ELEC0431 Electromagnetic Energy Conversion
Corrective of Exercises

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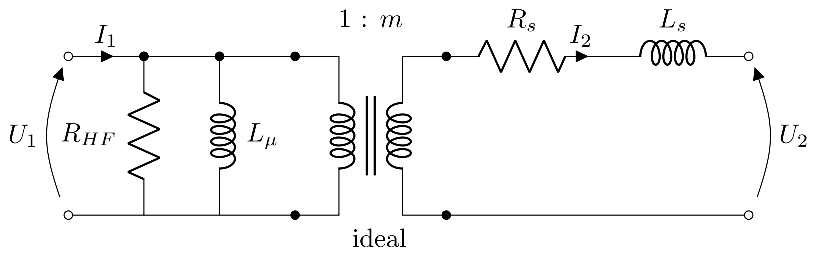


Figure 1: Single phase transformer equivalent circuit

1) $m = 5$
2) $R_l = 50\ \Omega$; $X_m = 6.3\ \Omega$
3) $R_s = 0.24\ \Omega$; $X_s = 0.32\ \Omega$
4) $\Delta U_2 = 4.61$ V; $U_2 = 95.2$ V
5) $P_2 = 916$ W
6) $I_1 = 62.3$ A
7) $\eta = 95.6$ %
8) $U_1' = 80$ V
9) $I_{1o}' = 0.8$ A
10) $R_s' = 15\ m\Omega$; $X_s' = 20\ m\Omega$
11) RC = 2.39 ms
12) $R_m = 5.41\ \Omega$; $\eta_m = 99.2$ %

4 AC synchronous machines

Exercise 10: Constant air gap alternator

1. Physical explanations and advantages

   The three synchronous alternators are fixed to the same shaft.

   The permanent magnet alternator is the smallest of the three machines (regarding power) and produces some three-phase current alternating current as it rotates.
   The current produced is then rectified (it becomes DC current) and used for the excitation of the second machine.

   The second machine (here the inversed alternator) is used to increase the power between the small permanent magnet alternator and the main alternator.

   It is called an inversed alternator because the fixed part contains the excitation winding (DC current) (normally in the rotor for a conventionnal machine) and the moving part contains the three-phase windings (normally in the stator for a conventionnal machine).

   The three-phase currents are then rectified and used for the excitation of the main alternator. Remark that the rectifier is also rotating along the inversed alternator.

   The main alternator rotor is then excited with the excitation current I_e and the mechanical power applied on the shaft is transfered into three-phase electrical power in the conductors (a, b, c, n).

   Advantages :

   The permanent magnet machine allows an autonomous start (no excitation current is required for this machine). Also, this machine is brushless, meaning that no spark are produced when it is rotating. This is an important point for the design of safe aircraft. A brushed DC generator could not be used instead of the permanent magnet alternator because it would create sparks.

   The electrical connections of the inversed alternator are simpler and do not require any connecting ring either for the excitation winding or the three-phase windings

2. Express f in terms of $\dot{\theta}_e$, k_m and p

   $$\dot{\theta} = k_m\,\dot{\theta}_e \tag{1}$$

   where $\dot{\theta}$ is the machine synchronous speed ; $k_m = 2{,}67$ is the gearbox ratio ; $\dot{\theta}_e$ is the aircraft reactor speed.

   Then,

   $$f = p\,\dot{\theta} = p\,k_m\,\dot{\theta}_e \qquad (\dot{\theta} \text{ and } \dot{\theta}_e \text{ expressed in turn/s}) \tag{2}$$

   $$f = p\,\frac{\dot{\theta}}{60} = \frac{p\,k_m\,\dot{\theta}_e}{60} \qquad (\dot{\theta} \text{ and } \dot{\theta}_e \text{ expressed in rpm}) \tag{3}$$

   $$f = \frac{p\,\dot{\theta}}{2\,\pi} = \frac{p\,k_m\,\dot{\theta}_e}{2\,\pi} \qquad (\dot{\theta} \text{ and } \dot{\theta}_e \text{ expressed in rad/s}) \tag{4}$$

3. Deduce p, f_{min} and f_{max}

The alternator frequency is 370 Hz when its rotation speed is 11 000 rpm. 11 000 rpm = 183,3 turn/s. Then, $p = \frac{f}{\dot{\theta}} = \frac{370}{183{,}3} = 2,018$ and the number of pair of poles $p = 2$.

$\dot{\theta}_{e,min} = 4160$ rpm and $\dot{\theta}_{e,max} = 9000$ rpm. Then,

$$f_{min} = \frac{p\,k_m\,\dot{\theta}_{e,min}}{60} = 370,24Hz \tag{5}$$

$$f_{max} = \frac{p\,k_m\,\dot{\theta}_{e,max}}{60} = 801Hz \tag{6}$$

4. Justify the relevance of working at variable frequency

- The high frequency (higher the industrial 50 Hz) enables the use of smaller components (L and C).
- The mechanic is easier, there is only one gearbox with one ratio.
- The rotation speed of the reactor can vary even if there are only two pairs of poles.

5. Nominal current I_{sn}

$$I_{sn} = \frac{S_n}{3\,V} = \frac{150\,000}{3 \cdot 115} = 434,783 \text{ A} \tag{7}$$

6. Express $e_s(t)$, the emf and deduce E_s

Considering $\dot{\theta}$ in rad/s,

$$\phi = \Phi_M \;\cos\big(p(\dot{\theta}\,t - \theta_0)\big) \tag{8}$$

$$e_s(t) = -\frac{d\phi}{dt} \tag{9}$$

$$= p\,\dot{\theta}\;\Phi_M\;\sin\big(p(\dot{\theta}\,t - \theta_0)\big) \tag{10}$$

$$= 2\pi f\;\Phi_M\;\sin\big(p(\dot{\theta}\,t - \theta_0)\big) \tag{11}$$

$$= E_m\;\sin\big(p(\dot{\theta}\,t - \theta_0)\big) \tag{12}$$

$$\tag{13}$$

and

$$E_s = \frac{E_m}{\sqrt{2}} = \sqrt{2}\pi\,f\,\Phi_M \tag{14}$$

7. Coil factor k_b, induced emf E and E wrt I_e

The coil factor k_b is a global coefficient that takes some non idealities into account, such as : the leakage flux between the rotor and stator, the angular section of the turns of a phase.

$$E = k_b\;N_s\;E_s \tag{15}$$

with

$$E_s = \sqrt{2}\;\pi\;f\;\Phi_M \tag{16}$$

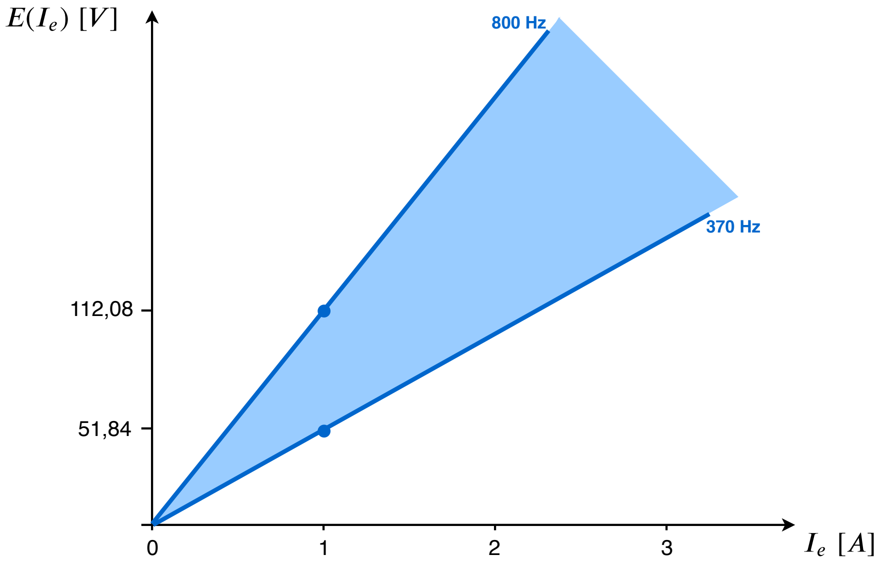


Figure 2: Emf as a function of the excitation current.

Also, in the linear range,

$$\Phi_M = \frac{\Phi_{M0}}{I_{e0}} I_e \tag{17}$$

Then,

$$\begin{aligned}
E &= k_b\ N_s\ \sqrt{2}\ \pi\ f\ \frac{\Phi_{M0}}{I_{e0}}\ I_e && (18)\\
&= 0.85\ 16\ \sqrt{2}\ \pi\ 370\ \frac{0,00684}{2,95}\ 1 = 51,84\,V\ \ (\text{for } f_{min}{=}370\text{ Hz}) && (19)\\
&= 0.85\ 16\ \sqrt{2}\ \pi\ 800\ \frac{0,00684}{2,95}\ 1 = 112,08\,V\ (\text{for } f_{max}{=}800\text{ Hz}) && (20)
\end{aligned}$$

8. Total fluxes $\Psi_{a,b,c}$

$$\begin{aligned}
\Psi_a &= L_s\ i_a + M_s\ i_b + M_s\ i_c + M_{af}\ I_e && (21)\\
&= L_s\ i_a + M_s\ i_b + M_s\ i_c + M\ \cos(p\,\theta)\ I_e && (22)\\
& && (23)\\
\Psi_b &= M_s\ i_a + L_s\ i_b + M_s\ i_c + M_{bf}\ I_e && (24)\\
&= M_s\ i_a + L_s\ i_b + M_s\ i_c + M\ \cos(p\,\theta - \frac{2\pi}{3})\ I_e && (25)\\
& && (26)\\
\Psi_c &= M_s\ i_a + M_s\ i_b + L_s\ i_c + M_{cf}\ I_e && (27)\\
&= M_s\ i_a + M_s\ i_b + L_s\ i_c + M\ \cos(p\,\theta - \frac{4\pi}{3})\ I_e && (28)
\end{aligned}$$

9. $\nu_{a,b,c}$

$$\nu_a = -R_s\ i_a\ - \frac{d\Psi_a}{dt}\quad ;\quad \nu_b = -R_s\ i_b\ - \frac{d\Psi_b}{dt}\quad ;\quad \nu_c = -R_s\ i_c\ - \frac{d\Psi_c}{dt} \tag{29}$$

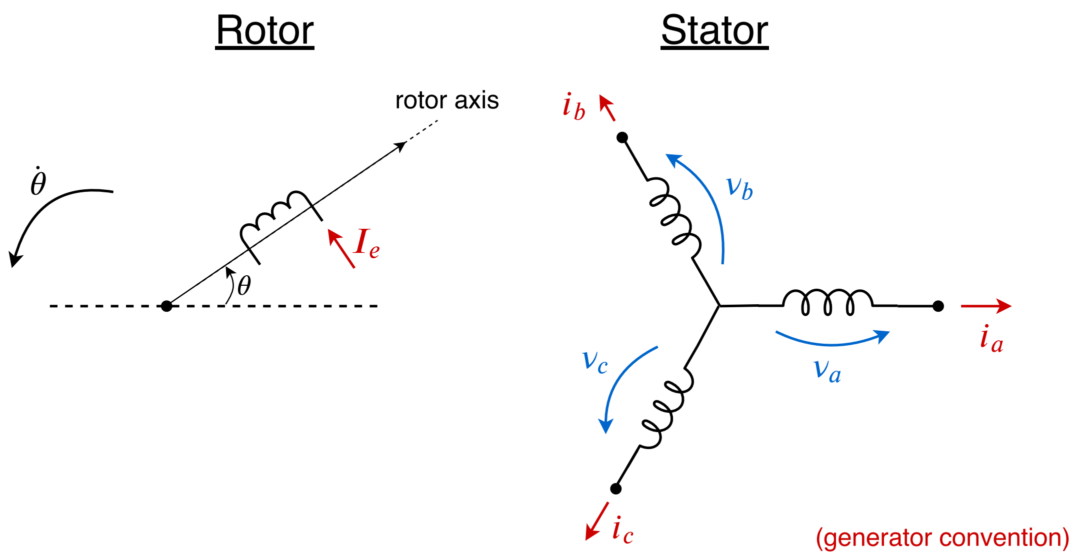


Figure 3: Rotor and stator windings.

With R_s the resistance of each phase and $\Psi_{a,b,c}$ the flux linkage of each phase.

10. Show that ...

$$\Psi_a(t) = L_s\, i_a(t) + M_s\, i_b(t) + M_s\, i_c(t) + M\, \cos(p\,\dot{\theta}\,t)\, I_e \tag{30}$$
$$= L_s\, i_a(t) + M_s\, \big(i_b(t) + i_c(t)\big) + M\, \cos(p\,\dot{\theta}\,t)\, I_e \tag{31}$$

With $\quad i_b(t) + i_c(t) = -i_a(t) \qquad <=> \qquad i_a(t) + i_b(t) + i_c(t) = 0$

Then,

$$\Psi_a(t) = (L_s - M_s)\, i_a(t) + M\, \cos(p\,\dot{\theta}\,t)\, I_e \tag{32}$$
$$= \quad \mathcal{L} \quad i_a(t) + M\, \cos(p\,\dot{\theta}\,t)\, I_e \tag{33}$$
$$\tag{34}$$
$$-\frac{d\Psi_a(t)}{dt} = -\mathcal{L}\,\frac{di_a(t)}{dt} + p\,\dot{\theta}\,M\,\sin(p\,\dot{\theta}\,t)\, I_e \tag{35}$$
$$-\frac{d\Psi_a(t)}{dt} = -\mathcal{L}\,\frac{di_a(t)}{dt} + e_a \tag{36}$$

is obtained by defining, $\mathcal{L}$: the cyclic impedance and e_a : the internal emf of phase a.
Finally, one can get,

$$\nu_a = -R_s\, i_a(t) - \frac{d\Psi_a(t)}{dt} = e_a - R_s\, i_a(t) - \mathcal{L}\,\frac{di_a(t)}{dt} \tag{37}$$

The same derivation remains valid for phases b and c.

