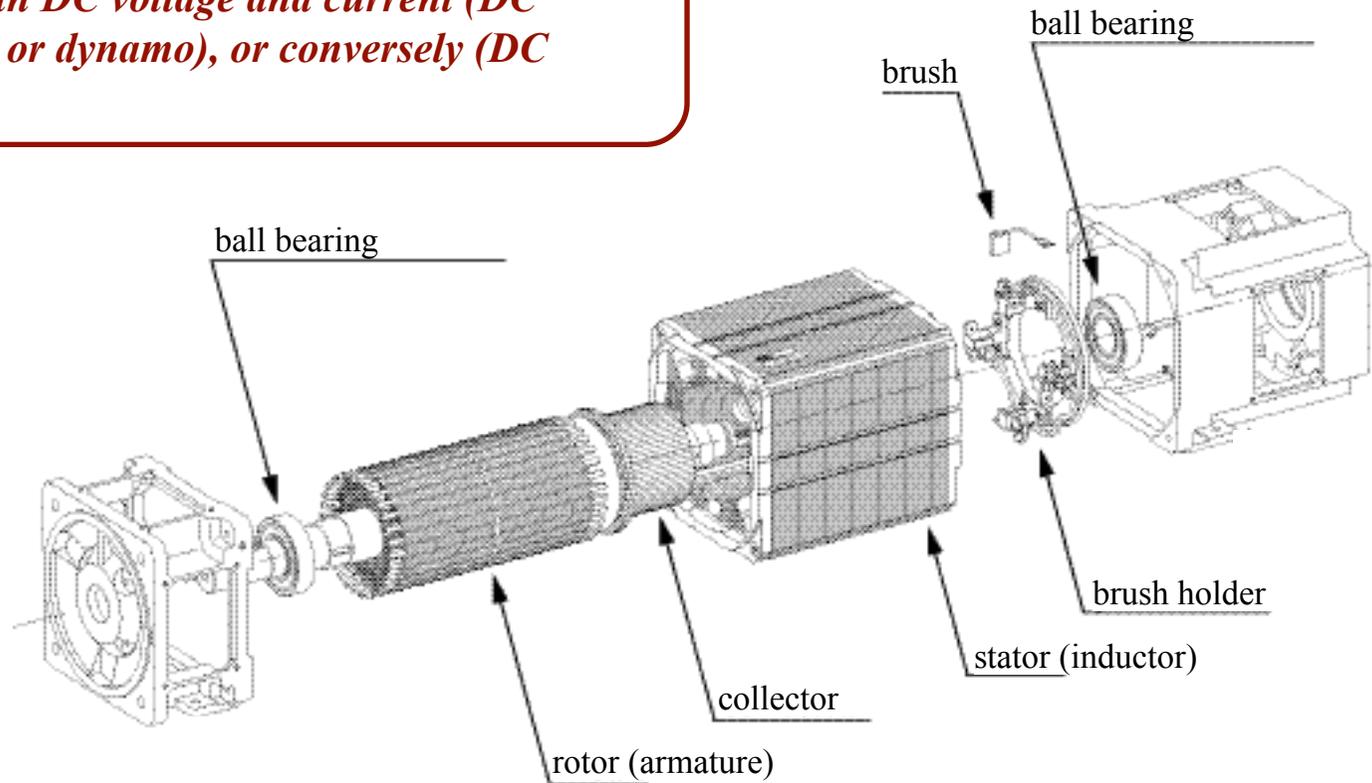
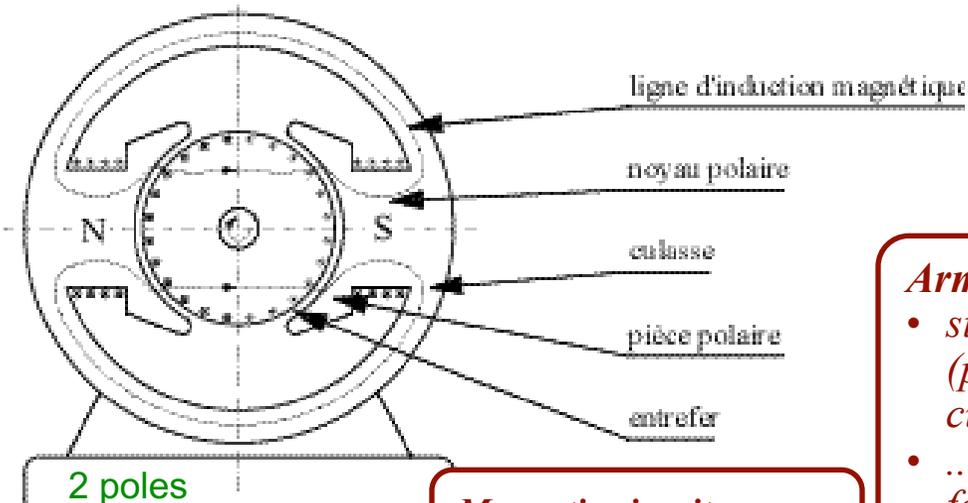


DC machines

Transforms mechanical energy into electric energy with DC voltage and current (DC generator or dynamo), or conversely (DC motor)



DC generators

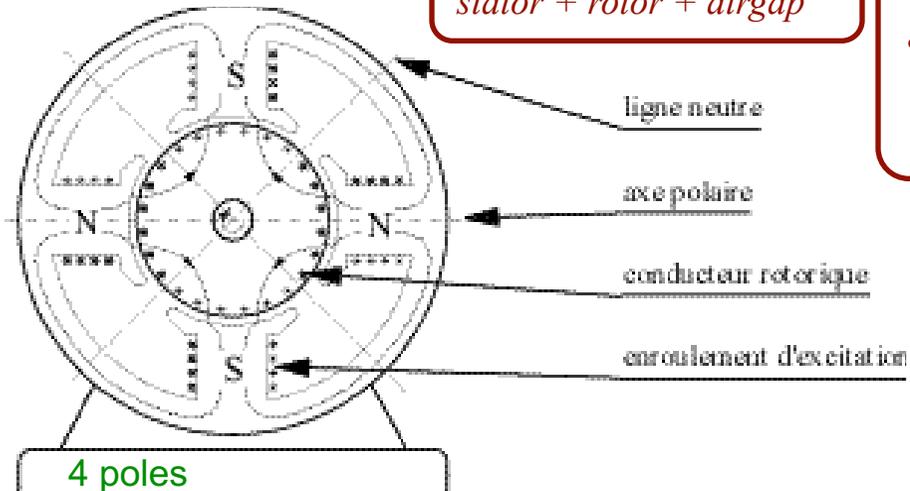


Inductor or stator: 2p poles with excitation windings carrying DC current

Armature or rotor :

- *stack of thin magnetic sheets (some tenth of a mm) (perpendicular to the machine axis to reduce eddy currents) ...*
- *... supporting conductors in which electromotive forces (e.m.f.s) appear when the armature rotates ($\mathbf{e} = \mathbf{v} \times \mathbf{b}$) ...*
- *... these e.m.f.s are time-varying and change sign each time the collector crosses a neutral line (bisector between 2 successive poles)*

*Magnetic circuit:
stator + rotor + airgap*

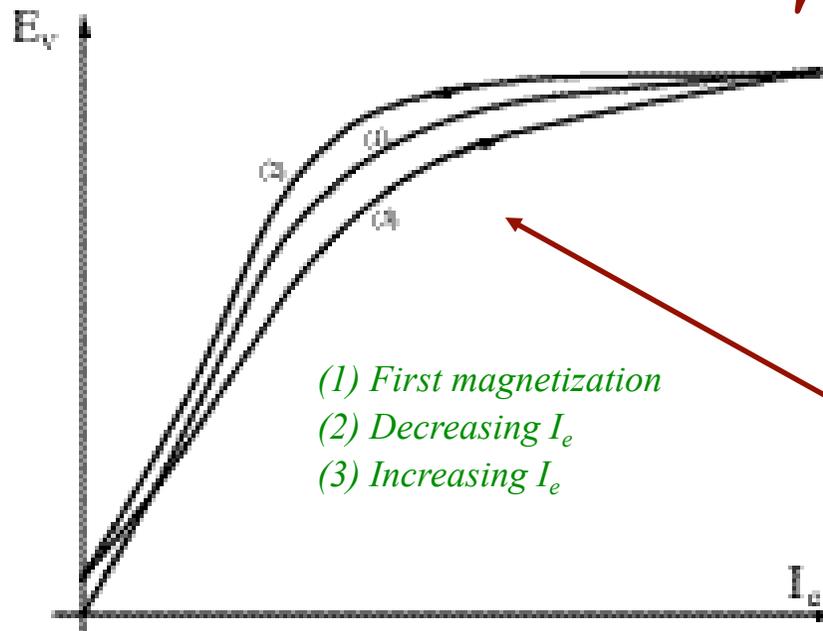


Collector: copper strips isolated from each other, and connected to equidistant points of the armature winding. Fixed brushes slide on the collector and rectify (mechanically) the e.m.f.s

No-load characteristic

No-load characteristic

Variation of the voltage E_v as a function of the excitation current I_e , at constant speed and with no delivered current



- (1) First magnetization
- (2) Decreasing I_e
- (3) Increasing I_e

$$E_v = f(I_e) \quad \text{with} \quad \begin{cases} \text{speed } \dot{\theta} = \text{constant} \\ I_a = 0 \end{cases}$$

$$E_v = k_E \dot{\theta} \Phi_v(I_e)$$

Magnetic flux produced by the inductor and seen by the armature winding

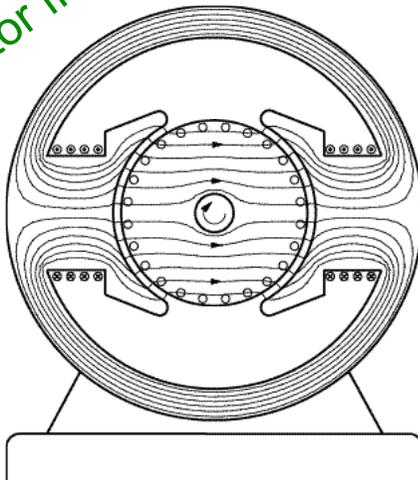
Nonlinear with hysteresis

Armature reaction

Armature reaction (magnetic)

