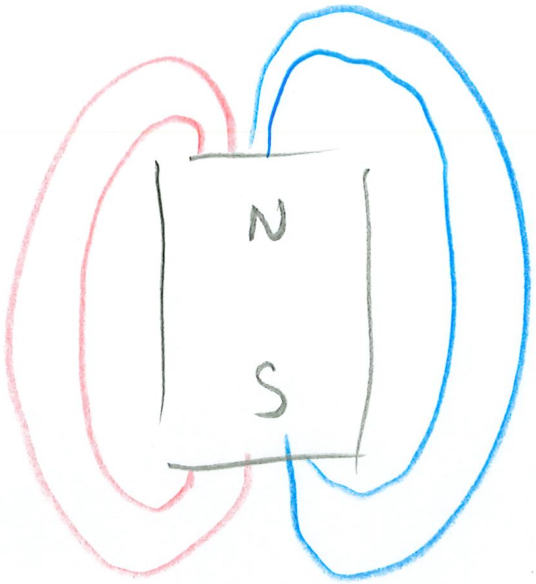
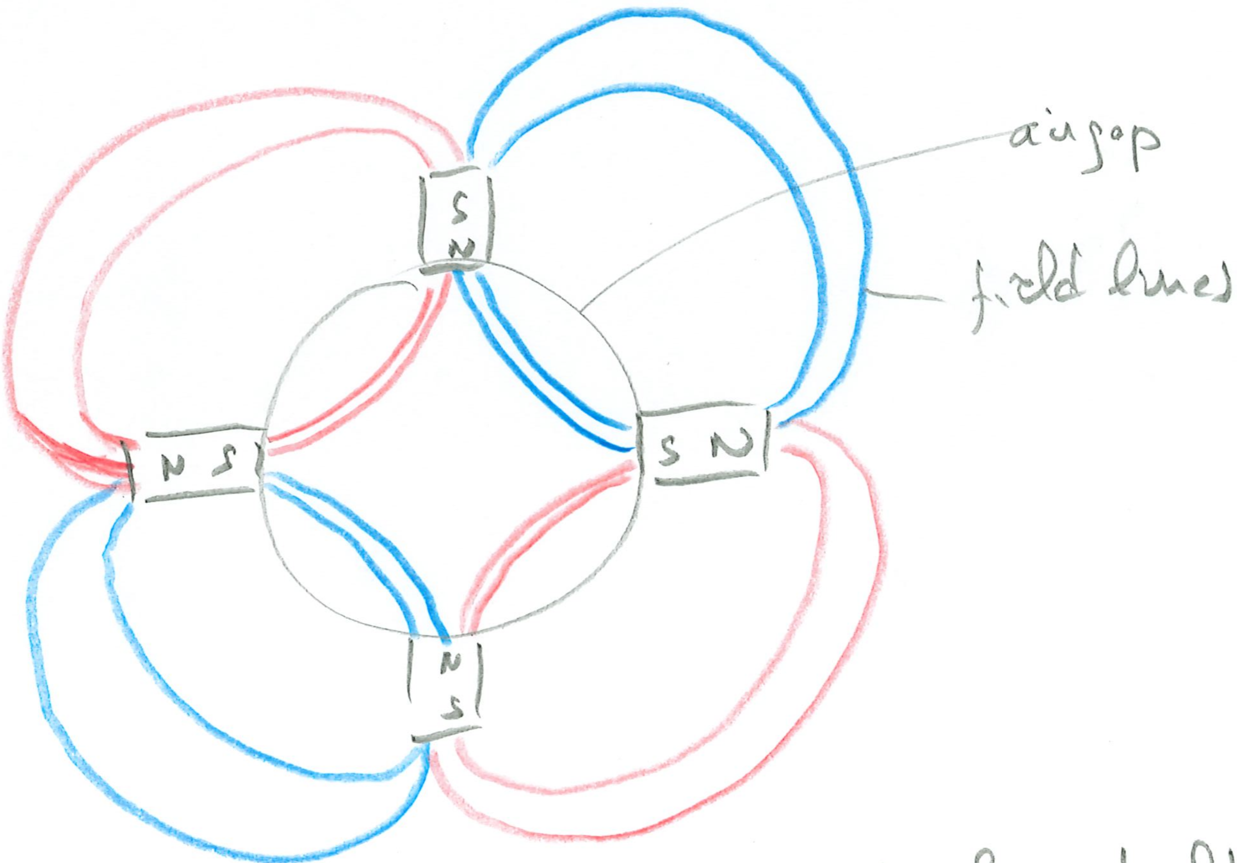
<u>Permanent Magnet</u> 1 pole pair



