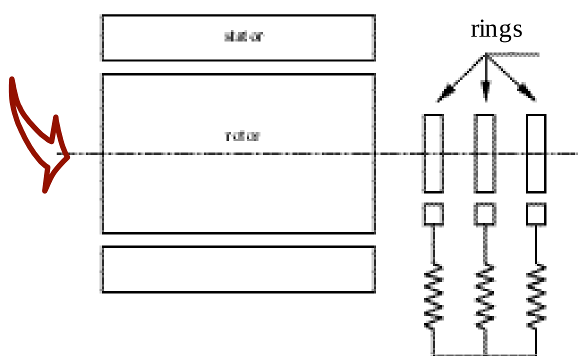


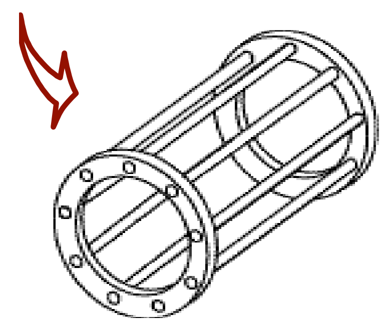
***Stator**: magnetic circuit and polyphased winding (usually 3-phase), with p pairs of poles, with polyphased currents*

***Rotor**: magnetic circuit and electric circuit...*

*... or made of copper or aluminum bars, short-circuited at each extremity of the rotor → **squirrel-cage rotor***

*... either made of a polyphased winding, (with star connection in order to avoid currents between rotor phases) connected to short-circuit rings on the rotor axis; brushes connect these rings to external resistances → **wound rotor***





General principle

Wound rotor machine initially at rest ...

$g = 1$

$g \overset{\text{def}}{=} \text{slip}$

When 3-phase currents of pulsation ω flow in the stator windings they create a rotating magnetic flux density with p pairs of poles, rotating with angular speed ω/p.

Rotation speed

$$\dot{\theta} = (1-g)\frac{\omega}{p}$$

$$0 \le g \le 1$$

Angular speed of the stator rotating field with respect to the rotor

$$\frac{\omega}{p} - \dot{\theta} = \frac{g\,\omega}{p}$$

This field induces 3-phased e.m.f.s of pulsation ω in the (stationary) rotor windings, and thus 3-phase currents of pulsation ω.

e.m.f.s and induced currents with pulsation gω, and torque

The interaction betweem the rotating stator flux density and the induced currents in the rotor creates a torque, which puts the rotor into movement

$g = 0$

Synchronous speed

$$\dot{\theta}_s = \frac{\omega}{p}$$

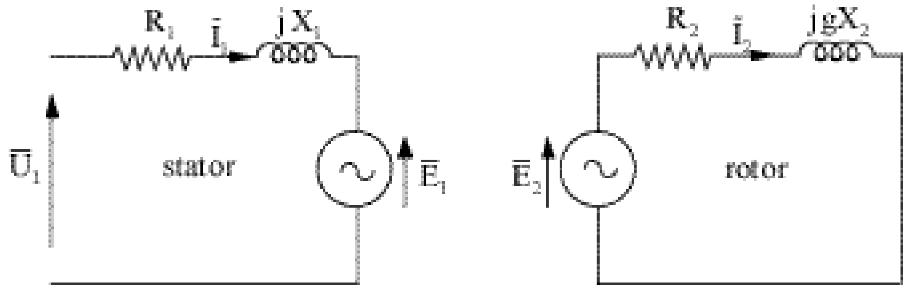
no e.m.f., no current, no torque

(asynchronous machine, or induction machine)

General equations

Stator equation *For one phase* **Rotor equation**

Rotating magnetic flux density in the airgap leads to e.m.f. E_1 in stator winding, with pulsation ω



(a) (b)

Rotating magnetic flux density in the airgap leads to e.m.f. E_2 in rotor winding, with pulsation $g\omega$.

$$\overline{U}_1 = \overline{E}_1 + (R_1 + j\,X_1)\,\overline{I}_1$$

X_1 = *Leakage reactance of one stator phase*

R_1 = *Resistance of a stator phase*

$$\overline{E}_2 = (R_2 + j\,g\,X_2)\,\overline{I}_2$$

X_2 = *Leakage reactance of a rotor phase at pulsation ω*

R_2 = *Resistance of a rotor phase*

$$\overline{E}_1 = -\,j\,\omega\,k_1\,\overline{\Phi}_r \qquad \overline{E}_2 = -\,j\,g\,\omega\,k_2\,\overline{\Phi}_r$$

Link equation

$$\frac{\overline{E}_1}{\overline{E}_2} = \frac{n_{tr}}{g}$$

$k_1 / k_2 = k'_1 / k'_2 = n_{tr}$ ...