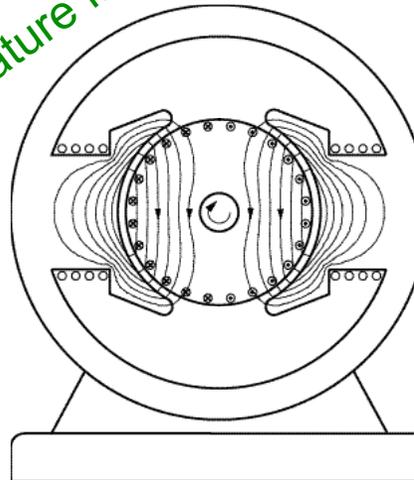
Magnetic phenomena due to the currents in the armature

Inductor field



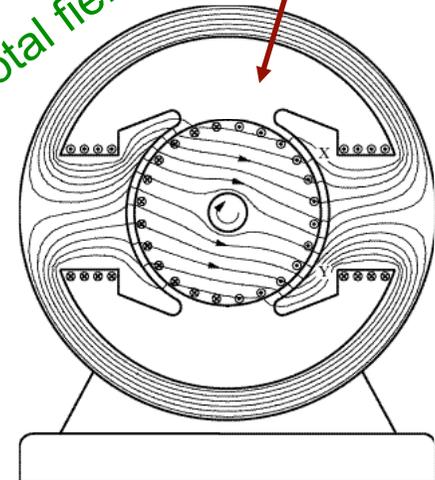
+

armature field



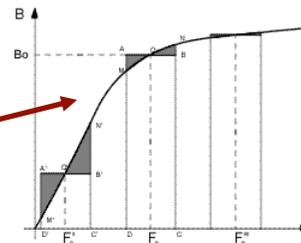
=

Total field



*1. Neutral line shifted (rotated) in the rotation direction
⇒ decrease of the e.m.f.*

*2. Local magnetic field reduction (entry part) and increase (exit part) not compensated due to nonlinearity
⇒ flux and e.m.f. reduction (+ incr. p_{mag})*



DC machines



$$E = E_v - \psi(I_a)$$

1. and 2.

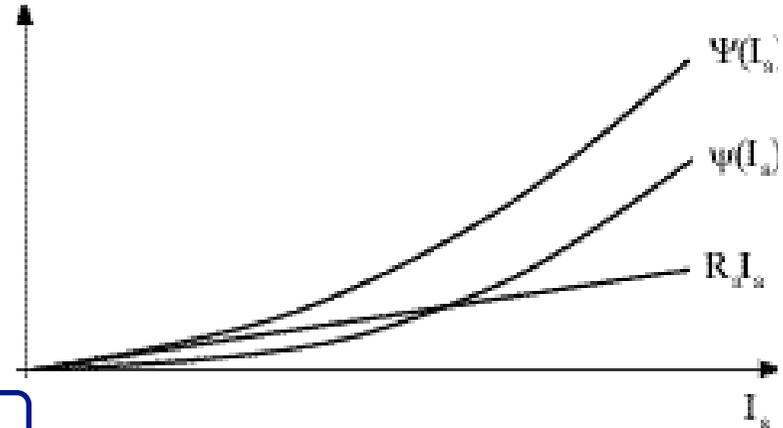
e.m.f. with load armature reaction

$$\psi(I_a) = k_E \dot{\theta} \Delta\Phi(I_a)$$

Armature reaction

Total armature reaction

$$\Psi(I_a) = \psi(I_a) + R_a I_a$$



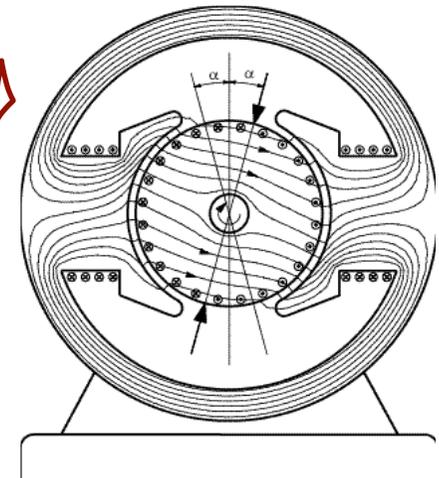
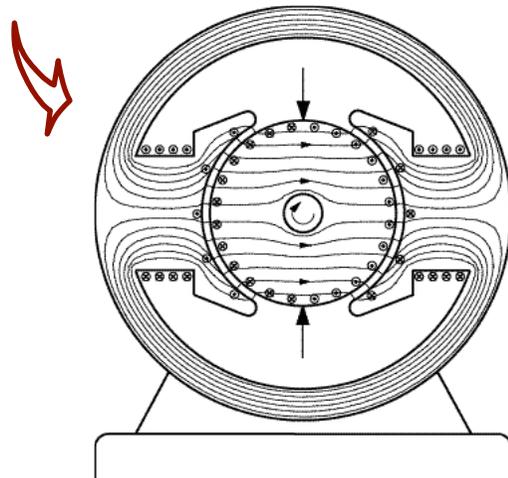
Compensating winding

Reduction of the armature reaction

Shift of the brushes w.r.t. neutral axis

disadvantages:

- for a single value of I_a
- shift direction depends on rotation direction
- shift direction depends on operating mode (generator or motor)



Exterior characteristics

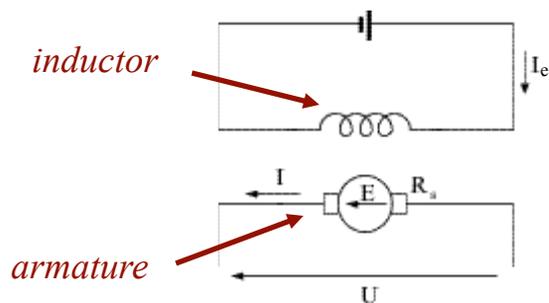
Exterior characteristic of a generator

Variation of delivered voltage U in terms of the delivered current I , at constant speed and excitation circuit

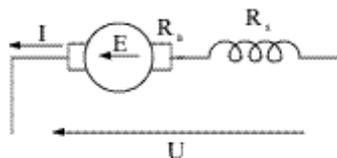
$$U = f(I) \quad \text{with} \quad \begin{cases} \text{speed } \dot{\theta} = \text{constant} \\ \text{fixed excitation circuit} \end{cases}$$

Excitation type...

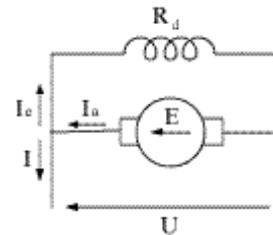
independent



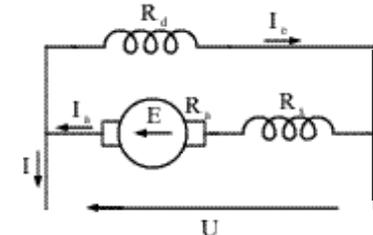
series



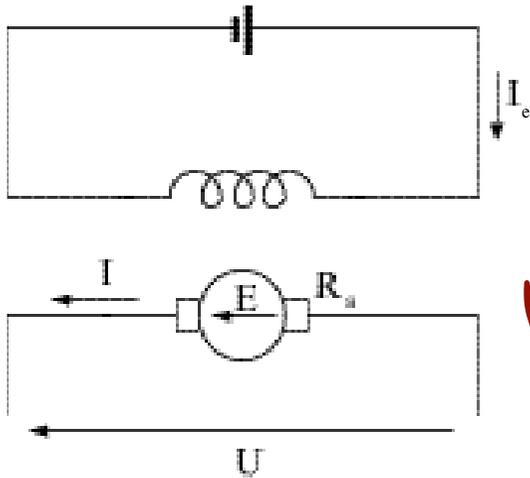
shunt



compound



Independent excitation generator

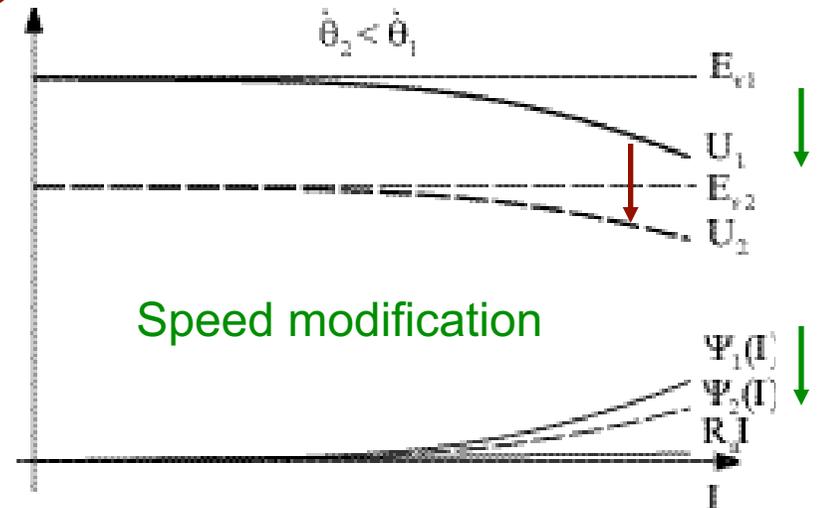
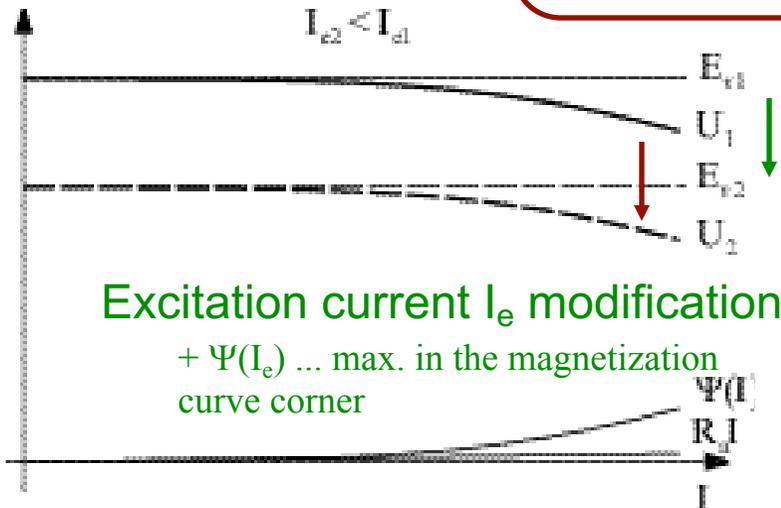
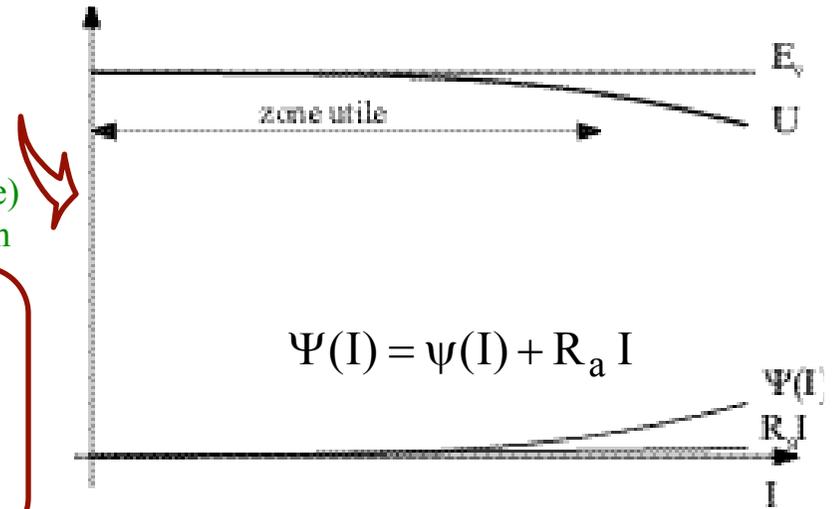