11. Compute λ for $I_e = 0{,}4$; 3 and 5,4 A

$$\omega = 2\pi\, f = 2\,\pi\left(\frac{2 \times 11100}{60}\right) = 2324,78 \text{ rad/s} \tag{38}$$

I_e	E	λ
0,4	21,2	0,02279787
3	137	0,01964345
5,4	148	0,011789256

Table 1: λ parameter computation.

$$\lambda = \frac{E}{\omega \, I_e} \tag{39}$$

12. E wrt I_r for $f_{min} = 370$ Hz and $f_{max} = 770$ Hz

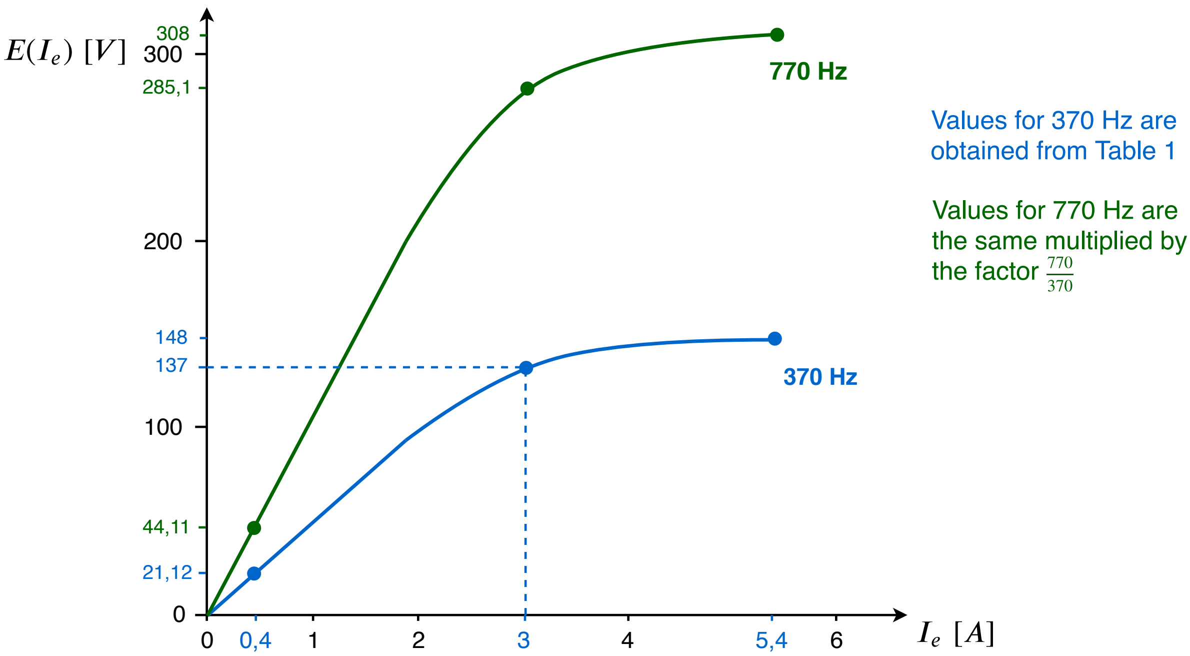


Figure 4: Emf as a function of the excitation current in the non linear case.

13. Calculate the synchronous reactance X_s for the linear part of the curve

The curve is linear for $I_e \in [0; 2]$. One can take the values corresponding to $I_e = 1,6$ A in order to minimize the measurement error and still remain in the linear part.

For $I_e = 1,6$ A, the no load voltage is $E = 84{,}8$ V and the short circuit current is $I_s = 379$ A.

$$Z_s = R_s + j\,X_s \tag{40}$$

$$Z_s = \frac{E}{I_s} = \frac{84,8}{379} = 0,2237\Omega \tag{41}$$

Taking the winding resistance $R_s = 0{,}4$ mΩ into account.

$$X_s = \sqrt{Z_s^2 - R_s^2} = 0,223699\Omega \tag{42}$$

Then, L_s = 96,22 μH and it is clear that the resistance R_s can be neglected in the electrical model of the machine.

14. Plot I_s wrt I_e for f_{min} = 370 Hz and f_{max} = 770 Hz

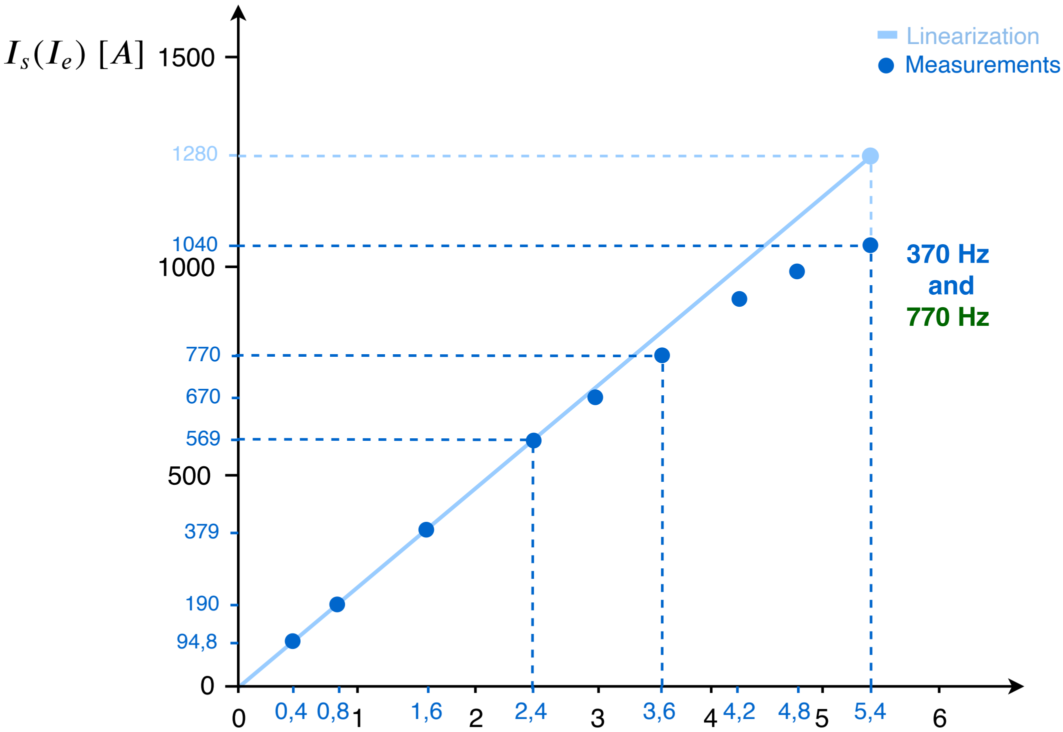


Figure 5: Short circuit current as a function of the excitation current.

As,

$$E = \lambda\,\omega\,I_e \qquad \text{(obtained from question 11)} \tag{43}$$

and

$$I_s = \frac{E}{X_s} \qquad \text{(obtained from question 13)} \tag{44}$$

One can deduce that :

$$I_s = \frac{E}{X_s} = \frac{\lambda\,\omega\,I_e}{\omega\,L_s} = \frac{\lambda}{L_s}\,I_e \tag{45}$$

Therefore, the short circuit current I_s is proportionnal to the excitation current I_e (less true out of the linear zone, but considered linear as λ does not vary much) and the short circuit current I_s does not depend of the frequency.

15. Resisitve load at 500 Hz with I_e = 2 A

Table 1 of the statement provides an emf of 106 V for I_e = 2 A and a frequency of 370 Hz. At 500 Hz, the emf becomes E = 106 $\frac{500}{370}$ = 143,24 V.

(a)

$$\overline{I} = \frac{\overline{E}}{R_L + j\,X_s} \tag{46}$$

$$I = \frac{E}{\sqrt{R_L^2 + X_s^2}} = \frac{143,24}{\sqrt{0,5^2 + (2\pi\,500\,96,22 \times 10^{-6})^2}} = 245,17 \text{ A} \tag{47}$$

$$V = R_L\,I = 0,5\;245,17 = 122,59 \text{ V} \tag{48}$$

(b)

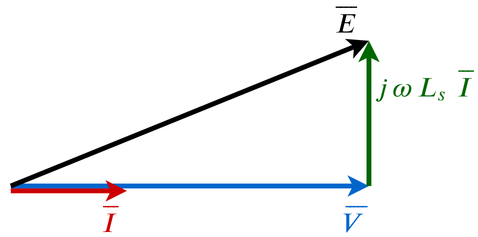


Figure 6: Phasor diagram for the purely resistive load.

(c) The load current is proportionnal to the frequency. Then, if the frequency increases, both the output voltage and current increase.

16. Resistive-inductive load

(a)

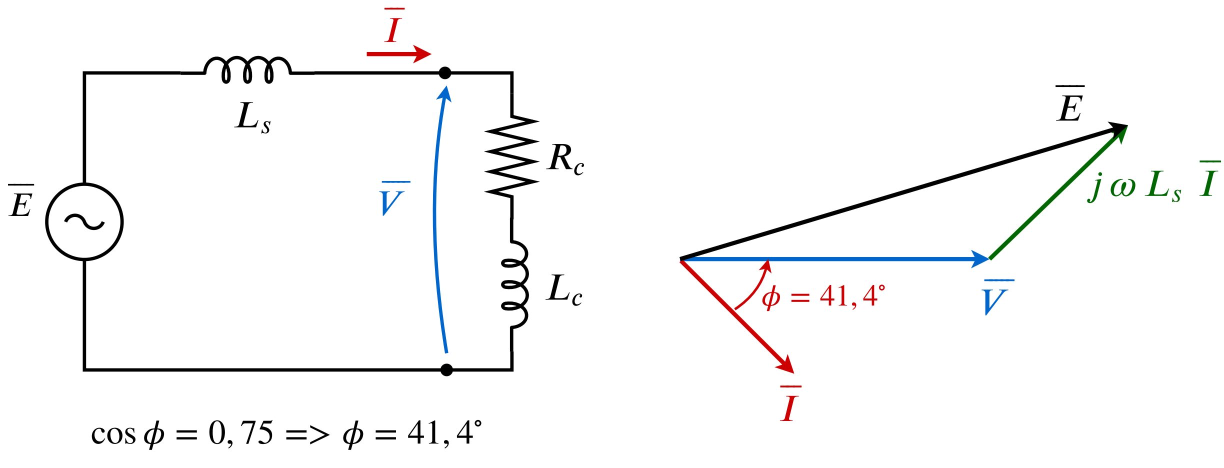


Figure 7: Phasor diagram for the resistive-inductive load.

(b)

Use

$$E = \omega\,\lambda\,I_e \tag{49}$$

$$I = \frac{E}{\sqrt{R_c^2 + \Big(\omega(L_c + L_s)\Big)^2}} \tag{50}$$

$$V = \sqrt{R_c^2 + (\omega\,L_c)^2}\;I \tag{51}$$

To expresss

$$I = \frac{\omega\,\lambda\,I_e}{\sqrt{R_c^2 + \Big(\omega(L_c + L_s)\Big)^2}} \tag{52}$$

$$V = \sqrt{R_c^2 + (\omega\,L_c)^2}\;\frac{\omega\,\lambda\,I_e}{\sqrt{R_c^2 + \Big(\omega(L_c + L_s)\Big)^2}} \tag{53}$$

(c)
With

$$P_{3\phi} = C_r\,\dot{\theta} \tag{54}$$

$$\dot{\theta} = \frac{\omega}{p} \tag{55}$$

$$P_{3\phi} = 3\;P_{1\phi} \tag{56}$$

$$P_{1\phi} = R_c\;I^2 = R_c\;\frac{(\omega\,\lambda\,I_e)^2}{R_c^2 + \Big(\omega(L_c + L_s)\Big)^2} \tag{57}$$

One finally obtain

$$C_r = 3\,p\;R_c\;\frac{\omega\,(\lambda\,I_e)^2}{R_c^2 + \Big(\omega(L_c + L_s)\Big)^2} \tag{58}$$

(d)
$R_c = 0{,}5\ \Omega$, $L_c = 150\ \mu$H and $L_s = 96{,}224\ \mu$H.

	I_e	λ	E	I	V	C_r
$f_{min} = 370$ Hz	0,4	0,02300512	21,2	27,89	17	1
$\omega_{min} = 2324{,}8$ rad/s	3	0,01982202	137	180,5	109,88	41,92
	5,4	0,01189643	148	194,73	118,7	48,93
$f_{max} = 800$ Hz	0,4	0,02300512	45,84	34,34	31	0,7
$\omega_{min} = 5026{,}5$ rad/s	3	0,01982202	296,22	221,9	200	29,39
	5,4	0,01189643	320	239,7	216,8	34,3

Table 2: λ parameter computation.

(e)
A higher frequency means a lower power factor.

$$PF = \frac{R_c}{\sqrt{R_c^2 + \omega^2\,L_c^2}} \tag{59}$$

Exercise 11: Three-phase turbo-alternator

Exercise 12: Alternator and synchronous condenser

1) $\phi = 27°$; $I = 4.16$ kA
2) $Q =$ -66 kvar
3) Graph

5 AC asynchronous machines

Exercise 13: Asynchronous motor 1

1. Synchronous speed $\dot{\theta}_s$, number of pairs of poles p, nominal slip g_n

The synchronous speed $\dot{\theta}_s$ is close to the nominal rotation speed $\dot{\theta}_n$ = 1 460 rpm. Then, $\dot{\theta}_s$ = 1 500 rpm.

