Electromagnetic Energy Conversion ELEC0431

AC reminder

15 March 2022

Florent Purnode (florent.purnode@uliege.be) – Nicolas Davister (ndavister@uliege.be)

Montefiore Institute, Department of Electrical Engineering and Computer Science,
University of Liège, Belgium

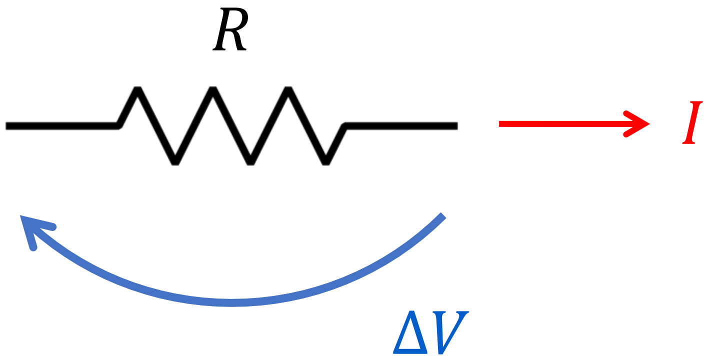
Basics of DC circuits: $V = RI$

In general, three fundamental values:

- The tension V [volt]
- The current I [amp]
- The impedance R [Ω]

And the relation:

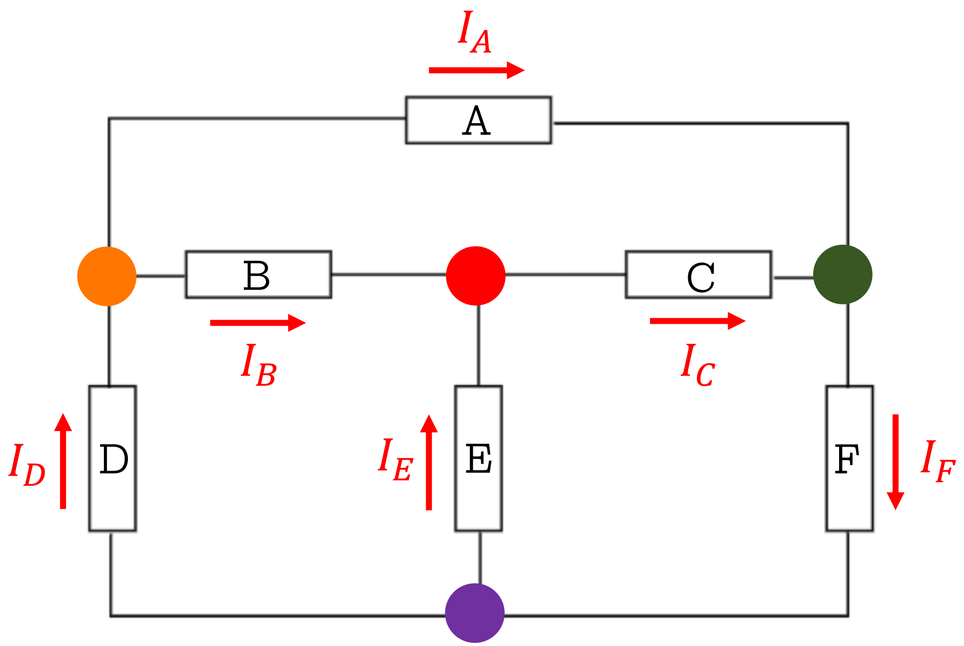
$$V = R\,I$$



Two other important laws: Kirchhoff's laws

Basics of DC circuits: Kirchhoff's first law*

At any junction in the electrical circuit, the sum of currents flowing into the junction is equal to the sum of currents flowing out of the junction



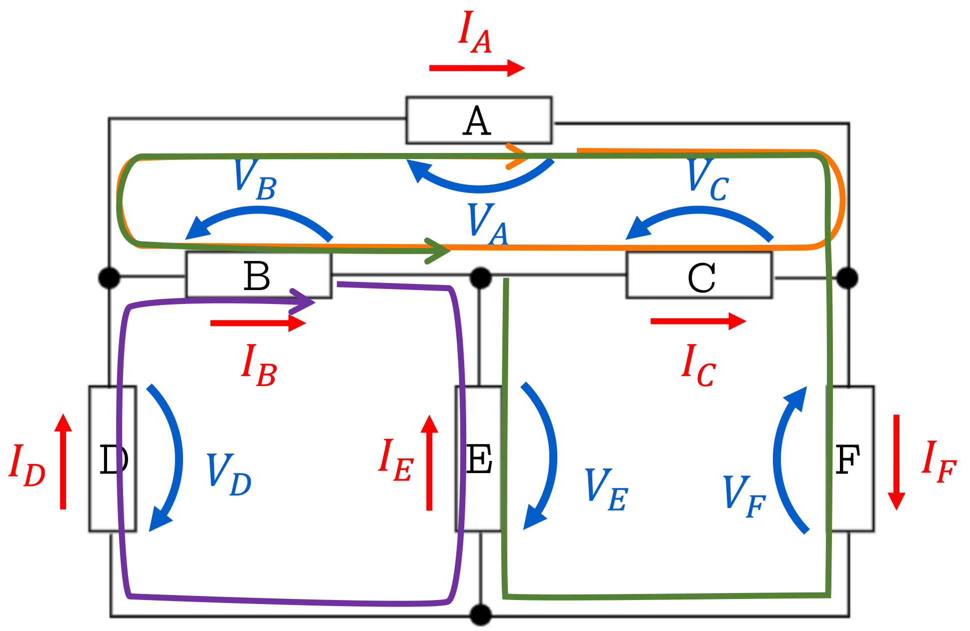
- $I_D = I_A + I_B$
- $I_B + I_E = I_C$
- $I_A = I_C + I_F$
- $I_F = I_E + I_D$

The values I can be negative. The important is to choose a convention and to stick with it !

