ELEC0431 Electromagnetic Energy Conversion Exercises

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Contents

1 Phasors and power in the sinusoidal steady state

Exercise 1: Voltage distribution

The circuit of Figure 1 presents a resistive-inductive load powered with an AC generator of sinusoidal voltage (230 V, 50 Hz). Find the voltages accross R and L (magnitude and phase angle) and represent all the voltages on a phasor diagram.
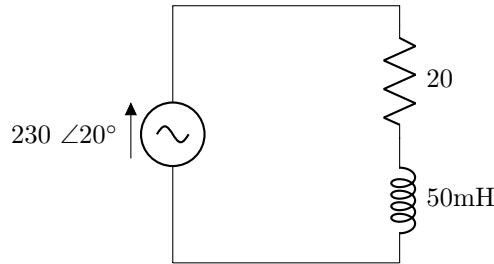
Figure 1: Resistive-inductive circuit.

Exercise 2: Reactive power compensation

Your colleague suggests to add a 50 μF capacitor in parallel of the RL load. It is supposed to compensate the reactive power consumed by the inductive load. Is it a good idea ? If so, what would be the exact value of C needed ?
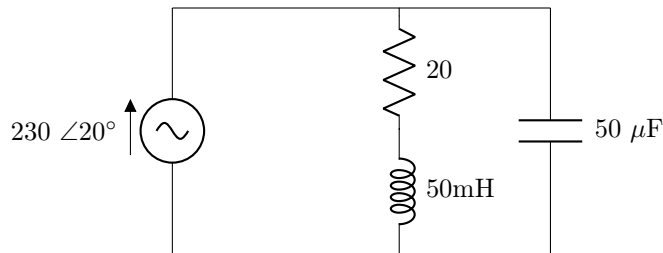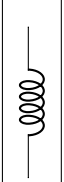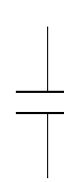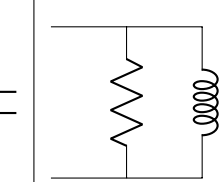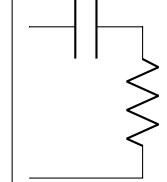
Figure 2: Inductive circuit with compensation capacitor.

Exercise 3: One-port small quiz

Fill the cells of the table below with the most appropriate answer among:

=0   <0   >0   =1   <1   +∞   -∞

one-port:				
active power consumed				
reactive power produced				
cos ϕ				
tan ϕ				

Exercise 4:   2-Ports characterization

It is asked to characterize the 2-ports of Figure 3. In that context, two tests have been performed: a short circuit test and an open circuit test.

- 559 mV and 1.118 A are measured at the access 1 while the access 2 is shorted (short circuit test).

- 5 V and 4.472 A are measured at the access 1 while the access 2 is left open (open circuit test).

1. Determine the value of R and L with the information provided below.
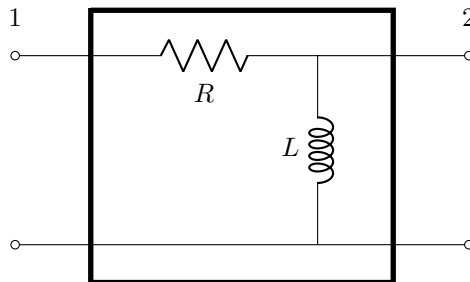
Figure 3: 2-Ports circuit

The 2-ports could be fully caracterized by only one of the two tests if the active power was measured during the tests. The active power can be measured with a wattmeter.

2. Which test would be necessary ?

3. During that test, an active power of 9.99392 W has been measured. Prove that it gives the correct value of R and L.

2   Power in three-phase systems

Exercise 5:   Electrical Heater

Consider an electrical heater that dissipates 15 kW of power when connected to a three-phase power system of 208 V. As a first approximation, the heater is modelled as a purely resistive three-phase load.
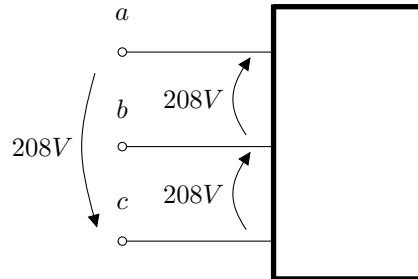
Figure 4: Three-phase Electrical Heater

1. If no additionnal information is provided about the voltage, does the 208 V correspond to the peak or the RMS value ?

2. Compute the line current if the resistive loads are connected in **Y**.

3. If the resistors are connected in **Y**, compute the resistance of each.

4. Compute the line current if the resistive loads are connected in Δ.

5. If the resistors are connected in Δ, compute the resistance of each.

3 Magnetic circuits & Transformers

Exercise 6: Reluctance computation

Consider an inductor made of an iron core (as described in Figure 5) and a 60 turns winding, wounded around the central leg.
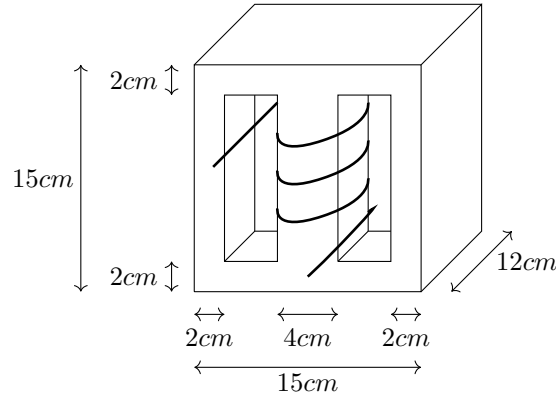
Figure 5: Magnetic circuit of the inductor

1. Draw an equivalent magnetic circuit of the inductor;

2. Compute the total reluctance of this circuit, considering a relative permeability μ_r of 1500 for the iron. Deduce the inductance from it;

3. Do the same computation as in the previous steps, but now considering a constant air gap of $0.1mm$ in each leg;

Exercise 7: Four secondaries single-phase transformer

In Belgium, most of the railways are powered using Direct Current (DC) $3\,\mathrm{kV}$ voltage. High speed train lines are however supplied with Alternating Current (AC) $25\,\mathrm{kV}$ $50\,\mathrm{Hz}$ (single-phase) voltage, requiring the use of high power single phase transformers. In this exercise, an input transformer of a locomotive is considered. It contains four secondary identical windings, as detailed in Fig. 6. A nominal Root Mean Square (RMS) voltage of $U_{1n} = 25\,\mathrm{kV}$ with nominal frequency $f = 50\,\mathrm{Hz}$ is supplied to the primary winding with an apparent power $S_n = 5.6\,\mathrm{MVA}$.