$$I = I_a$$

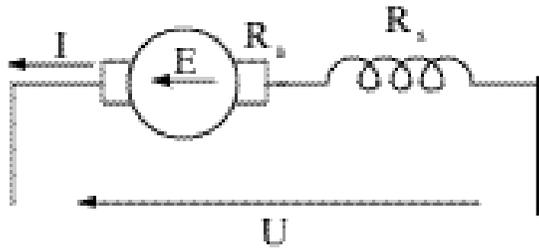
$$U = E_v(I_e) - \psi(I) - R_a I$$

$R_a \approx 0.1 \Omega$ (110V/50A machine)
Compensated armature reaction

Delivered voltage quasi independent of delivered current → Voltage source



Series excitation generator

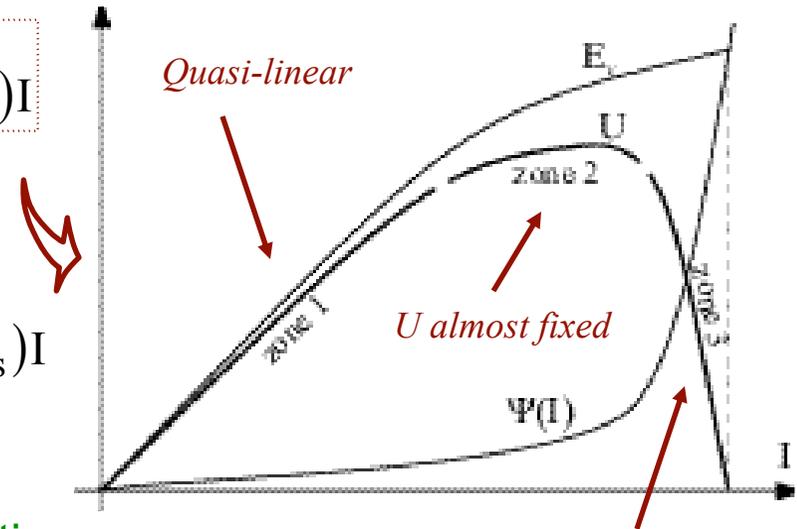


$$I = I_a = I_e$$

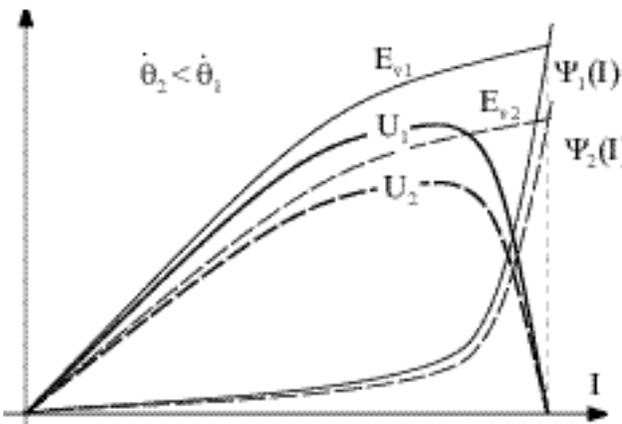
$$U = E_v(I) - \psi(I) - (R_a + R_s)I$$

$R_s \ll$ since $I_e = I$ is high
coherent: section $>$, $n_s <$

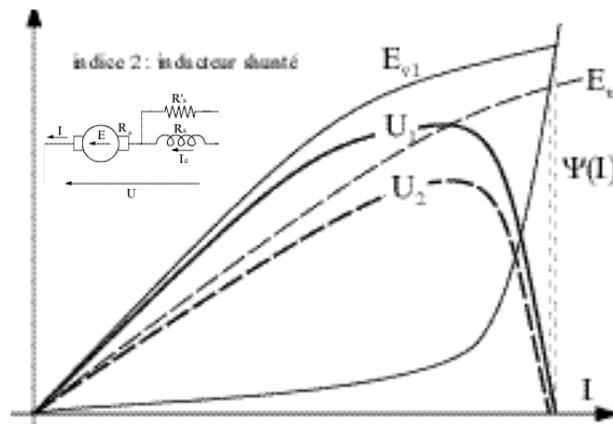
$$\Psi(I) = \psi(I) + (R_a + R_s)I$$



Speed modification



Inductor shunting

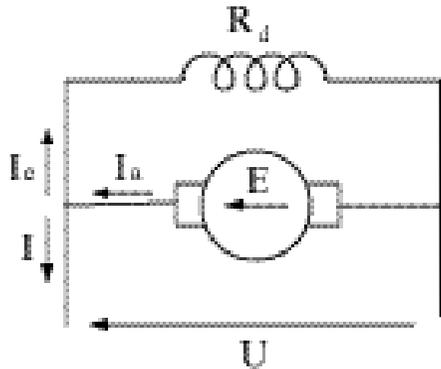


I almost constant

useful zone

Current source

Shunt excitation generator



$$I = I_a - I_e$$

$$U = E_v(I_e) - \psi(I_a) - R_a I_a$$

$$U = R_d I_e$$

$$\Psi(I_a) = \psi(I_a) + R_a I$$

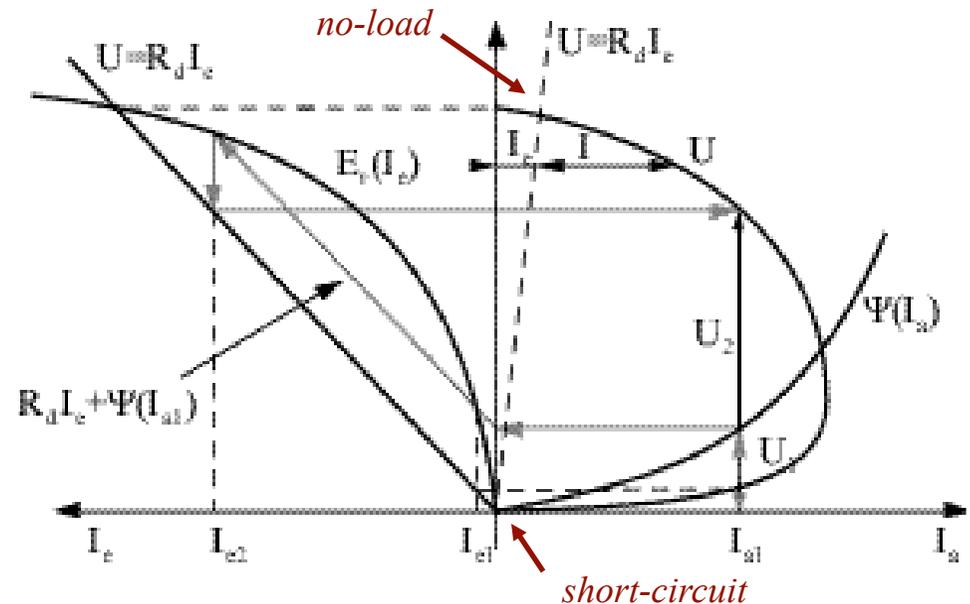
$R_d \gg$ to reduce Joule losses
 $I_e < \Rightarrow n_s >$