*Assuming localised electrical circuit (which is true for all the exercises presented in this class)

Basics of DC circuits: Kirchhoff's second law*

Around any closed loop in a circuit, the directed sum of potential differences across components is zero



$$V_E - V_D - V_B = 0$$

$$V_C + V_B - V_A = 0$$

$$V_E + V_F + V_A - V_B = 0$$

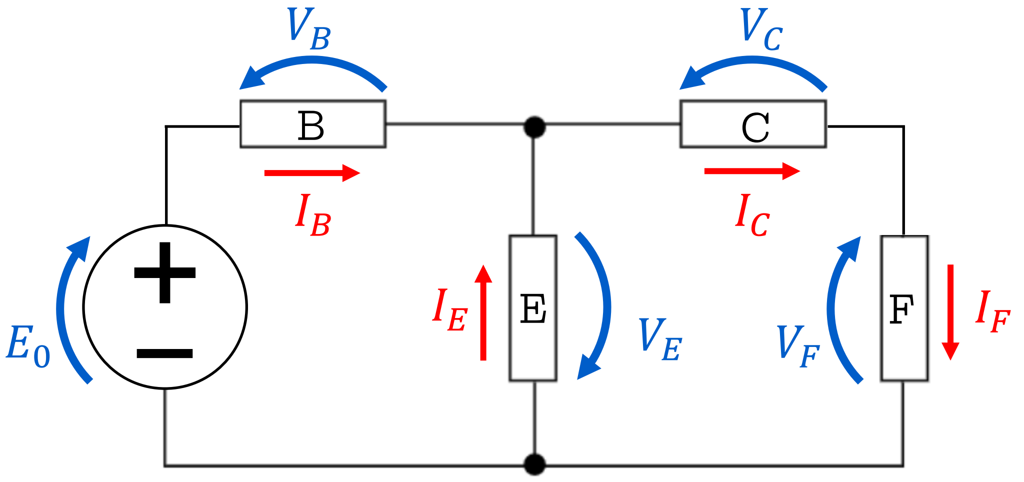
…

The values V can be negative. The important is to choose a convention and to stick with it !

*Assuming localised electrical circuit (which is true for all the exercises presented in this class)

Basics of DC circuits: Applying Kirchhoff's laws

$E_0 = 6V, R_B = 25\Omega, R_C = 8\Omega, R_E = 10\Omega, R_F = 2\Omega$



Kirchhoff's first law:

- $I_B + I_E = I_C$
- $I_C = I_F$

Kirchhoff's second law:

- $E_0 \; - V_B + V_E = 0$

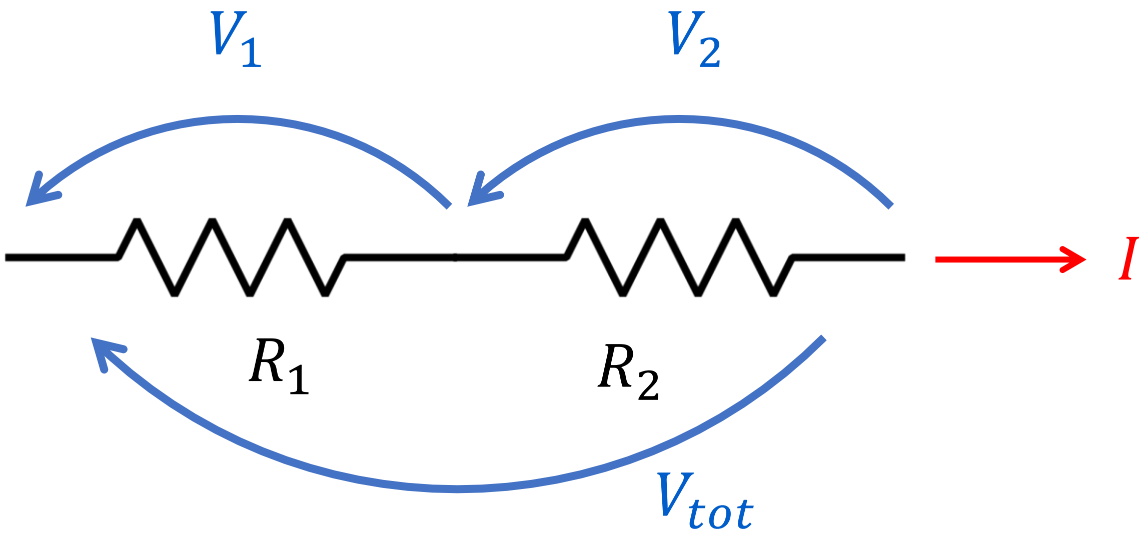
  $\Rightarrow \mathrm{E_0 - I_B R_B + I_E R_E}$
- $-V_E - V_C - V_F = 0$

  $\Rightarrow I_E R_E + I_C R_C + I_F R_F = 0$

Answer:

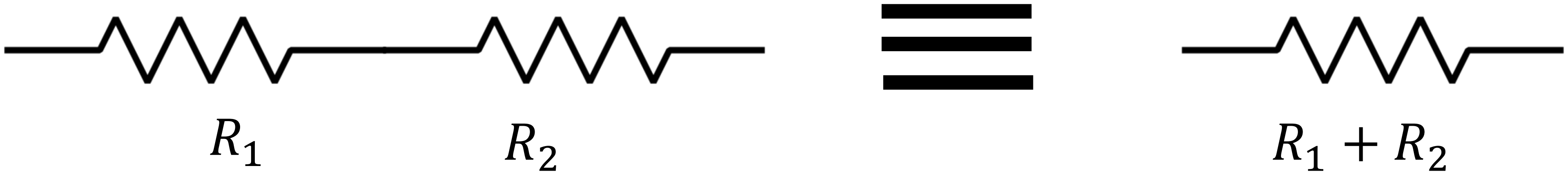
$I_B = 0.2A, \;\; I_C = 0.1A, \;\; I_E = -0.1A, \;\; I_F = 0.1A$