To characterize the transformer two tests have been performed:

- Using open secondary windings, the transformer generates a voltage $U_{2o} = 1.36\,\mathrm{kV}$ at one secondary winding, for a current drawn at the primary $I_{1o} = 1.25\,\mathrm{A}$, and an active consumed power $P_{1o} = 6.8\,\mathrm{kW}$;

- Using short-circuited secondary windings, the transformer consumes an active power $P_{1s} = 25\,\mathrm{kW}$, considering that a reduced voltage of $37.1\,\%$ of U_{1n} was applied to the primary winding to maintain the secondary winding currents to their nominal values I_{2n}.

1. Calculate the transformer ratio n (for a single secondary winding);

2. Determine the nominal RMS secondary current I_{2n} and primary current I_{1n};

3. Compute the power factor $\cos\phi_{1o}$ for the first test (open secondary windings) and deduce the phase shift ϕ_{1o} of the current at the primary winding with respect to the primary winding voltage;

4. Give the reactive power Q_{1o} for the first test (open secondary windings);
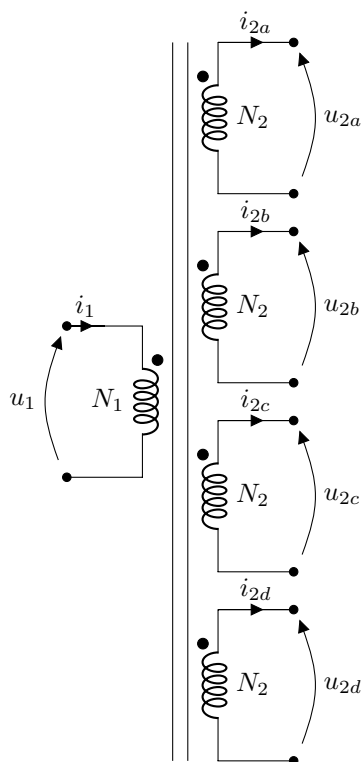
Figure 6: Four secondaries single-phase transformer.

5. Considering a simplified circuit of the transformer model, calculate the resistance R_{H+F} (related to the core losses) and the magnetizing inductance L_μ, seen from the primary. One can also calculate R'_{H+F} and L'_μ, the same components seen from one secondary winding;

6. Compute the RMS current intensity I_{2s} in a secondary winding for the second test (shorted secondary windings), compute the primary winding voltage U_{1s}, and calculate the values of the resistance R and of the inductance L of the primary winding in the equivalent model. One can also calculate R' and L', seen from one secondary winding;

7. Considering that L' is chosen to large enough to provide sufficient smoothing at the input of single-phase rectifiers, compare the values of R' and $X' = L'\omega$ (for ω the angular frequency corresponding to f) and propose a simplified version of the equivalent model of the transformer.

The nominal regime is now considered by applying the nominal voltage U_{1n} at the primary winding and connecting a load at each secondary winding, drawing a RMS current $I_2 = 1.03\,\text{kA}$ with a power factor $\cos\phi_2$, the current being ahead on the voltage. The currents i_{2k} in each secondary winding k are aimed to be in phase with the voltage u_{2ko} of the considered secondary winding.

8. Build the corresponding Fresnel diagram, clearly identifying the load voltage u_{2k};

9. Compute the phase shift ϕ_2 of the current i_{2k} with respect to u_{2k}, and deduce the load power factor $\cos\phi_2$;

10. Compute the RMS voltage value U_2 appearing at each secondary winding;

11. Compute the reactive power Q_2 drawn by the load at each secondary winding, and the reactive power Q_1 at the primary winding;

12. Compute the active power P_2 drawn by the load at each secondary winding, and the active power P_1 at the primary winding;

13. Check that the power factor of the primary winding is almost equal to 1;

14. Compute the transformer efficiency η.

Exercise 8: Three-phase transformer

Three-phase power transformers are commonly used to adapt power line voltages and to provide some galvanic insulation between two parts of an electrical grid. The three-phase transformer, described by the normalized scheme in Fig. 7, is connected to a balanced three-phase network of composed voltages u_{AB}, u_{BC}, u_{CA} of RMS voltage U_1 on the primary side, whereas on the secondary side, a three-phase balanced system of composed voltages u_{ab}, u_{bc}, u_{ca} of RMS voltage U_2 is obtained.
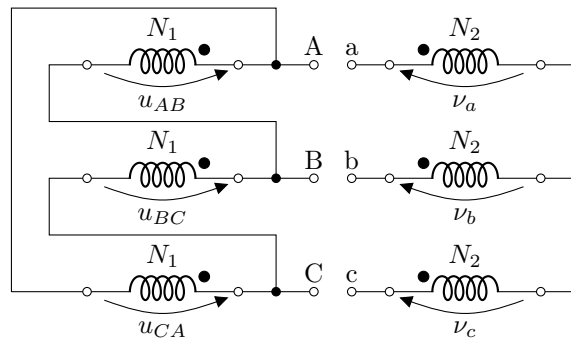
Figure 7: Three-phase transformer.

The line current intensities in the primary and secondary windings are respectively denoted I_1 and I_2. The transformer has the following characteristics:

- Apparent nominal power $S_n = 250\,\text{kVA}$;

- Composed primary winding RMS voltages $U_{1n} = 5.2\,\text{kV}$;

- Nominal frequency $f_n = 50\,\text{Hz}$;

and ferromagnetic losses are neglected. To characterize the transformer two tests have been performed:

- Using open secondary windings, the transformer generates a composed voltage of RMS value $U_{2o} = 400\,\text{V}$ at each secondary winding, for an applied composed nominal voltage of RMS value U_{1n};

- Using short-circuited secondary windings, a composed voltage of RMS value $U_{1s} = 600\,\text{V}$ is applied at each primary winding for a total primary power $P = 7.35\,\text{kW}$, producing line current of RMS intensity $I_{2s} = 350\,\text{A}$.

1. Calculate the transformer ratio n;

2. For the first test condition (open secondary windings), draw a Fresnel diagram including the primary composed voltages u_{AB}, u_{BC}, u_{CA}, the direct secondary voltage ν_a, ν_b, ν_c and the secondary composed voltages u_{ab}, u_{bc}, u_{ca};

3. Express and compute the 'single-phase' transformer ratio $n_c = \frac{N_1}{N_2}$ according to n;

4. Given that the transformer is composed of 3 cores of section $A_c = 5\,\text{dm}^2$, and that the magnetic field amplitude is $B_m = 1.2\,\text{T}$, compute the number of turns N_1 of each primary winding and deduce the value of the number of turns of each winding N_2;
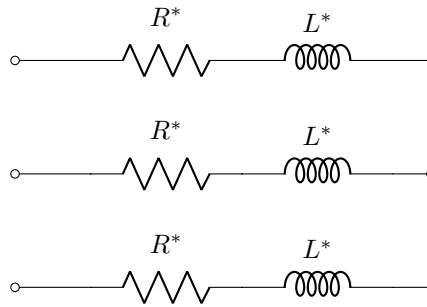
Figure 8: First load connected to the secondary side of the three-phase transformer.