Electrical Machine with two pole pairs ($p = 2$)



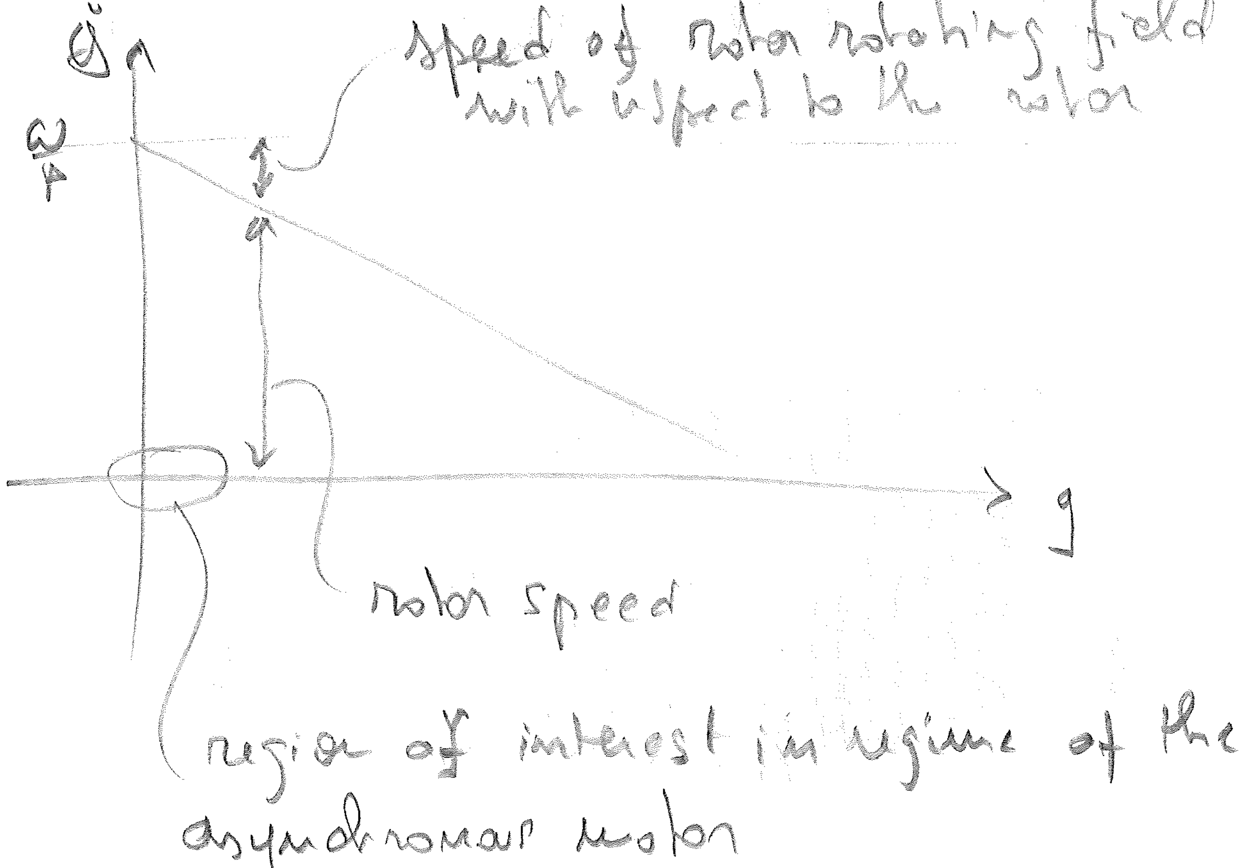
The colors (red blue) correspond to how fieldlines are colored in ONELAB models of machines