Picou construction



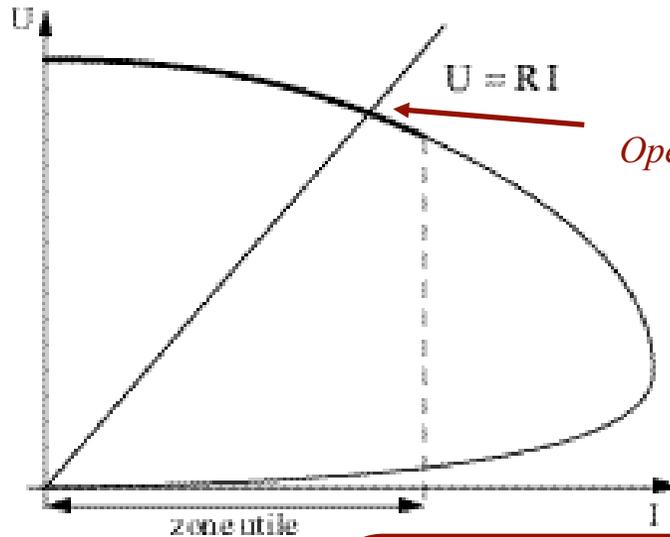
$E_v(I_e)$ & $\Psi(I_a)$ known
 $R_d I_e = U(I_e)$
 For I_{a1} (point by point procedure)
 $\rightarrow \Psi(I_a) \rightarrow \Psi(I_a) + R_d I_e \equiv E_{v1} \text{ \& \ } E_{v2}$
 $\rightarrow I_{e1} \text{ \& \ } I_{e2} \rightarrow U_1 \text{ \& \ } U_2$

$$I = I_a - I_e = I_a - \frac{U}{R_d}$$



Shunt excitation generator

Exterior characteristic



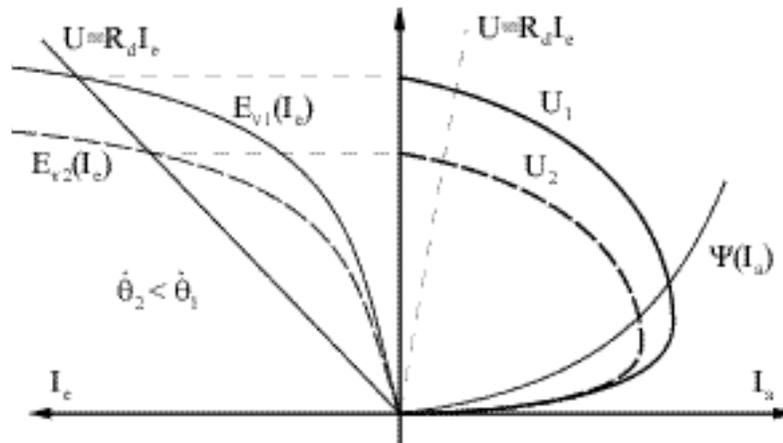
Operating point of the generator driving a resistance R

*Delivered voltage
almost independent of
the delivered current
→ Voltage source*

*... the voltage varies however more
than for the generator with
independent excitation*

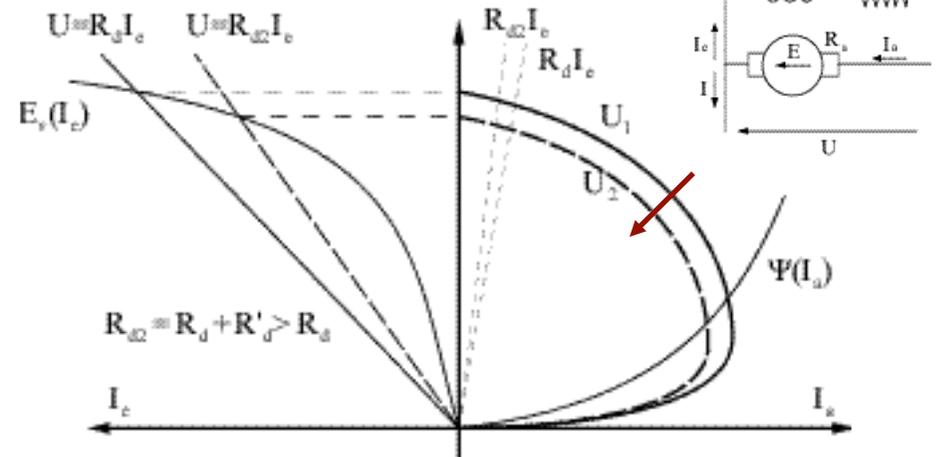
Shunt excitation generator

Speed modification



*If the speed is too low or if R_d is too large
→ no operating point*

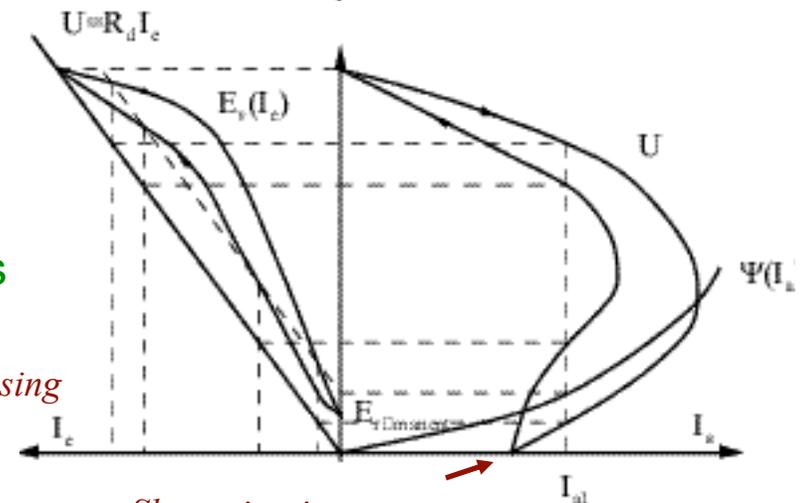
Excitation circuit modification



$$R_{d2} = R_d + R'_d > R_d$$

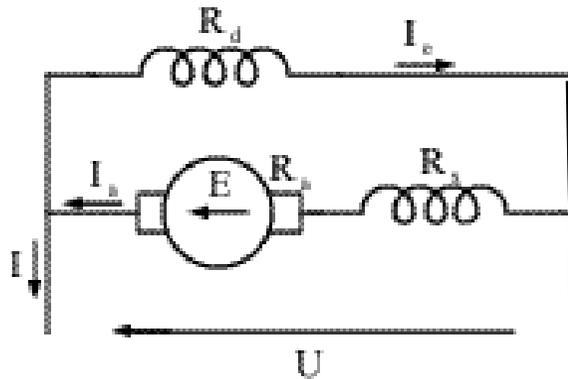
Effect of hysteresis

*2 branches :
 I_c increasing and decreasing*



Short-circuit current

Compound excitation generator

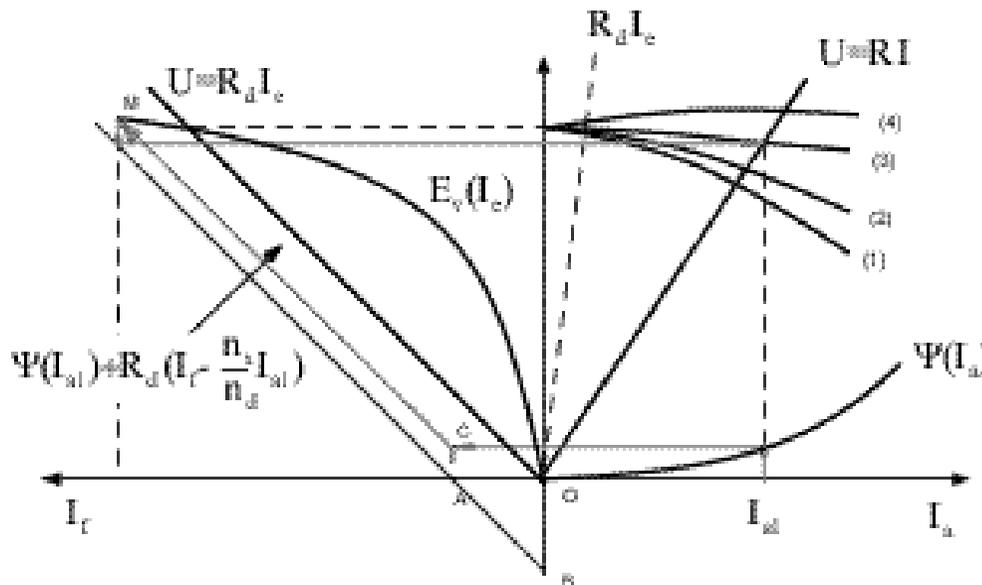


Mixed excitation: shunt inductor and series inductor wound on the same poles

$$\text{m.m.f.} = n_d I_e \pm n_s I_a = n_d \left(I_e \pm \frac{n_s}{n_d} I_a \right) = n_d I_f$$

$$U = E_v(I_f) - \Psi(I_a)$$

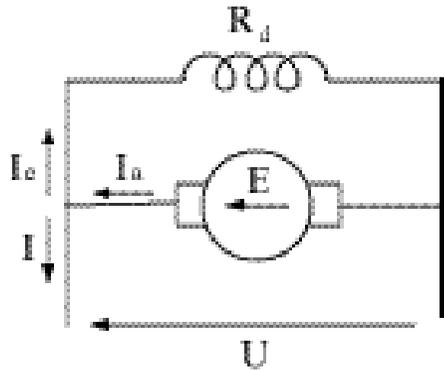
$$U = R_d I_e = R_d \left(I_f \mp \frac{n_s}{n_d} I_a \right)$$



- (4) hypercompound ($n_s \gg 1$)
- (3) concordant compound (same direction m.m.f.)
- (2) shunt dynamo
- (1) antagonist compound (opposite m.m.f.)

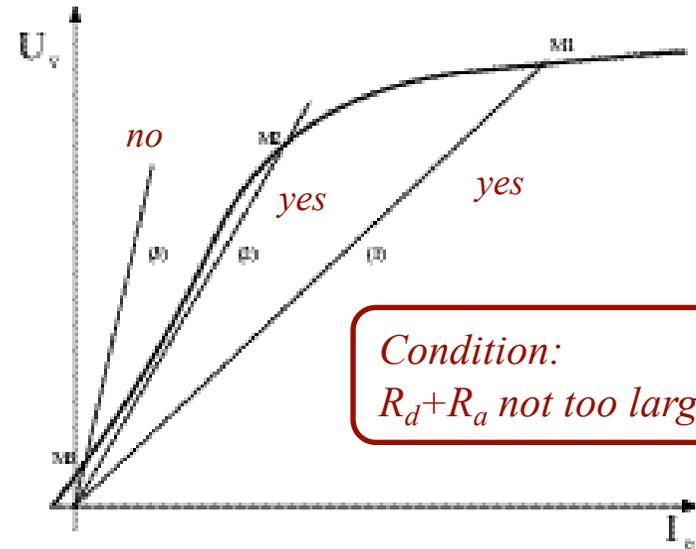
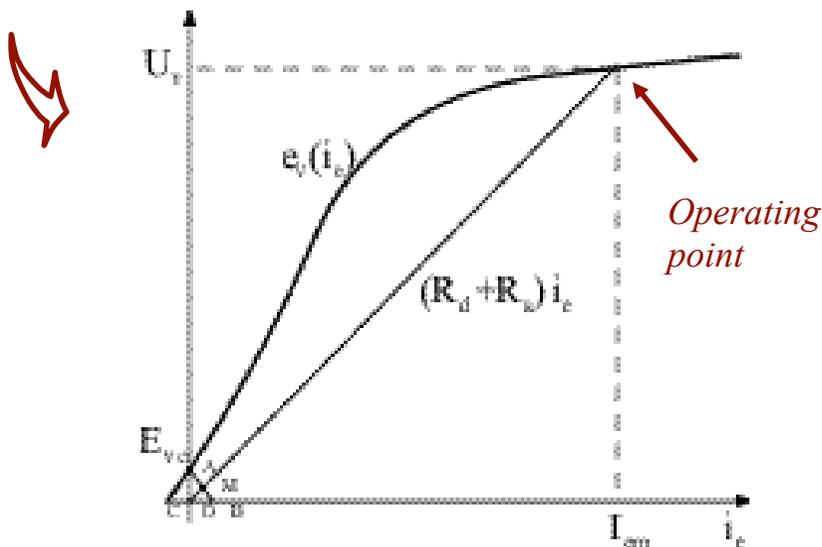
$$E_v(I_f) = \Psi(I_a) + R_d \left(I_f \mp \frac{n_s}{n_d} I_a \right)$$