5. Using a simple single-phase equivalent model (leak resistance and inductance moved to the secondary windings), provide the Thévenin's model seen from a secondary winding and calculate the resistance R_{eq} (R') and the reactance X_{eq} (X') of this model;

The nominal regime is now considered by applying the composed nominal voltage U_{1n} at the primary windings and connecting a three-phase balanced load on the secondary side (detailed in Fig. 8). Each branch is composed of a resistor of value $R^* = 554\,\text{m}\Omega$ in series with a coil of value $L^* = 3.05\,\text{mH}$.

6. Calculate the power factor $\cos\phi_2$ of this load;

7. Draw the Fresnel diagram corresponding to the balanced single-phase equivalent model. Using an appropriate approximation, deduce the RMS values of the current intensities I_2 and the composed voltages U_2.;

8. Compute the power P_2 flowing from the transformer to the load;

9. Calculate the transformer efficiency η;

10. Another load is used (Fig. 9), compute the value of the resistance R° and the inductance L° such that this load is equivalent to the one detailed in Fig. 8.
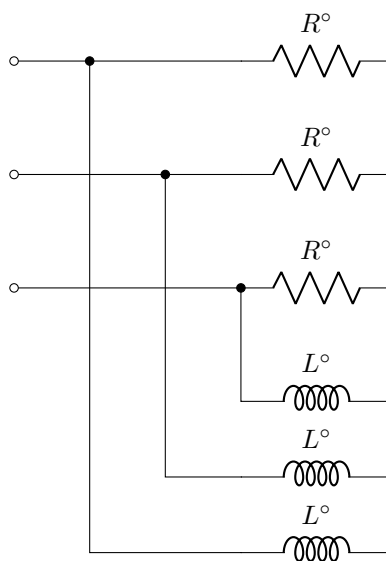

Figure 9: Second load connected to the secondary side of the three-phase transformer.

Exercise 9: Single-phase autotransformer

When a galvanic insulation is not required, due to its better efficiency, reduced cost and smaller size, the autotransformer is an interesting alternative to the classical transformer. Autrotransformers are also known to have larger short circuit currents which is not always suitable. Two tests are performed on the transformer, illustrated in Fig. 10:
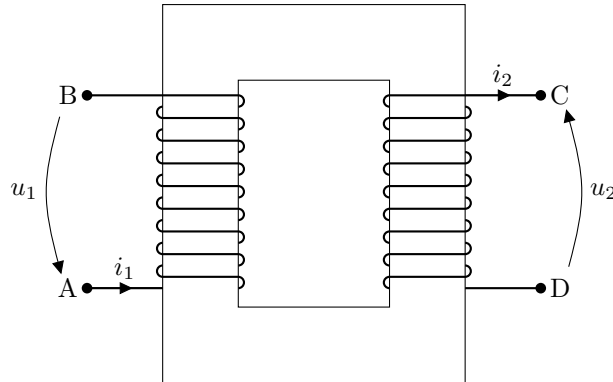
Figure 10: Single-phase transformer.

- Using open secondary winding, the transformer generates a voltage of RMS value $U_{2o} = 100\,\text{V}$ at the secondary winding, for an applied composed voltage of RMS value $U_1 = 20\,\text{V}$ for a drawn current intensity of RMS value $I_{1o} = 3.2\,\text{A}$ and a consumed power $P_{1o} = 8\,\text{W}$;

- Using short-circuited secondary winding, a voltage of RMS value $U_{1s} = 0.8\,\text{V}$ for a total power of $P_{1o} = 24\,\text{W}$ is measured, causing a current flow of RMS value $I_{2s} = 10\,\text{A}$ through the secondary winding.

Considering a simplified equivalent model of the transformer (leak resistance and inductance moved to the secondary windings):

1. Calculate the transformer ratio m;

2. Calculate the leakage resistance R_l and the leakage inductance L_m;

3. Compute the values of the resistance R_s and the reactance X_s corresponding to the inductance L_s of the secondary winding.

Using the transformer connected to a load on the secondary side drawing a current of RMS value $I_2 = 12\,\text{A}$ with a power factor $\cos\phi_2 = 0.8$ (the current is lagging the voltage), a RMS voltage of $U_1 = 20\,\text{V}$ is applied to the primary winding.

4. Calculate the RMS voltage U_2 appearing across the secondary winding by using a wise approximation of the voltage dropout ΔU_2 and justify that the approximation is relevant;

5. Deduce the active power P_2 provided to the load;

6. Calculate the RMS current I_1 on the primary side;

7. Compute the transformer efficiency η.

To turn the transformer into an autotransformer, the terminals A and D are connected together, such that the primary winding is located between C and B and the secondary between C and D (Fig. 11).

8. Compute the RMS voltage U_1' to be applied on the primary winding to reach a RMS voltage value of $U_{2o} = 100\,\text{V}$ at the terminals of the secondary winding;
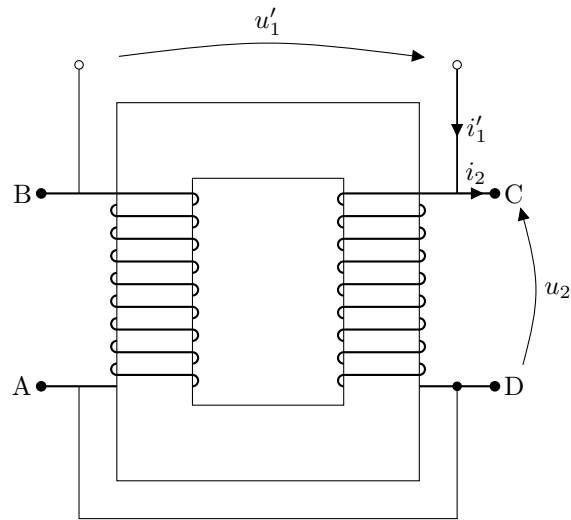
Figure 11: Single-phase autotransformer.