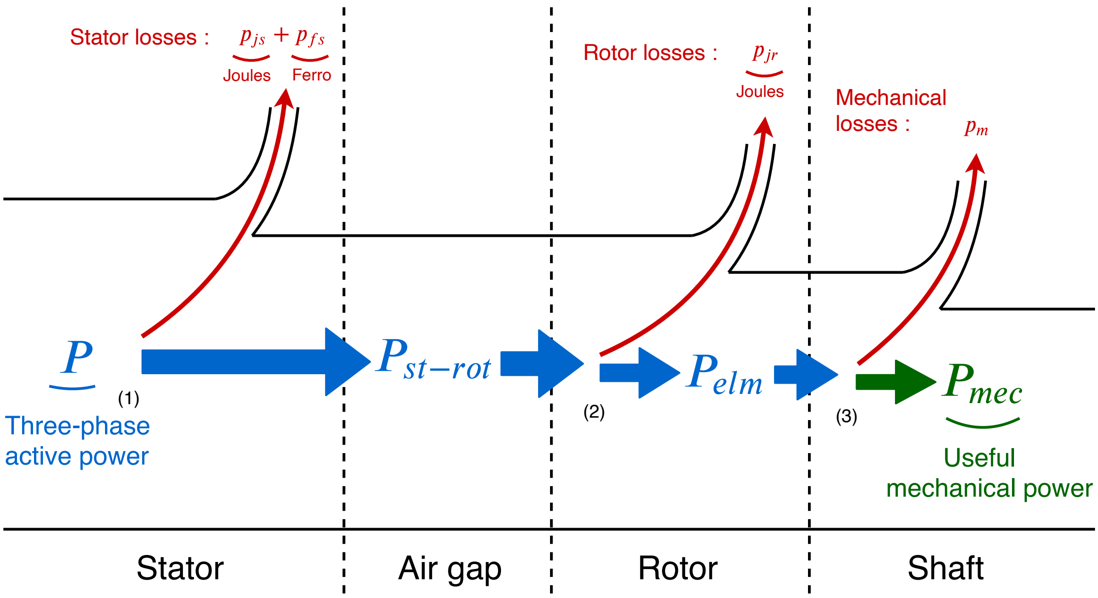
$$\dot{\theta}_s = \frac{1500}{60} = 25s^{-1} \tag{60}$$

$$p = \frac{f}{\dot{\theta}_s} = \frac{50}{25} = 2 \tag{61}$$

The slip corresponds to the relative difference between the synchronous speed $\dot{\theta}_s$ and the rotation speed $\dot{\theta}_n$ such that

$$\boxed{g_n = \frac{\dot{\theta}_s - \dot{\theta}_n}{\dot{\theta}_s} = \frac{1500 - 1460}{1500} = 2,67\%} \tag{62}$$

Recall of the power balance for an asynchronous motor



(1) $P_{st-rot} = P - p_{js} - p_{fs}$

(2) $P_{elm} = P_{st-rot} - p_{jr}$

(3) $P_{mec} = P_{elm} - p_m$

The shaft output power (here 5.5 kW)

Figure 8: Power balance of the asynchronous motor.

The ferromagnetic losses in the rotor can ben neglected since the frequency of the rotoric currents is much smaller than the grid frequency (*i.e.* the frequency of the statoric currents) : $g_n\, f << f$ and $p_{fr} \simeq 0$ (good approximation for small slip).

Understanding $\frac{R_2'}{g}$

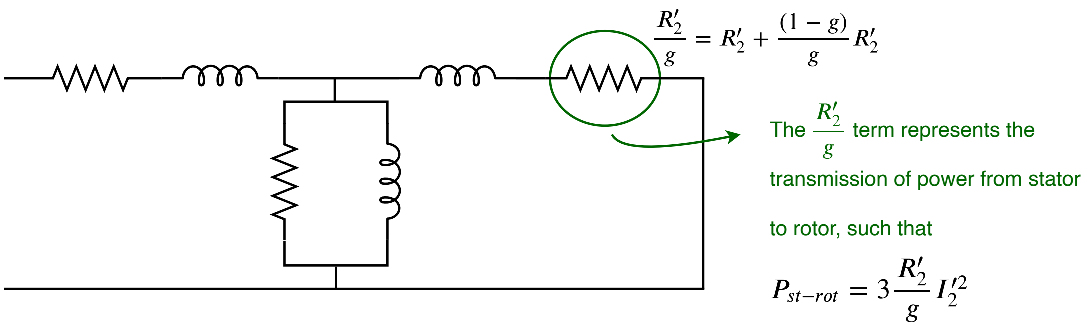


Figure 9: Power transmission between the stator and the rotor of the asynchronous motor.

$$P_{st-rot} = 3\,\frac{R_2'}{g}\,I_2'^2 \tag{63}$$

$$p_{jr} = 3\,R_2'\,I_2'^2 \tag{64}$$

$$P_{elm} = P_{st-rot} - p_{jr} = 3\,\frac{R_2'}{g}\,I_2'^2\,(1-g) \tag{65}$$

$$\tag{66}$$

Which leads to

$$\boxed{P_{elm} = (1-g)\,P_{st-rot}} \tag{67}$$

$$\boxed{p_{jr} = g\,P_{st-rot}} \tag{68}$$

2. Stator resistance R_s

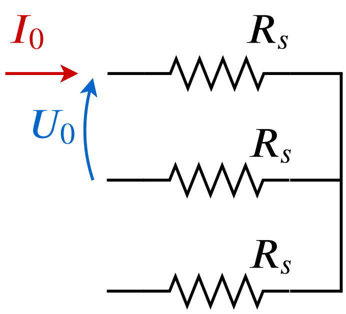


Figure 10: Stator model (star-shaped).

$$R_s = \frac{U_0}{2\,I_0} = \frac{20.6}{2 \cdot 10} = 1,03\ \Omega$$

3. $\underline{R_{HF} \text{ and } X_\mu}$

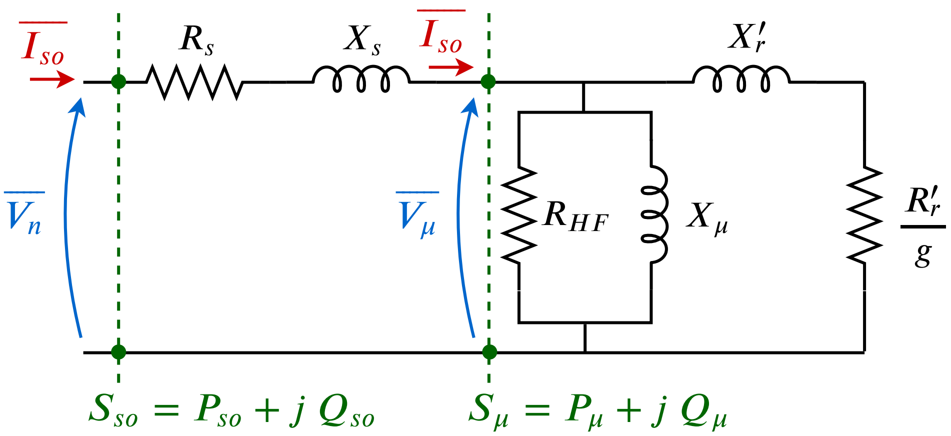


Figure 11: Equivalent circuit of the asynchronous motor running at nominal speed.

$$I_{so} = 3,07\ \ \text{A} \tag{69}$$
$$V_n = \frac{U_n}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230,94\ \ \text{V} \tag{70}$$
$$P_{so} = \frac{245}{3} = 81,667\ \ \text{W} \tag{71}$$
$$S_{so} = V_n\,I_{so} = 708,986\ \text{VA} \tag{72}$$
$$Q_{so} = \sqrt{S_{so}^2 - P_{so}^2} = 704,27\text{var} \tag{73}$$
$$P_\mu = P_{so} - R_s\,I_{so}^2 = 81,667 - 1,03 \cdot 3,07^2 = 71,96\ \text{W} \tag{74}$$
$$Q_\mu = Q_{so} - X_s\,I_{so}^2 = 704,27 - 1,03 \cdot 3,07^2 = 694,56\ \text{var} \tag{75}$$
$$S_\mu = \sqrt{P_\mu^2 + Q_\mu^2} = 698,28\ \text{VA} \tag{76}$$
$$V_\mu = \frac{S_\mu}{I_{so}} = 227,45\ \ \text{V (relatively close to } V_n) \tag{77}$$

At the rotor, the losses can be decomposed as

$$\frac{R'_r}{g} I_r'^2 = \underbrace{R'_r\,I_r'^2}_{\text{rotor Joule losses}} + \underbrace{\frac{(1-g)\,R'_r}{g} I_r'^2}_{\text{mechanical losses}} \tag{78}$$

You are told that the ferromagnetic losses equal the mechanical losses.

Then,

$$\underbrace{\frac{V_\mu^2}{R_{HF}}}_{\text{ferromagnetic losses}} \underset{=}{+} \underbrace{\frac{(1-g)\,R'_r}{g} I_r'^2}_{\text{mechanical losses}} = P_\mu \tag{79}$$

$$\rightarrow\ 2\,\frac{V_\mu^2}{R_{HF}} = P_\mu \tag{80}$$

$$\boxed{R_{HF} = 2\,\frac{V_\mu^2}{P_\mu} = 2\,\frac{227,45^2}{71,96} = 1437,88\Omega} \tag{81}$$

$$\boxed{X_\mu = 2\,\frac{V_\mu^2}{Q_\mu} = 2\,\frac{227,45^2}{694,56} = 74,99\Omega} \tag{82}$$

The losses seen from the magnetizing (μ) branch are $P_\mu = 71,96$ W in the equivalent circuit of one phase. Half of those losses account for the mechanical losses $p_{m,1\phi} = 35,98$ W and the other half account for the ferromagnetic losses $p_{f,1\phi} = 35,98$ W. The total mechanical and ferromagnetic losses of the three-phase motor correspond to

$$p_m = 3\,p_{m,1\phi} = 3 \cdot 35,98 = 107,94 \text{ W} \tag{83}$$

$$p_f = 3\,p_{f,1\phi} = 3 \cdot 35,98 = 107,94 \text{ W} \tag{84}$$

4. Nominal operating point : $P_{st-rot},\ p_{js},\ P$

The mechanical losses are considered independent of the rotation speed. Then, $p_m(1460 \text{ rpm}) = p_m(1500 \text{ rpm}) \simeq 108$ W.

From the equation (3) of the power balance

$$P_{mec} = P_{elm} - p_m \tag{85}$$

$$P_{elm} = P_{mec} + p_m = 5\,500 + 108 = 5\,608 \text{ W} \tag{86}$$

From (67)

$$P_{elm} = P_{st-rot}\,(1-g) \tag{87}$$

$$\boxed{P_{st-rot} = \frac{P_{elm}}{1-g} = \frac{5\,608}{1-0,02667} = 5\,761,6 \text{ W}} \tag{88}$$

The Joule losses in the stator are

$$\boxed{p_{js} = 3\,R_s\,I_{sn}^2 = 3 \cdot 1,03 \cdot 11^2 = 373,89 \text{ W}} \tag{89}$$

The ferromagnetic losses are kept constant

$$\boxed{p_f = 108 \text{ W}} \tag{90}$$

Finally, the three-phase active power can be computed as

$$\boxed{P = P_{st-rot} + p_{js} + p_f = 5761,6 + 373,4 + 108 = 6\,243 \text{ W}} \tag{91}$$

5. Find R'_r and L'_r (seen from the stator)

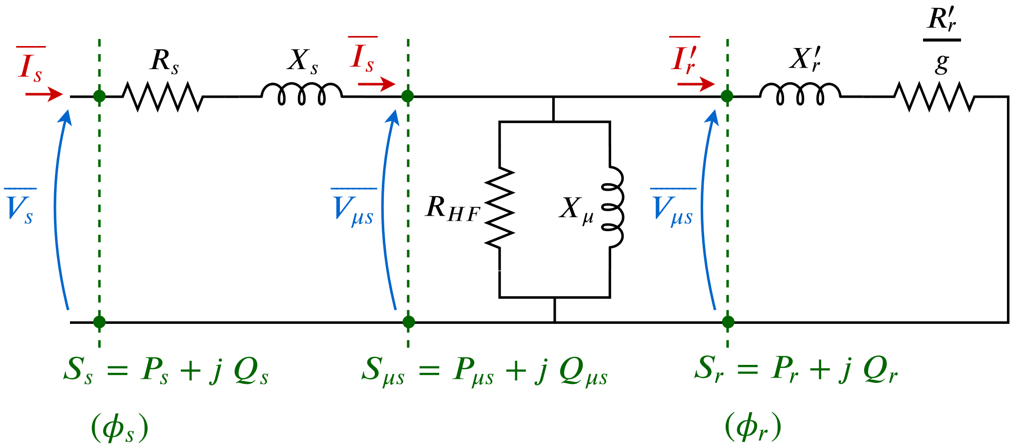


Figure 12: Equivalent circuit of the asynchronous motor in operation.