resulting m.m.f.

$$\overline{F}_r = \overline{F}_1 + \overline{F}_2 = -k'_1\,\overline{I}_1 + k'_2\,\overline{I}_2$$

with

$$\overline{\Phi}_r = \frac{\overline{F}_r}{R_{ep}}$$

resulting flux

$$\overline{E}_1 = j\,X_\mu \left(\overline{I}_1 - \frac{\overline{I}_2}{n_{tr}} \right)$$

X_μ = *Magnetizing reactance*

Equivalent circuit

$$\overline{U}_1 = \overline{E}_1 + (R_1 + j\,X_1)\,\overline{I}_1 \quad \text{(a)}$$

$$\frac{\overline{E}_2}{g} = \left(\frac{R_2}{g} + j\,X_2\right)\overline{I}_2$$

$$\frac{\overline{E}_1}{\overline{E}_2} = \frac{n_{tr}}{g}$$

$$\rightarrow \quad \overline{E}_1 = \left(n_{tr}^2\frac{R_2}{g} + j n_{tr}^2 X_2\right)\frac{\overline{I}_2}{n_{tr}}$$

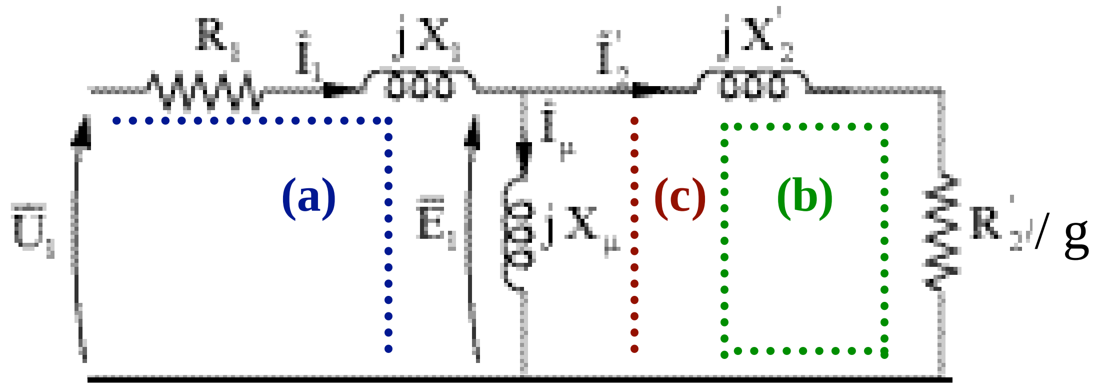
$$\overline{E}_1 = \left(\frac{R'_2}{g} + j\,X'_2\right)\overline{I}'_2 \quad \text{(b)}$$

$$\overline{E}_1 = j\,X_\mu\left(\overline{I}_1 - \frac{\overline{I}_2}{n_{tr}}\right)$$

$$\rightarrow \quad \overline{E}_1 = j\,X_\mu\left(\overline{I}_1 - \overline{I}'_2\right) \quad \text{(c)}$$

magnetizing current

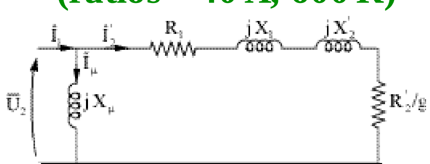
$$\overline{I}_\mu = \overline{I}_1 - \overline{I}'_2$$



Simplified equivalent circuit

$$R_1, R'_2, X_1, X'_2 << X_\mu$$

(ratios ~ 40 X, 600 R)