Self-starting generator



Self-starting is possible thanks to the remanent magnetization of the inductor

Example: shunt generator



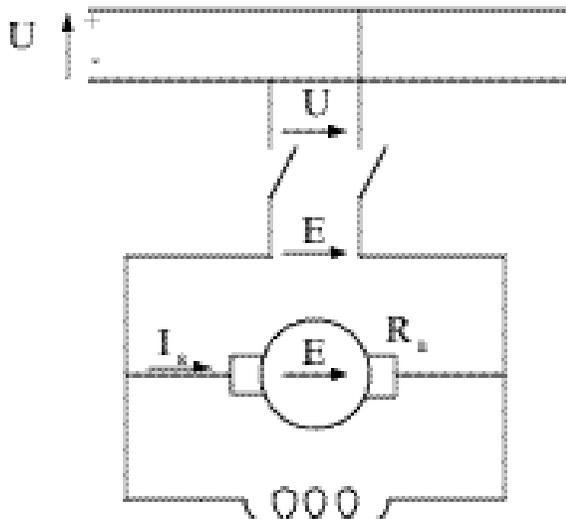
*Condition:
 $R_d + R_a$ not too large!*

DC network connection

Conditions :

$$E \approx U$$

E and U in opposition



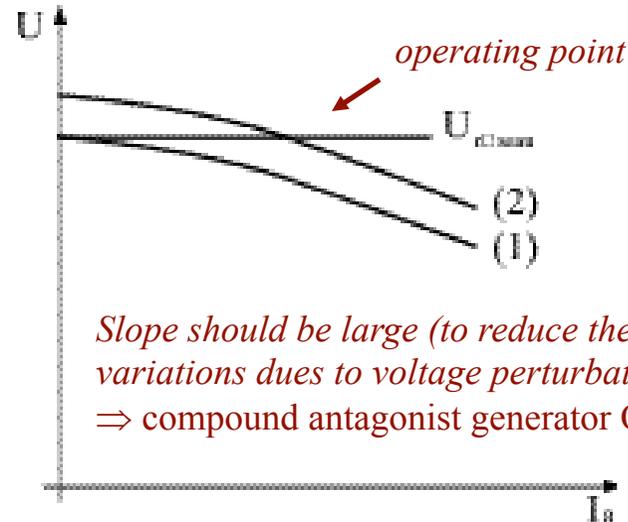
After connexion (1) :



$$I_a = \frac{E(I_e, I_a) - U}{R_a}$$

If $E \ll, I_a \gg !$

Then, increase E (2) \Rightarrow the generator produces energy



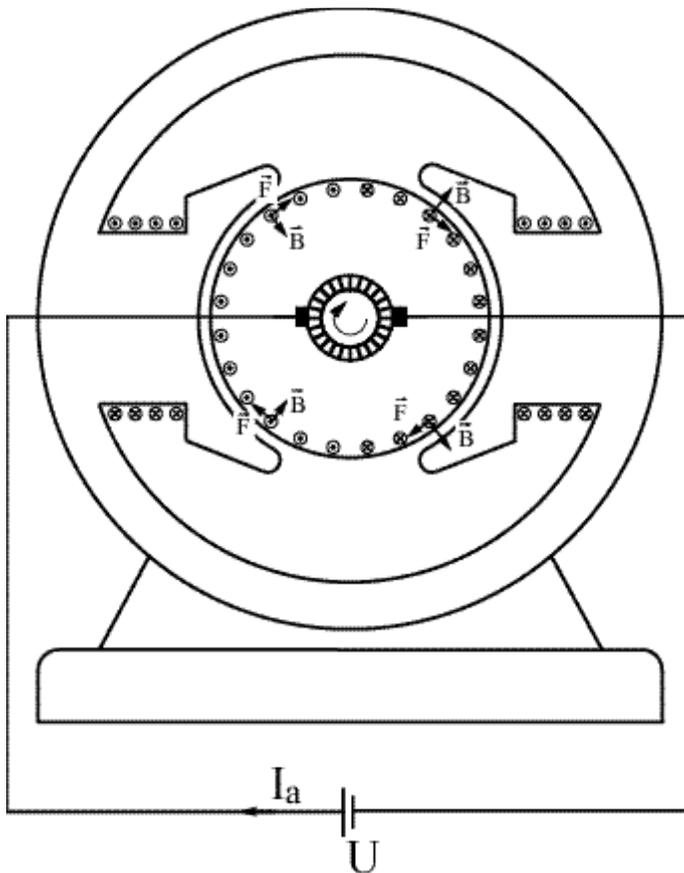
*Slope should be large (to reduce the current variations dues to voltage perturbations)
 \Rightarrow compound antagonist generator OK*

If E decreases

\Rightarrow the generator receives energy (motor for shunt and compound machines!)

DC motors

Main principle



Excitation current I_e and armature current I_a

The armature conductors are subjected to the magnetic flux density created by the inductor

... hence to the Laplace force

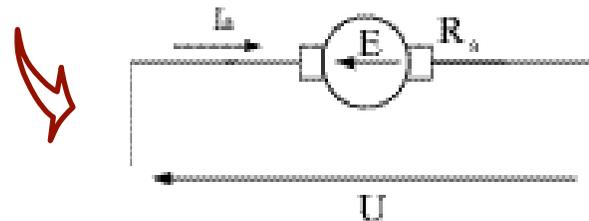
$$\mathbf{f} = \mathbf{j} \times \mathbf{b}$$

... hence to a torque that tends to make the armature rotate

Electromotive force (e.m.f.)

... in the armature conductors as soon as they rotate, opposed to the current

Total e.m.f. (E) on brushes is equal to the integral of the electromotive field along the armature conductors



$$U = E + R_a I_a$$

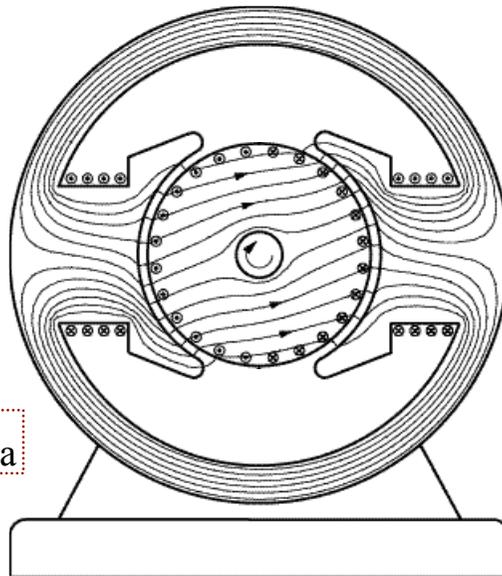
Armature reaction

$$E = E_v - \psi(I_a)$$

e.m.f. with load

armature reaction

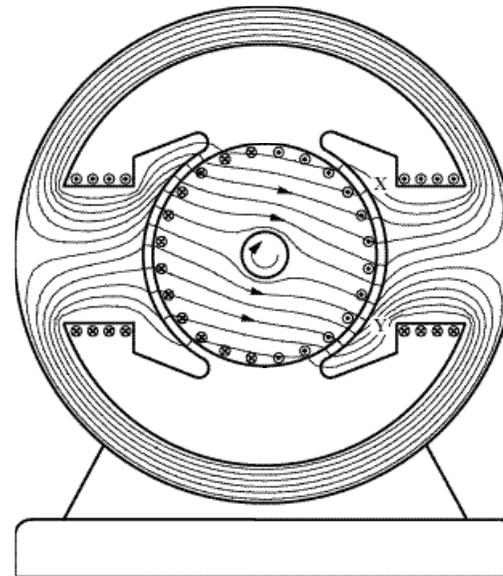
$$\psi(I_a) = k_E \dot{\theta} \Delta\Phi(I_a)$$



$$U = E + R_a I_a$$

DC motor

$$\Psi(I_a) = \psi(I_a) - R_a I_a$$



$$U = E - R_a I_a$$

DC generator

$$\Psi(I_a) = \psi(I_a) + R_a I_a$$

Total armature reaction

Motor torque

$$U = E + R_a I_a = E_v - \psi(I_a) + R_a I_a$$

$$\times I_a$$

$$U I_a = E I_a + R_a I_a^2 = (E_v - \psi(I_a)) I_a + R_a I_a^2$$

Electric power
provided to the
armature

Electromagnetic
power

Joule losses in the
armature

Electromagnetic torque

$$C = \frac{P_{elm}}{\dot{\theta}} = \frac{E I_a}{\dot{\theta}}$$



$$C = k_E \Phi(I_e, I_a) I_a = k_E [\Phi_v(I_e) - \Delta\Phi(I_a)] I_a$$

Mechanical characteristics

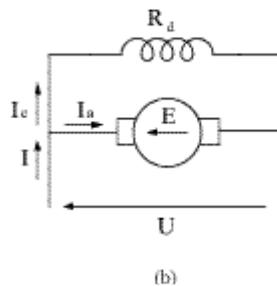
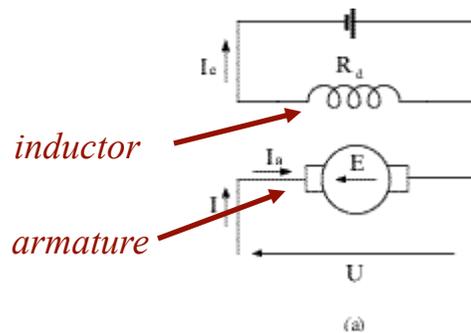
Mechanical characteristic of a motor

Motor speed in terms of the electromagnetic torque, with fixed voltage and excitation circuit

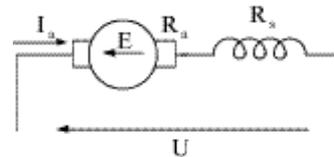
$$\dot{\theta} = f(C) \quad \text{with} \quad \begin{cases} U = \text{constant} \\ \text{fixed excitation circuit} \end{cases}$$

Excitation type...