In order to determine R'_r and X'_r, find P_r, Q_r and I'_r and deduce

$$R'_r = g\frac{P_r}{I'^2_r} \tag{92}$$

$$X'_r = \frac{Q_r}{I'^2_r} \tag{93}$$

Then,

$$P_s = \frac{P}{3} = 2\,081 \text{ W} \tag{94}$$

$$S_s = V_s\, I_s = 230,94 \cdot 11 = 2\,540,34 \text{ VA} \tag{95}$$

$$Q_s = \sqrt{S_s^2 - P_s^2} = 1\,457 \text{ var} \tag{96}$$

$$\phi_s = \arccos\left(\frac{P_s}{S_s}\right) = 35° \tag{97}$$

Accross the magnetizing branch,

$$P_{\mu s} = P_s - R_s\, I_s^2 = 2\,081 - 1,03 \cdot 11^2 = 1\,956,37 \text{ W} \tag{98}$$

$$Q_{\mu s} = Q_s - X_s\, I_s^2 = 1\,457 - 1.03 \cdot 11^2 = 1\,332,37 \text{ W} \tag{99}$$

$$S_{\mu s} = \sqrt{P_\mu^2 + Q_\mu^2} = 2\,366,9 \text{ VA} \tag{100}$$

$$V_{\mu s} = \frac{S_{\mu s}}{I_s} = \frac{2\,366,9}{11} = 215,2 \text{ V} \tag{101}$$

Then, on the rotor side of the equivalent circuit,

$$P_r = P_{\mu s} - \frac{V_{\mu s}^2}{R_{HF}} = 1\,956,37 - \frac{215,2^2}{1\,437,88} = 1\,923,2 \text{ W} \tag{102}$$

$$Q_r = Q_{\mu s} - \frac{V_{\mu s}^2}{X_\mu} = 1\,332,37 - \frac{215,2^2}{74,99} = 714,8 \text{ var} \tag{103}$$

$$S_r = \sqrt{P_r^2 + Q_r^2} = 2\,051,7\text{VA} \tag{104}$$

$$\phi_r = \arccos\left(\frac{P_r}{S_r}\right) = 20,43° \tag{105}$$

$$I'_r = \frac{S_r}{V_{\mu s}} = \frac{2\,052,24}{215,2} = 9,537 \text{ A} \tag{106}$$

Finally,

$$\boxed{R'_r = g\,\frac{P_r}{I'^2_r} = 0,02667\,\frac{1\,923,2}{9,537^2} = 0,564\ \Omega} \tag{107}$$

$$X'_r = \frac{Q_r}{I'^2_r} = \frac{714,8}{9,537^2} = 7,86\ \Omega \tag{108}$$

$$\boxed{L'_r = 25 \text{ mH}} \tag{109}$$

6. C_{mec}, C_{elm}, $\cos\phi_n$ and η_n (for the nominal operating point)

$$P_{mec} = C_{mec,n}\,\dot{\theta}_n \tag{110}$$

Then, the output torque is given by

$$C_{mec,n} = \frac{P_{mec}}{\dot{\theta}_n} = \frac{5\,500}{1\,460 \cdot \frac{2\,\pi}{60}} = 35,97 \text{ Nm} \tag{111}$$

The electromagnetic torque is

$$C_{elm,n} = \frac{P_{elm}}{\dot{\theta}_n} = \frac{5\,608}{1\,460 \cdot \frac{2\pi}{60}} = 36,7 \text{ Nm} \tag{112}$$

The $\cos\phi_n$ can be computed by two ways,

$$\cos\phi_n = \frac{P}{S_n} = \frac{6\,243}{\sqrt{3} \cdot 400 \cdot 11} = 0,818 \tag{113}$$

or

$$\phi_s = \phi_n => \cos\phi_n = 0,818 \tag{114}$$

Finally, the efficiency during nominal operation is

$$\eta_n = \frac{P_{mec}}{P} = \frac{5\,500}{6\,242} = 88,1\% \tag{115}$$

7. I_s and $\cos\phi$ for $\dot{\theta} = 0$ rpm

At 0 rpm, the motor is stalled, the slip becomes $g = 1$, $\frac{R'_r}{g} = R'_r$ and the equivalent circuit of the asynchronous motor becomes

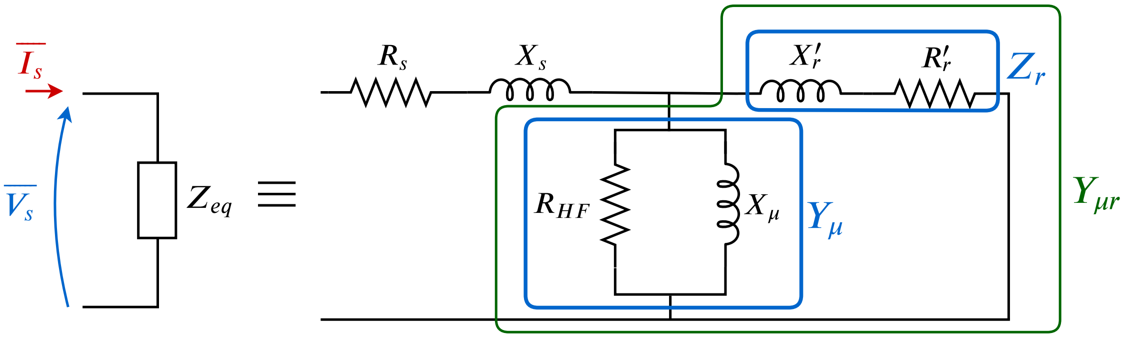


Figure 13: Equivalent circuit of the asynchronous motor when stalled.

$$Z_r = R'_r + j\ X'_r \tag{116}$$

$$Y_r = \frac{1}{Z_r} = \frac{1}{R'_r + j\ X'_r} \tag{117}$$

$$Y_\mu = \frac{1}{R_{HF}} + j\ \frac{1}{X_\mu} \tag{118}$$

$$Y_{\mu r} = Y_\mu + Y_r = \frac{1}{R_{HF}} + j\ \frac{1}{X_\mu} + \frac{1}{R'_r + j\ X'_r} \tag{119}$$

$$Z_{\mu r} = \frac{1}{Y_{\mu r}} = \frac{1}{\frac{1}{R_{HF}} + j\ \frac{1}{X_\mu} + \frac{1}{R'_r + j\ X'_r}} \tag{120}$$

$$Z_{eq} = R_s + j\ X_s + Z_{\mu r} = R_s + j\ X_s + \frac{1}{\frac{1}{R_{HF}} + j\ \frac{1}{X_\mu} + \frac{1}{R'_r + j\ X'_r}} \tag{121}$$

$$Z_{eq} = 1,03 + j\ 1,03 + \frac{1}{\frac{1}{1\,437{,}8} + j\ \frac{1}{75} + \frac{1}{0{,}564 + j\ 7{,}87}} = 9,969\angle 79,67^\circ \tag{122}$$

$$\boxed{\overline{I_s} = \frac{\overline{V_s}}{Z_{eq}} = \frac{230}{9,969\angle 79,67^\circ} = 23,07\angle \underbrace{-79,67^\circ}_{\phi}} \tag{123}$$

$$\boxed{\cos\phi = \cos(-79,67^\circ) = 0,179} \tag{124}$$

Exercise 14: Asynchronous motor 2

1. Explain the nameplate

Nominal active power : $P_n = 4,4$ kW

RMS voltages : $\underbrace{230}_{\Delta} \,/\, \underbrace{400}_{Y}$ V

Line currents : $\underbrace{15,5}_{\Delta} \,/\, \underbrace{9}_{Y}$ A

Nominal frequency : $f_n = 50$ Hz

Number of pairs of poles : 4 poles $\rightarrow$ 2 pairs of poles $\rightarrow p = 2$

2. Which coupling for a 230 V network ?

230 V network means that the RMS value of thez composed voltages is $U = 230$ V. Then, the armature (stator) of the machine should be connected in Δ.

3. Synchronous speed $\dot{\theta}_s$

$$\dot{\theta}_s = \frac{f_n}{p} = \frac{50}{2} = 25 \text{ Hz} = 1500 \text{ rpm} = 157 \text{ rad/s} \tag{125}$$

4. Stator resistance R_s

R_a corresponds to the total resitance measured between two terminals of the stator. Then, R_s, the resistance of one phase is two times smaller than $R_a = \frac{U_0}{I_0}$.

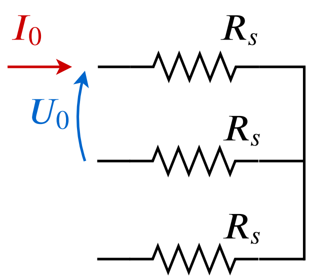


Figure 14: Stator model (star-shaped).

$$R_s = \frac{R_a}{2} = \frac{0,654}{2} = 0,327\ \Omega$$

5. Mechanical losses at synchronous speed

The asynchronous machine is put into potion by an external motor rotating at $\dot{\theta}_s$ and remains unpowered at the stator. Therefore, the mechanical power provided by the external potor exactly compensate the mechanical losses of the asynchronous machine : $p_m = 86$ W.

The mechanical losses depends on the rotation speed. The nominal rotation speed is close to the synchronous rotation speed (low slip), that is why the variation in mechanical losses can be neglected.

6. Determine R_{HF} and L_μ

During the no load test, the active power in the stator P_{so} is

$$P_{so} = \underbrace{p_{js}}_{\text{stator joule losses}} + \underbrace{p_f}_{\text{ferromagnetic losses}} + \underbrace{p_m}_{\text{mechanical losses}} \tag{126}$$

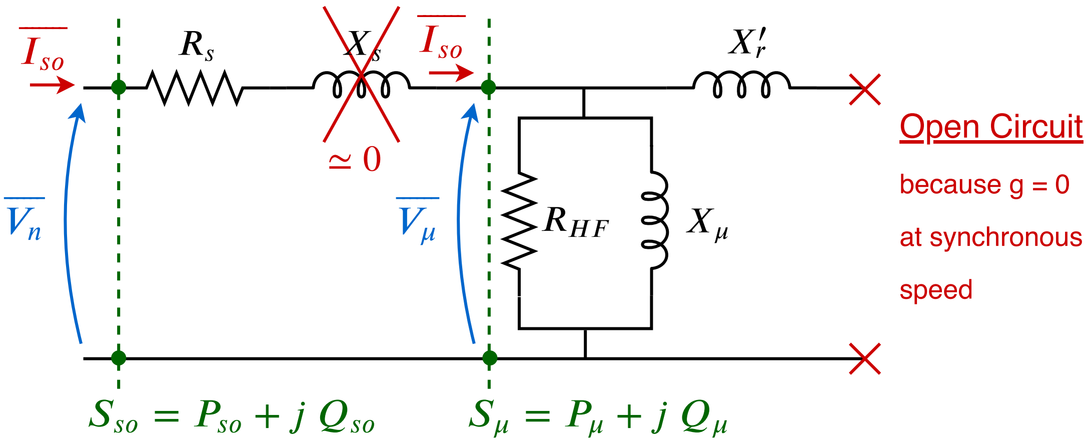


Figure 15: Equivalent circuit of the asynchronous motor running at synchronous speed.

$$I_{so} = 3,82 \ \ \text{A} \tag{127}$$

$$V_n = \frac{U_n}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132,79 \ \ \text{V} \tag{128}$$

$$P_{so} = \frac{300}{3} = 100 \ \ \text{W} \tag{129}$$

$$S_{so} = V_n\, I_{so} = 507,26 \text{ VA} \tag{130}$$

$$Q_{so} = \sqrt{S_{so}^2 - P_{so}^2} = 497,3\text{var} \tag{131}$$

$$P_\mu = P_{so} - R_s\, I_{so}^2 = 100 - 0,327 \cdot 3,82^2 = 95,23 \text{ W} \tag{132}$$

$$Q_\mu = Q_{so} - \underbrace{X_s}_{\simeq 0}\, I_{so}^2 = Q_{so} = 497,3 \text{ var} \tag{133}$$

$$S_\mu = \sqrt{P_\mu^2 + Q_\mu^2} = 506,34 \text{ VA} \tag{134}$$

$$V_\mu = \frac{S_\mu}{I_{so}} = \frac{506,34}{3,82} = 132,55 \ \ \text{V (relatively close to } V_n\text{)} \tag{135}$$

At this point, the mechanical losses are still taken into account and must be subtracted.

Then, the ferromagnetic losses correspond to

$$p_f = P_\mu - \frac{p_m}{3} = 95,23 - \frac{86}{3} = 66,56 \text{ W} \tag{136}$$

$$\boxed{R_{HF} = \frac{V_\mu^2}{P_f} = \frac{132,55^2}{66,56} = 264\ \Omega} \tag{137}$$

$$X_\mu = \frac{V_\mu^2}{Q_\mu} = \frac{132,55^2}{497,3} = 35,2\ \Omega \tag{138}$$

$$\boxed{L_\mu = \frac{X_\mu}{100\,\pi} = 0,112 \text{ H}} \tag{139}$$

7. Stalled rotor test

Stalled rotor $\rightarrow g = 1 \rightarrow \frac{R_r'}{g} = R_r'$

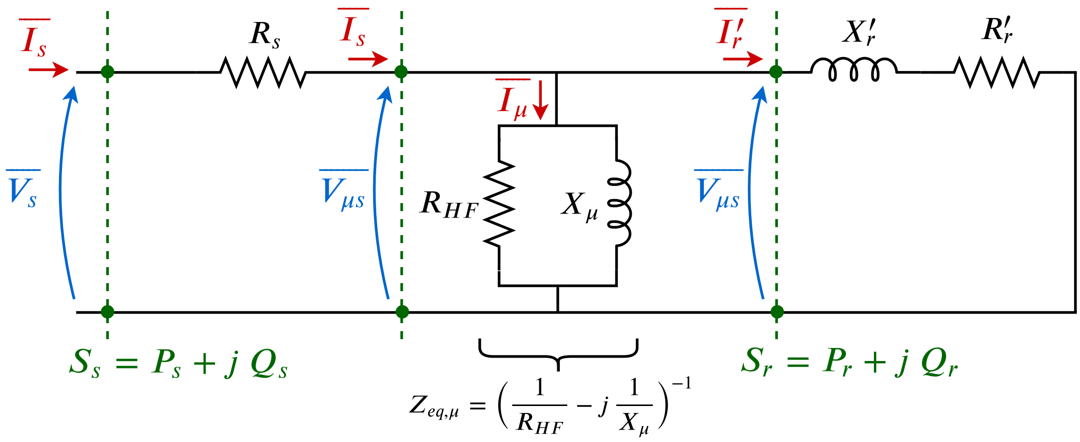


Figure 16: Equivalent circuit of the stalled asynchronous motor.

