Explicit Discontinuous Galerkin Methods for Wave Problems with Local Time-Stepping

MATH0471 – Spring 2025

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v1 (04/02/2025)

This project consists in studying a hyperbolic system of equations in its conservative form. Spatial discretization will be performed using the Discontinuous Galerkin (DG) method and Lagrange nodal basis functions on unstructured meshes. An explicit time-marching method will be chosen, with possibly different local time step sizes in order to achieve good computational efficiency on unstructured, locally refined meshes.

Since the DG method requires a more elaborate mesh data structure than the classical finite element method, the numerical scheme will be implemented with the help of the Gmsh SDK so that meshing, computation of surface integrals and postprocessing of results will be simplified.

The numerical scheme will first be developed, tested and validated for the solution of n_{eq} uncoupled scalar transport equations. Then, the coupled systems describing the propagation of physical (acoustic, shallow water, electromagnetic, ...) waves will be considered.

The project is organized as follows:

1. Students will be divided in 3 groups. Each group will write its own solver.
2. Three intermediate deadlines are given, with a mandatory (but not graded) 8-page progress report that should detail the computer implementation and the mathematical, numerical and physical experiments.
3. The final report (about 60 pages) will present the method and numerical results, the computer implementation and a detailed analysis of physical experiments on non-trivial configurations.
4. An oral presentation of the main project results will be organized during the June exam session; individual theoretical and practical questions will be asked to each member of each group.

Important dates:

1. **Wednesday March 12th: Intermediate deadline #1: DG method for transport equations**: Implementation of the DG method using Lagrange shape functions

of arbitrary order for n_{eq} uncoupled scalar transport equations in 2D. The implementation should take advantage of the Gmsh library for creating and/or reading the mesh, computing values of shape functions and Jacobians, as well as exporting results. At this stage, the time integration should be performed using a simple forward-Euler method.

2. **Wednesday April 2nd: Intermediate deadline #2: Extension to the chosen physics**: Extension of the code to handle the chosen physical problem (acoustic, shallow water, electromagnetic, ...); testing and performance analysis of the solver on simple verification cases.

3. **May 7th: Intermediate deadline #3: Implementation and test of local time stepping strategy and realistic applications**: Modification of the code to handle the local time stepping strategy, and application to realistic physical problems.

4. **May 16th: Final deadline:** Final report and code.

5. **June exam session:** Oral presentation of the projects.

The full source code and report in PDF format should be tagged in the ULiège Gitlab for each deadline. Each group should send an email to r.boman@uliege.be and cgeuzaine@uliege.be for each deadline once the code and report are ready to be reviewed.