Basics of DC circuits: resistors in series

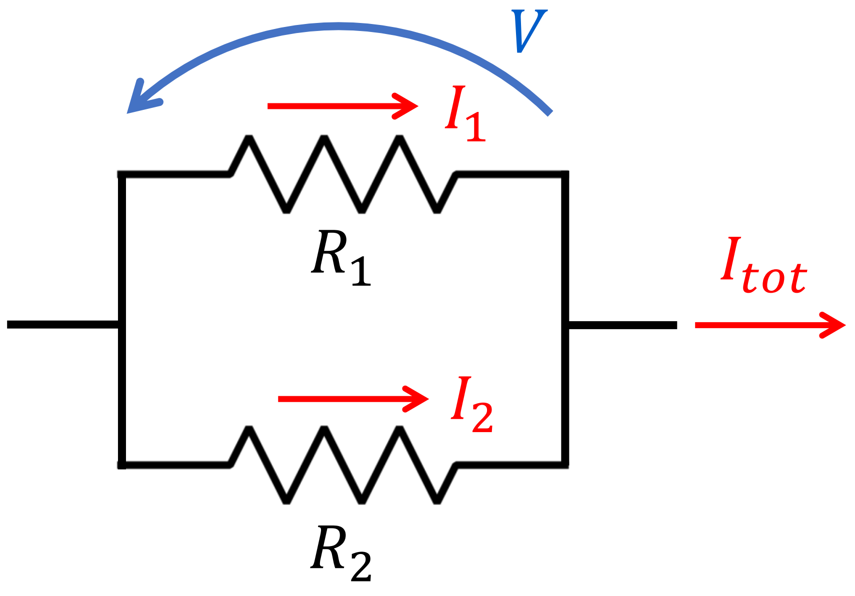


$$V_{tot} = V_1 + V_2$$
$$= I\,R_1 + I\,R_2$$
$$= I\,(R_1 + R_2)$$

Two resistors R_1 and R_2 in series act like one resistor $R_{tot} = (R_1 + R_2)$

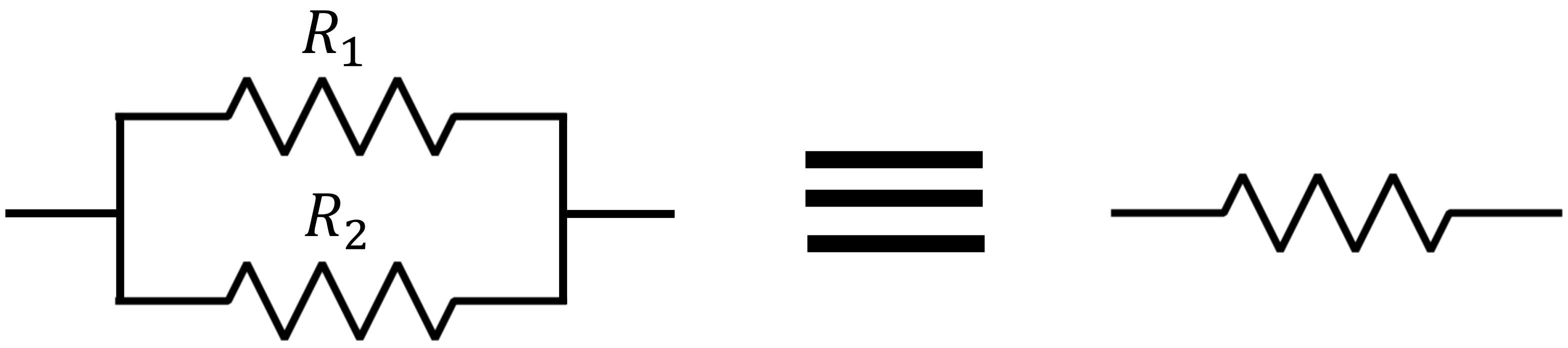


Basics of DC circuits: resistors in parallel



$$I_{tot} = I_1 + I_2$$
$$= \frac{V}{R_1} + \frac{V}{R_2}$$
$$\Rightarrow V = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} I$$

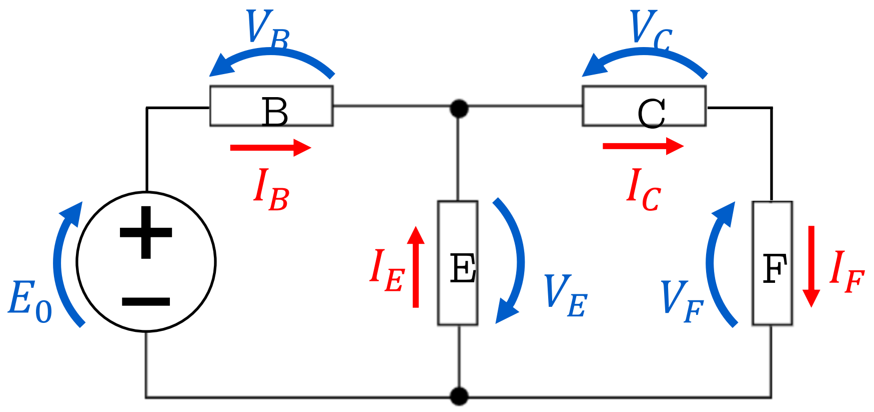
Two resistors R_1 and R_2 in parallel act like one resistor $R_{tot} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$



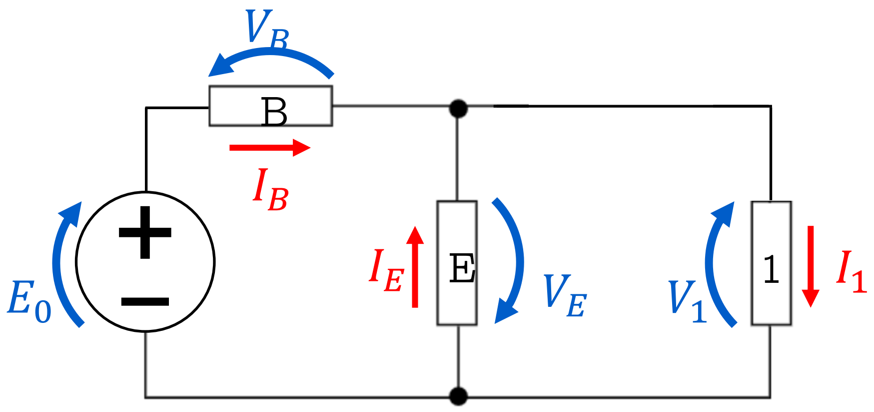
$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$