$$P_s = \frac{374}{3} = 124,67 \ \text{ W} \tag{140}$$
$$Q_s = \frac{1090}{3} = 363,3 \text{ var} \tag{141}$$
$$S_s = \sqrt{P_s^2 + Q_s^2} = 384,1 \text{ VA} \tag{142}$$
$$V_s = \frac{U_s}{\sqrt{3}} = \frac{57,5}{\sqrt{3}} = 33,2 \ \text{ V} \tag{143}$$
$$\overline{I_s} = \frac{P_s - j\, Q_s}{V_s} = \frac{124,67 - j\ 363,3}{33,2} = 11,57\angle - 71,06° \tag{144}$$
$$I_s = 11,57 \ \text{ A} \tag{145}$$
$$V_\mu = V_s - R_s\, I_s = 33,2 - 0,327 \cdot 11,57 = 29,42 \text{ V} \tag{146}$$
$$\overline{I_\mu} = \frac{\overline{V_\mu}}{Z_{eq,\mu}} = \frac{\overline{V_\mu}}{\left(\frac{1}{R_{HF}} - j\,\frac{1}{X_\mu}\right)^{-1}} = \frac{29,42}{\left(\frac{1}{264} - j\,\frac{1}{35,2}\right)^{-1}} = 0,843\angle - 82° \tag{147}$$
$$\overline{I_r'} = \overline{I_s} - \overline{I_\mu} = (11,57\angle - 71,06°) \ - (0,843\angle - 82°) = 10,74\angle - 70,21° \tag{148}$$
$$I_r' = 10,74 \text{ A} \tag{149}$$
$$P_r = P_s - R_s\, I_s^2 - \frac{V_\mu^2}{R_{HF}} = 124,67 - 0,327 \cdot 11,57^2 - \frac{29,42^2}{264} = 77,62 \text{ W} \tag{150}$$
$$Q_r = Q_s - \frac{V_\mu^2}{X_\mu} = 363,3 - \frac{29,42^2}{35,2} = 338,71 \text{ var} \tag{151}$$
$$\boxed{R_r' = \frac{P_r}{I_r'^2} = \frac{77,62}{10,74^2} = 0,673\ \Omega} \tag{152}$$
$$\boxed{X_r' = \frac{Q_r}{I_r'^2} = \frac{338,71}{10,74^2} = 2,93\ \Omega} \tag{153}$$

8. Express I_s in terms of V_s, R_s, R_r', g and X_r'.

Neglecting the magnetizing brnahc and the magnetizing current I_μ,

$$\overline{I_s} = \frac{\overline{V_s}}{R_s + \frac{R_r'}{g} + j\ X_r'} \tag{154}$$
$$\boxed{I_s = \frac{V_s}{\sqrt{\left(R_s + \frac{R_r'}{g}\right)^2 + X_r'^2}}} \tag{155}$$

9. Transmitted power from stator to rotor.

$$P_{st-rot} = 3\,\frac{R_r'}{g}\ I_s^2 = 3\,\frac{R_r'}{g}\ \frac{V_s^2}{\left(R_s + \frac{R_r'}{g}\right)^2 + \left(X_r'\right)^2} \tag{156}$$

10. Show that C_{elm} is maximum for $g_m$and compute C_{elm}.

As $P_{elm} = (1-g)\, P_{st-rot}$ and $\dot{\theta} = (1-g)\, \dot{\theta}_s$, the torque can be expressed as

$$C_{elm} = \frac{P_{elm}}{\dot{\theta}} = \frac{P_{st-rot}}{\dot{\theta}_s} \tag{157}$$

$$C_{elm} = \frac{P_{st-rot}}{\dot{\theta}_s} = 3\, \frac{V_s^2}{\dot{\theta}_s}\, \frac{\frac{R_r'}{g}}{\left(R_s + \frac{R_r'}{g}\right)^2 + \left(X_r'\right)^2} \tag{158}$$

$$\frac{d\, C_{elm}}{d\, g} = 3\, \frac{R_r'\, V_s^2}{\dot{\theta}_s} \left(- \frac{\frac{1}{g^2} \left(\left(R_s + \frac{R_r'}{g}\right)^2 \,+\, (X_r')^2 \right) + \frac{1}{g}\, 2\, \left(R_s + \frac{R_r'}{g}\right) \left(-\frac{R_r'}{g^2}\right)}{\left(\left(R_s + \frac{R_r'}{g}\right)^2 \,+\, (X_r')^2 \right)^2} \right) \tag{159}$$

$$\frac{d\, C_{elm}}{d\, g} = 3\, \frac{R_r'\, V_s^2}{g^2\, \dot{\theta}_s} \left(\frac{R_s^2 - \left(\frac{R_r'}{g}\right)^2 + X_r'^2}{\left(\left(R_s + \frac{R_r'}{g}\right)^2 \,+\, (X_r')^2 \right)^2} \right) \tag{160}$$

From that expression, $\frac{d\, C_{elm}}{d\, g} = 0$ if $R_s^2 - \left(\frac{R_r'}{g}\right)^2 + X_r'^2 = 0$
Meaning, for a slip such that

$$\boxed{g_{max} = +\frac{R_r'}{\sqrt{R_s^2 + X_r'^2}}} \quad \text{(positive slip for the motor)} \tag{161}$$

And for such a slip, g_{max}, the maximum electromagnetic torque is

$$\boxed{C_{elm,max} = 3\, \frac{V_s^2}{\dot{\theta}_s}\, \frac{\sqrt{R_s^2 + X_r'^2}}{\left(R_s + \sqrt{R_s^2 + X_r'^2}\right)^2 + X_r'^2} = 3\, \frac{V_s^2}{2\, \dot{\theta}_s}\, \frac{1}{R_s + \sqrt{R_s^2 + X_r'^2}}} \tag{162}$$

Neglecting the stator resistance, R_s, the result is

$$C_{elm,max} = 3\, \frac{V_s^2}{\dot{\theta}_s}\, \frac{1}{X_r'} \quad \text{(same as theory)} \tag{163}$$

11. <u>Plot C_{elm} wrt g for $V_s = V_n$, $V_s = \frac{V_n}{\sqrt{2}}$ and $V_s = \frac{V_n}{2}$.</u>

At $V_s = V_n$ = 132,79 V ; the torque is

$$C_{elm,max} = 3\,\frac{132,79^2}{1\,500 \cdot \frac{2\pi}{60}}\,\frac{\sqrt{0,327^2 + 2,84^2}}{\left(0,327 + \sqrt{0,327^2 + 2,84^2}\right)^2 + 2,84^2} = 52,86 \text{ Nm} \tag{164}$$

At $V_s = \frac{V_n}{\sqrt{2}}$ = 93,9 V ; the torque is

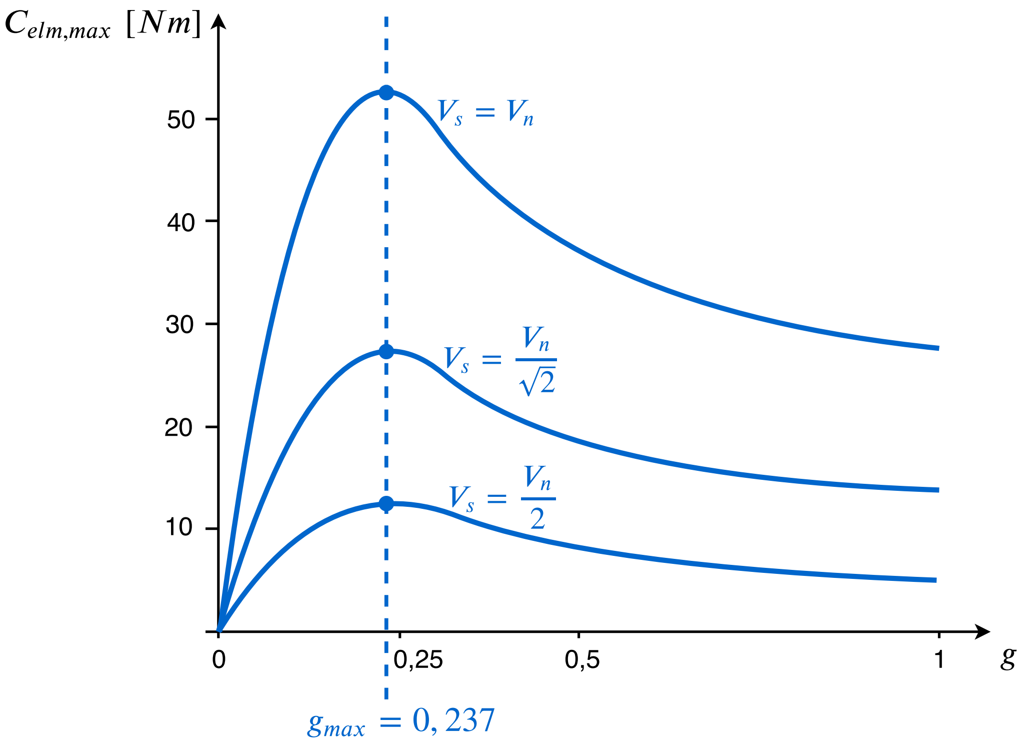
$$C_{elm,max} = 3\,\frac{93,9^2}{1\,500 \cdot \frac{2\pi}{60}}\,\frac{\sqrt{0,327^2 + 2,84^2}}{\left(0,327 + \sqrt{0,327^2 + 2,84^2}\right)^2 + 2,84^2} = 26,43 \text{ Nm} \tag{165}$$

At $V_s = \frac{V_n}{2}$ = 66,4 V ; the torque is

$$C_{elm,max} = 3\,\frac{66,4^2}{1\,500 \cdot \frac{2\pi}{60}}\,\frac{\sqrt{0,327^2 + 2,84^2}}{\left(0,327 + \sqrt{0,327^2 + 2,84^2}\right)^2 + 2,84^2} = 13,21 \text{ Nm} \tag{166}$$

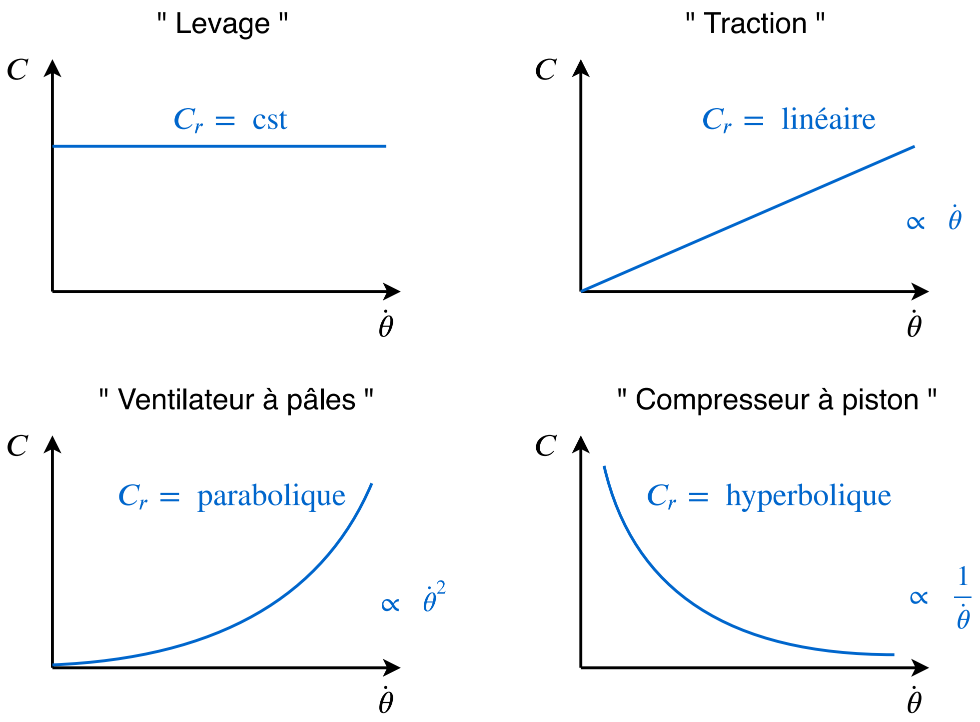
All maximum torques occur at the same slip

$$g_{max} = \frac{0,677}{\sqrt{0,327^2 + 2,84^2}} = 0,237 \tag{167}$$



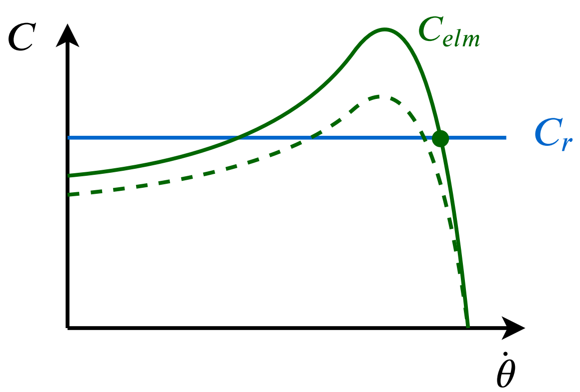
12. Why the control of the voltage is not suited for speed variations at a constant torque.

First of all, the mechanical behavior of different loads (pump, car, fan...) can be separated in 4 main categories.

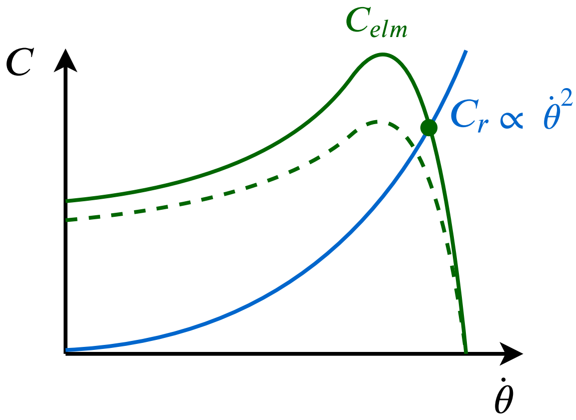


In the first case, decreasing the voltage enables to decrease the speed, but with very small impact. Moreover, the machine must remain in the stable region which is narrow.

