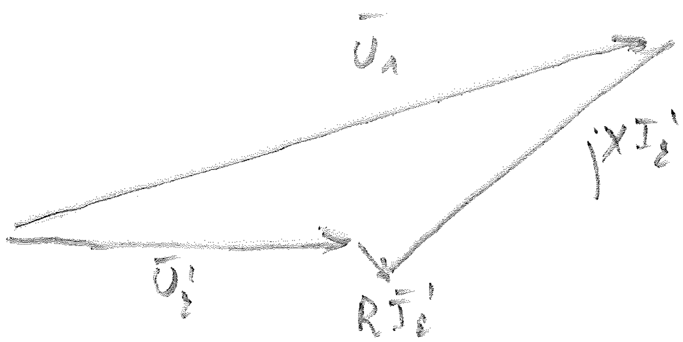
ELEC0431 (24/12/2023) 1

Exterior characteristic of the transformer

- Source $\bar{U}_1$ at the primary is fixed $\bar{U}_1 = \bar{E}_1$; $jX_\mu \bar{I}_\mu$
- What is the effect of connecting different kinds of loads (R, L, C)?

1°) Resistive load R_L

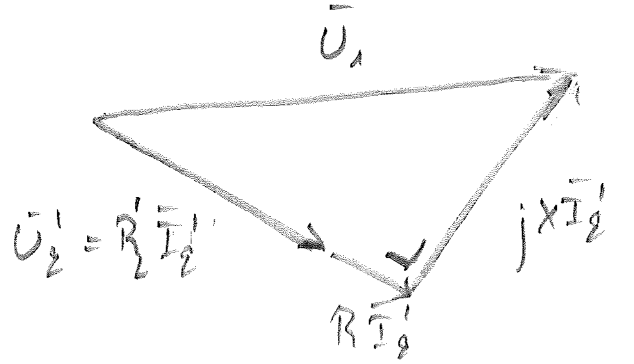
- Connecting a load at a circuit's terminal (here the secondary of the transformer) means giving a relationship between $\bar{U}_2'$ and $\bar{I}_2'$.
- This relationship closes the system of equations so that one can now calculate all currents and all voltages and draw the ~~phasor~~ phasor diagram of the considered working point / with the considered passive load
- Resistive load $\Rightarrow$ $\bar{U}_2' = R_L' \bar{I}_2'$



is NOT the phasor diagram of a transformer with a resistive load

WHY?

This is the phasor diagram of a transformer with a resistive load



Now for the rest of the discussion on the exterior characteristic, one remembers that:

- $R \ll X$ at least a factor 10 both in Ω
- phasor diagrams contain anyway inaccuracies due to the simplifications made along the way
- phasor diagrams are useful if they are simple

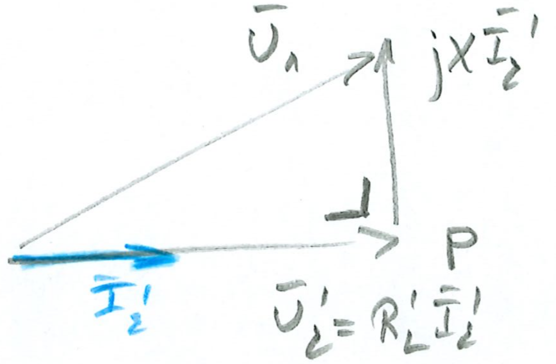
$\Rightarrow$ one neglects $R\bar{I}_2'$

- X (leakage) is the main characteristic that makes a transformer a <u>real</u> transformer (if one also neglects $X \ll X_\mu$ the transformer is <u>ideal</u>!)