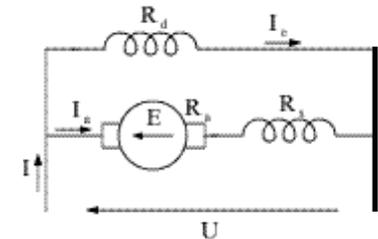
independent or shunt



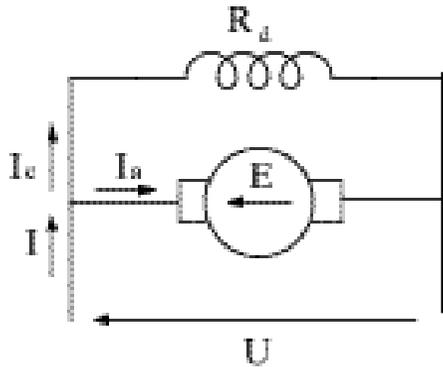
series



compound



Shunt excitation motor

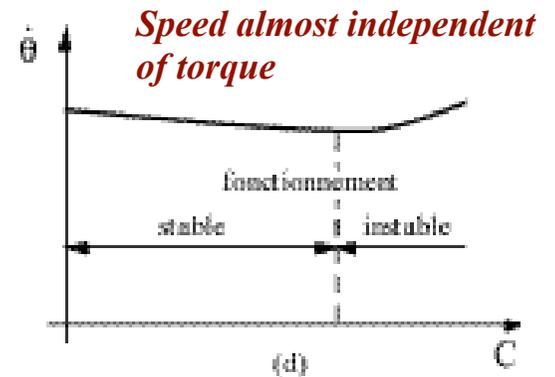
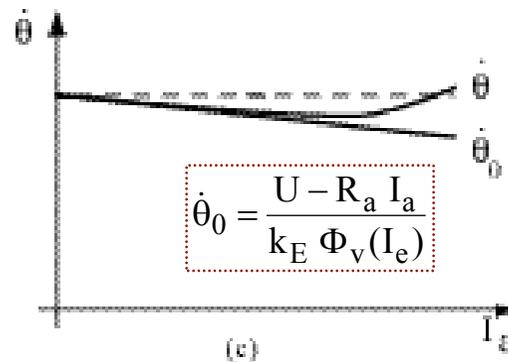
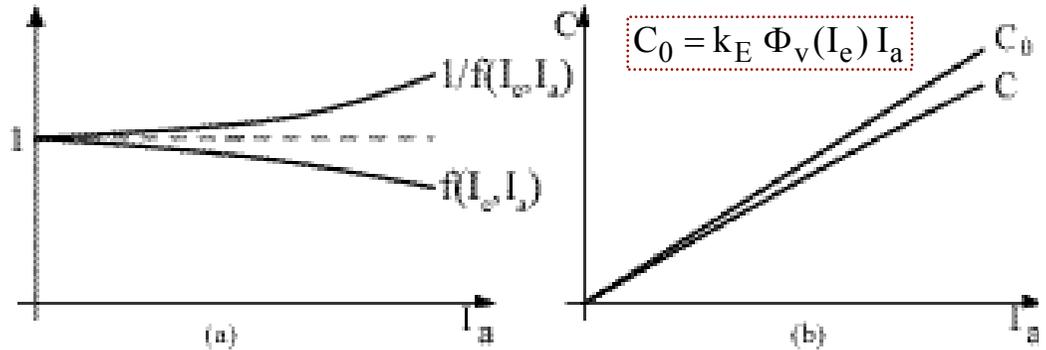


$$C = k_E [\Phi_v(I_e) - \Delta\Phi(I_a)] I_a = C_0 f(I_e, I_a)$$

with $(I_e \text{ constant})$

$$f(I_e, I_a) = \frac{\Phi_v(I_e) - \Delta\Phi(I_a)}{\Phi_v(I_e)} \leq 1$$

C_0 = torque produced by the motor if there was no armature reaction

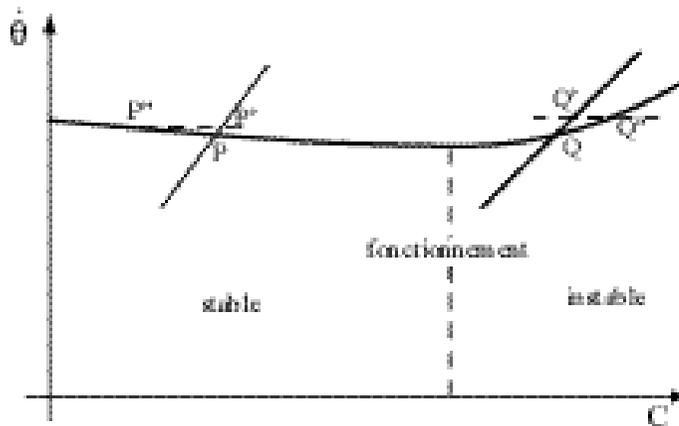


$$U = E + R_a I_a = k_E \dot{\theta} [\Phi_v(I_e) - \Delta\Phi(I_a)] + R_a I_a$$

$$\dot{\theta} = \frac{U - R_a I_a}{k_E [\Phi_v(I_e) - \Delta\Phi(I_a)]} = \dot{\theta}_0 \frac{1}{f(I_e, I_a)}$$

Shunt excitation motor

Stable and unstable zones



Small perturbation: e.g. speed increase



From P:

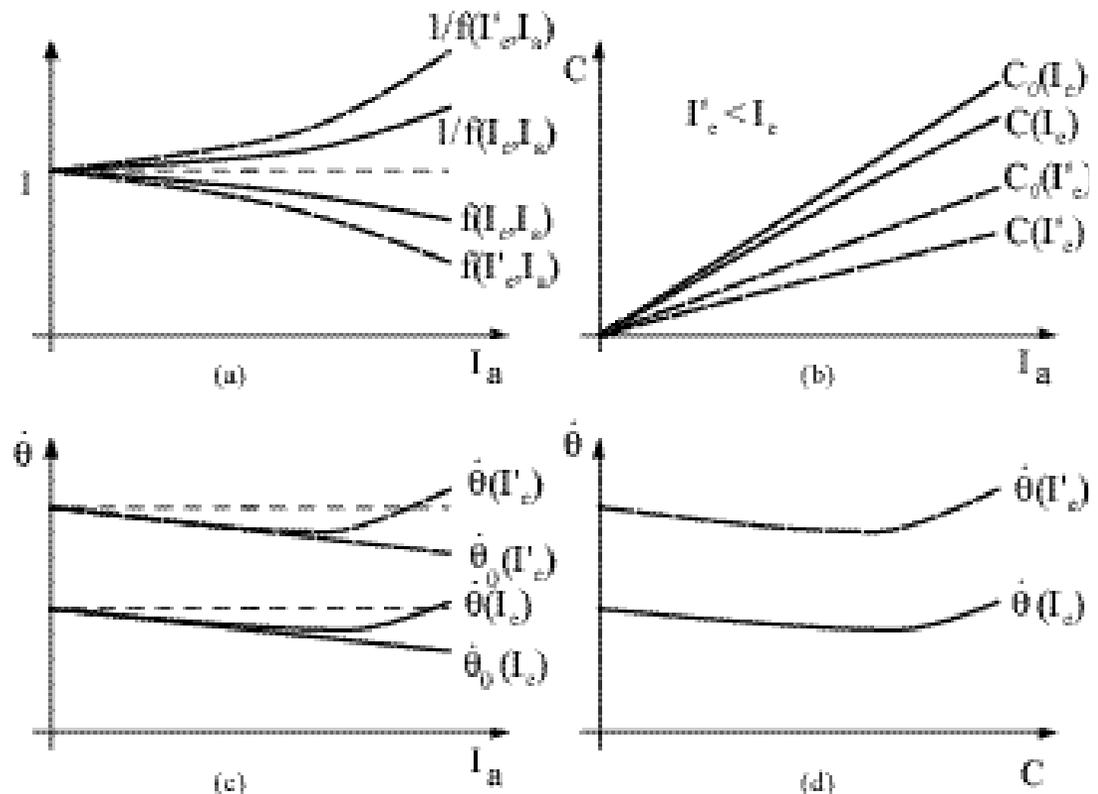
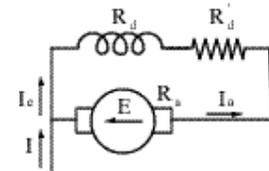
motor torque $P'' <$ resisting torque P'
 \Rightarrow speed decreases, back to P \Rightarrow **stable**



From Q:

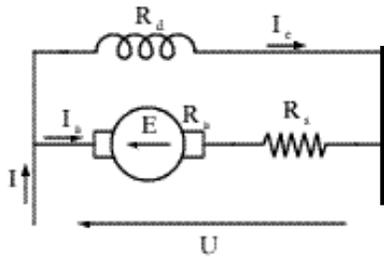
motor torque $Q'' >$ resisting torque Q'
 \Rightarrow speed increases! \Rightarrow **unstable**

Influence of I_e



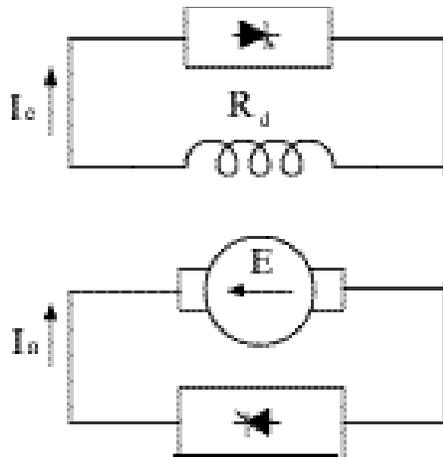
Limited speed range (saturation)