→ The voltage control is not suited for speed variation (at a constant resistive torque).

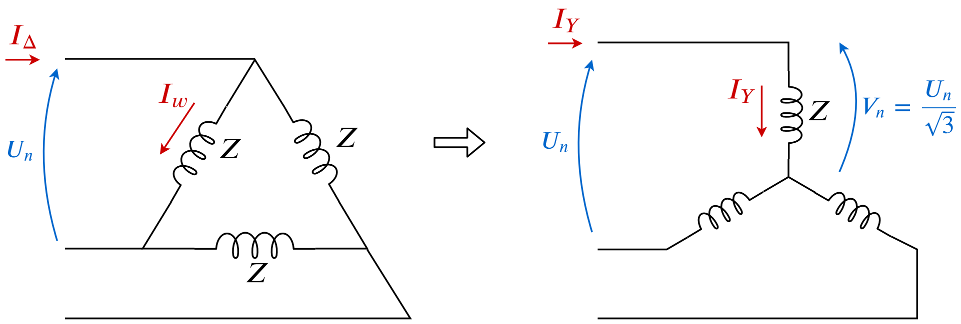


In the other case, if the load is a fan (such as in this exercise), the speed can be easily controlled only by varying the voltage.



13. Comparison without and with star/delta starter.

The use of the star/delta connection allows to reduce the inrush current by a factor of 3.



Winding current : $I_w = \frac{I_\Delta}{\sqrt{3}} = \frac{U_n}{Z}$

Winding current : $I_Y = \frac{V_n}{Z} = \frac{U_n}{\sqrt{3}\,Z} = \frac{I_\Delta}{\sqrt{3}}\frac{1}{\sqrt{3}} = \frac{I_\Delta}{3}$

Exercise 15: Wind turbine

1) $\Gamma_i = \frac{U_s^2}{R_r+R}\frac{\dot{\theta}_s-\dot{\theta}}{\dot{\theta}_s^2}$

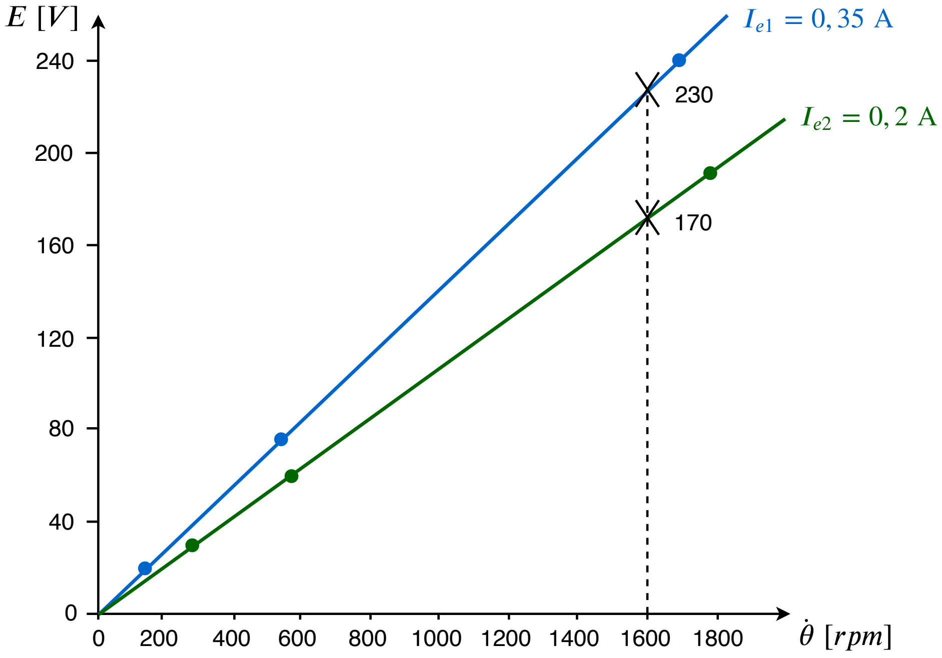
2) $\dot{\theta}$ = 159 rad/s at 2 MW with R = 0 and g = -1.25 % ;
$\dot{\theta}$ = 172 rad/s et 4 MW with R = 9 $m\Omega$; g = -9.23 %

6 DC machines

Exercise 16: DC brushed motor

1. Plot E wrt $\dot{\theta}$ for different excitation current (I_{e1} and I_{e2}).

   In both cases, the emf E is proportionnal to the rotation speed. The excitation current I_{e1} is 75 % higher than I_{e2} ($\frac{I_{e1}}{I_{e2}} = 1,75$ (*)) whereas the ratio of the emf is lower than 1,75 due to the saturation.



2. Show that the flux Φ is nont proportionnal to the excitation current I_e.

   First, the electromotive force E is proportionnal to the flux Φ : $E = k\,\dot{\theta}\,\Phi$.
   Then, at 1600 rpm, $\frac{E_1}{E_2} = \frac{230}{170} = 1,35$ such that $\frac{\Phi_1}{\Phi_2} = 1,35$ (**).

   Since (*) $\neq$ (**), the flux Φ is not proportionnal to the excitation current I_e. This is due to the saturation phenomenon.

3. Plot the voltage E wrt the excitation current I_e and justify.

   The curve $E(I_e)$ behaves as the $\mathcal{B}(\mathcal{H})$. The behaviour is linear for low excitation current I_e and the saturation occurs at some point, when I_e increases more.

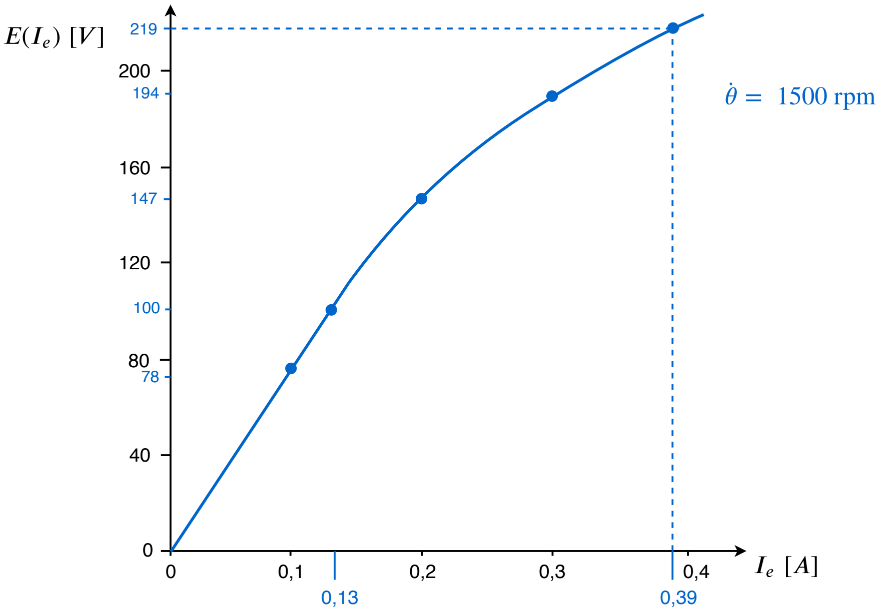


Figure 17: DC motor emf as a function of the excitation current.

4. Draw the equivalent circuit of the motor.

The total power consumed by the motor corresponds to

$$P = U\,I + U_e\,I_e \tag{168}$$

The joule losses in the stator are

$$p_{je} = R_e\,I_e^2 = U_e\,I_e \tag{169}$$

and the joule losses in the rotor are

$$p_{ja} = R\,I^2 \tag{170}$$

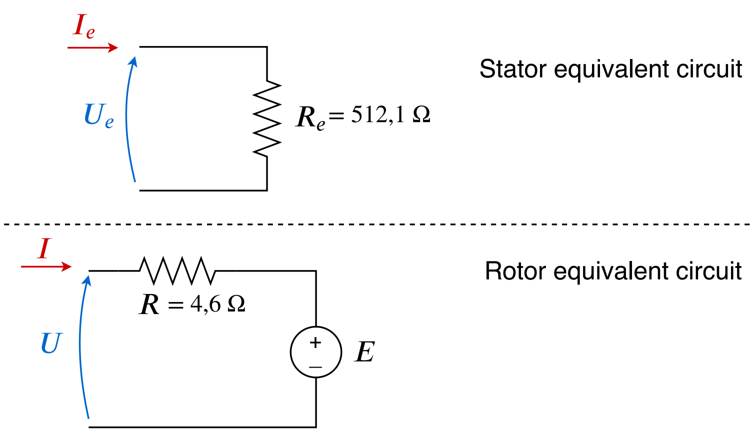


Figure 18: Equivalent circuit of the DC motor.

5. Plot the collective losses p_c wrt the excitation current I_e.

$$\underbrace{p_c}_{\text{collective losses}} = \underbrace{p_m}_{\text{mechanical losses}} + \underbrace{p_f}_{\text{ferromagnetic losses}} \tag{171}$$

For a no load test, the useful mechanical power (available et the shaft) is 0. Meaning that all the electrical power P, injected into the machine, is consumed by the losses.

$$P = p_{ja} + p_{je} + p_c \tag{172}$$
$$p_c = P - p_{ja} - p_{je} \tag{173}$$

Remark that,

$$P = U\,I + U_e\,I_e \tag{174}$$

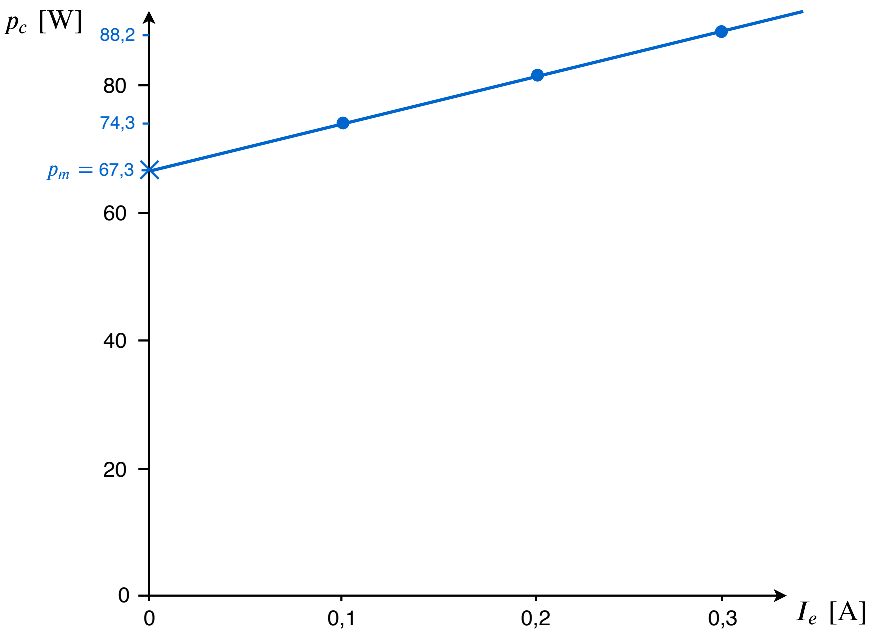
which leads to

$$\boxed{p_c = U\,I - R\,I^2} \tag{175}$$

Then, the collective losses can be computed for different escitation currents.

I_e [A]	U [V]	I [A]	p_c [W]
0,1	85	0,92	74,3
0,2	151	0,56	83,1
0,3	198	0,45	88,2

Table 3: Collective losses p_c as a function of I_e.



6. Determine the mechanical losses p_m.

For $I_e = 0$, there are no ferromagnetic losses. Then, only the mechanical losses p_m occur. Considering that the collective losses depend linearly on the excitation current, one can deduce p_m such that

$$p_m = 88,2 - \frac{88,2 - 74,3}{0,3 - 0,1} \cdot 0,3 = 67,3 \text{ W} \tag{176}$$

7. Calculate the emf E_o and deduce $I_{e,o}$, the corresponding excitation current.

As the hole is drilled, one can measure $I_o = 3$ A and $U_o = 212$ V. Then,

$$E_o = U_o - R\,I_o = 212 - 4,6 \cdot 3 = 198 \text{ V} \tag{177}$$

From the $E(I_e)$ characteristics, one can deduce $I_{e,o} = 0{,}31$ A .

8. Compute the shaft output power, P_u.

The power balance and other power equations of the machine can be established as :

$$P = P_u + p_{ja} + p_{je} + p_c \tag{178}$$
$$P = U_o\,I_o + U_e\,I_e \tag{179}$$
$$p_{ja} = R\,I_o^2 \tag{180}$$
$$p_{je} = U_e\,I_e \tag{181}$$

Then,

$$P_u = P - p_{ja} - p_{je} - p_c \tag{182}$$
$$= U_o\,I_o + U_e\,I_e - R\,I_o^2 - U_e\,I_e - p_c \tag{183}$$
$$= U_o\,I_o - R\,I_o^2 - p_c \tag{184}$$
$$= 212 \cdot 3 - 4,6 \cdot 3^2 - 89 = 506 \text{ W} \tag{185}$$

9. Deduce the resistive torque C_r.

During the nominal regime, the two torques balance each other such that

$$\underbrace{C_u}_{\text{useful torque}} = \underbrace{C_r}_{\text{resistive torque}} \tag{186}$$

$$C_u = \frac{P_u}{\dot{\theta}} = \frac{506}{1\,500 \cdot \frac{2\pi}{60}} = 3,22 \text{ Nm} \tag{187}$$

Exercise 17: Regenerative braking

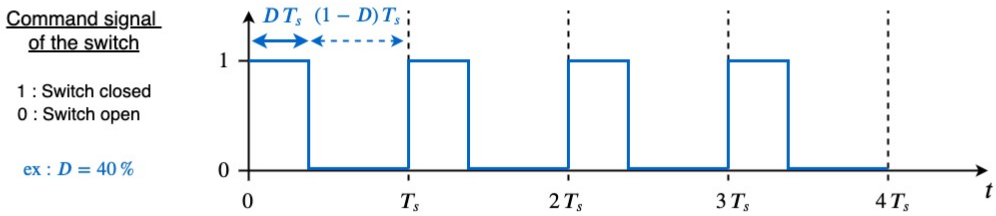
Basic principle of the switched DC-DC converter

The switches are supposed ideal. Meaning that they act as perfect short-circuits when closed and perfect open-circuits when open. In practice, transistors are used and can not be considered as ideal (*i.e.* ON resistance, OFF resistances,...).