Slide 2

$$g = \frac{\frac{\omega}{p} - \dot{\theta}}{\frac{\omega}{p}}$$

$\frac{\omega}{p} \equiv$ synchronous speed

$$\dot{\theta} = (1-g)\,\frac{\omega}{p}$$



Ex $\frac{\omega}{p} = 1500$ rpm (50 Hz $p=2$)

$\dot{\theta} = 1440$ rpm

$$g = \frac{1500-1440}{1500} = 4\% = 0.04$$

Slide 3

$$\bar{E}_1 = -j\omega n_1 \bar{\Phi}_1$$

$$= -j\omega n_1 \left(- \frac{n_1' \bar{I}_1 + n_2' \bar{I}_2}{\mathcal{R}} \right) \qquad \mathcal{R} \equiv R_{ep}$$

$$= j\omega \frac{n_1 n_1'}{\mathcal{R}} \bar{I}_1 - j\omega \frac{n_1 n_2'}{\mathcal{R}} \bar{I}_2'$$

$$= j\omega \frac{n_1 n_1'}{\mathcal{R}} \left(\bar{I}_1 - \frac{n_2'}{n_1'} \bar{I}_2' \right)$$

$$= j\omega X_\mu \left(\bar{I}_1 - \frac{\bar{I}_2}{m_{t1}} \right) \qquad m_{t1} = \frac{n_1'}{n_2'}$$

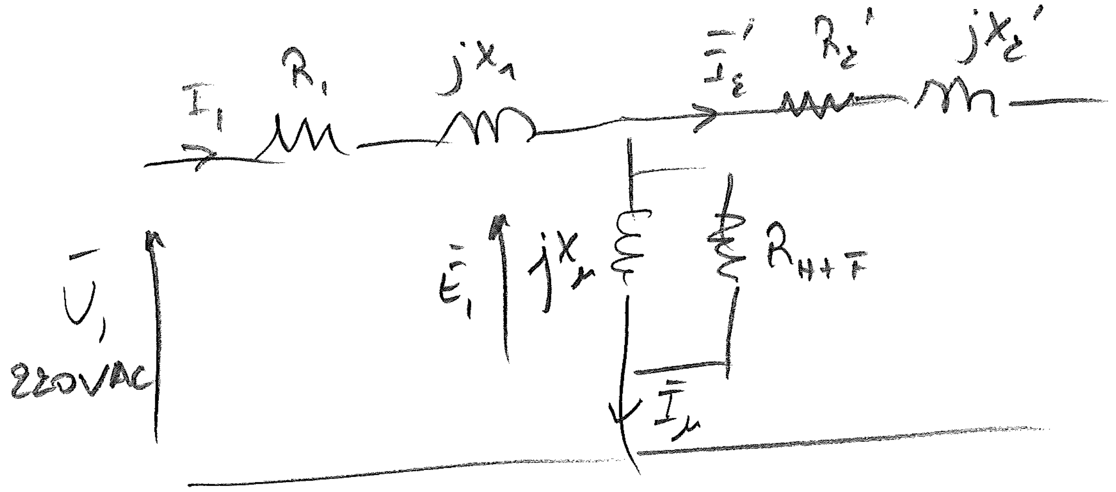
$$X_\mu = \frac{n_1 n_1'}{\mathcal{R}}$$

$$\bar{E}_1 = j\omega X_\mu \left(\bar{I}_1 - \bar{I}_2' \right) \qquad \bar{I}_2' = \frac{\bar{I}_2}{m_{t1}}$$

$$= j\omega X_\mu \bar{I}_\mu$$

The phasor algebra for the asynchronous machine is this very similar to that of the transformer.