Basics of DC circuits: Applying Kirchhoff's laws

$$E_0 = 6V, R_B = 25\Omega, R_C = 8\Omega, R_E = 10\Omega, R_F = 2\Omega$$

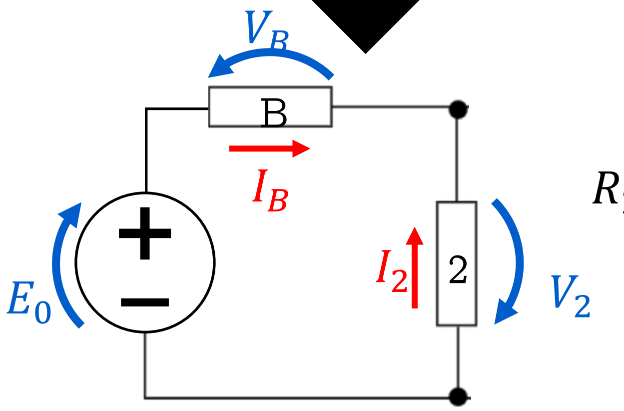


R_C and R_F in series



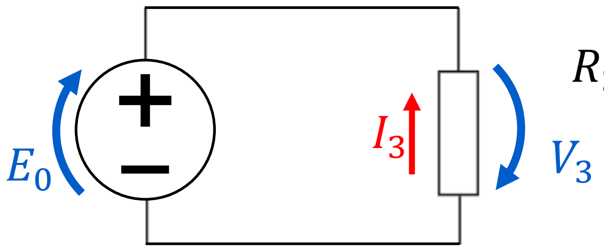
$$R_1 = R_C + R_F = 8 + 2 = 10\ \Omega$$

R_E and R_1 in parallel



$$R_2 = \left(\frac{1}{R_1} + \frac{1}{R_E}\right)^{-1} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} = 5\Omega$$

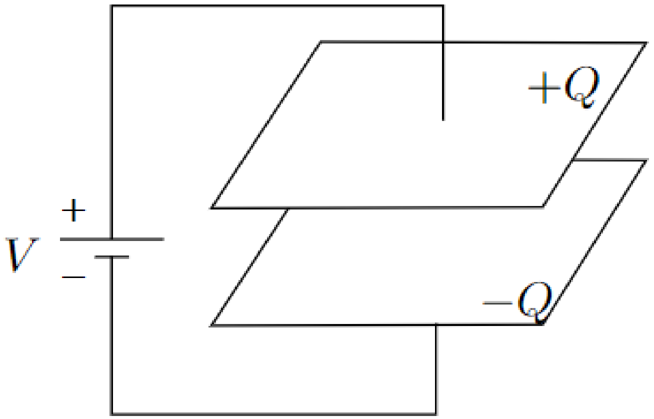
R_B and R_2 in series



$$R_3 = R_2 + R_B = 5 + 25 = 30\ \Omega$$

$$\Rightarrow I_B = \frac{E_0}{R_3} = \frac{6}{30} = 0{,}2\ A$$

AC circuits: the capacitor



Two parallel conductive plates of surface S, placed a distance d apart and carrying charges equal and opposite $\pm Q$.

The charges give rise to a potential difference V between the plates.

The capacity of the capacitor is defined as:

$$C = \frac{Q}{V}$$

AC circuits: the capacitor

$C = \frac{Q}{V}$ If V increases ➔ Q increases ➔ a movement of charges is a current

$$q(t) = \int_{-\infty}^{t} i(x)dx$$

⬇

$$u(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \quad \& \quad i(t) = C\frac{du(t)}{dt}$$

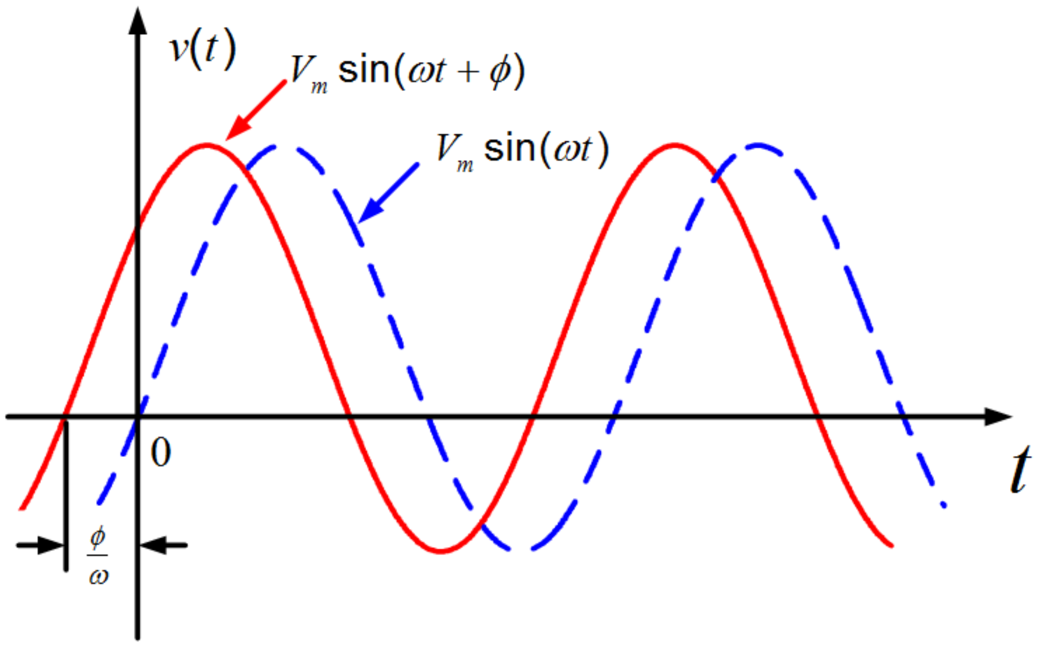
AC circuits: the inductor

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} u(\tau)d\tau$$

$$u(t) = L\frac{di(t)}{dt}$$

AC circuits: the inductor

In “Electromagnetic energy conversion”, we deal only with sinusoidal excitations:



What does it gives for:

- A resistor ?
- A capacitor ?
- An inductor ?