So for a resistive load, one has

$$\bar U_1 = \bar U_2' + jX\bar I_2' \quad \text{with} \quad \bar U_2' = R_L' \bar I_2'$$

<u>phasors</u>

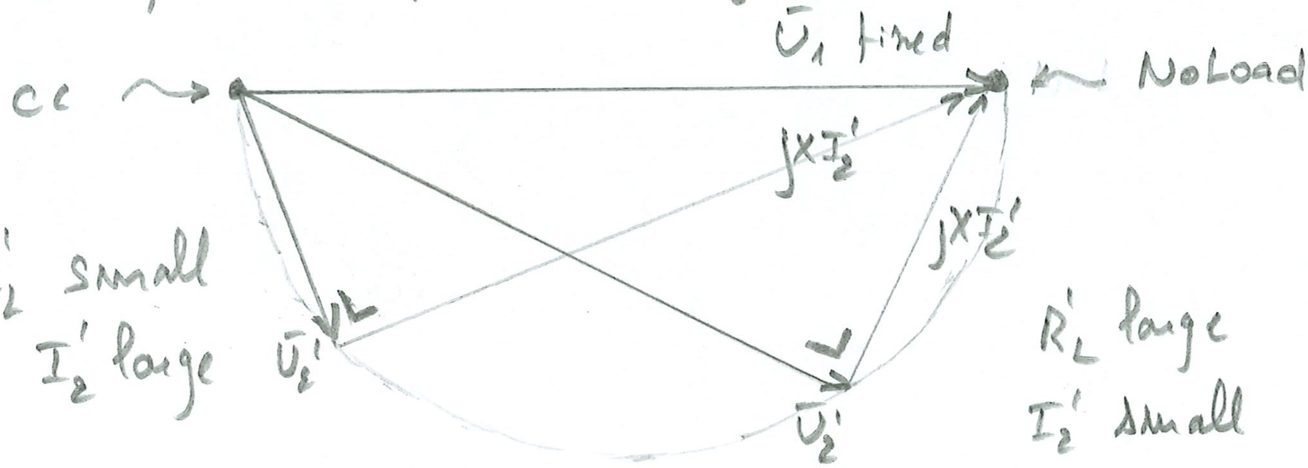


$\varphi_2 = 0$
because purely resistive load

Pythagora: $U_1^2 = (U_2')^2 + (XI_2')^2$

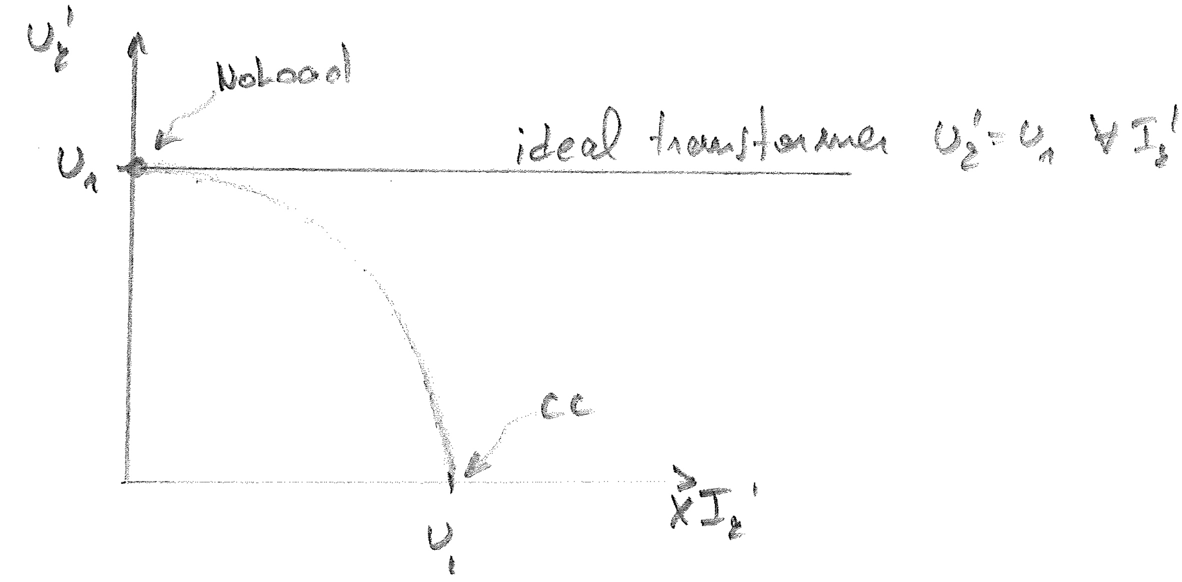
with $|\bar Z| = Z$ as notation for the modulus of a phasor.