Slide 5 Per Phase



$\bar{I}_2' = 0$
no load
$g = 0$

Realistic values

$X_\mu = 20\,\Omega \quad R_1 = 0.1\,\Omega \quad R_{H+F} = 800\,\Omega \quad X_1 = 0.5\,\Omega$

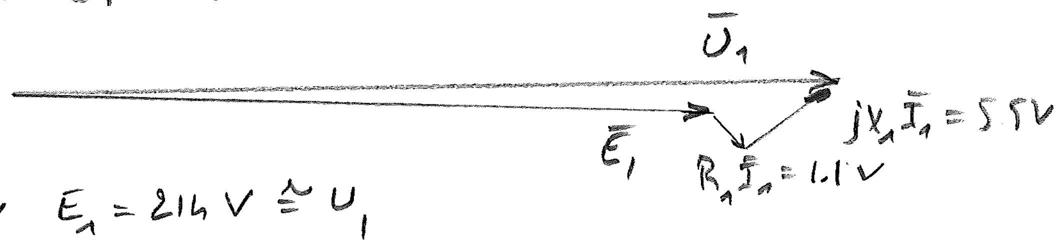
No Load $\quad Z_{TOT} = R_1 + jX_1 + \left(\frac{1}{jX_\mu} + \frac{1}{R_{H+F}}\right)^{-1}$

which is too complicated already for a "mental picture" model. But one notices that

$$Z_{TOT} \cong jX_\mu \Rightarrow \bar{I}_1 = \frac{\bar{U}_1}{jX_\mu}$$

$$\Rightarrow I_1 \cong \frac{220\,V}{20\,\Omega} = 11\,A$$

$$\Rightarrow \bar{E}_1 = \bar{U}_1 - (R_1 + jX_1)\bar{I}_1$$



$U_1 = 220\,V \quad E_1 = 214\,V \cong U_1$

This example intends to show how good/bad are the approximations done routinely in this course.

Slide 8 : Stable and unstable zone

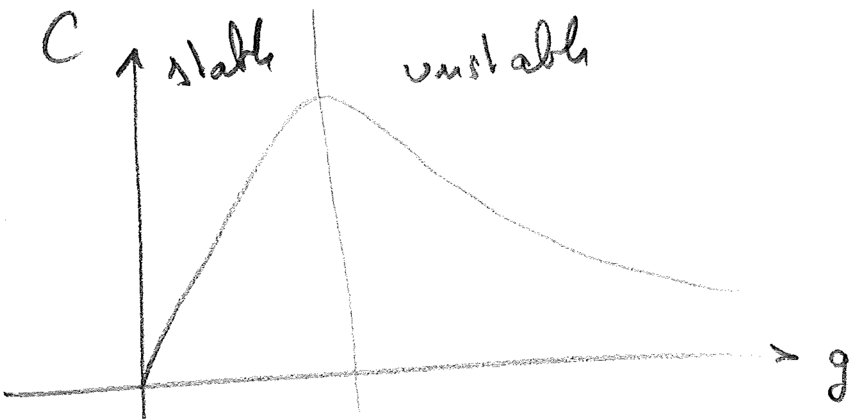
$$\frac{\partial C_{res}}{\partial \delta_{in}} < 0 \Rightarrow \text{Stability}$$

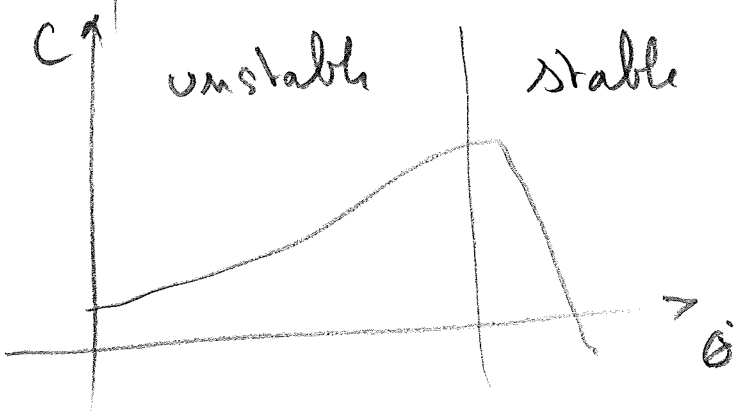
(Cf synchronous machine Slide 14)