Equivalent circuit parameters

Experimental determination of the parameters

No load test

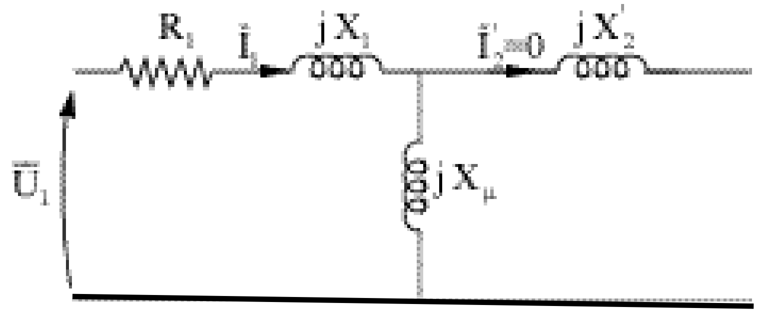
$g = 0$

Under nominal voltage and at synchronous speed

$$I_1 \ll \Rightarrow P_{\text{Joule stator}} \ll$$

$$P_v = P_{mag} \qquad Q_v = 3\frac{U_1^2}{X_\mu}$$

magnetic losses in the stator laminations



Short-circuit test

$g = 1$

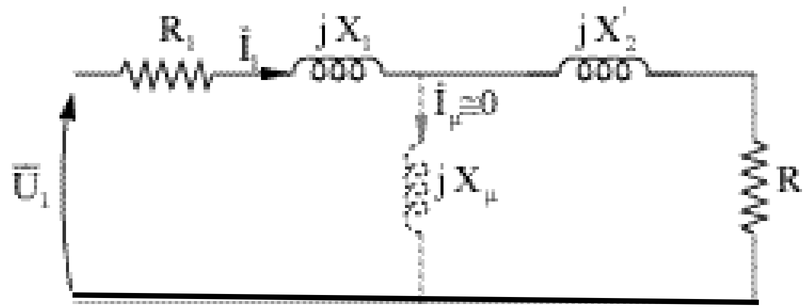
Under reduced voltage at at zero speed (stationnary rotor)

$$U_1 \ll \Rightarrow I_\mu \text{ and } P_{mag} \ll$$

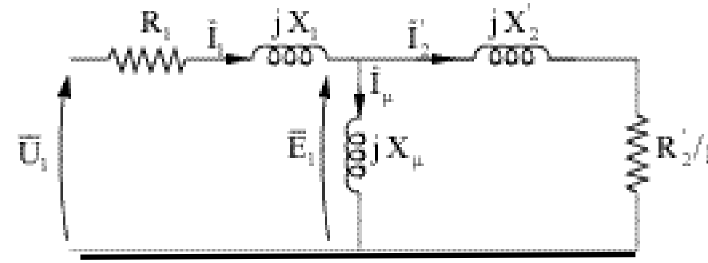
$$P_{cc} = 3\left(R_1 + R'_2\right) I_1^2 \qquad Q_{cc} = 3\left(X_1 + X'_2\right) I_1^2$$

in the resistances of the stator and rotor windings

in the leakage reactances of the stator and the rotor



Power, torque and efficiency



Electric power

$$P = 3\, U_1\, I_1 \cos\varphi = 3\left(R_1\, I_1^2 + \frac{R'_2}{g}\, I'^2_2 \right)$$

Stator	Absorbed power
	Stator Joule losses $3\, R_1\, I_1^2$
Airgap	Power transmitted from stator to rotor
Rotor	Rotor Joule losses $3\, R'_2\, I'^2_2$
	Electromagnetic power

$$P_{st\to rot} = P - 3\, R_1\, I_1^2 = 3\, \frac{R'_2}{g}\, I'^2_2$$

$$P_{elm} = P_{st\to rot} - 3\, R'_2\, I'^2_2 = 3\, \frac{1-g}{g}\, R'_2\, I'^2_2$$

$$C_{elm} = \frac{P_{elm}}{\dot{\theta}} = \frac{3\, p}{\omega}\, \frac{R'_2}{g}\, I'^2_2$$

Electromagnetic torque

$$\eta_{stat} = \frac{P_{st\to rot}}{P}$$

Stator efficiency (1)

Rotor efficiency (2)

$$\eta_{rot} = \frac{P_{elm}}{P_{st\to rot}} = \frac{3\, \frac{1-g}{g}\, R'_2\, I'^2_2}{3\, \frac{1}{g}\, R'_2\, I'^2_2} = 1 - g$$

$$\eta_{mec} = \frac{P_{mec}}{P_{elm}}$$

Mechanical efficiency (3)

Asynchronous motor efficiency (1)x(2)x(3)

$$\eta = (1-g)\, \eta_{stat}\, \eta_{mec} < 1 - g$$

Mechanical characteristic

Mechanical characteristic

Evolution of the electromagnetic torque C as a function of the slip g (or the rotation speed), for a given stator voltage U_1 and pulsation ω

$$C = \frac{3\,p}{\omega}\,\frac{R'_2}{g}\,I'^2_2$$

$$C = f(g) \quad \text{with} \quad \begin{cases} U_1 = \text{constant} \\ \omega = \text{constant} \end{cases}$$

with $$I'_2 = \frac{E_1}{\sqrt{\left(\frac{R'_2}{g}\right)^2 + X'^2_2}} \approx \frac{U_1}{\sqrt{\frac{R'^2_2}{g^2} + X'^2_2}}$$