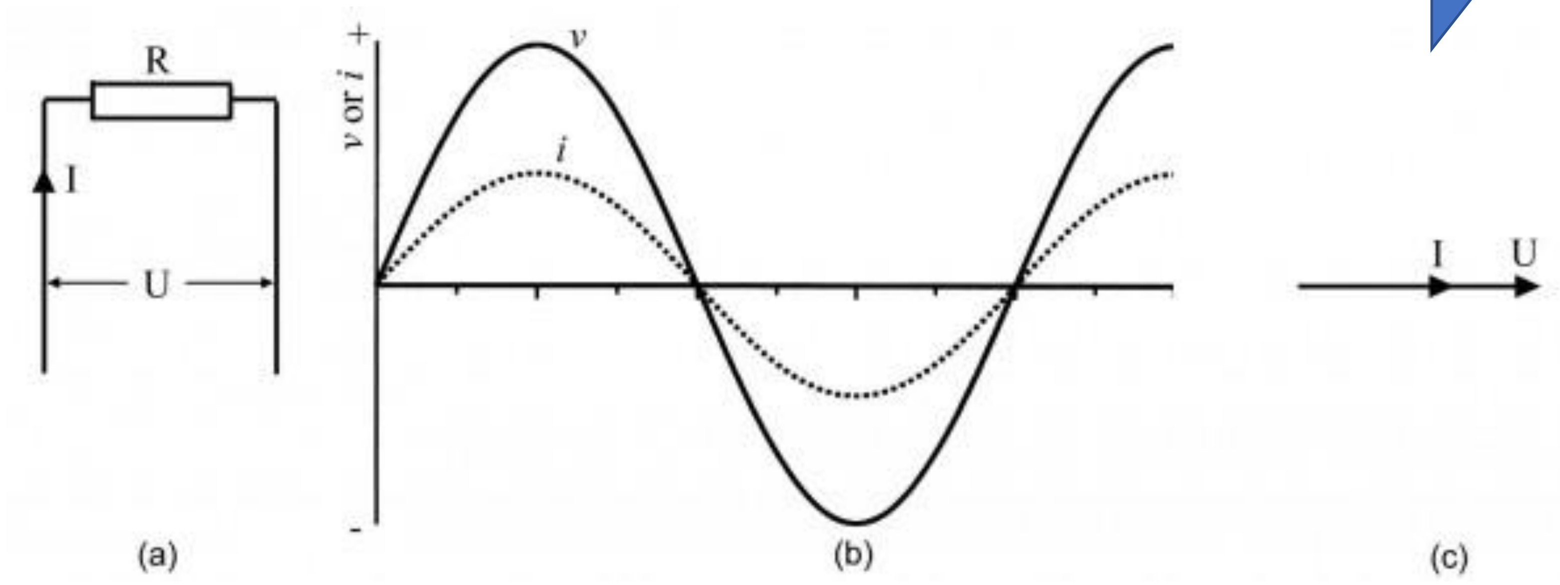
AC circuits: sinusoidal excitations and resistor

We consider a cosine wave $I(t) = I_m \cos(2\pi f t)$.

For a resistor, the constitutive law is $V = RI$

➔ $V(t) = R\, I(t) = R\, I_m \cos(2\pi f t)$

V and I are in phase



AC circuits: sinusoidal excitations and capacitor

We consider a cosine wave $V(t) = V_m \cos(2\pi f t)$.

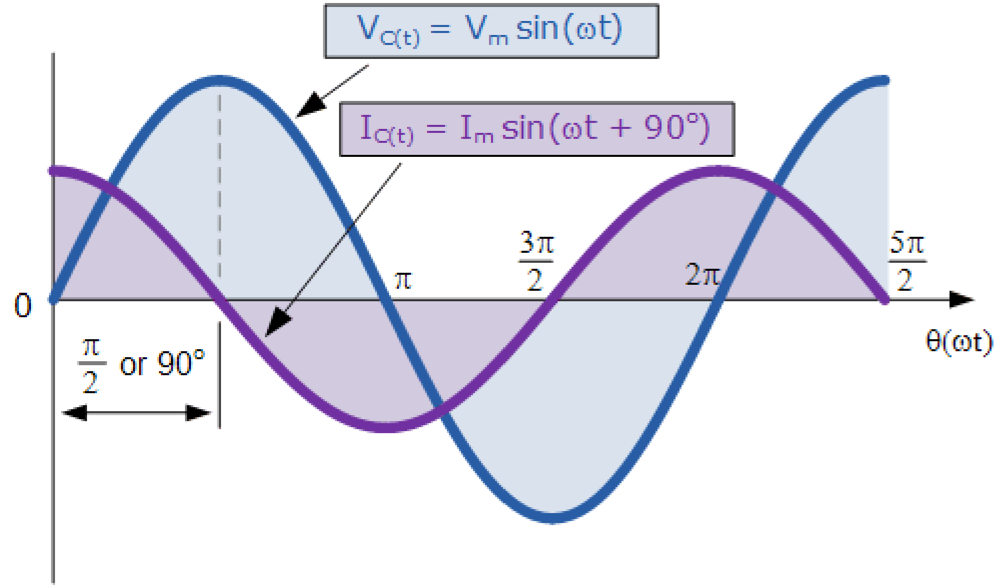
For a capacitor, the constitutive law is $I(t) = C\frac{dV(t)}{dt}$

➔ $I(t) = C\frac{dV(t)}{dt} = C\frac{d}{dt}(V_m \cos(2\pi f t)) = -C\,2\,\pi\,f\,V_m \sin(2\pi f t) = 2\pi f C V_m \cos\left(2\pi f t + \frac{\pi}{2}\right)$