Now, what if one changes the value of R_L'?



The locus of the tip of the $\bar U_2'$ phasor is a half circle.

Exterior characteristic



cc: $X I_2' = U_1$ and $U_2' = 0$

$$\Rightarrow (I_2')_{cc} = \frac{U_1}{X}$$

No Load: $U_1 = U_2'$ and $I_2' = 0$

Remark: Short circuit (cc) does not mean $I_2' \to \infty$ but rather

$$I_2' = \frac{U_1}{X}$$

As X is a small impedance, this current is already large, and in particular larger than the nominal current of the transformer.

A transformer is not designed to operate at the cc working point.

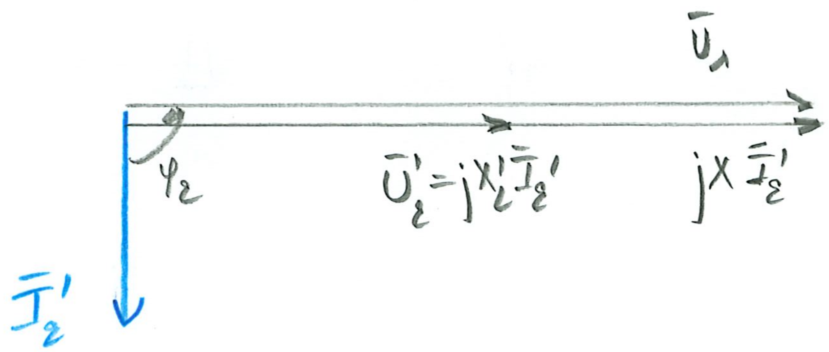
2°) Inductive Load X_L'

(Don't mix up X and X_L')

$\bar{U}_2' = jX_L'\bar{I}_2'$ and $\bar{U}_1 = \bar{U}_2' + jX\bar{I}_2'$

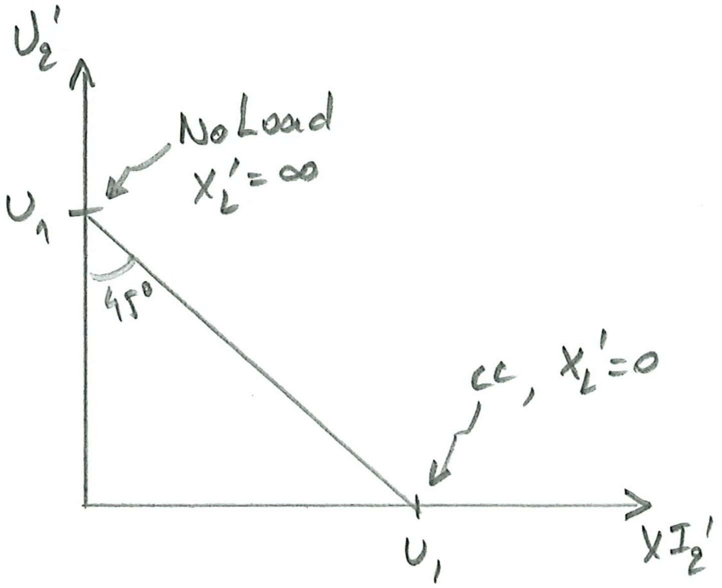
Phasors

$\varphi_2 = +\frac{\pi}{2}$ because purely inductive load



$$U_1^2 = (U_2' + XI_2')^2 \Rightarrow U_1 = U_2' + XI_2'$$

Exterior characteristic



$(I_2')_{cc} = \frac{U_1}{X}$

3°) Capacitive load C

<u>Remark</u> $Z = R + jX = \frac{1}{j\omega C} \Rightarrow X = \frac{-1}{\omega C} < 0$