$$\frac{\partial C_{motor}}{\partial \dot{\theta}} < 0 \Rightarrow \text{Stability}$$

$$\dot{\theta} = (1-g)\frac{\omega}{p}$$

$$\frac{\partial C_{motor}}{\partial g} > 0 \Rightarrow \text{Stability}$$





Flux

$$\phi_n = l_\mu I_\mu = \frac{X_\mu I_\mu}{\omega} = \frac{E_1}{\omega} \cong \frac{U_1}{\omega}$$

* In an induction motor connected to the net ($U_1 = cst$, $\omega = cst$) the flux is always the same, whatever the load.
* The flux is also constant if the ratio U_1/ω is maintained constant (e.g. for speed control, slide 9)

Active power (slide 6)

$$P = 3 U_1 I_1 \cos\varphi = P_{J\,stator} + \overbrace{P_{stator \to rotor}}^{C\frac{\omega}{p}}$$

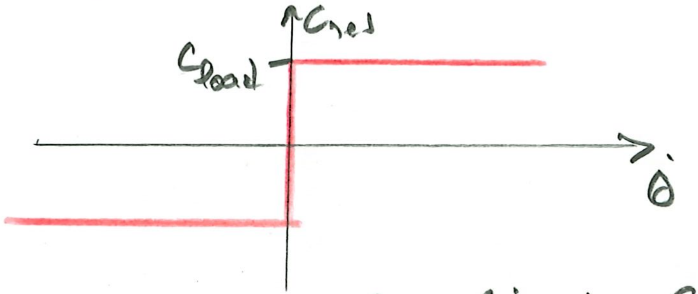
$$= 3\left(R_1 I_1^2 + \frac{R_2'}{g} I_2'^2\right)$$

$$P = P_{J\,stator} + P_{J\,rotor} + \overbrace{P_{elm}}^{C\dot{\theta}}$$

$$= 3\left(R_1 I_1^2 + R_2' I_2'^2 + R_2'\left(\frac{1-g}{g}\right) I_2'^2\right)$$

Start up (slide 10)

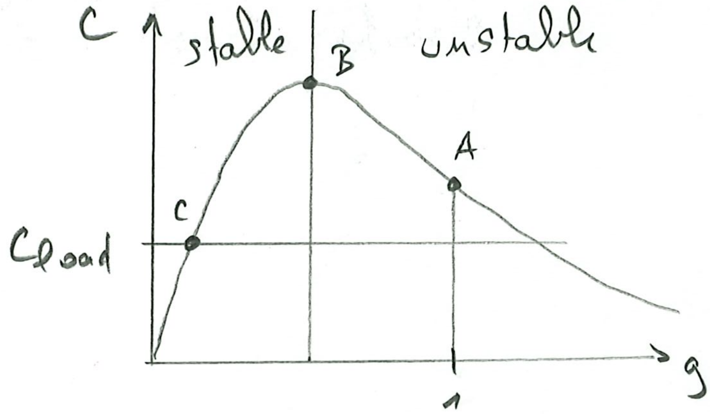
$$C_{motor} - C_{res}(\dot{\theta}) = I\ddot{\theta} \qquad I = \text{inertia}$$



(*) $C_{motor} < C_{load} \Rightarrow C_{res}(\dot{\theta}=0) = C_{motor} \Rightarrow I\ddot{\theta} = 0$

the motor does not start

(*)