➔ $I_m \cos(2\pi f t - \frac{\pi}{2}) = 2\pi f C\, V_m \cos(2\pi f t)$

➔ $V(t) = \frac{1}{2\pi f C} I_m cos(2\pi f t - \frac{\pi}{2})$

V lags I by 90°



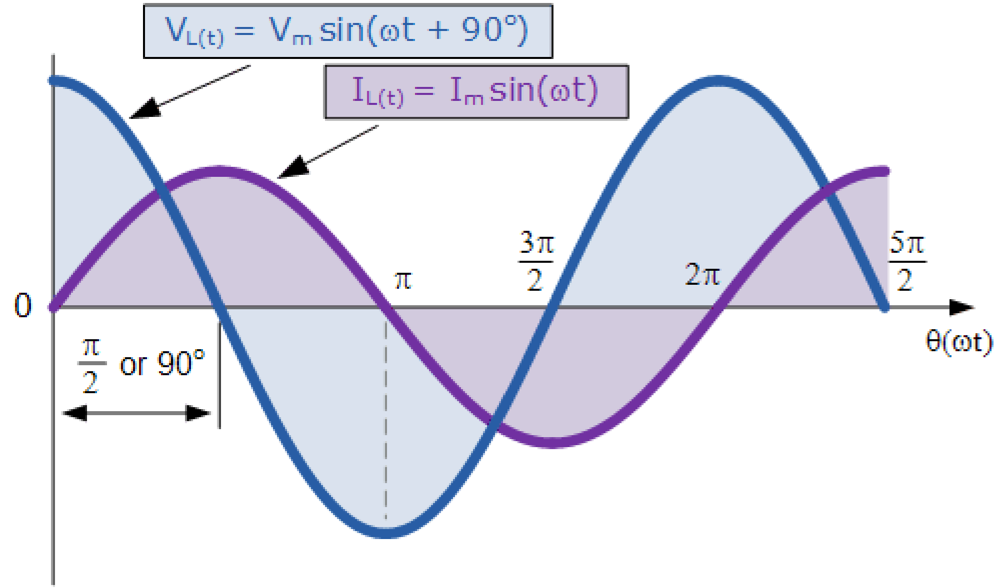
AC circuits: sinusoidal excitations and capacitor

We consider a cosine wave $\mathrm{I(t)} = I_m \cos(2\pi f t)$.

For an inductor, the constitutive law is $\mathrm{V}(t) = L\frac{dI(t)}{dt}$

➔ $\mathrm{V}(t) = L\frac{dI(t)}{dt} = L\frac{d}{dt}(\, I_m \cos(2\pi f t)) = -L\, 2\,\pi\, f\, I_m \sin(2\pi f t) = 2\pi f L\, I_m \cos\left(2\pi f t + \frac{\pi}{2}\right)$

I lags V by 90°



AC circuits: introduction to phasors

We found three relations:

- $V_R(t) = R\, I_m \cos(\omega t)$
- $V_C(t) = \frac{1}{\omega C}\, I_m cos(\omega t - \frac{\pi}{2})$
- $V_L(t) = \omega L\, I_m cos(\omega t + \frac{\pi}{2})$

where $\omega = 2\pi f$

Let's now introduce phasors ☺

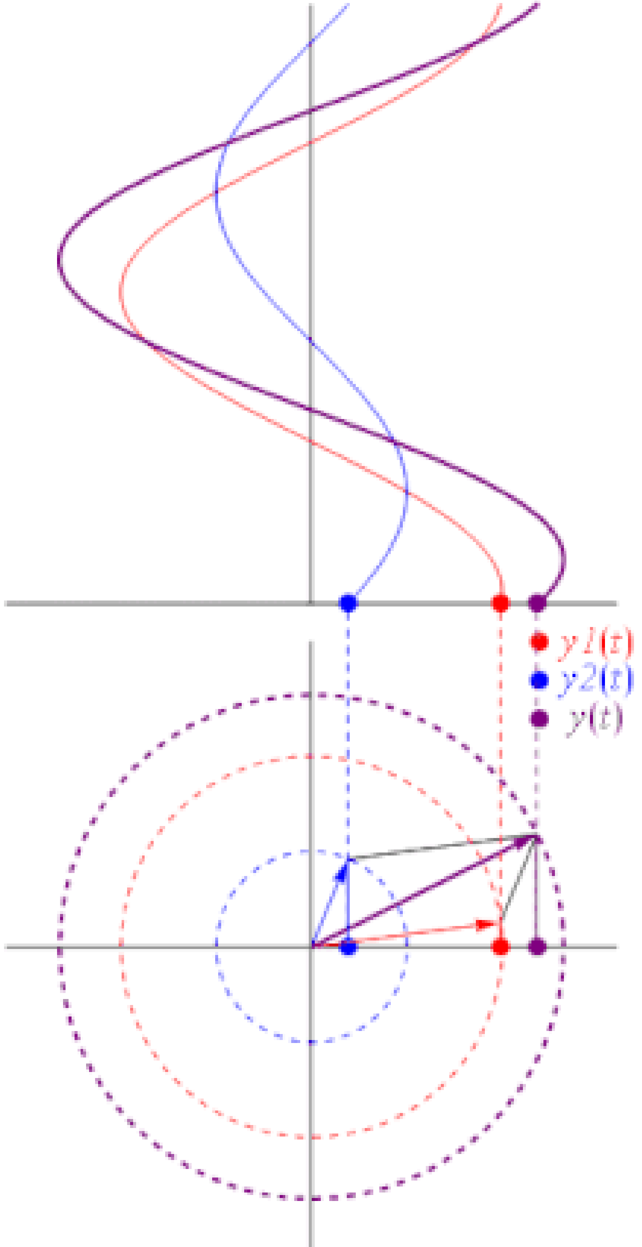
$$I(t) = I_m \cos(\omega t + \varphi) = \sqrt{2}\, I \cos(\omega t + \varphi)$$

$$= \mathbb{R}\left(\sqrt{2}\, I\ e^{j(\omega t + \varphi)}\right) = \mathbb{R}(\sqrt{2}\, I\ e^{j\varphi}\ e^{j\omega t})$$

$$e^{\pm j\theta} = \cos(\theta) \pm \sin(\theta)$$

➡ For a given frequency f (and thus given pulsation ω), the phasor of $I(t)$ is a complex number with an amplitude I and phase angle φ.