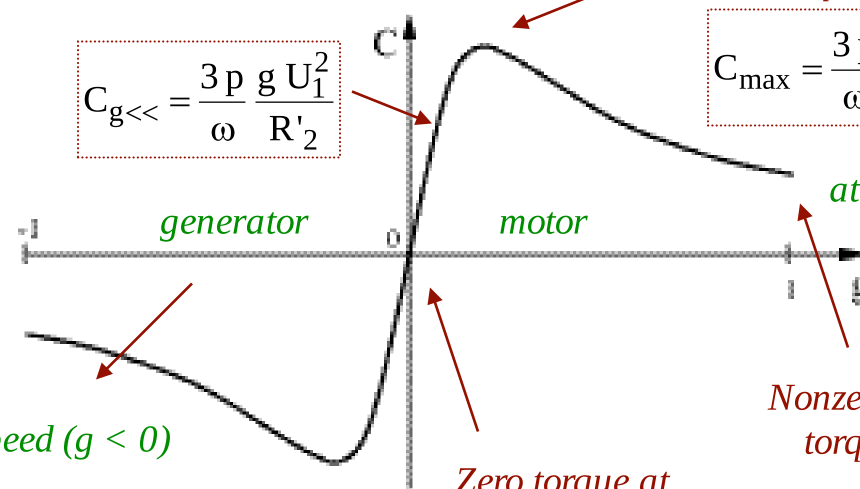
$$C = \frac{3\,p}{\omega}\,R'_2\,\frac{g\,U_1^2}{R'^2_2 + g^2\,X'^2_2}$$

Maximum torque $\neq f(R'_2)$

$$C_{max} = \frac{3\,p}{\omega}\,\frac{U_1^2}{2\,X'_2}$$

at $$g = \pm\frac{R'_2}{X'_2}$$

$$C_{g<<} = \frac{3\,p}{\omega}\,\frac{g\,U_1^2}{R'_2}$$



above synchronous speed (g < 0)

Zero torque at synchronous speed (g = 0)

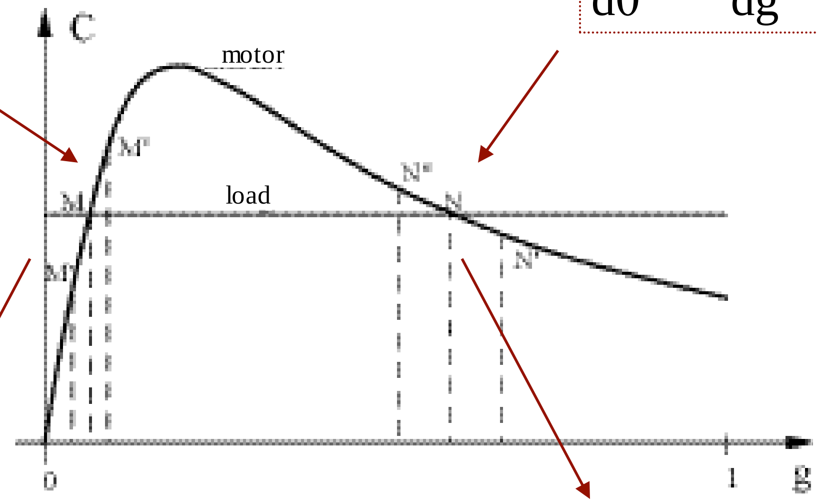
Nonzero startup torque (g = 1)

Stability zone

$$\frac{dC}{d\dot{\theta}} = -\frac{dC}{dg} < 0$$

$$\frac{dC}{d\dot{\theta}} = -\frac{dC}{dg} > 0$$

Stable



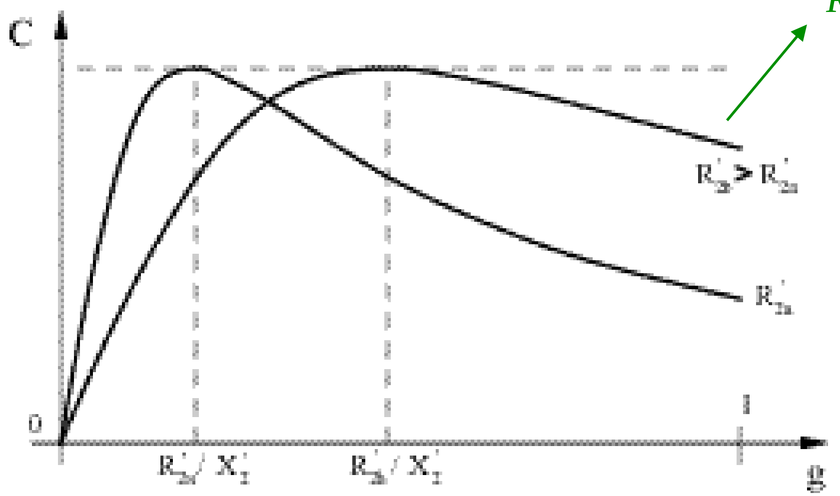
Unstable

A decrease (resp. increase) in speed, or an increase (resp. decrease) in the slip g, leads to an increase (resp. decrease) of the motor torque, from point M→M' (resp. M→M''). The machine will thus accelerate (resp. decelerate) to reach back point M, where the motor and resistant torques match
⇒ ***stable***.

