FEM-BEM Coupling

MATH0471 – Spring 2022

This project consists in studying the multiphysic problem of electrostatically actuated mechanical systems such as MEMS. The underlying coupled elastic/electrostatic problem will be discretized using a coupled finite element/boundary element method (FEM/BEM). The numerical scheme will first be developed, tested and validated for the solution of the uncoupled elastic (solved using FEM) and electrostatic (solved using BEM) equations. Then, a FEM/BEM coupling strategy will be investigated. The code will be applied to several geometrical configurations, from a simple beam to more sophisticated microsystem shapes.

The project is organized as follows:

1. Students will be divided in 2 groups. Each group will write its own solvers. Within each group, one subgroup will be in charge of developing the FEM solver and one subgroup will be in charge of the BEM solver. The group as a whole will be responsible for the coupling.
2. Three intermediate deadlines are given, with a mandatory (but not graded) 8-page progress report that should detail the computer implementation and the mathematical, numerical and physical experiments.
3. The final report (about 60 pages) will present the method and numerical results, the computer implementation and a detailed analysis of physical experiments on non-trivial configurations.
4. An oral presentation of the main project results will be organized during the June exam session; individual theoretical and practical questions will be asked to each member of the groups.

Important dates:

1. **Wednesday March 16th: Intermediate deadline #1 (independent FEM and BEM solvers)**. Implementation of the two-dimensional finite element method using triangular and quadrangular Lagrange elements for the elastic equation with homogeneous and inhomogeneous Dirichlet and Neumann boundary conditions. Implementation of the two-dimensional boundary element method using constant shape

functions for the electrostatic equation with homogeneous and inhomogeneous Dirichlet and Neumann boundary conditions. The implementation should take advantage of the Gmsh library for creating and/or reading the mesh, computing values of shape functions and Jacobians, as well as exporting results; and the Eigen library for numerical linear algebra. The code should be validated against reference solutions on simple geometries.

2. **Wednesday April 20th: Intermediate deadline #2 (Coupled FEM/BEM solver)**. Computation of the electrostatic force and coupling of the FEM and BEM solvers.

3. **Wednesday May 4th: Intermediate deadline #3 (Applications)**. Tests and analysis of physical results on more sophisticated geometries.

4. **Wednesday May 20th: Final deadline**. Final report and code.

5. **June session: Exam**. Oral presentation of the projects.

The full source code should be tagged in the ULiège GitLab for each deadline, and should be directly configurable and compilable on the CECI clusters. The reports in PDF format should also be associated to this tag on GitLab for each deadline.