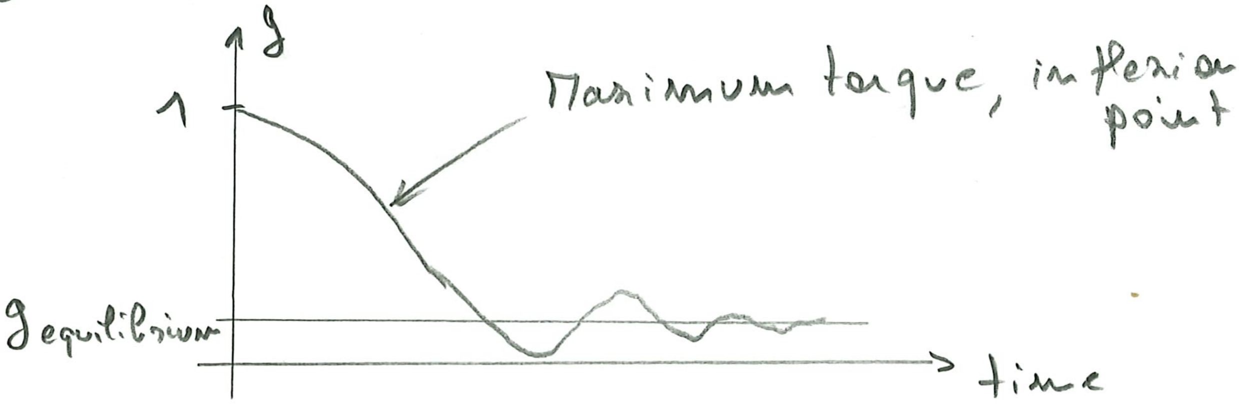
A $\quad C_{motor} > C_{load} \Rightarrow I\ddot{\theta} > 0 \Rightarrow$ the motor starts

$\dot{\theta}$ increases, g decreases

B $\quad C_{motor} - C_{load}$ is maximum.

acceleration $\ddot{\theta}$ is maximum

C $\quad C_{motor} = C_{load} \Rightarrow$ stable equilibrium point

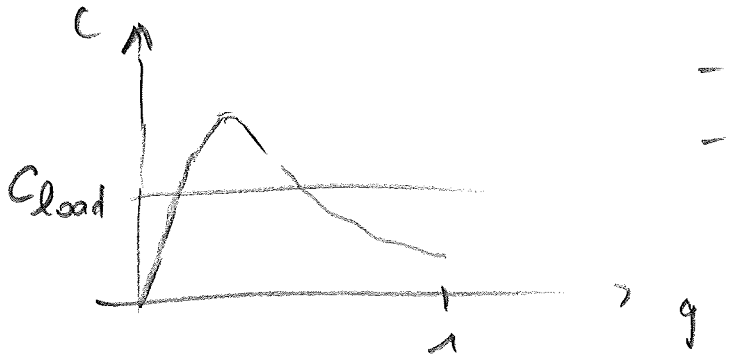


Two contradictory conditions for the start-up of the induction motor

- limit the start-up current

$$I_d = \frac{U_1}{\sqrt{(R_1 + R_2')^2 + (X_1 + X_2')^2}}$$

- have a sufficiently large start-up torque



- does not start
- I_d very large

(*) With a normal squirrel cage motor, no action is possible at the rotor.
The best one can do is to start in star connection