Shunt excitation motor



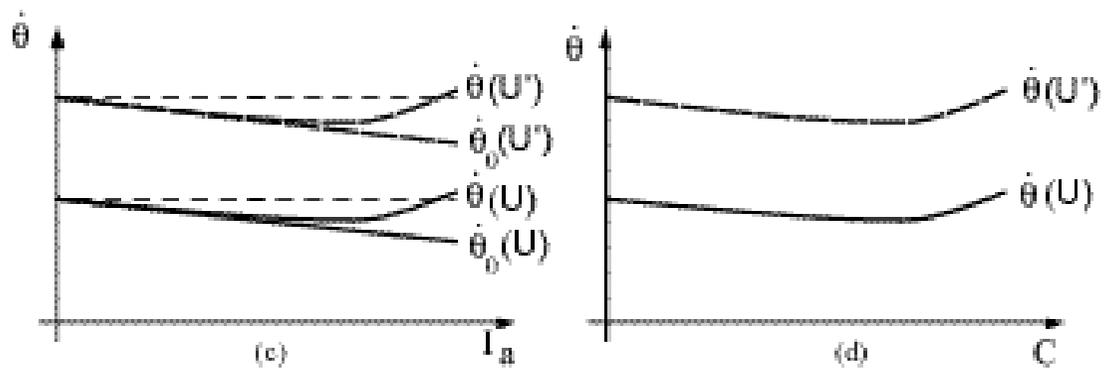
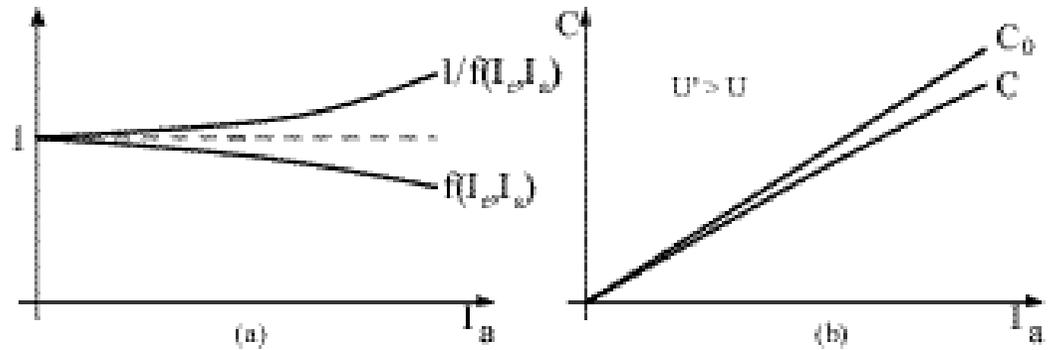
Poor efficiency!

+ power electronics...



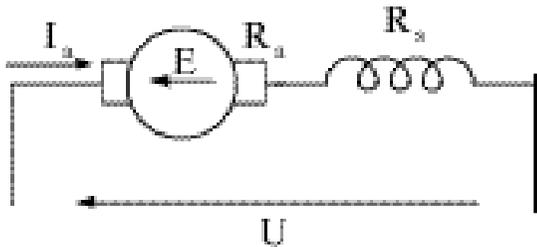
Influence of the voltage U

$$\dot{\theta} \approx \frac{U}{k_E \Phi_v(I_e)}$$



High dynamic torque control (since $\lambda_a \ll 1$)

Series excitation motor



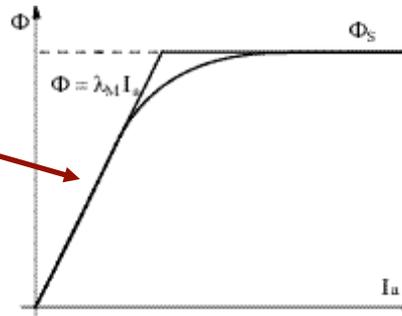
$$C = k_E \Phi(I_a) I_a$$

$$U = k_E \dot{\theta} \Phi(I_a) + (R_a + R_s) I_a$$

$$C \approx k_E \lambda_M I_a^2$$

$$U \approx k_E \lambda_M \dot{\theta} I_a + (R_a + R_s) I_a$$

Non saturated machine



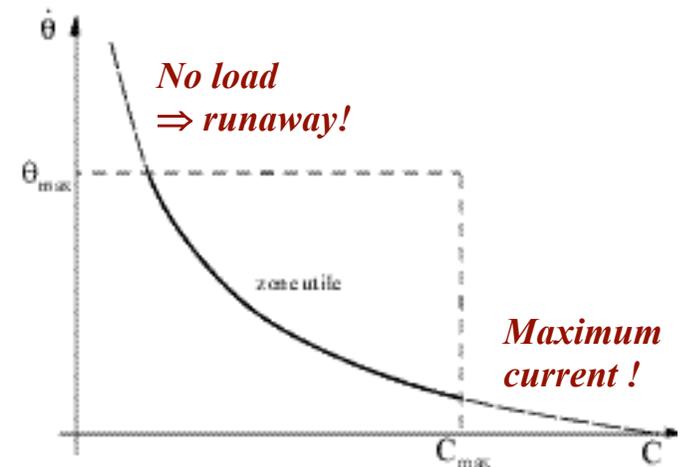
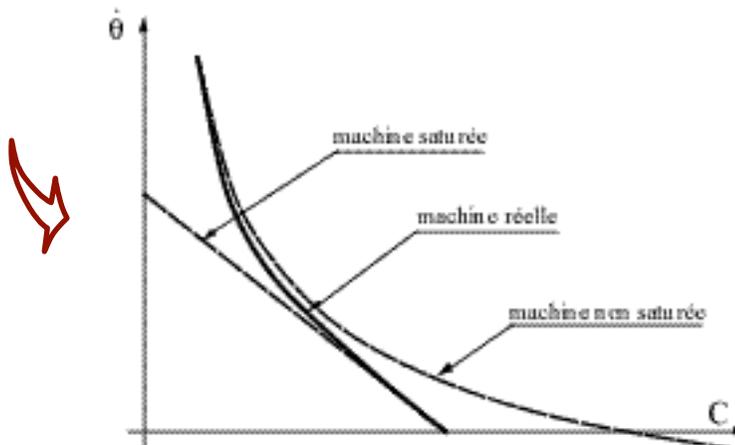
Saturated machine

$$C \approx k_E \Phi_S I_a$$

$$U \approx k_E \Phi_S \dot{\theta} + (R_a + R_s) I_a$$

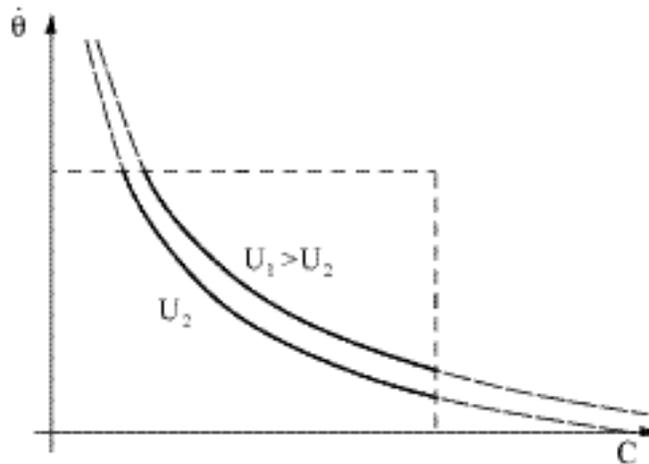
$$C \approx k_E \lambda_M \frac{U^2}{(k_E \lambda_M \dot{\theta} + R_a + R_s)^2} \approx \frac{1}{k_E \lambda_M} \frac{U^2}{\dot{\theta}^2}$$

$$C \approx k_E \Phi_S \frac{U - k_E \Phi_S \dot{\theta}}{R_a + R_s}$$



Series excitation motor

Influence of the voltage source U



+ power electronics...

Typical use

Electric traction and lifts (large startup torque)

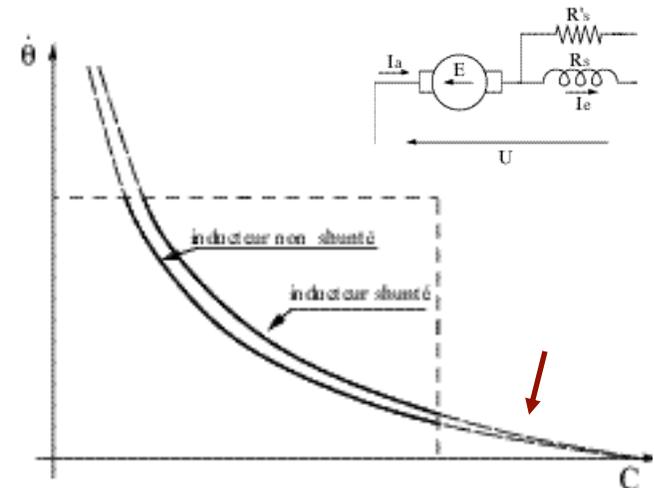
Shunting the inductor

$$I_e = \frac{R'_s}{R_s + R'_s} I_a \leq I_a$$

$$\lambda'_M = \frac{R'_s}{R_s + R'_s} \lambda_M \leq \lambda_M$$

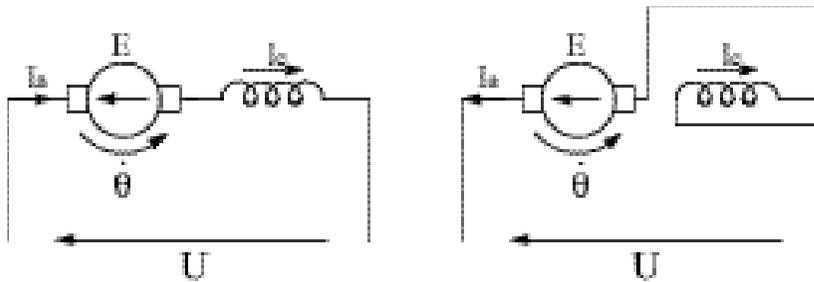


$$\Phi(I_e) \approx \lambda_M I_e = \lambda'_M I_a \leq \lambda_M I_a = \Phi(I_a)$$