A decrease (resp. increase) in speed, or an increase (resp. decrease) in the slip g, leads to a decrease (resp. increase) of the motor torque, from point N→N' (resp. N→N''). The machine will thus decelerate (resp. accelerate), further increasing the mismatch between the resistant and motor torques (resp. leading to an evolution towards the stable stable N''→M)
⇒ ***unstable***.

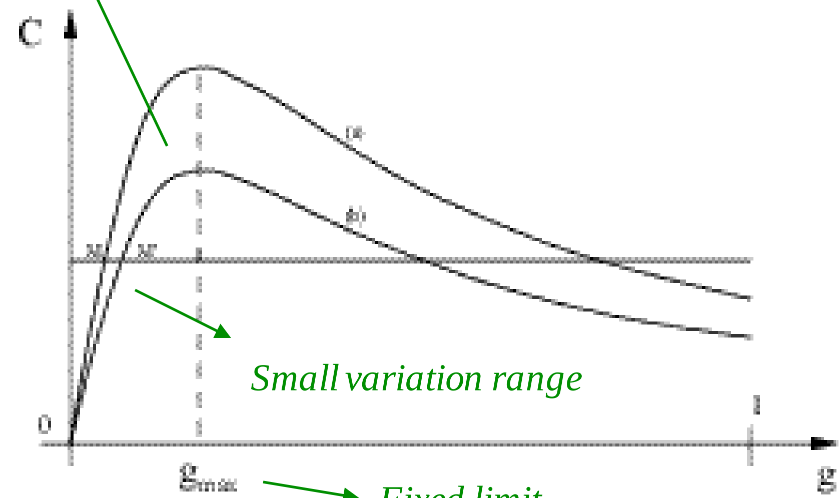
Speed control

Modification of the rotor resistance R'$_2$



Reduction of speed at the cost of efficiency ($\eta < 1-g$)

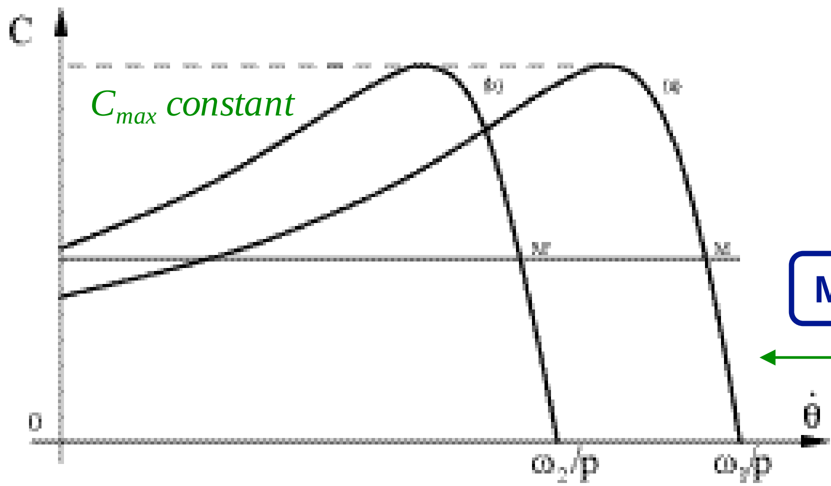
Modification of the voltage U$_1$



Small variation range

Fixed limit

Modification of the frequency

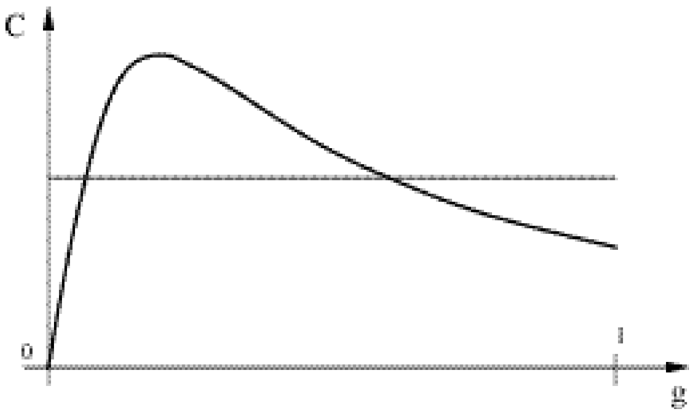


C_{max} constant

Interest in keeping the magnetic flux (hence U_1/ω) constant...