- Start-up torque divided by 3
- Start-up phase current ——— $\sqrt{3}$

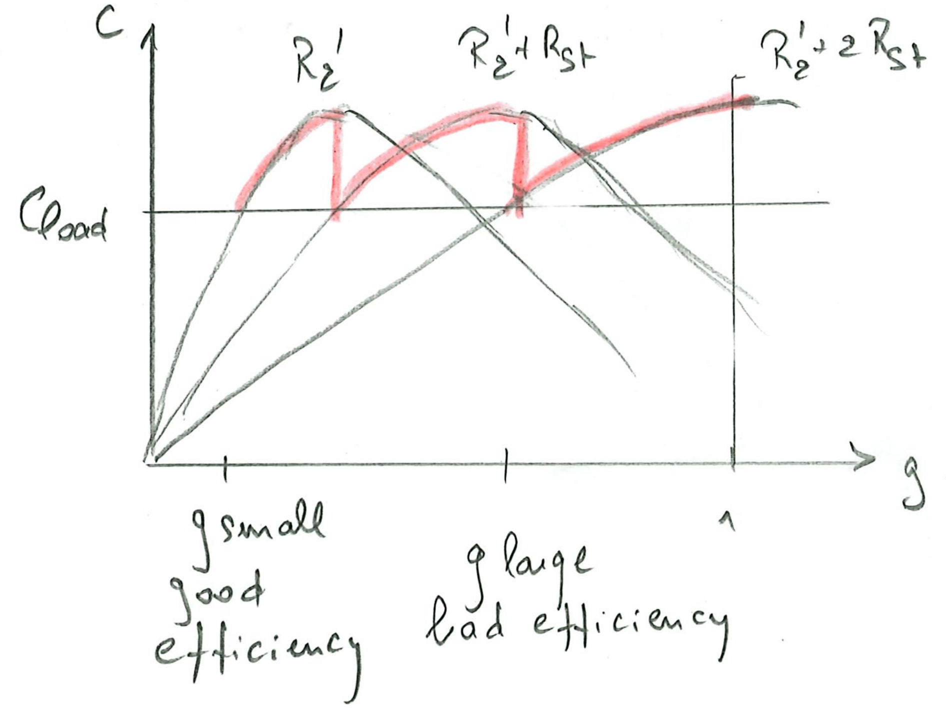
and then to switch in Δ (Slide 10)

(*) With a wound rotor
(rotor of a synchronous machine with short-circuited rings)

Here the solution is to insert resistances R_{st} in series with the rotor windings

$$I_d(R_{st}) = \frac{U_1}{\sqrt{(R_1+R_2'+R_{st})^2+(X_1+X_2')}} \ll I_d(R_{st}=0)$$

and to remove them progressively



But we do not want rings and tedious connection mechanisms

→ clever squirrel cage rotor

Slide 11

$$C_{start} = \frac{3p}{\omega} R_2' \frac{U_1^2}{R_2'^2 + X_2'^2} \simeq \frac{3p}{\omega} R_2' \frac{U_1^2}{X_2'^2}$$

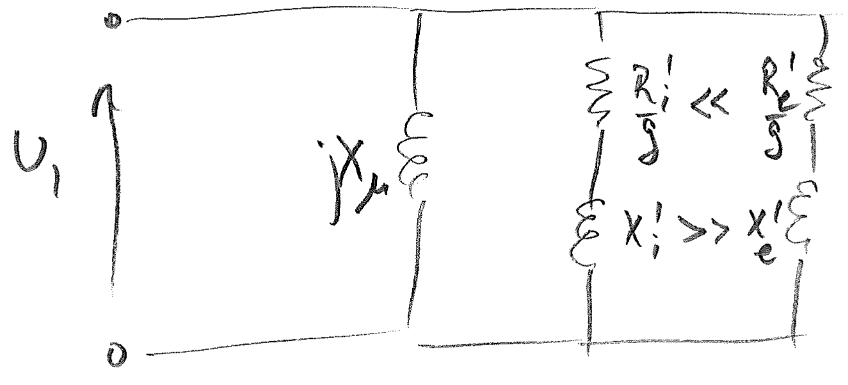
is proportional to R_2'

Slope $\partial_g C(g=0) = \frac{3p}{\omega} \frac{U_1^2}{R_2'}$

is inversely proportional to R_2'

Double cage

regime $(g \ll 1)$ Startup $(g=1)$



At start up, $R_e' \gg R_i'$ but $X_i' \gg X_e'$

ensures that I_e' is not $\ll I_i'$, i.e., $I_e' \simeq I_i'$

In regime, $\frac{R_e'}{g} \gg \frac{R_i'}{g} \gg X_i' \gg X_e'$ and $I_i' \gg I_e'$

Speed Control (slide 9)

Speed control means
How can one modify the rotation speed $\dot{\theta}$ corresponding to a given resistant torque (load) C_{res} ?