9. Calculate the RMS current I'_{1o} drawn at the primary winding in the case of an open secondary winding;

10. Neglecting the open circuit magnetomotive force, provide the Thévenin's model of the secondary winding including the electromotive force as well as the impedance components namely R'_s and X'_s;

11. The autotransformer is now loaded by a resistor R in parallel with a capacitor C, calculate the value of the product RC required to maintain the open circuit voltage RMS value;

12. In the same situation, determine the resistor value R_m maximizing the autotransformer efficiency and compute the resulting efficiency η_m.

4   AC synchronous machines

Exercise 10:   Constant air gap alternator

Several alternators are used on airplanes which, coupled to the reactors, feed all the necessary onboard electrical grids. Those alternators are characterized by the higher frequency of the generated voltage and currents compared to alternators coupled to 50 Hz or 60 Hz electrical grids. Moreover, due the the variable speed of the airplane reactors, the delivered frequency is not constant. The considered constant air gap three-phase alternator and its rotor winding are coupled following a star shape. Magnetic leakage, saturation, hysteresis and Eddy currents will be neglected.

For a rotating speed of the alternator shaft of $\dot{\theta} = 11\,100$ RPM, the frequency of the delivered voltages and currents is $f = 370$ Hz for a nominal apparent power $S_n = 150$ kVA and a direct voltage of RMS value $V_n = 115$ V. The rotation speed of the reactor $\dot{\theta}_e$ varies from 4160 RPM to 9000 RPM. The alternator is therefore coupled to the reactor through a gear box of ratio $k_m = \frac{\dot{\theta}}{\dot{\theta}_e} = 2.67$.

The excitation of the alternator is controlled such that the output voltage of the alternator is 115 V (direct voltage or 200 V for the composed voltage). This excitation consists of an inversed alternator coupled with a permanent magnet alternator (Fig. 12).

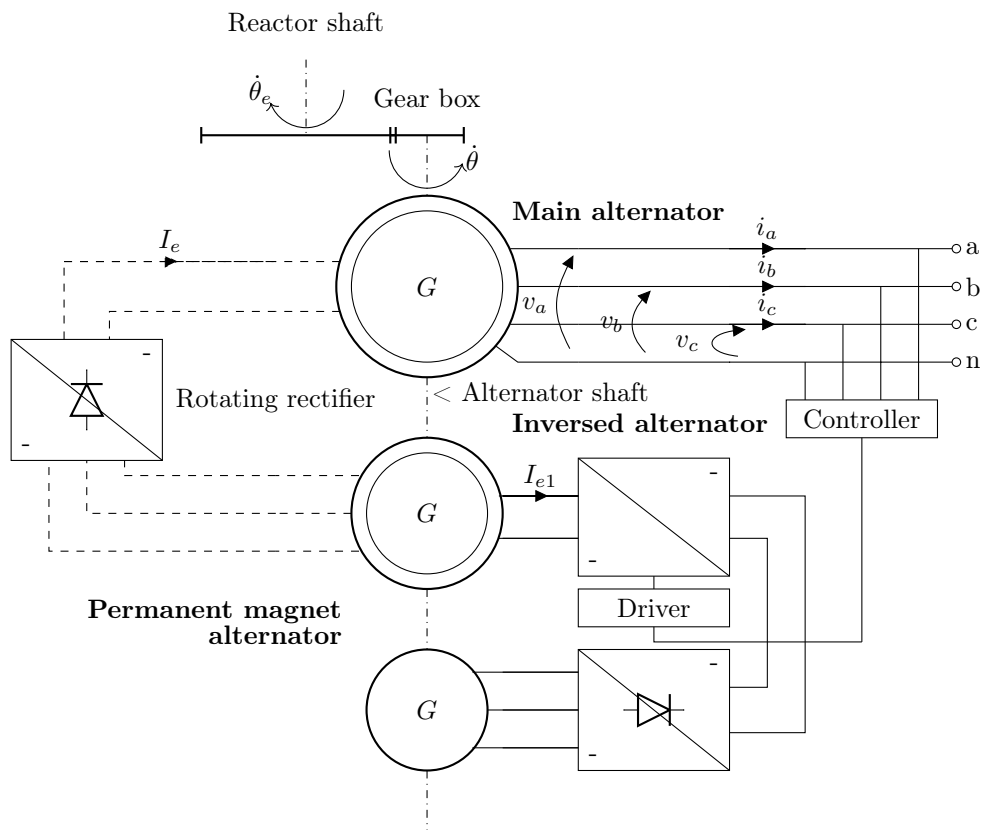Figure 12: Excitation system of the alternator.

1. Explain how the excitation system works. What are the main advantages of such a system?

2. Express the frequency of the generated voltages and currents f with respect to the rotation speed of the reactor n_e, the gear box ratio k_m and the number of pairs of poles of the alternator p;

3. Deduce the number of pair of poles, as well as the minimal and maximal values $f_{\min}$, $f_{\max}$ of the generated voltages and currents;

4. For an airplane, justify the relevance of a system working at a variable frequency in the targeted range;

5. Calculate the nominal RMS current I_{sn} of the line currents of the alternator;

6. The flux generated by a pole is:

$$\phi = \Phi_M \cos\left(p\left(\dot{\theta}t - \theta_0\right)\right),$$

where Φ_M is the flux amplitude, p the number of pairs of poles, Ω the speed of rotation, t the time variable and θ_0 the initial angular position of the rotor. Express the electromotive force e_s induced in a single turn of the rotor with respect to Φ_M, f, t and θ_0. Deduce the RMS value E_s of e_s with respect to Φ_m and f;

7. The RMS value E of the induced electromotive force in a phase is $E = k_b N_s E_s$ where $k_b = 0.850$ is the coil factor and $N_s = 16$ is the number of turn per phase. The magnetic circuit is built using laminations allowing to reach a maximal magnetic field corresponding to a flux amplitude $\Phi_{M0} = 6.84\,\text{mWb}$ and a current $I_{e0} = 2.95\,\text{A}$. Where does the coil factor comes from? Express the RMS value E of the electromotive force induced in each phase with respect to k_b, N_s, Φ_{M0}, I_{e0}, I_e and f. Plot E with respect to I_e in the range between $f_{\min}$ and $f_{\max}$ and conclude.