$$\Phi_r = \lambda_\mu \, I_\mu = \frac{X_\mu \, I_\mu}{\omega} = \frac{E_1}{\omega} \approx \frac{U_1}{\omega}$$

Asynchronous motor startup



Startup torque (g = 1)

$$C = \frac{3\,p}{\omega} R'_2 \frac{g\,U_1^2}{R'^2_2 + X'^2_2}$$

usually quite small, since $R'_2 <<$

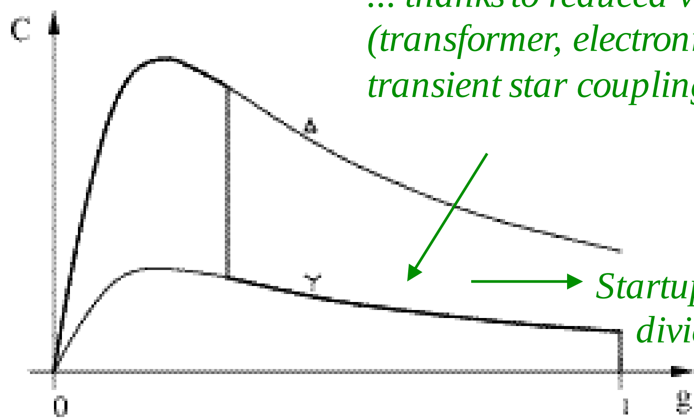
Sartup current

$$I_d = \frac{U_1}{\sqrt{(R_1 + R'_2)^2 + (X_1 + X'_2)^2}}$$

very large, should be limited…

Squirrel cage motor

… thanks to reduced voltage (transformer, electronic converter, transient star coupling)

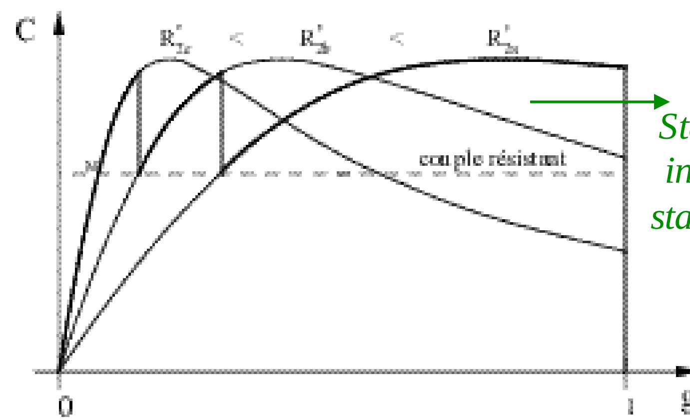


Startup torque divided by 3

Startup possible only for small resistant torque at low speed (e.g. pumps, ventilators, air conditioners)

Wound rotor

… resistances in series with the rotor winding (progressively removed)



Startup torque increases and startup current decreases

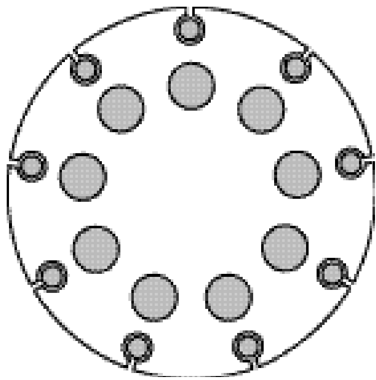
Special asynchronous motors

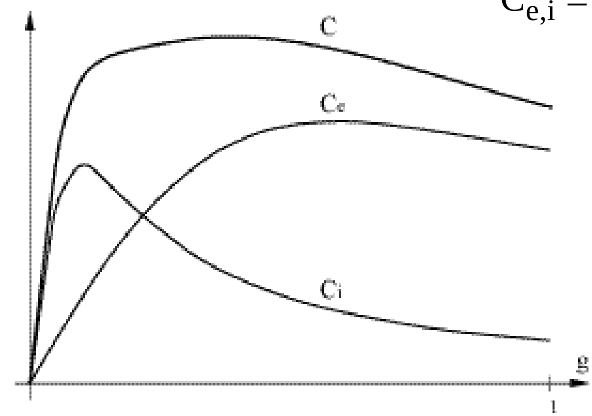
Double cage rotor

Exterior (e) and interior (i)

$R'_e >> R'_i$

$X'_e << X'_i$





$$C_{e,i} = \frac{3\,p}{\omega} R'_{e,i} \frac{g\,U_1^2}{R'^2_{e,i} + g^2\,X'^2_{e,i}}$$