With
- modification of R_2'
- ———————— U_1

one buys a bad / limited speed control with a significant loss of efficiency

The good way to control speed is to modify U_1 and ω so as to maintain the ratio U_1/ω constant, i.e constant flux in the machine. Power electronics is needed for that.

$$C_{max} = \frac{3p}{\omega} \frac{U_1^2}{2X_2'} \quad \text{(cf slide 7)}$$

$$= \frac{3p}{\omega} \frac{U_1^2}{2\omega L_{2f}'} = \frac{3p}{2L_{2f}'} \left(\frac{U_1}{\omega}\right)^2$$

remains constant all speeds

$$g = 1 - \frac{p\dot{\theta}}{\omega}$$

$$\partial_{\dot{\theta}} C(g=0) = \partial_g C(g=0)\, \partial_{\dot{\theta}} g$$

$$= \frac{C_{gen}}{g} \text{ (slide 7) } \partial_{\dot{\theta}} g$$

$$= \frac{3p}{\omega} \frac{U_1^2}{R_2'} \left(-\frac{p}{\omega}\right) = \frac{-3p^2}{R_2'} \left(\frac{U_1}{\omega}\right)^2$$

remains constant at all speeds

The slips corresponding to C_{max} is

$$g_{max} = \pm \frac{R_2'}{X_2'} \quad \text{(slide 7)}$$

$$\dot{\theta}_{max} = (1 - g_{max}) \frac{\omega}{p} = \left(1 \mp \frac{R_2'}{X_2'}\right) \frac{\omega}{p}$$

$$= \frac{\omega}{p} \mp \underbrace{\frac{R_2'}{pL_{2f}'}}_{\text{constant for all speeds}}$$

Slide 12 : Circle diagram

⊛ $P = 3U_1 I_1 \cos\varphi = 3U_1\, AM \Rightarrow AM = \frac{P}{3U_1}$

⊛ $Q = 3U_1 I_1 \sin\varphi = 3U_1 (O'O + OA)$

$= Q_{X_\mu} + Q_X = 3X_\mu I_\mu^2 + 3X I_2'^2$

$\Rightarrow 3U_1\, OA = 3X I_2'^2 \Rightarrow OA = \frac{X I_2'^2}{U_1}$

⊛ $\bar{U}_1 = jX \bar{I}_2' + \left(R_1 + \frac{R_2'}{g}\right) \bar{I}_2'$

$\Rightarrow \frac{\bar{U}_1}{jX} = \bar{I}_2' + \frac{R_1 + \frac{R_2'}{g}}{X} \bar{I}_2'$

The constant phasor $\frac{\bar{U}_1}{jX}$ is the sum of two orthogonal phasors

$\Rightarrow$ the locus of the tip of $\bar{I}_2'$ is a circle of diameter $\frac{U_1}{X}$

* $P_{JS} = 3 R_1 I_1^2 \simeq 3 R_1 I_2'^2$ with $I_2'^2 = \frac{U_1}{X} OA$

$$= 3 R_1 \frac{U_1}{X} OA$$

$$\Rightarrow \frac{P_{JS}}{3U_1} = \frac{R_1}{X} OA = AB$$

is thus represented by a straight line starting from O and with slope $\frac{R_1}{X}$

* $P_{JR} = 3 R_2' I_2'^2 = 3 R_2' \frac{U_1}{X} OA$

$$\Rightarrow \frac{P_{JR}}{3U_1} = \frac{R_2'}{X} OA = BC$$

* $P_{elm} = 3 R_2' I_2'^2 \left(\frac{1-g}{g}\right) = 3 R_2' \left(\frac{1-g}{g}\right) \frac{U_1}{X} OA$

$$\Rightarrow \frac{P_{elm}}{3U_1} = \frac{R_2' \left(\frac{1-g}{g}\right)}{X} OA = CM$$

* $BM = BC + CM = \frac{R_2'}{gX} OA$

$$BC = \frac{R_2'}{X} OA$$

$$\Rightarrow g = \frac{BC}{BM}$$