The stator of the machine is composed of three-phase windings whose phases are noted a, b and c, while the rotor is composed of a inductor winding f (Fig. 13). Each phase has an impedance composed of a resistance R_s, a self inductance L_s and a mutual inductance M_s with respect to each other phase. The mutual inductances between each phase and the inductor phase have a sinusoidal pulsation with respect to the rotation angle θ:

$$M_{af} = M \cos(p\theta)$$
$$M_{bf} = M \cos(p\theta - \frac{2\pi}{3})$$
$$M_{cf} = M \cos(p\theta + \frac{2\pi}{3})$$

8. Express the total fluxes Ψ_a, Ψ_b and Ψ_c crossing the phase windings a, b and c with respect to the flowing current intensities i_a, i_b and i_c, the excitation current intensity I_e, the self inductance L_s, the mutual inductance M_s, M and the angle $p\theta$;

9. Express the voltages ν_a, ν_b and ν_c across the phase windings a, b, c with respect to i_a, i_b and i_c, the total flux derivatives Ψ_a, Ψ_b and Ψ_c and R_s;

10. Show that the direct voltages of the stator can be written:

$$\nu_a = e_a - R_s i_a - \mathcal{L}\frac{di_a}{dt}$$
$$\nu_b = e_b - R_s i_b - \mathcal{L}\frac{di_b}{dt}$$
$$\nu_c = e_c - R_s i_c - \mathcal{L}\frac{di_c}{dt}$$

Express the electromotive forces e_a, e_b and e_c with respect to M, I_e, ω, t, p and θ_0, and their common RMS value E with respect to M, I_e and ω. Explain the significance of $\mathcal{L}$ and reexpress it in terms of L_s and M_s.

The single-phase equivalent model of Behn-Eschenburg is now considered with $R_s = 0.4\,\text{m}\Omega$. To characterize the alternator two tests have been performed:
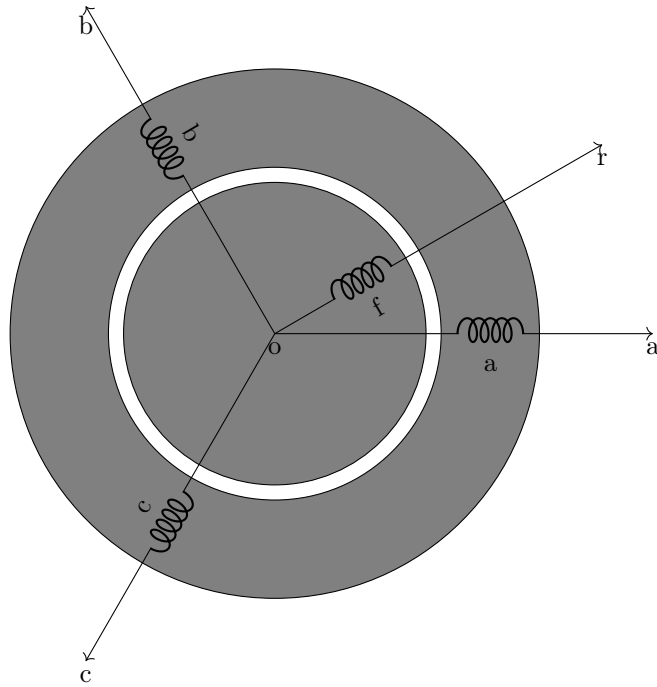
Figure 13: Electrical representation of the alternator windings.

- Using open stator windings, at the speed of rotation $\dot\theta = 11\,100\,\mathrm{RPM}$, the RMS direct voltage values have been measured with respect to the RMS current intensity I_e flowing through the inductor (Table 1);

- Using short-circuited stator windings, at the speed of rotation $\dot\theta = 11\,100\,\mathrm{RPM}$, the RMS current intensity I_c have been measured with respect to the RMS current intensity I_e flowing through the inductor (Table 1).

11. Knowing that $E = \lambda\omega I_e$, compute the value of the coefficient λ for $I_e = 0.4\,,3.0\,,5.4\,\mathrm{A}$;

12. Plot the open stator windings curve, E with respect to I_e, for $f_{\min} = 370\,\mathrm{Hz}$ and $f_{\max} = 770\,\mathrm{Hz}$;

13. Calculate the synchronous reactance X_s for the linear part of the curve;

Table 1: Alternator open and short-circuited stator windings test measurements.

I_e [A]	E [V]	I_s [A]
0.4	21.2	94.8
0.8	42.2	190
1.2	63.6	284
1.6	84.8	379
2	106	474
2.4	122	569
3	137	670
3.6	143	770
4.2	145	860
4.8	147	948
5.4	148	1040

14. Plot the short-circuited stator windings curve, I_s with respect to I_e, for $f_{\min} = 370\,\text{Hz}$ and $f_{\max} = 770\,\text{Hz}$;

15. The alternator is connected to a star-shaped load composed of 3 resistors of value $R_L = 0.5\,\Omega$ working at a frequency $f = 500\,\text{Hz}$ for an excitation current $I_e = 2\,\text{A}$.

    (a) Calculate the stator RMS current and voltage values I_s and V_s;
    (b) Sketch the Behn-Eschenburg diagram;
    (c) Explain how I_s and V_s vary when the frequency increases;

16. Working at constant I_e, a balanced inductive load is now considered with a corresponding impedance $Z_c = R_c + \jmath\omega L_c$ for each phase.

    (a) Sketch the Behn-Eschenburg diagram for a power factor $\cos\phi = 0.75$;
    (b) Express the stator RMS voltage V_s with respect to λ, ω, R_c, L_c, L_s and I_e;
    (c) Express the resistive torque C_r with respect to p, λ, ω, R_c, L_c, L_s and I_e;
    (d) Knowing that $R_c = 0.5\,\Omega$ and $L_c = 150\,\mu\text{H}$, compute I_s, V_s and C_r for $f_{\min}$ and $f_{\max}$ for $I_e = 0.4\,\text{A}$, $3.0\,\text{A}$ and $5.4\,\text{A}$;
    (e) How does the frequency variation influence the load power factor?

Exercise 11:   Three-phase turbo-alternator

Turbo-alternators are alternators coupled to turbines allowing to convert the mechanical power of a moving fluid (steam or liquid) to electrical power. In this exercise a turbo alternator, built with a star-shaped rotor, has the following nominal characteristics:

- Power $P_n = 600\,\text{MW}$,
- Frequency $f_n = 50\,\text{Hz}$,
- Speed of rotation $\dot{\theta}_n = 3000\,\text{RPM}$,
- Power factor $\cos\phi_n = 0.9$,
- RMS value of the composed voltages $U_n = 20\,\text{kV}$,
- Ferromagnetic losses $p_f = 543\,\text{kW}$,
- Mechanical losses $p_m = 1.35\,\text{MW}$,
- Rotor resistance $R_e = 0.17\,\Omega$,
- Excitation system efficiency $\eta_e = 0.92$,
- Stator phase resistance $R = 2.3\,\text{m}\Omega$.