Startup (g=1):

$R'_{i,e} << gX'_e << gX'_i$

$\Rightarrow C_e >> C_i$

Nominal speed (g <<):

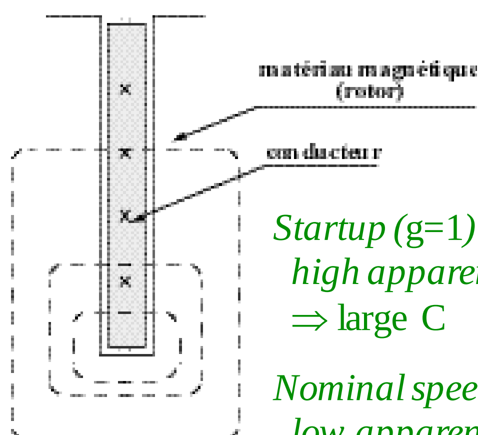
$R'_e >> R'_i >> g\,X'_{i,e}$

$\Rightarrow C_i >> C_e$

Skin effect rotor

Elongated rectangular rotor conductors

$\Rightarrow$ same behavior as double cage



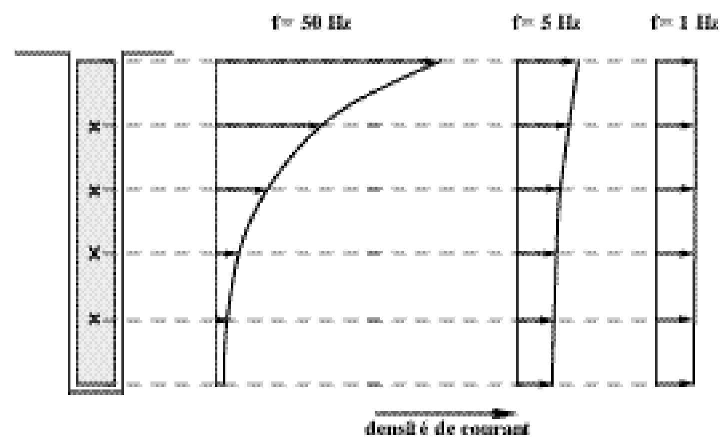
Startup (g=1):

high apparent resistance

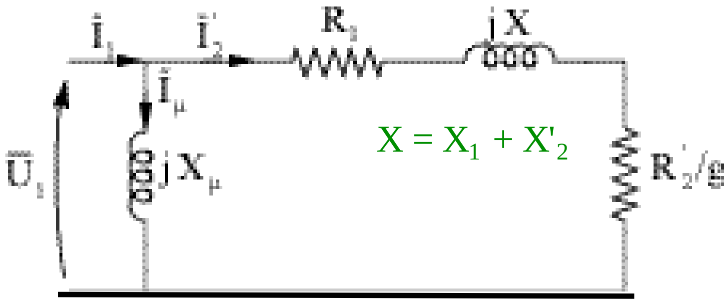
$\Rightarrow$ large C

Nominal speed (g <<):

low apparent resistance



Circle diagram



$X = X_1 + X'_2$

$\bar{U}_1 = jX\,\bar{I}'_2 + (R_1 + R'_2/g)\,\bar{I}'_2$

$\boxed{\bar{U}_1 = \text{constant}}$ *(0)*

$$\frac{\bar{U}_1}{jX} = \bar{I}'_2 + \frac{R_1 + R'_2/g}{jX}\bar{I}'_2$$

⇒ Locus of I'_2 is a circle of diameter U_1/X

(2)-(3)