AC circuits: “phasor is good, phasor is life”



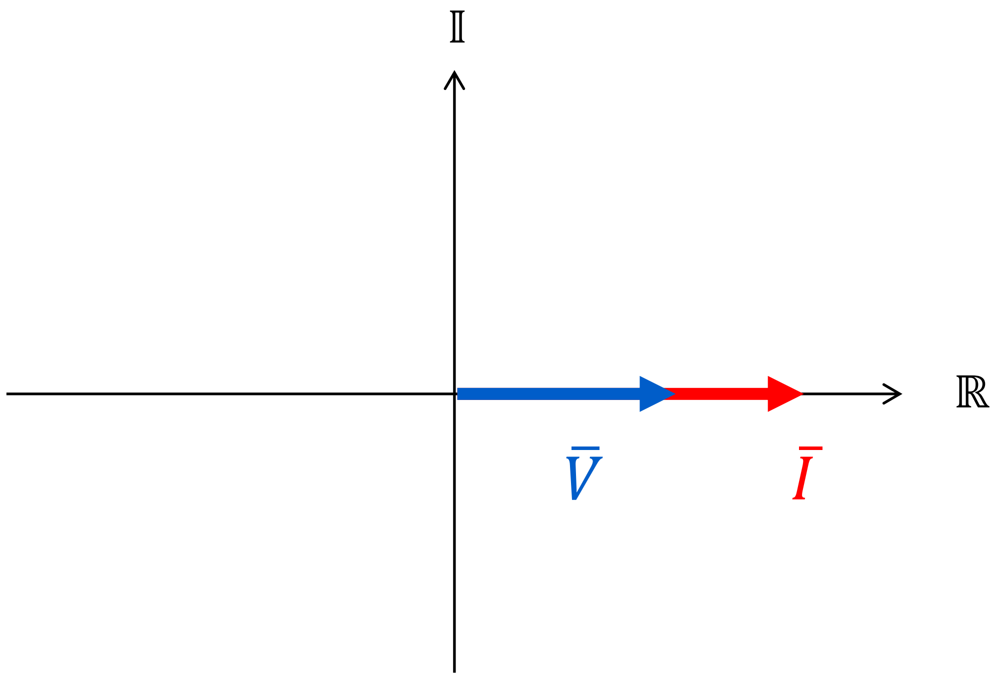
For signals evolving at a same frequency, phasors are like a snapshot representation containing all the necessary information (phase angle and amplitude) and allowing easy computations.

AC circuits: phasor for a resistor

What do we get when we apply phasors to our relations ?

- $V_R(t) = R\, I_m \cos(\omega t)$
- $V_C(t) = \frac{1}{\omega C}\, I_m cos(\omega t - \frac{\pi}{2})$
- $V_L(t) = \omega L\, I_m cos(\omega t + \frac{\pi}{2})$

$$I(t) = I_m \cos(\omega t + \varphi) \Rightarrow \bar{I} = I\, e^{j\varphi}$$



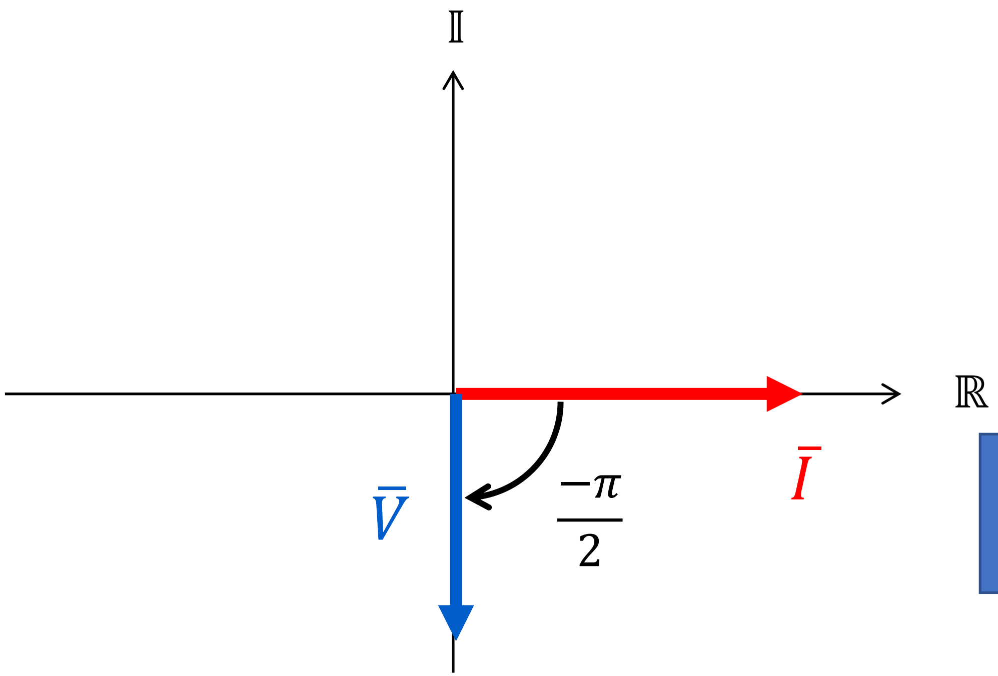
Through the resistor, we have a current $I(t) = I_m \cos(\omega t)$ which generates a voltage $V(t) = R\, I_m \cos(\omega t)$

AC circuits: phasor for a capacitor

What do we get when we apply phasors to our relations ?