To characterize the turbo-alternator three tests have been performed:

- Using open stator windings, at the nominal speed of rotation $\dot{\theta}_n$, the RMS direct voltage values have been measured with respect to the RMS current intensity I_e flowing through the inductor (Table 2);

- Using short-circuited stator windings, at the nominal speed of rotation $\dot{\theta}_n$, using an excitation current of RMS value $I_e = 1.18\,\text{kA}$ has allowed a current flow in each phase winding of the stator reaching the half of the RMS nominal value;

- Using an inductive load, an excitation current of RMS value $I_e = 2.085\,\text{kA}$ has allowed a current flow in each phase winding of the stator reaching the half of the RMS nominal value. Also, the output voltage was measured as half the nominal voltage.

Table 2: Alternator open circuit stator windings test measurements. (Voltages measured between two phases.

I_e [A]	E_v [kV]
400	5.2
700	9.1
963	11.5
1200	13
1450	14
1900	15

1. Calculate the nominal RMS intensity I_n of the stator currents;

2. Compute the total losses and the turbo-alternator efficiency at the nominal operating point, knowing the RMS excitation current value is $I_e = 3.2\,\text{kA}$;

3. Calculate the needed mechanical power for each of the considered test;

4. Calculate the (unsaturated) synchronous reactance X_s of the turbo-alternator.

Using Potier diagram with the experimental measurements

5. Calculate the leak partial reactance X_f of the stator;

6. Calculate the equivalence ratio $\frac{\delta}{\gamma} = \frac{I_e}{I}$ between the intensity of the current in the rotor I_e and the current in the stator I;

7. Neglecting resistive losses in the rotor, plot Potier diagram for the nominal operating point and verify that, with a value of the resulting excitation current $I_r = 1.64\,\text{kA}$, the corresponding intensity I_e of the excitation current is equal to $I_e = 3.2\,\text{kA}$;

8. Draw the internal lag δ and give its value;

9. Compute the RMS value E_s of the synchronous electromotive force;

10. Calculate the saturated synchronous reactance X_s.

Exercise 12: Alternator and synchronous condenser

A synchronous condenser is a DC excited synchronous motor, whose rotating shaft is not connected to any load. By controlling its field (or the excitation current), using a voltage regulator, the condenser is able to generate or absorb reactive power as needed to adjust the voltage on the power grids, or to improve the power factor.

- Synchronous speed: $428\,\text{RPM}$ for 14 poles,

- Star-shape coupling with a voltage $V = 8.95\,\text{kV}$ between phase and neutral,

- Nominal intensity $I_n = 6.33\,\text{kA}$,

- Apparent nominal power $S_n = 170\,\text{MVA}$,

- Nominal synchronous reactance $X_n = 1.2\,\Omega$ at the nominal frequency $f_n = 50\,\text{Hz}$.

The machine is used as an alternator, providing active power $P = 100\,\text{MW}$ and reactive power $Q = 50\,\text{Mvar}$ to the power grid.

1. Calculate $\cos\phi$, ϕ and the lines current intensities I.

The machine is now turned into a freely spinning motor, keeping the excitation current constant and assuming $P \approx 0\,\text{W}$,

2. Calculate the reactive power $Q < 0$ provided by the motor (and thus consumed by the motor).

Going back in the alternator mode, the machine is working at a constant power $P_n = 100\,\text{MW}$ and exchanges reactive power from $Q_{\min} = -100\,\text{Mvar}$ to $Q_{\max} = 100\,\text{Mvar}$,

3. Between $Q_{\min}$ and $Q_{\max}$, plot I with respect to the electromotive force E.

5 AC asynchronous machines

Exercise 13:   Asynchronous motor 1

A star-shaped asynchronous motor of a high-pressure washer cleaner has the following nominal characteristics:

- Power $P_n = 5.5\,\text{kW}$,

- RMS composed voltage value $U_{sn} = 400\,\text{V}$,

- Frequency $f_n = 50\,\text{Hz}$,

- RMS line current intensities $I_{sn} = 11\,\text{A}$,

- Speed of rotation $\dot{\theta}_n = 1460\,\text{RPM}$.

Assume that the stator reactance X_s is equal to the stator resistance R_s. Using a single-phase equivalent model of the asynchronous motor when needed,

1. Calculate the synchronous speed of rotation n_s, the number of pair of poles of the motor and the nominal slip g_n;

2. Determine the value of the stator resistance R_s given that a current of RMS value $I_0 = 10\,\text{A}$ flows when a voltage of RMS value $U_0 = 20.6\,\text{V}$ is applied;

3. At the nominal operating point, without mechanical load, the motor draws a current of RMS value $I_{so} = 3.07\,\text{A}$ for an active power $P_{so} = 245\,\text{W}$. Calculate the overall losses and calculate the resistance modelling ferromagnetic losses R_f and the statoric inductance L_s, asumming that mechanical losses equals ferromagnetic losses;

4. At the nominal operating point, calculate the transmitted power from the stator to the rotor and the Joules losses in the stator p_{js} and deduce the total consumed power P;

5. Calculate the rotoric resistance R'_r and the leak inductance L'_r seen from the stator;

6. At the nominal operating point, calculate the mechanical torque Γ_{un} and the electromagnetic torque Γ_n, the power factor $\cos\phi_n$ and the efficiency η_n;

7. Compute the RMS value I_s of the line currents, and the power factor $\cos\phi$ at a rotation speed of $0\,\text{RPM}$.

Exercise 14:   Asynchronous motor 2

On the nameplate of an asynchronous motor of a fan used in an air handling unit, the following characteristics are read:

$$\boxed{4.4\,\text{kW}; 230/400\,\text{V}; 15.5/9\,\text{A}; 50\,\text{Hz}; 4\,\text{poles}}$$

Using a single-phase equivalent model of the asynchronous motor:

1. Explain the meaning of each element on the nameplate;

2. The motor is used on a $230\,\text{V}$ network, explain which winding coupling should be used for the stator;

3. Calculate the synchronous speed of rotation $\dot{\theta}_s$;

4. Given that the (DC) resistance value measured between two stator terminals is $R_a = 0.654\,\Omega$, compute the value of the statoric resistance R_s of the equivalent single-phase model;

5. A calibrated motor is used to rotate the shaft of the unpowered considered motor, upto reaching the synchronous speed, at which the calibrated motor consumes $86\,\text{W}$. Calculate the mechanical losses of the motor and explain why assuming that these mechanical losses remain constant is a good approximation;

6. At the nominal operating point, without mechanical load, the motor draws a current of RMS value $I_{so} = 3.82\,\mathrm{A}$ for an active power $P_{so} = 300\,\mathrm{W}$. Calculate the resistance modelling ferromagnetic losses R_{H+F} and the statoric inductance L_μ;

7. The rotor shaft of the motor is stalled while a voltage of RMS value $U_{sc} = 57.5\,\mathrm{V}$ is applied for a consumed three-phase active power $P_{sc,3\phi} = 374\,\mathrm{W}$ and three-phase reactive power $Q_{sc,3\phi} = 1.09\,\mathrm{kvar}$. Calculate the rotoric resistance R'_r and the leak inductance X'_r seen from the stator.