Active power

$P = 3\,U_1\,I_1 \cos\varphi = 3\,U_1\,AM$

$$AM = \frac{P}{3\,U_1}$$ *(5)*

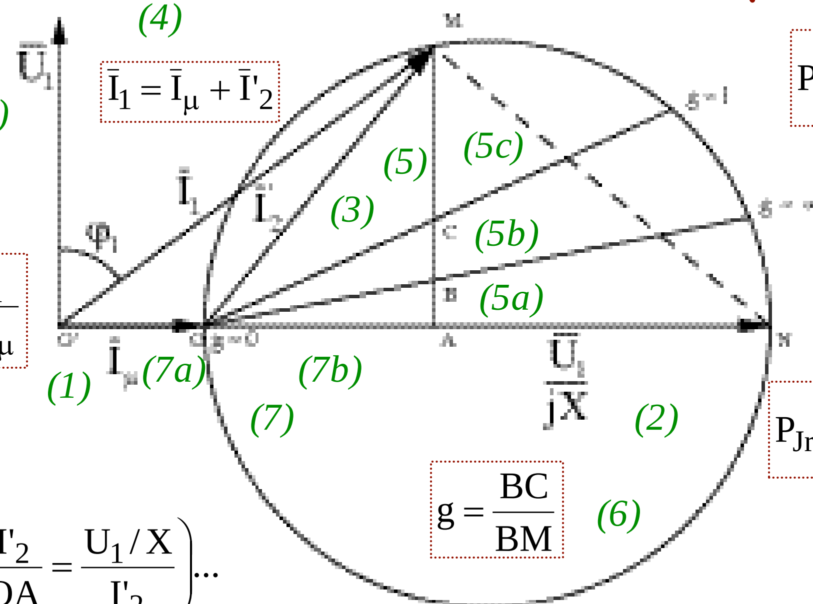
Reactive power

$Q = 3\,U_1\,I_1 \sin\varphi = 3\,U_1\,O'A$

$$O'A = \frac{Q}{3\,U_1}$$ *(7) = (7a) + (7b)*

$$O'A = \frac{Q_{X_\mu} + Q_X}{3\,U_1} = O'O + OA \quad \left(\frac{I'_2}{OA} = \frac{U_1/X}{I'_2}\right)\ldots$$

Reactive power in X_μ and X



$$g = \frac{BC}{BM}$$ *(6)*

Stator Joule losses

$$P_{Js} = 3\,R_1\,I'^2_2 = 3\,R_1\,OA\,\frac{U_1}{X}$$

$$AB = \frac{P_{Js}}{3\,U_1} = \frac{R_1}{X}\,OA$$

(5a)

Rotor Joule losses

$$P_{Jr} = 3\,R'_2\,I'^2_2 = 3\,R'_2\,OA\,\frac{U_1}{X}$$

$$BC = \frac{P_{Jr}}{3\,U_1} = \frac{R'_2}{X}\,OA$$

(5b)

Electromagnetic power

$\boxed{CM}$ *(5c)*

Operating modes

The machine

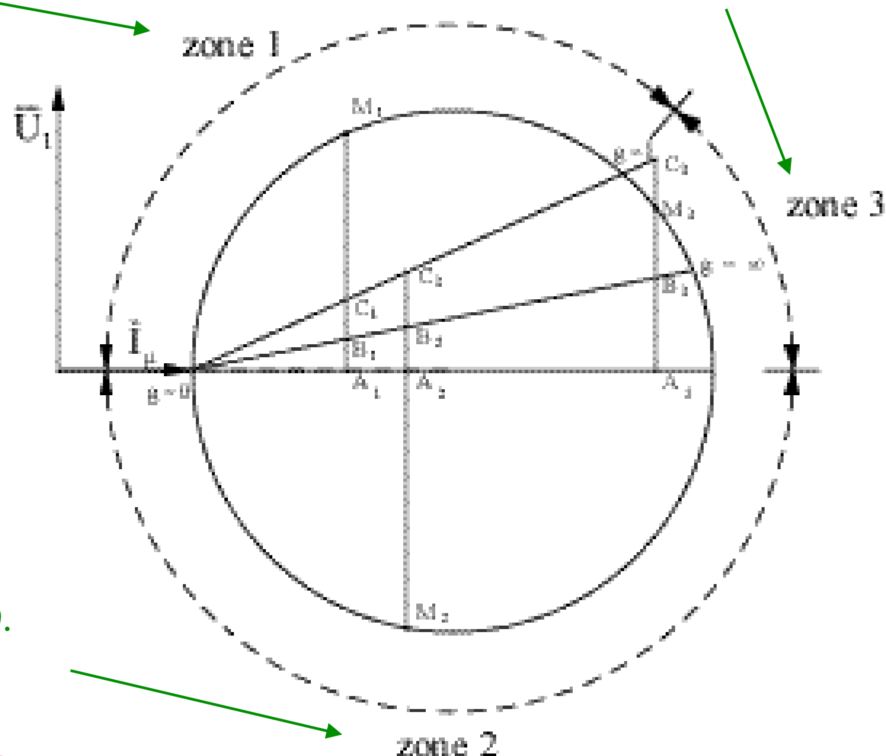
- *absorbs electric power ($A_1M_1 \geq 0$) ;*
- *produces mechanical power ($C_1M_1 \geq 0$).*

Asynchronous motor

The machine

- *absorbs electric power ($A_3M_3 \geq 0$) ;*
- *absorbs mechanical power ($C_3M_3 \leq 0$).*

Radiator



The machine

- *produces electric power ($A_2M_2 \leq 0$) ;*
- *absorbs mechanical power ($C_2M_2 \leq 0$).*

Asynchronous generator