Series excitation motor

Braking



$$C = k_E \Phi(I_a) I_a$$

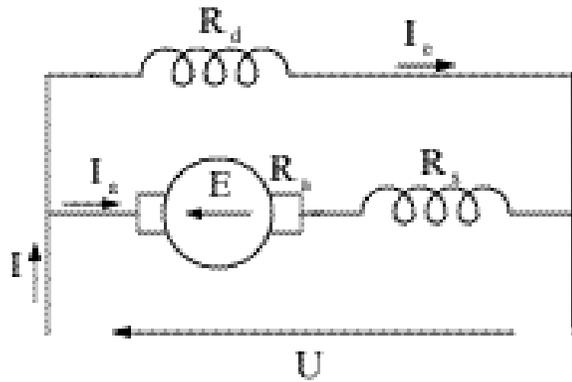
Change the sign of the torque to work as a brake

Electric power changes sign (recovers energy)

Different modes

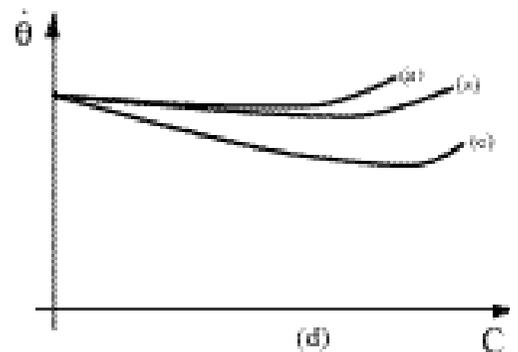
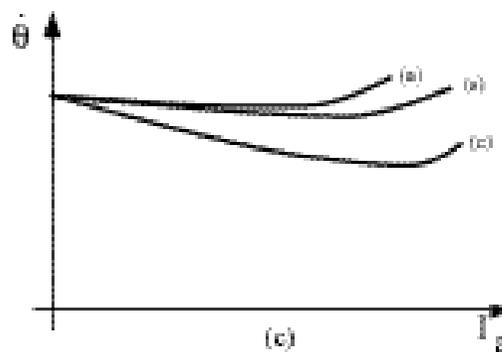
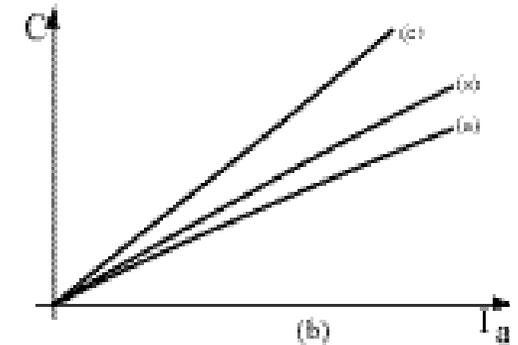
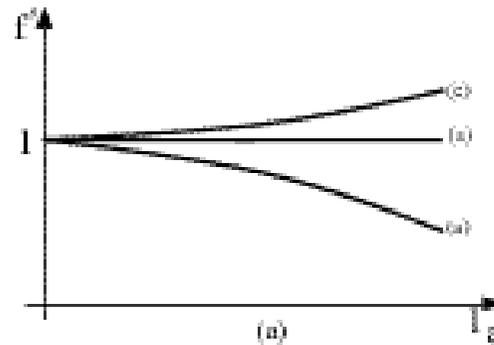


Compound excitation motor



Mixed excitation: shunt and series inductor wound on the same poles

$$\begin{aligned} \text{m.m.f.} &= n_d I_e \pm n_s I_a \\ &= n_d \left(I_e \pm \frac{n_s}{n_d} I_a \right) \\ &= n_d I_f \end{aligned}$$



DC motor startup

Zero speed at startup \Rightarrow zero e.m.f. E

$$I_a = \frac{U - E}{R_a} = \frac{U - k_E \dot{\theta} \Phi}{R_a}$$

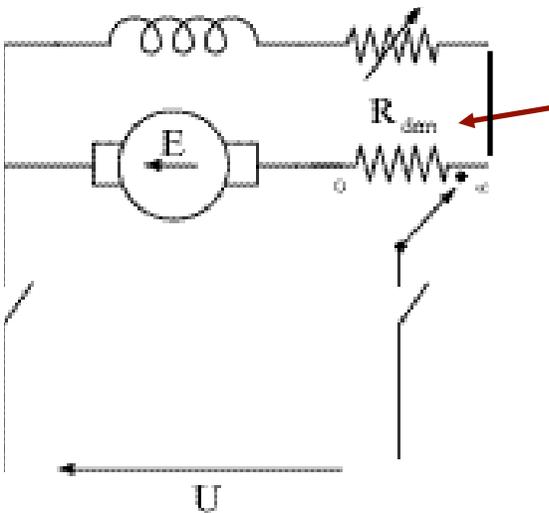
Induced current I_a limited only by the armature resistance R_a

$$I_a = \frac{U}{R_a} \gg$$

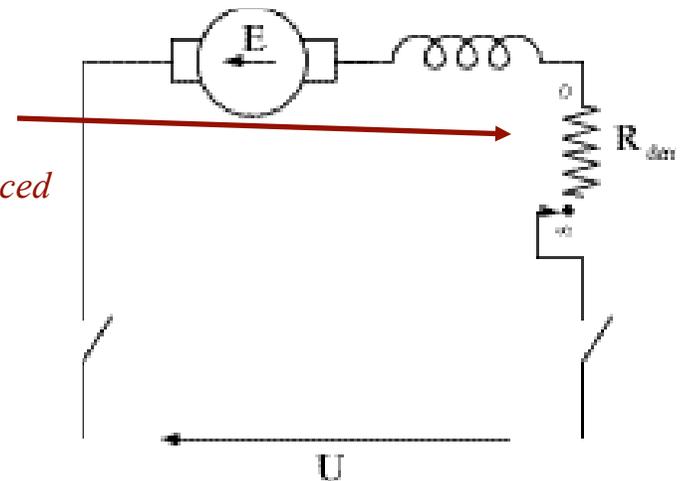
Startup rheostat in series with the armature (to limit I_a)

(One allows $I_{as} = 1.5 I_{an}$)

Shunt motor

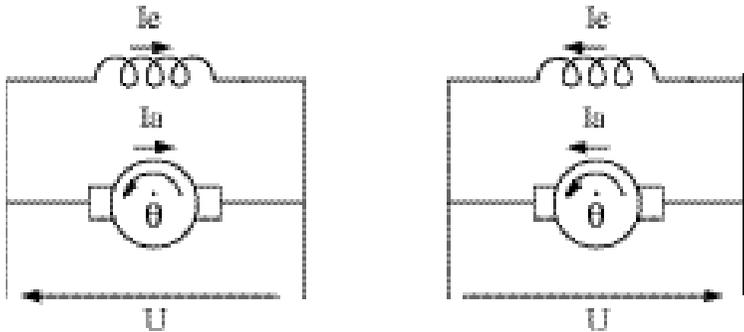


Series motor



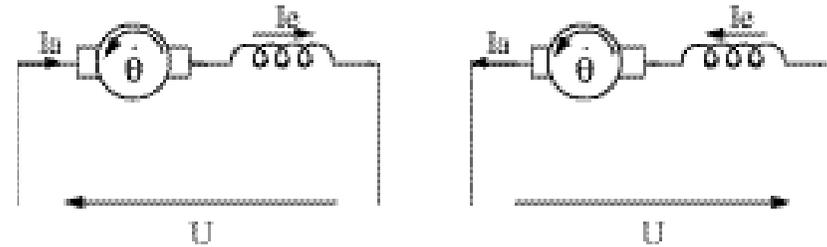
Inverting the rotation direction

Shunt motor



Series motor

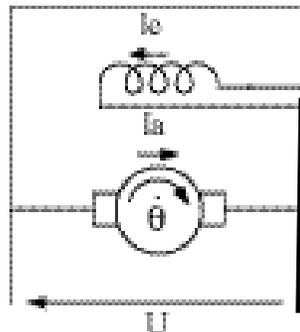
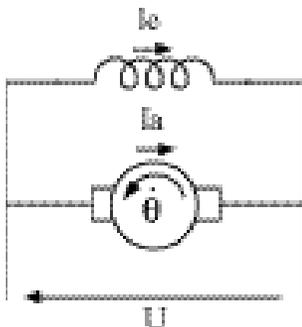
Same direction



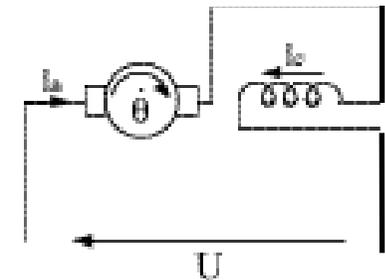
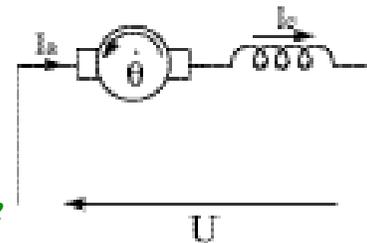
$$C = k_E \Phi(I_e) I_a$$

Modify the direction of the current in the excitation circuit w.r.t. the rotor

Torque changes sign



Opposite direction



Losses in DC machines

❖ Mechanical losses

- friction losses in bearings ($\div v$) ($v = \text{speed}$)
- windage losses ($\div v^2$)
- friction losses from brushes on the collector ($\div v$)

❖ Magnetic losses

- eddy current losses in armature ($\div v^2$, $\div b_{\max}^2$)
- hysteresis losses in armature ($\div v$, $\div b_{\max}^{1.5 \rightarrow 2}$)

❖ Electric losses

- Joule losses in armature, inductor and brushes ($\div I^2$, function of temperature)

❖ Supplementary losses

- due to skin effect in the rotor and sparks at brushes/collector contact
- increased magnetic losses due to the magnetic reaction