A direct voltage of value V_s and frequency f is applied on each phase of the motor.

8. Using single-phase equivalent model of the asynchronous motor, express the RMS current value I_s in terms of V_s, R_s R'_r, g et X'_r;

9. Calculate the transmitted power from the stator to the rotor;

10. Calculate the electromagnetic torque C_{elm} and give the maximal reachable torque $\Gamma_{\max}$ after showing that C_{elm} is maximal for a slip value g_{max};

11. Plot C with respect to g for an applied voltage V_s equal to V_n, $\frac{V_n}{\sqrt{2}}$ and $\frac{V_n}{2}$;

12. Explain why a control on the rotor voltages is not suitable for speed variation for load having constant resistive torque;

13. To limit the peak current when starting the motor, a star/delta starter is frequently used. Assuming that this transient mode is much more longer compared to period corresponding to the frequency f of the applied voltages, calculate the RMS current values of the line currents compared to those drawn by using a star/delta starter.

Exercise 15: Wind turbine

Due to the variations of wind conditions, coupling a synchronous generator, needing a constant speed of rotation, is not feasible. An asynchronous generator (hypersynchronous) is therefore generally used to allow more flexibility in the speed of rotation of the blades. A variable speed wind turbine is studied in this exercise. It is possible to tune the slip by using a wound rotor (not a squirrel cage) typically from $0\,\%$ to $10\,\%$ by acting on the resistances R of the rotor. During wind gust, the value R is increased, to increase the slip value (in absolute value, since it is negative) allowing thus to increase the speed of rotation and smoothen the power transmitted to the electrical grid.

The studied wind turbine has a 4-poles asynchronous generator of nominal power $P_n = 800\,\mathrm{kW}$ connected to an electrical grid having a composed voltage $U_{sn} = 690\,\mathrm{V}$. The resistance values R of the rotor can be tuned from $0\,\mathrm{m\Omega}$ to $9\,\mathrm{m\Omega}$, in addition to the resistance value of the windings $R_R = 3\,\mathrm{m\Omega}$.

1. Express the input shaft torque Γ_i with respect to the speed of rotation n ;

2. Calculate the operating point (speed and slip) of the generator for $R = 0\,\mathrm{m\Omega}\,, P = 2\,\mathrm{MW}$ and $R = 9\,\mathrm{m\Omega}\,, P = 4\,\mathrm{MW}$.

6   DC machines

Exercise 16:   DC brushed motor

The motor of a hammer drill has the following characteristics:

- independant excitation DC machine,

- 2 poles (1 pair),

- Nominal power $P_n = 800\,\mathrm{W}$,

- Nominal speed of rotation $\dot{\theta}_n = 1500\,\mathrm{RPM}$,

- Nominal power voltage $U_n = 220\,\mathrm{V}$,

- Nominal rotor current intensity $I_n = 4.6\,\mathrm{A}$,

- Nominal stator current intensity $I_{en} = 0.35\,\mathrm{A}$.

Using two different excitation currents I_e, the electromotive force has been determined for different rotation speeds (Tables 3 et 4).

Table 3: Electromotive force with respect to the rotation speed for an excitation current $I_{e1} = 0.35\,\mathrm{A}$.

n [RPM]	E [V]
1670	240
1510	220
1380	200
1040	150
820	120
510	75
110	20
0	0

Table 4: Electromotive force with respect to the rotation speed for an excitation current $I_{e1} = 0.20\,\mathrm{A}$.

n [RPM]	E [V]
1800	186
1450	150
1150	120
850	90
560	60
260	30
0	0

1. Plot E with respect to $\dot{\theta}$ for I_{e1} and I_{e2} and justify the shape of the curves;

2. Show that the flux Φ is not proportional to the excitation current intensity I_e.

Maintaining the nominal speed of rotation, the electromotive force is measured for different excitation currents I_e (Table 5).

3. Plot E with respect to I_e and justify the shape of the curves.

Table 5: Electromotive force with respect to the excitation current at nominal constant rotation speed.

I_e [A]	E [V]
0.39	219
0.35	210
0.33	204
0.31	198
0.3	194
0.28	188
0.26	179
0.24	168
0.22	158
0.2	147
0.18	137
0.17	130
0.14	107
0.13	100
0.11	87
0.1	78
0.08	65
0.07	56

Table 6: Voltage across the rotor U and current drawn in the rotor I for different excitation currents value I_e.

I_e [A]	U [V]	I [A]
0.4	222	0.43
0.35	213	0.44
0.3	198	0.45
0.25	176	0.48
0.2	151	0.56
0.15	120	0.66
0.1	85	0.92

Some measurements have allowed to quantify the stator resistance value, which is $R_e = 512.1\,\Omega$ and the rotor resistance value is $R = 4.6\,\Omega$.

4. Draw the equivalent model of the motor.

A test at constant nominal speed has been performed to measure the voltage across the rotor U and the current drawn in the rotor I for different excitation currents value I_e (Table 6).

5. Plot the collective (i.e. ferromagnetic plus mechanical) losses p_c with respect to I_e;

6. For the linear part of the curve, determine the mechanical losses p_m at the nominal speed of rotation.

A hole is drilled using the drill. The nominal speed of rotation remains constant while the rotor draws a current of $I_0 = 3\,\text{A}$ when a voltage $U_0 = 212\,\text{V}$ is measured on the rotor terminals.

7. Calculate the electromotive force and deduce the value of the excitation current I_{e0};

8. Compute the shaft output power P_u;

9. Deduce the resistive torque C_r induced by the drilling process.

Exercise 17: Regenerative braking

Hybrid electric vehicles are generally provided with regenerative braking, allowing to load onboard battery when the vehicle is braking or when the vehicle acts as a driving load. In this exercise, the DC motor, having an electromotive force E and internal resistance $R = 0.5\,\Omega$ is connected (when the regenerative breaking is active) to a battery delivering a current I under the voltage $V = 100\,\text{V}$ using a chopper DC-DC converter (Fig. 14).
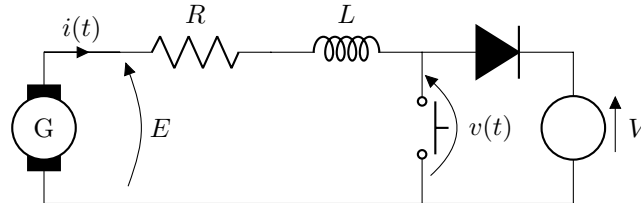

Figure 14: DC-DC converter for regenerative braking.