- $V_R(t) = R\, I_m \cos(\omega t)$
- $V_C(t) = \frac{1}{\omega C}\, I_m cos(\omega t - \frac{\pi}{2})$
- $V_L(t) = \omega L\, I_m cos(\omega t + \frac{\pi}{2})$

$$I(t) = I_m \cos(\omega t + \varphi) \Rightarrow \bar{I} = I\, e^{j\varphi}$$



Through the capacitor, we have a current $I(t) = I_m \cos(\omega t)$ which generates a voltage $V(t) = \frac{1}{\omega C}\, I_m \cos(\omega t - \frac{\pi}{2})$

In an complex graph, this is equivalent to multiply $\bar{I}$ by

$$\frac{-j}{\omega C} = \frac{1}{j\omega C}$$

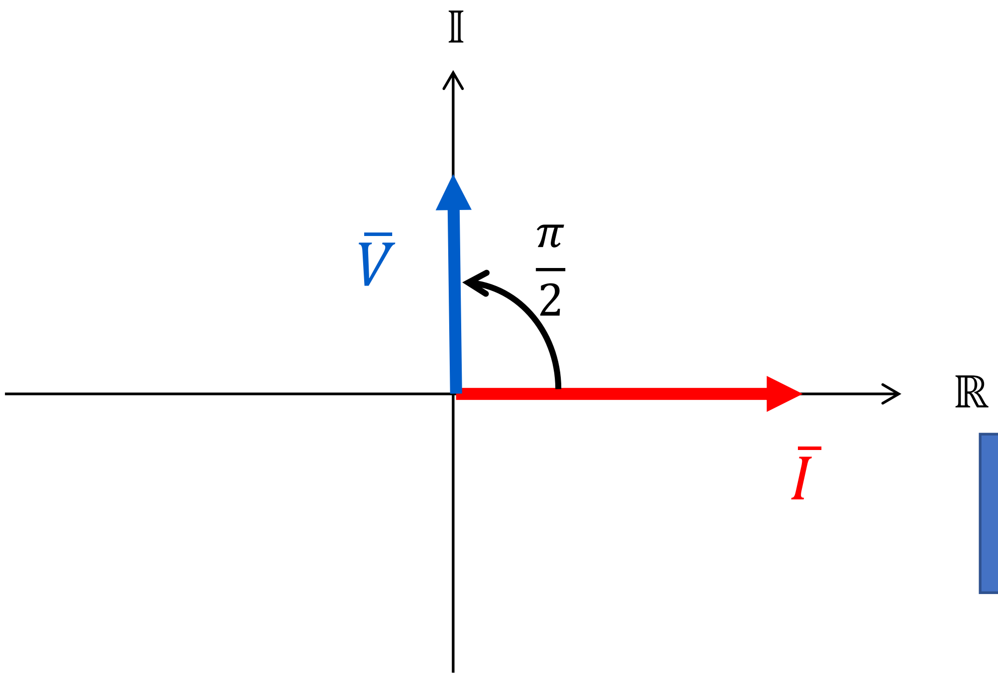
AC circuits: phasor for an inductor

What do we get when we apply phasors to our relations ?

- $V_R(t) = R\, I_m \cos(\omega t)$
- $V_C(t) = \frac{1}{\omega C}\, I_m cos(\omega t - \frac{\pi}{2})$
- $V_L(t) = \omega L\, I_m cos(\omega t + \frac{\pi}{2})$

$$I(t) = I_m \cos(\omega t + \varphi) \Rightarrow \bar{I} = I\, e^{j\varphi}$$

Through the inductor, we have a current $I(t) = I_m \cos(\omega t)$ which generates a voltage $V(t) = \omega L\, I_m \cos(\omega t + \frac{\pi}{2})$



In an complex graph, this is equivalent to multiply $\bar{I}$ by $j\omega L$

AC circuits: Impedance

For DC circuits, we have the relation $V = RI$. This relation is adapted for AC and phasors with:

$$\bar{V} = Z\,\bar{I}$$

where:

- $Z = R$ for a resistor
- $Z = \frac{1}{j\omega C}$ for a capacitor
- $Z = j\omega L$ for an inductor

If you know DC and phasors (complex numbers) you know AC.

Exercise 1: voltage distribution

The circuit of Figure 1 presents a resistive-inductive load powered with an AC generator of sinusoidal voltage (230 V, 50 Hz). Find the voltages accross R and L (magnitude and phase angle) and represent all the voltages on a phasor diagram.

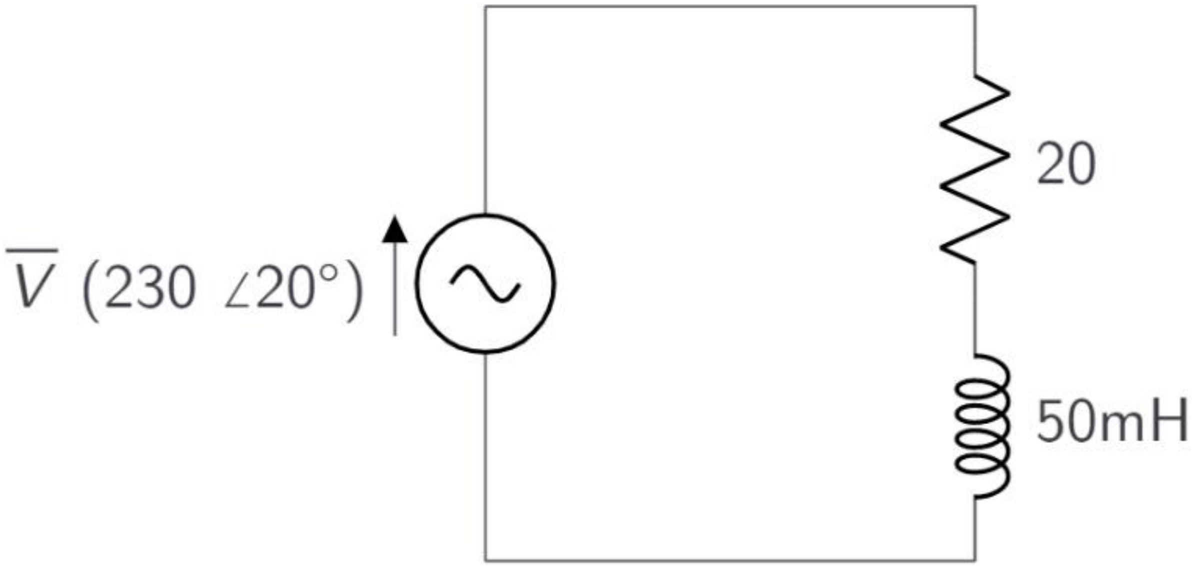


Figure: Resistive-inductive circuit.