Diodes are also supposed ideal even if, in practice, every diode will present a voltage drop when the current flows.

For a switched-mode converter, one can consider that each period of time T_s (T_s : the switching period) is divided into 2 sub-intervals : During sub-interval 1, the switch (transistor) is closed and during sub-interval 2, the switch is open.

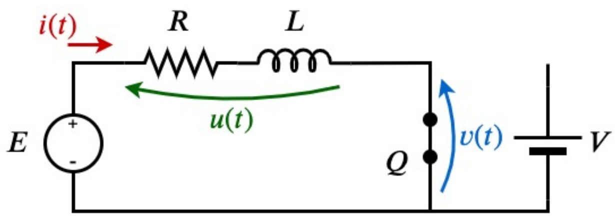
The duty cycle (D) corresponds to the proportion of T_s during which the switch remains closed.



Therefore, 2 sub-circuits exist (one for each sub-interval) and the converter's behavior can be summarized as following.

Sub-interval 1

$D\,\%$ of T_s (The switch is conducting)



$$v_D = 0$$

$$u_D = E \qquad (>0)$$

$$i_D = \left(i_0 - \frac{E}{R}\right)e^{-\frac{R}{L}t} + \frac{E}{R}$$

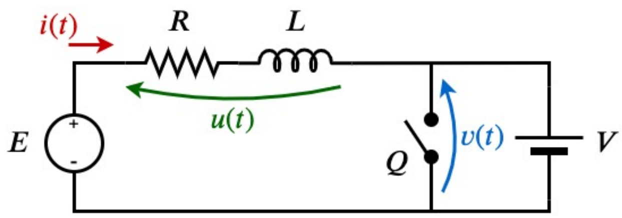
Assumption :

R is small compared to L .
The exponential can be replaced by a first order function considering that $\frac{R}{L}t$ is close to 0 .

$$i_D \simeq i_0 + \frac{E}{L}\,t$$

Sub-interval 2

$(1-D)\,\%$ of T_s (The diode is conducting)



$$v_{(1-D)} = V$$

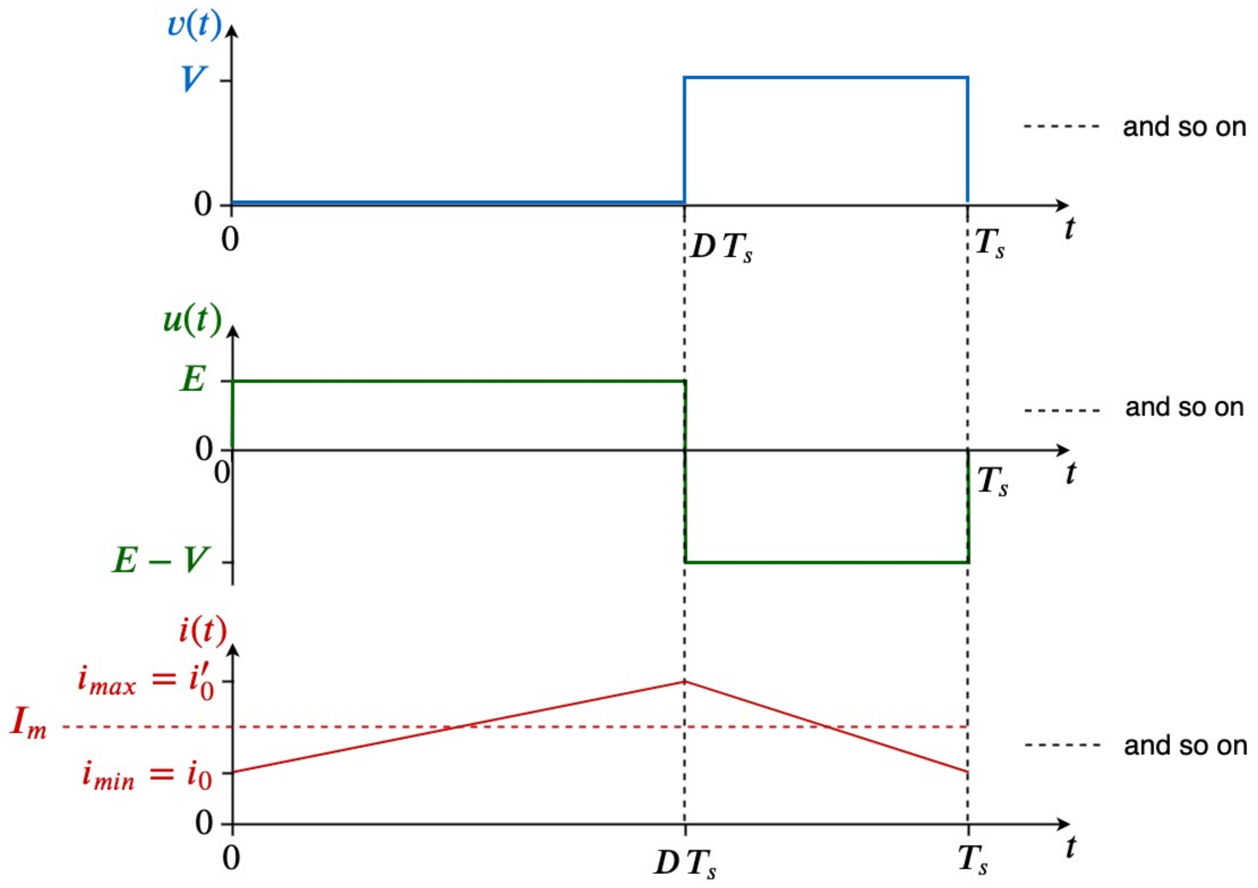
$$u_{(1-D)} = E - V \qquad (<0)$$

$$i_{(1-D)} = \left(i'_0 - \frac{E-V}{R}\right)e^{-\frac{R}{L}t} + \frac{E-V}{R}$$

Assumption :

R is small compared to L .
The exponential can be replaced by a first order function considering that $\frac{R}{L}t$ is close to 0 .

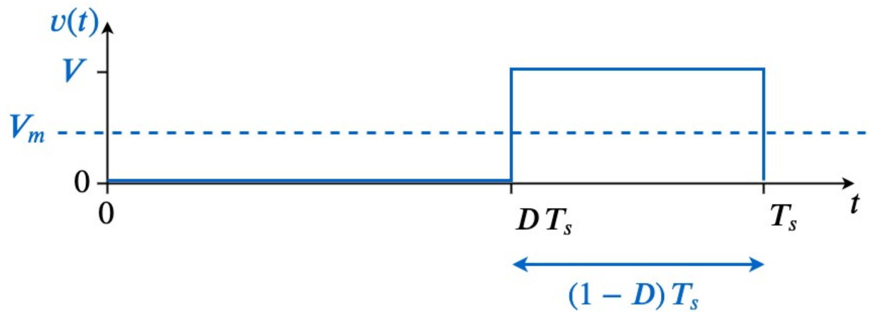
$$i_{(1-D)} \simeq i'_0 + \frac{(E-V)}{L}\,t'$$



As the inductance L is designed with a high value, the inductor current $i(t)$ does not vary much over a switching period (indeed, the goal of the inductance is to smoothen the current). The inductor current can be correctly approximated by its mean value I_m.

On average over a switching period the inductance current remains constant such that : $i(t) \simeq I_m$

1. Find the mean value of $v(t)$, here denoted V_m.



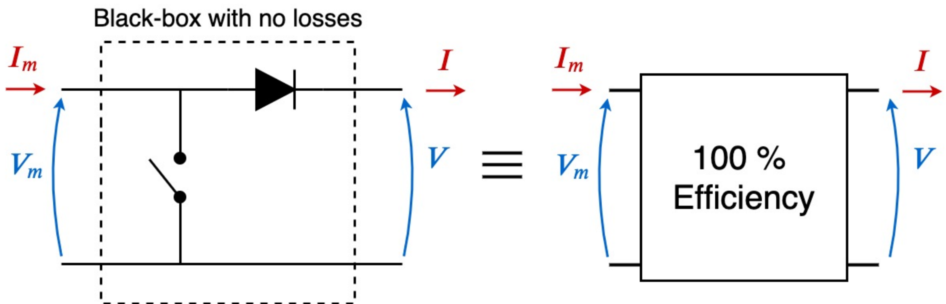
Over the switching period T_s, the mean value of $v(t)$ can be defined as

$$V_m = \frac{1}{T_s} \int_0^{T_s} v(t)\, dt = \frac{1}{T_s} \int_{D\,T_s}^{T_s} V\, dt = \frac{1}{T_s}(1-D)\; V\; T_s \tag{188}$$

$$\boxed{V_m = (1-D)\, V} \tag{189}$$

2. Find the link between the output current I and the input current I_m.

The average input current I_m is the current through the inductor. This current can be linked to the output current by writing the power balance around the semiconductors (diode and switch).



As the semiconductors are considered lossless, the power balance can directly be written as

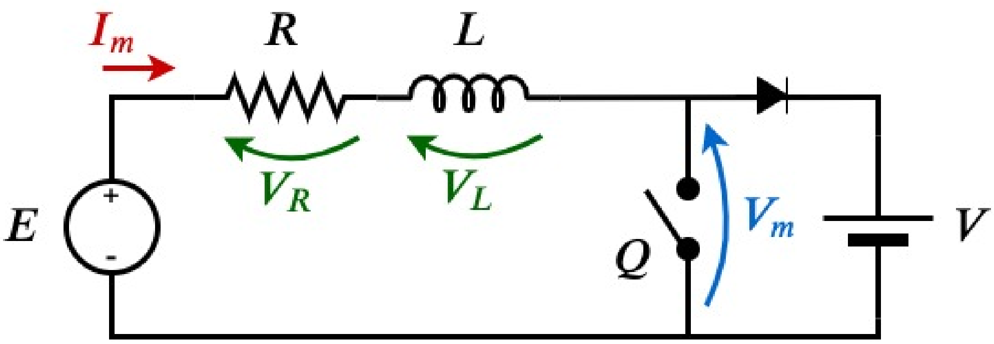
$$V_m\, I_m = V\; I \tag{190}$$

Using the result of the previous question, one can find the link between the input and output currents :

$$\boxed{I = I_m\; (1-D)} \tag{191}$$

3. Express the voltage V with respect to I_m, E, R and D.

Considering the time-averaged equivalent circuit,



one can apply the Kirchhoff Voltage Law to relate the switch average voltage V_m to the input voltage E :

$$V_m = E - V_R - V_L \tag{192}$$

Remark that the average voltage of the inductance is 0 : $V_L = 0$ because the inductor average current is constant over time. This leads to

$$V_m = E - V_R \tag{193}$$
$$V_m = E - R\,I_m \tag{194}$$

The relationship between the switch average voltage V_m and the output voltage V is already known as $V_m = (1-D)\,V$, which finally leads to

$$\boxed{V = \frac{E - R\,I_m}{(1-D)}} \tag{195}$$

4. Compute the duty cycle D allowing to obtain $V_m = 60$ V.

The input voltage is $V = 100$ V and $V_m = (1-D)\,V$. Then, the duty cycle can be expressed as :

$$D = 1 - \frac{V_m}{V} \tag{196}$$

$$\boxed{D = 1 - \frac{60}{100} = 0,4} \tag{197}$$

5. Compute the average braking current I_m for $E = 70$ V, $V = 100$ V and $V_m = 60$ V.

The duty cycle has already been computed for $V = 100$ V and $V_m = 60$ V. The value remains $D = 0,4$.

Then, from question 3, one can use the expression

$$V = \frac{E - R\,I_m}{(1-D)} \tag{198}$$

to isolate the average inductor current, such that :

$$\boxed{I_m = \frac{E - (1-D)\,V}{R} = \frac{70 - 0,6 \cdot 100}{0,5} = 20 \text{ A}} \tag{199}$$

6. Braking power $E\,I_m$ and the braking torque for $\dot{\theta} = 955$ rpm.

$$\boxed{E\,I_m = 70 \cdot 20 = 1400 \text{ W}} \tag{200}$$

$$\boxed{C_m = \frac{E\,I_m}{\dot{\theta}} = \frac{1400}{955 \cdot \frac{2\,\pi}{60}} = 14 \text{ Nm}} \tag{201}$$

Exercise 18: DC generator-motor mechanical coupling

1) $emf = 519{,}075$ V
2) $p = 17{,}11$ W
3) $P = 9603$ W
4) $C = 88.17$ Nm
5) $E_m = 499$ V ; $\dot{\theta} = 1000$ rpm
6) $I_m = 420$ A
7) $C = 1910$ Nm
8) Ratio = 15,7 %

7 Electronic control system

Exercise 19: DC-DC buck converter

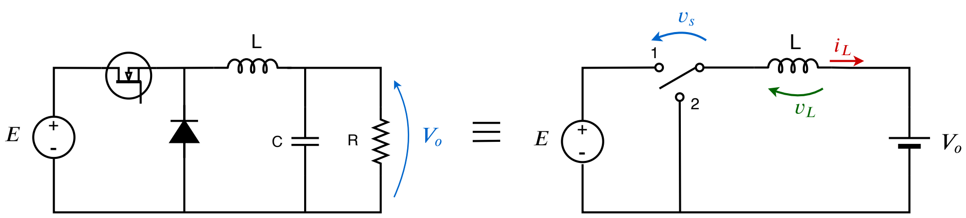
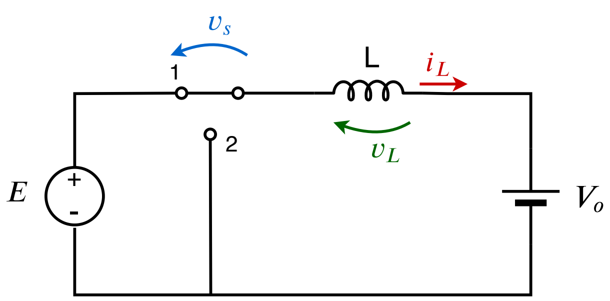


Figure 19: Ideal buck converter equivalent circuit.