1. Find the mean value of $v(t)$: V_m;

2. Find the link between the mean input current I_m and the mean output current I;

3. Express the voltage V with respect to I_m, E, R and D, the duty cycle;

4. Compute the duty cycle allowing to obtain $V_m = 60\,\text{V}$;

5. Compute the mean braking current I_m when the motor delivers an electromotive force $E = 70\,\text{V}$ for $V_m = 60\,\text{V}$;

6. Calculate the braking power EI_m and the braking torque C_m if the motor speed of rotation is $\dot{\theta} = 955\,\text{RPM}$.

Exercise 18: DC generator-motor mechanical coupling

Two identical DC machines, rated $P_{u,n} = 200\,\text{kW}$, $V_n = 520\,\text{V}$, $I_n = 420\,\text{A}$, $n_n = 1000\,\text{RPM}$, are tested by coupling their shafts together, so that one will act as a generator to supply power to the other, which will act as a motor and drive the generator. This is a common test procedure to avoid the use of high-power electrical supplies and heavy loading rigs. The two rotors are connected in parallel (Fig. 15) while both machines are separately excited by manually adjusting the excitation current. To startup, the excitation current is set to its nominal value while the applied voltage V is progressively increased to V_n to reach the unloaded speed $n_u = 1040\,\text{RPM}$. The couple generator-motor draws 37 A from the DC source, which is mainly due the torque required to overcome windage, friction, and iron losses. Knowing the the armature resistance value of each machine is $R = 0.05\,\Omega$ and ignoring commutation losses (brush volt-drop):

1. Calculate the electromotive force at no-load;

2. Compute the power loss at no-load;

3. Compute the mechanical power losses;

4. Calculate the torque at no-load.

The excitation current of the generator is now reduced to reach the nominal value of the output current I_n.

5. Calculate the speed of rotation of each machine;

6. Calculate the current drawn by the motor;

7. Calculate the shaft output torque;

8. Compare the power supplied by the DC source to the nominal power of each machine and conclude about the relevance of such a test and about the load level reached during this test.
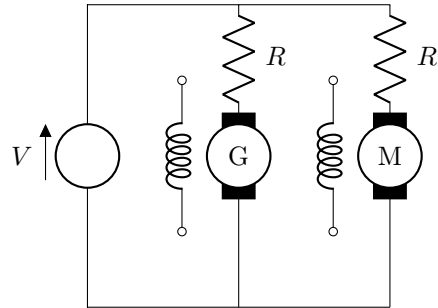

Figure 15: DC generator-motor mechanical coupling.

7 Electronic control system

Exercise 19:   DC-DC buck converter

DC-DC converters are used to adapt two different voltage levels. Typically, in an electric car, the battery voltage is set to $E = 302$ V, whereas the auxiliaries (lights, cigar lighter, window and wiper motors, ...) are working with $V_o = 12$ V. A DC-DC buck converter is used to reduce the battery high voltage to the lower value (12 V) ensuring high efficiency. The DC-DC buck converter can be modelled by the following circuit.
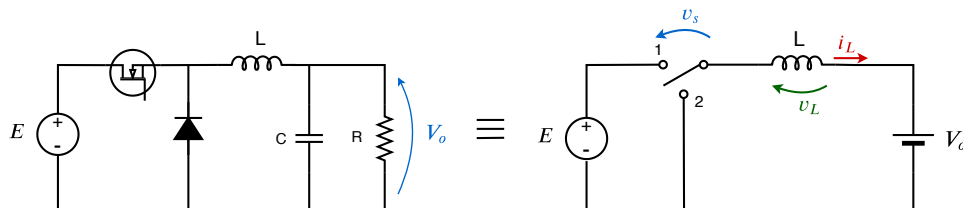
1. Find the waveforms of the voltage across the ideal switch (v_s) and the voltage across the inductance (v_L).

2. Deduce the inductance current waveform from it.

Now, suppose that the average voltage across the inductance is 0 during a switching period (corresponding to a steady-state condition).
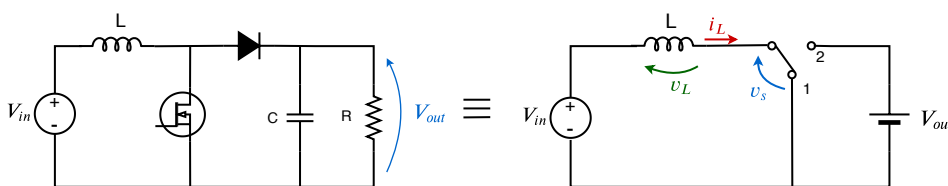
3. Express the ratio $\frac{V_o}{E}$ in terms of the duty cycle D.

4. Give the value of D in this situation.

The current ripple Δi is defined as the absolute difference between the maximum of current (during a switching period) and the average current I (over the same switching period).

5. Find the expression of the inductor current ripple Δi_L in terms of V_o, E, D, T_s and L.

6. Estimate the inductor current ripple Δi_L for a switching frequency $f_s = 1000$ Hz and an inductance of 50 mH. Compare the value of the current ripple to the value of the output current if the auxiliaries draw 12 W.

Exercise 20:   DC-DC boost converter

DC-DC converters are used to adapt two different voltage levels. In some electronic calculator, the battery voltage is set as $V_{in} = 3$ V, whereas the electronic parts work under $V_{out} = 9$ V. A DC-DC boost converter is used to increase the battery low voltage to the higher value (9 V) ensuring high efficiency. The DC-DC boost converter can be modelled by the following circuit.

1. Find the waveforms of the voltage across the ideal switch (v_s) and the voltage across the inductance (v_L).

2. Deduce the inductance current waveform from it.

Now, suppose that the average voltage across the inductance is 0 during a switching period (corresponding to a steady-state condition).

3. Express the ratio $\frac{V_{out}}{V_{in}}$ in terms of the duty cycle D.

4. Give the value of D in this situation.

The current ripple Δi is defined as the absolute difference between the maximum of current (during a switching period) and the average current I (over the same switching period).

5. Find the expression of the inductor current ripple Δi_L in terms of V_{out}, V_{in}, D, T_s and L.

6. Estimate the inductor current ripple Δi_L for a switching frequency $f_s = 30$ kHz and an inductance of 75 mH. Compare the value of the current ripple to the value of the output current if the system draws 15 mW.