<u>Here</u> $X_L' = \frac{-n^2}{\omega C} < 0$

$\bar{U}_2' = j X_L' \bar{I}_2' \Rightarrow |\bar{U}_2'| = U_2' = -X_L' I_2'$

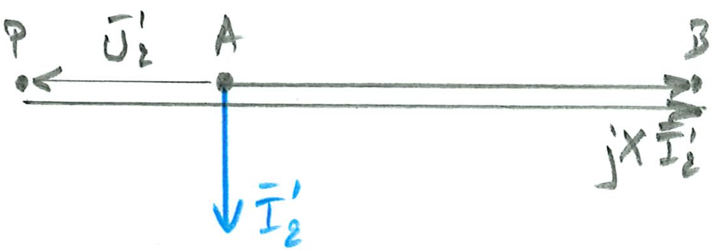
$\bar{U}_1 = \bar{U}_2' + jX\bar{I}_2' = j(X_L + X)\bar{I}_2'$

$\Rightarrow U_1 = |X_L' + X| \, I_2'$

$$\Rightarrow \begin{cases} U_1 = (X_L' + X) I_2' & \text{if } (X_L' + X) \geq 0 \\ U_1 = -(X_L' + X) I_2' & \text{if } (X_L' + X) \leq 0 \end{cases}$$

* $X_L' + X \geq 0 \qquad \frac{n^2}{\omega C} \leq X$

$U_1 = (X_L' + X) I_2' = -U_2' + X I_2' \Rightarrow U_2' = -U_1 + X I_2'$



$\bar{I}_2'$ is ahead of $\bar{U}_2'$ by $\frac{\pi}{2}$ $(\varphi_2 = -\frac{\pi}{2})$

CC: $P \to A$

$X_L = 0$ and $C = \infty \qquad I_2' = \frac{U_1}{X} \qquad U_1 = U_2'$

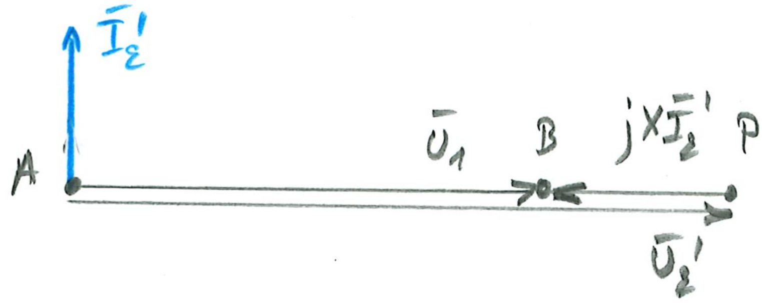
Resonance: $X_L + X = 0, \ \frac{n^2}{\omega C} = X$

$P \to \infty$ towards left

$I_2' = \frac{U_1}{X_L' + X} \longrightarrow \infty$

(*) $X_L' + X \le 0 \qquad \frac{M^2}{\omega C} \ge X$

$$U_1 = -(X_L' + X) I_2' = U_2' - X I_2' \Rightarrow U_2' = U_1 + X I_2'$$



$\bar{I}_2'$ is ahead of $\bar{U}_2'$ by $\frac{\pi}{2}$ $(\varphi_2 = -\frac{\pi}{2})$

Noload: $P \to B$, $X_L' \to \infty$ and $C = 0$

$I_2' = 0$, $U_1 = U_2'$

Resonance: $X_L' + X = 0$

$P \to \infty$ towards right

$I_2' = \frac{U_1}{X_L' + X} \to \infty$

resonance: $\frac{M^2}{\omega C} = X$

Exterior characteristic