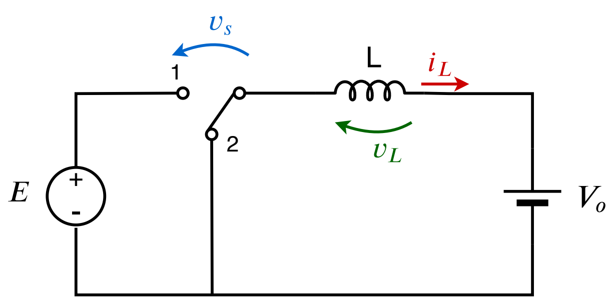
1. Find the waveforms of the voltages v_s and v_L.

During DT_s :

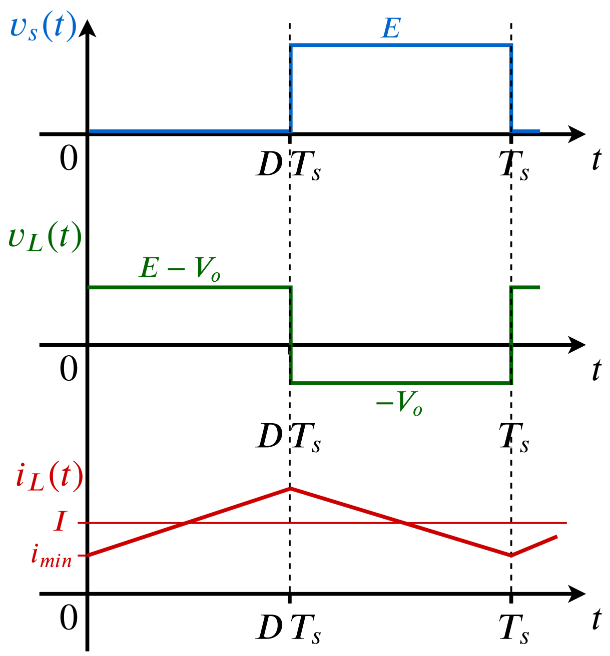


$v_s = 0$ $\qquad$ $v_L = E - V_o$

During $(1-D)T_s$:



$v_s = E$ $\qquad$ $v_L = -V_o$



$$v_L = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{v_L}{L}$$

Constant during each sub-interval

2. Deduce the waveform of the inductance current i_L.

See the figure below. The current in the inductance behaves as

$$v_L(t) = L\,\frac{di_L(t)}{dt} \tag{202}$$

$$i_L(t_2) = i_L(t_1) + \int_{t_1}^{t_2} \frac{v_L(t)}{L}\,dt \tag{203}$$

The voltage across the inductance remains constant during each sub-interval. The current first increases linearly during $D\,T_s$ and then decreases linearly during $(1-D)\,T_s$.

Remark that the steady-state condition imposes that the current must return to its initial value i_{min} at the end of the switching period. With this condition, the successive switching periods have the same waveforms and the steady-state condition can be considered.

$$\underbrace{i_L(T_s)}_{=i_{min}} = \underbrace{i_L(0)}_{=i_{min}} + \underbrace{\int_0^{T_s} \frac{v_L(t)}{L}\,dt}_{=0} \tag{204}$$

The steady-state condition can be fulfilled if

$$\int_0^{T_s} v_L(t)\,dt = 0 \tag{205}$$

3. Express the ratio $\frac{V_o}{E}$ in terms of the duty cycle D.

On average, the inductance voltage is 0

$$\int_0^{T_s} v_L(t)\,dt = 0 \tag{206}$$

$$\int_0^{T_s} v_L(t)\,dt = \int_0^{D\,T_s} (E - V_o)\,dt + \int_{D\,T_s}^{T_s} (-V_o)\,dt \tag{207}$$

$$= (E - V_o)\,D\,T_s + (-V_o)\,(1-D)\,T_s = 0 \tag{208}$$

Simplifying the switching period T_s leads to

$$(E - V_o)\,D + (-V_o)\,(1-D) = 0 \tag{209}$$

$$D\,E - V_o = 0 \tag{210}$$

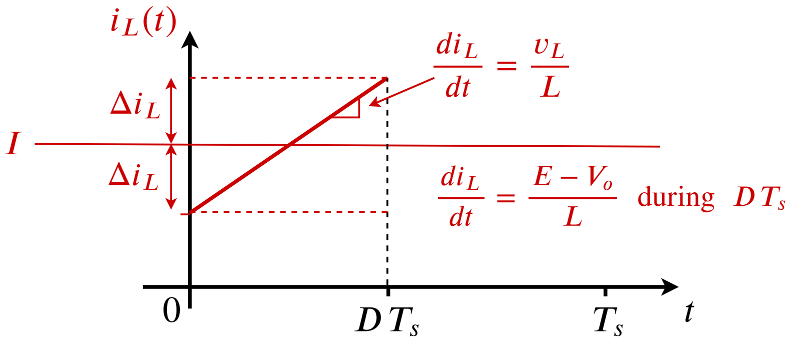
$$\boxed{\frac{V_o}{E} = D} \tag{211}$$

4. Give the value of D in this situation.

In this case,

$$\boxed{D = \frac{V_o}{E} = \frac{12}{302} = 3,973\%} \tag{212}$$

5. Find the expression of the inductor current ripple Δi_L in terms of V_o, E, D, T_s and L.



The ripple can be computed during the first sub-interval by observing that the slope $\frac{di_L}{dt}$ is equal to $\frac{E-V_o}{L}$:

$$2\,\Delta i_L = \frac{E - V_o}{L}\,D\,T_s \tag{213}$$

$$\boxed{\Delta i_L = \frac{E - V_o}{2\,L}\,D\,T_s} \tag{214}$$

6. Estimate the inductor current ripple Δi_L and compare it to the output current.

$$\Delta i_L = \frac{E - V_o}{2\,L}\,D\,T_s = \frac{302 - 12}{2 \cdot 0,05}\,\frac{12}{302} \cdot 0,001 = 0,11523 \text{ A} \tag{215}$$

For a 12 W output power, the output current is

$$I_o = \frac{P_o}{V_o} = \frac{12}{12} = 1 \text{ A} \tag{216}$$

Then the ripple is 12 % of the output current which is small enough.

Exercise 20: DC-DC boost converter

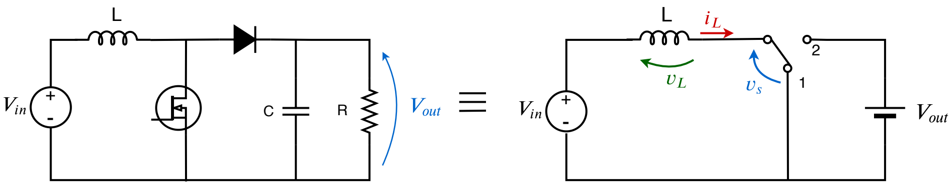
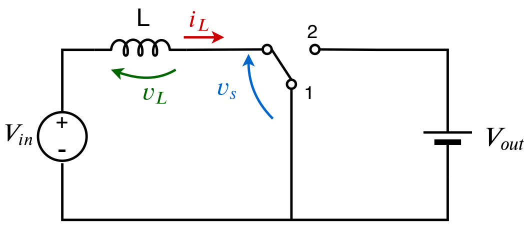


Figure 20: Ideal boost converter equivalent circuit.

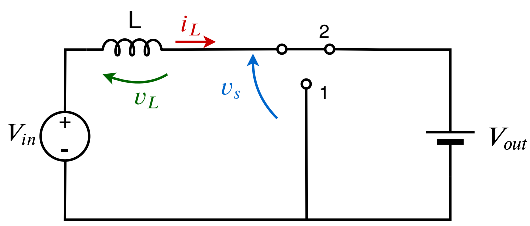
1. Find the waveforms of the voltages v_s and v_L.

During DT_s :

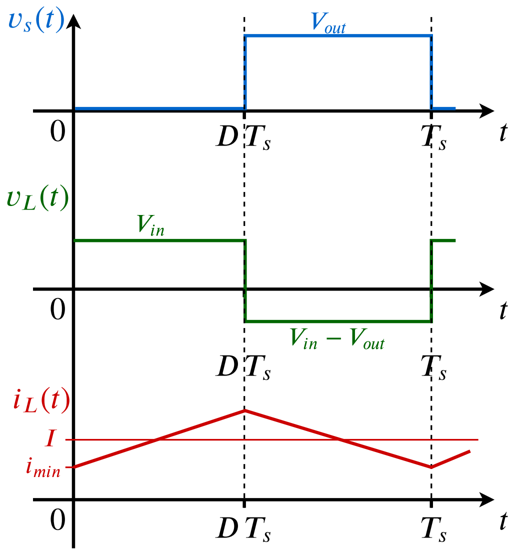


$v_s = 0$ $\quad$ $v_L = V_{in}$

During $(1 - D)T_s$:



$v_s = V_{out}$ $\quad$ $v_L = V_{in} - V_{out}$



$v_L = L\dfrac{di_L}{dt}$ $\quad$ Constant during each sub-interval

$\hookrightarrow \dfrac{di_L}{dt} = \dfrac{v_L}{L}$

2. Deduce the waveform of the inductance current i_L.

See the figure below. The current in the inductance behaves as

$$v_L(t) = L\,\frac{di_L(t)}{dt} \tag{217}$$

$$i_L(t_2) = i_L(t_1) + \int_{t_1}^{t_2} \frac{v_L(t)}{L}\,dt \tag{218}$$

The voltage across the inductance remains constant during each sub-interval. The current first increases linearly during $D\,T_s$ and then decreases linearly during $(1-D)\,T_s$.

Remark that the steady-state condition imposes that the current must return to its initial value i_{min} at the end of the switching period. With this condition, the successive switching periods have the same waveforms and the steady-state condition can be considered.

$$\underbrace{i_L(T_s)}_{=i_{min}} = \underbrace{i_L(0)}_{=i_{min}} + \underbrace{\int_0^{T_s} \frac{v_L(t)}{L}\,dt}_{=0} \tag{219}$$

The steady-state condition can be fulfilled if

$$\int_0^{T_s} v_L(t)\,dt = 0 \tag{220}$$

3. <u>Express the ratio $\frac{V_o}{E}$ in terms of the duty cycle D.</u>

On average, the inductance voltage is 0

$$\int_0^{T_s} v_L(t)\,dt = 0 \tag{221}$$

$$\int_0^{T_s} v_L(t)\,dt = \int_0^{D\,T_s} (V_{in})\,dt + \int_{D\,T_s}^{T_s} (V_{in} - V_{out})\,dt \tag{222}$$

$$= (V_{in})\,D\,T_s + (V_{in} - V_{out})\,(1-D)\,T_s = 0 \tag{223}$$

Simplifying the switching period T_s leads to

$$(V_{in})\,D + (V_{in} - V_{out})\,(1-D) = 0 \tag{224}$$

$$V_{in} - (1-D)\;V_{out} = 0 \tag{225}$$

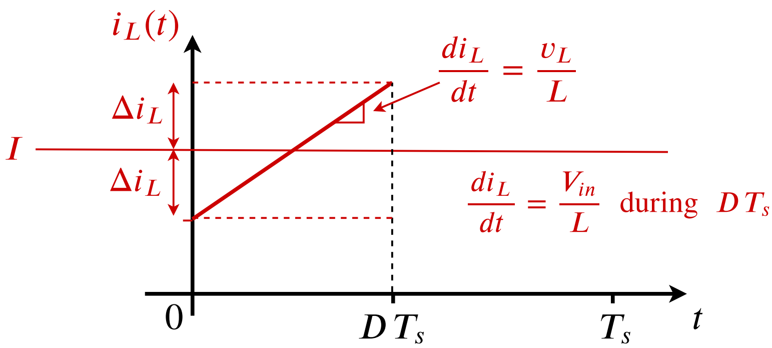
$$\boxed{\frac{V_{out}}{V_{in}} = \frac{1}{1-D}} \tag{226}$$

4. <u>Give the value of D in this situation.</u>

In this case,

$$\boxed{D = 1 - \frac{V_{in}}{V_{out}} = \frac{2}{3} = 66,6\%} \tag{227}$$

5. Find the expression of the inductor current ripple Δi_L in terms of V_o, E, D, T_s and L.



The ripple can be computed during the first sub-interval by observing that the slope $\frac{di_L}{dt}$ is equal to $\frac{V_{in}}{L}$:

$$2\,\Delta i_L = \frac{V_{in}}{L}\,D\,T_s \tag{228}$$

$$\boxed{\Delta i_L = \frac{V_{in}}{2\,L}\,D\,T_s} \tag{229}$$

6. Estimate the inductor current ripple Δi_L and compare it to the output current.

$$\Delta i_L = \frac{V_{in}}{2\,L}\,D\,T_s = \frac{3}{2\cdot 0,075}\cdot\frac{2}{3}\cdot\frac{1}{30\,000} = 0,44\text{ mA} \tag{230}$$

For a 15 mW output power, the output current is

$$I_o = \frac{P_o}{V_o} = \frac{0,015}{9} = 1,67\text{ mA} \tag{231}$$

Then the ripple is 27 % of the output current.